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# Employing wave variables for coordinated control of robots with distributed control architecture

Christian Ott and Yoshihiko Nakamura
Department of Mechano-Informatics, University of Tokyo
ott@ynl.t.u-tokyo.ac.jp

*Abstract*— **By controlling complex robotic systems one often has to cope with the situation that different sub-systems are interfaced and controlled by different computers. In this paper the problem of coordinated control of such a system with distributed control structure is addressed. In particular one must handle the transmission delays in the communication between the different computers, which can be considered small but not negligible, since also small delays in the transmission of power variables violate the passivity and therefore may lead to instability. In this paper the wave variables concept is applied to handle the delays and is used in combination with a virtual inertia for designing a Cartesian compliance controller. Therefore, in particular the steady state properties of the wave variable based communication is of interest and leads for the case of small delays to the analogy with a flexible joint robot. In a second step the virtual inertia is eliminated in order to approximate the desired closed loop behavior better. Finally, some simple planar simulations are presented which validate the proposed approach.**

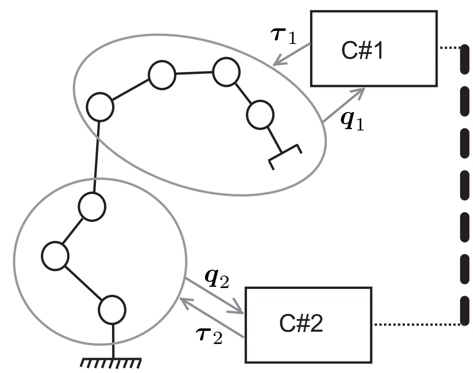


Fig. 1. Considered control architecture: Two computers C#1 and C#2 are interfaced with different subsystems.

## I. INTRODUCTION

For the design of real-time controllers one usually assumes that all the control algorithms can be implemented in one computer. If the computation is sufficiently fast such that the used sampling rates are much faster than the relevant modes of the system, one can design the controller as a continuous system and can ignore the time-discrete nature of the controller implementation in practice.

But in reality one often has to cope with the situation that some sub-systems are interfaced and controlled by different computers (Figure 1). One example would be a humanoid robot in which the upper body is controlled by one computer and the lower body is controlled by a second computer. As a second common example one can think of a mobile manipulator in which the robotic arm is controlled by one computer while the mobile part is controlled by a second one. For designing a coordinated control law one then has to establish some kind of communication between the different computers which consequently introduces small time delays in the control system. If the communication is designed with care these delays should be quite small, i.e. in the range of a few $ms$, and can be considered as constant. But since a delay jeopardized the passivity of the sub-systems, also small delays may lead to instability and should be considered by the control engineer [1].

The effect of time delay has extensively been studied in the realm of teleoperation, where one has two robots, a master and a slave, and the goal is to control the slave robot remotely via the master [2]. Scattering approaches allow to incorporate the time delay in a passivity based analysis [3], [4]. This lead to the concept of wave variables which have been introduced by Niemeyer and Slotine [5], [6]. Extensions to variable delays are treated in [7]. Another connection between teleoperation and passivity is given in [8], [9] in which the "Time Domain Passivity Control" from Hannaford and Ryu [10] is applied.

In the robotics literature on decentralized control of multiple manipulators [11], [12] on the other hand the issues of time delay are often neglected.

In this paper we show how to apply the wave variables concept in the design of a coordinated control law for a system with distributed architecture. The wave variables approach has been chosen because of its close relation to passivity for constant time delay. This allows to design the complete closed loop system based on a passivity based approach which guarantees advantageous robustness properties with respect to a large class of model uncertainties.

The motivation of this work is an application for a robotic wheelchair robot. For this robot the mobile part including an articulated seat will be interfaced to one real-time computer and an on-board arm will be controlled by a second real-time computer.

The paper is organized as follows. In Section II we first formulate the considered control problem. Then, after presenting the main design idea in Section III a short review of the wave variables concept is given in Section IV. The controller design using a virtual inertia and the analogy to the dynamics of a flexible joint robot is presented in Section V. In Section VI it will be shown how to the additional virtual inertia can be avoided. Some simulation results are presented in Section VII. Finally, Section VIII concludes the paper with a short summary.

## II. PROBLEM FORMULATION

The general model of a robot with $n$ joints can be written in the form

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{h}(\boldsymbol{q},\dot{\boldsymbol{q}}) + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{\tau}_c + \boldsymbol{\tau}_{ext}, \qquad (1)$$

with the generalized coordinates denoted as $\boldsymbol{q} \in \mathbb{R}^n$. The matrix $\boldsymbol{M}(\boldsymbol{q}) \in \mathbb{R}^{n\times n}$ is the symmetric and positive definite inertia matrix, $\boldsymbol{h}(\boldsymbol{q},\dot{\boldsymbol{q}})$ contains the centrifugal and Coriolis forces, and $\boldsymbol{g}(\boldsymbol{q})$ represents the gravity torques which are related to the gravity potential $V_g(\boldsymbol{q})$ via $\boldsymbol{g}(\boldsymbol{q}) = (\partial V_g(\boldsymbol{q})/\partial \boldsymbol{q})^T$. The control torques are given by $\boldsymbol{\tau}_c$, while $\boldsymbol{\tau}_{ext}$ contains external forces and torques between the robot and the environment. Notice that motor friction is not explicitly considered in this work. However, in practice an additional friction compensation in the control action clearly will improve the performance.
In this paper we assume a distributed control architecture using two computers C#1 and C#2 which are associated with two sub-systems of the robot, see Fig. (1). We assume that on each computer we can measure a subset $\boldsymbol{q}_1 \in \mathbb{R}^{n_1}$ and $\boldsymbol{q}_2 \in \mathbb{R}^{n_2}$ of the $n = n_1 + n_2$ coordinates $\boldsymbol{q} = (\boldsymbol{q}_1^T, \boldsymbol{q}_2^T)^T$. Accordingly, the control torques are split up into $\boldsymbol{\tau}_1 \in \mathbb{R}^{n_1}$ and $\boldsymbol{\tau}_2 \in \mathbb{R}^{n_2}$, i.e. $\boldsymbol{\tau}_c = (\boldsymbol{\tau}_1^T, \boldsymbol{\tau}_2^T)^T$. Any feedback path of the joint angles $\boldsymbol{q}_2$ to the joint torque $\boldsymbol{\tau}_1$, as well as feedback from $\boldsymbol{q}_1$ to $\boldsymbol{\tau}_2$, will be affected by the constant communication delay $T_d$ between the two computers.
As a control problem we consider the achievement of a desired Cartesian compliance behavior. Therefore, we define some Cartesian coordinates describing the pose of the end-effector via

$$\boldsymbol{r} = \boldsymbol{f}(\boldsymbol{q}_1, \boldsymbol{q}_2) \in \mathbb{R}^6 \ . \qquad (2)$$

The Jacobian matrix of the forward kinematics map with respect to $\boldsymbol{q}_1$ and $\boldsymbol{q}_2$ will be denoted by $\boldsymbol{J}(\boldsymbol{q}_1,\boldsymbol{q}_2) = \begin{bmatrix}\partial(\boldsymbol{f}(\boldsymbol{q}_1,\boldsymbol{q}_2))/\partial\boldsymbol{q}_1 & \partial(\boldsymbol{f}(\boldsymbol{q}_1,\boldsymbol{q}_2))/\partial\boldsymbol{q}_2\end{bmatrix}$. The desired compliance shall be given by a symmetric and positive definite stiffness matrix $\boldsymbol{K}_d \in \mathbb{R}^{6\times 6}$, a positive definite Cartesian damping matrix $\boldsymbol{D}_d \in \mathbb{R}^{6\times 6}$, and by a virtual equilibrium pose $\boldsymbol{r}_d \in \mathbb{R}^6$.
By neglecting the time delay in the data transfer between the computers C#1 and C#2 one could implement a compliance controller in a straight-forward way as

$$\begin{aligned} \boldsymbol{\tau}_c &= \begin{pmatrix}\boldsymbol{\tau}_1\\ \boldsymbol{\tau}_2\end{pmatrix} = \boldsymbol{g}(\boldsymbol{q}) - \boldsymbol{D}(\boldsymbol{q}_1,\boldsymbol{q}_2)\dot{\boldsymbol{q}} + \\ &\quad \boldsymbol{J}(\boldsymbol{q}_1,\boldsymbol{q}_2)^T\boldsymbol{K}_d(\boldsymbol{r}_d - \boldsymbol{f}(\boldsymbol{q}_1,\boldsymbol{q}_2)) \ , \\ \boldsymbol{D}(\boldsymbol{q}_1,\boldsymbol{q}_2) &:= \boldsymbol{J}(\boldsymbol{q}_1,\boldsymbol{q}_2)^T\boldsymbol{D}_d\boldsymbol{J}(\boldsymbol{q}_1,\boldsymbol{q}_2) \end{aligned} \qquad (3)$$

In the considered case with distributed control architecture the delay may be considered as small but not negligible and, thus, the control law (3) can not be implemented in the present form.

## III. BASIC DESIGN IDEA

In order to design a compliance controller similar to (3) for a system with distributed control architecture, we propose a procedure as follows:
1.) Introduce some "virtual" generalized coordinates $\boldsymbol{q}_v \in \mathbb{R}^{n_2}$ according to a simple second order dynamics of the form

$$\boldsymbol{M}_v\ddot{\boldsymbol{q}}_v = \boldsymbol{\tau}_{v,1} - \boldsymbol{\tau}_{v,2}, \qquad (4)$$

with a virtual inertia matrix $\boldsymbol{M}_v \in \mathbb{R}^{n_2\times n_2}$ and two virtual input torques $\boldsymbol{\tau}_{v,1} \in \mathbb{R}^{n_2}$ and[1] $\boldsymbol{\tau}_{v,2} \in \mathbb{R}^{n_2}$. This dynamics is to be implemented on computer C#1.
2.) Design a controller for $\boldsymbol{\tau}_2$ and $\boldsymbol{\tau}_{v,2}$ which makes the virtual coordinates $\boldsymbol{q}_v$ and the real coordinates $\boldsymbol{q}_2$ follow each other as good as possible under consideration of the communication delay between the computers C#1 and C#2.
3.) Design a compliance control law, to be implemented on computer C#1 via $\boldsymbol{\tau}_1$ and $\boldsymbol{\tau}_{v,1}$, based on the coordinates $\boldsymbol{q}_1(t)$ and $\boldsymbol{q}_v(t)$ instead of using $\boldsymbol{q}_1(t)$ and the delayed variables $\boldsymbol{q}_2(t - T_d)$.

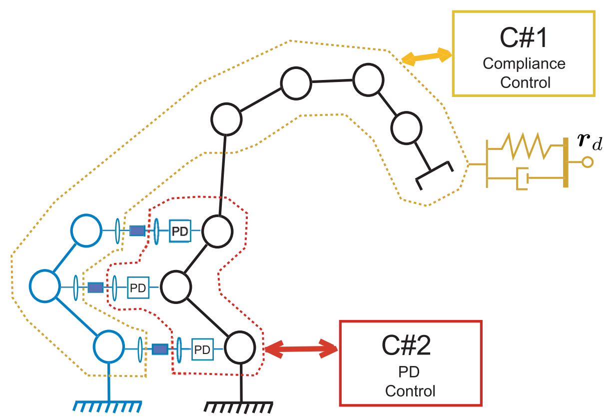


Fig. 2. Considered control approach: The system is augmented by virtual coordinates $\boldsymbol{q}_v$ which are coupled to the coordinates $\boldsymbol{q}_2$ by means of a teleoperation controller. The Cartesian compliance is then realized on C#1 without direct feedback of $\boldsymbol{q}_2$.

The whole procedure is illustrated in Figure 2. The idea behind introducing additional virtual coordinates $\boldsymbol{q}_v$ (and their dynamics (4)) in step 1 is that these coordinates shall serve as a "substitute" for the coordinates $\boldsymbol{q}_2$ in the computation of the compliance control law implemented on computer C#1 (step 3). Moreover, the second order dynamics (4) allows for a simple physical interpretation in terms of a mechanical system which will facilitate the design of the compliance controller in section V by utilizing an analogy with another type of underactuated mechanical systems. However, in section VI it will be shown that the introduction of these additional dynamics is not strictly necessary and can indeed be avoided in a modified controller design.
With regard to the stability properties of the closed loop system all the controller parts will be designed by utilizing

[1]The negative sign for $\boldsymbol{\tau}_{v,2}$ was chosen at this point such that the presentation in section IV fits better to the usual sign convention for two-ports in the teleoperation literature.

passivity based control approaches. This facilitates the interconnection of the controllers from step 2 and step 3 in a robust way.

## IV. WAVE VARIABLES

In this section we briefly review the concept of wave variables as it was introduced and discussed in [5], [6]. In teleoperation one has to cope with transmission delays between a master and a slave robot. These delays obstruct the application of passivity based control approaches and must be treated carefully. Based on the scattering approach it was shown in [5] that one can transform the input and output variables force and velocity, which are related to physical power, into another set of variables which describe the power flow in and out of the system.

The notation of *passivity* as used in nonlinear control theory is related to an input and an output of the system[2]. Let us assume that we have a system which is passive with respect to some input forces $\boldsymbol{F}$ and collocated output velocities $\dot{\boldsymbol{x}}$ such that the physical energy stored in the system can be considered as the storage function [13]. Then, the wave variables are defined by

$$\boldsymbol{u} = \frac{b\dot{\boldsymbol{x}} + \boldsymbol{F}}{\sqrt{2b}} , \tag{5}$$

$$\boldsymbol{v} = \frac{b\dot{\boldsymbol{x}} - \boldsymbol{F}}{\sqrt{2b}} , \tag{6}$$

where the scalar parameter $b > 0$ is the so-called wave impedance [6]. In the new coordinates the physical power is given by $P = \boldsymbol{F}^T\dot{\boldsymbol{x}} = \frac{1}{2}\boldsymbol{u}^T\boldsymbol{u} - \frac{1}{2}\boldsymbol{v}^T\boldsymbol{v}$, from which one can see that the terms $\frac{1}{2}\boldsymbol{u}^T\boldsymbol{u}$ and $\frac{1}{2}\boldsymbol{u}^T\boldsymbol{u}$ correspond to the ingoing and outgoing power. The inverse mapping, i.e. the computation of the power variables from the wave variables can be performed by

$$\dot{\boldsymbol{x}} = \frac{1}{\sqrt{2b}}(\boldsymbol{u} + \boldsymbol{v}) , \tag{7}$$

$$\boldsymbol{F} = \sqrt{\frac{b}{2}}(\boldsymbol{u} - \boldsymbol{v}) . \tag{8}$$

The use of wave variables for teleoperation originates from the fact that by using these transformations time delay becomes a passive operation, i.e. the 2-port block depicted in Figure 3 is passive. By connecting this ”passivated” transmission delay with passive systems on the right and left handed side one thus automatically achieves a passive closed loop dynamics.

Besides the passivity property, two other advantageous properties of the wave variable based communication will be important in the following. Firstly, it does not assume impedance or admittance causality on either sides of the transmission line and thus is compatible with both force and position controllers. Secondly, in steady state, i.e. with zero velocities and the forces equal on both sides of the transmission, the wave communication behaves statically like a spring with stiffness $K_t = \frac{b}{T_d}$ such that for small delays the communication looks like a rigid connection [6]. Clearly, this physical analogy to the behavior of a spring holds only in the sense of a steady state result as pointed out by Niemeyer in [6]. Furthermore, a more detailed analysis reveals that the wave communication also takes characteristics of an inertia $M_t = bT_d$ in certain situations [14]. Therefore, the above analogy with a static spring should not be used as the only design criterion for choosing $b$.

Step 2 from Section III, i.e. the coupling of the virtual coordinates $\boldsymbol{q}_v$ and the coordinates $\boldsymbol{q}_2$, can then be solved by applying the ”basic wave teleoperator” from [6]. Therefore, $\boldsymbol{\tau}_2$ is chosen as a PD controller of the form

$$\boldsymbol{\tau}_2 = -\boldsymbol{K}(\boldsymbol{q}_2 - \boldsymbol{q}_{2,d}) - \boldsymbol{B}(\dot{\boldsymbol{q}}_2 - \dot{\boldsymbol{q}}_{2,d}) , \tag{9}$$

where $\dot{\boldsymbol{q}}_{2,d}$ is the output of the wave transmission and can be derived via (5) and (9) as

$$\dot{\boldsymbol{q}}_{2,d} = (b + \boldsymbol{B})^{-1}\left(\sqrt{2b}\boldsymbol{u}_2 + \boldsymbol{B}\dot{\boldsymbol{q}}_2 + \boldsymbol{K}(\boldsymbol{q}_2 - \boldsymbol{q}_{2,d})\right) . \tag{10}$$

According to (6) the virtual control input $\boldsymbol{\tau}_{v,2}$ is given by

$$\boldsymbol{\tau}_{v,2} = b\dot{\boldsymbol{q}}_v - \sqrt{2b}\boldsymbol{v}_v \tag{11}$$

and the transmitted wave signals $\boldsymbol{u}_2(t) = \boldsymbol{u}_v(t - T_d)$ and $\boldsymbol{v}_v(t) = \boldsymbol{v}_2(t - T_d)$ are computed based on (5) and (6) via

$$\boldsymbol{u}_v = (b\dot{\boldsymbol{q}}_v + \boldsymbol{\tau}_{v,2})/\sqrt{2b} , \tag{12}$$

$$\boldsymbol{v}_2 = (b\dot{\boldsymbol{q}}_2 - \boldsymbol{\tau}_2)/\sqrt{2b} . \tag{13}$$

In the telerobotic literature it is well known that the integration of $\dot{\boldsymbol{q}}_{2,d}$ may lead to position drift due to numerical errors. Due to lack of space these issues are not discussed in detail in this paper. Instead the reader may refer to [6] for some extensions of the basic wave teleoperator by which one can counteract the numerical drift.

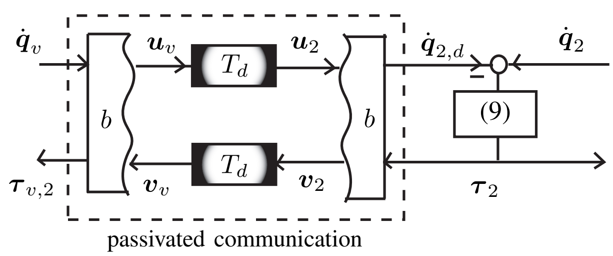


Fig. 3. Basic wave teleoperator consisting of the passivated communication as well as a PD controller for coupling the coordinates on both sides of the transmission.

## V. COMPLIANCE CONTROL

In this section a Cartesian compliance controller (to be implemented on computer C#1) is designed which assumes that the virtual coordinates $\boldsymbol{q}_v$ and the physical coordinates $\boldsymbol{q}_2$ are coupled via the basic wave teleoperator from Figure 3. The design approach will aim at a passive controller, which does not require delayed feedback of $\boldsymbol{q}_2$, but can be implemented via feedback of $\boldsymbol{q}_1$ and $\boldsymbol{q}_v$ only. The system structure for this situation is shown in Figure 4.

[2] A system $\dot{\boldsymbol{z}} = \boldsymbol{f}(\boldsymbol{z}, \boldsymbol{u})$ with input $\boldsymbol{u}$ and output $\boldsymbol{y}$ is said to be passive if there exists a non-negative function $S(\boldsymbol{z})$, the so-called *storage function*, such that $S(\boldsymbol{z}(t)) - S(\boldsymbol{z}(0)) \leq \int_0^t \boldsymbol{y}(s)^T\boldsymbol{u}(s)ds$ holds for all $t > 0$ [13].

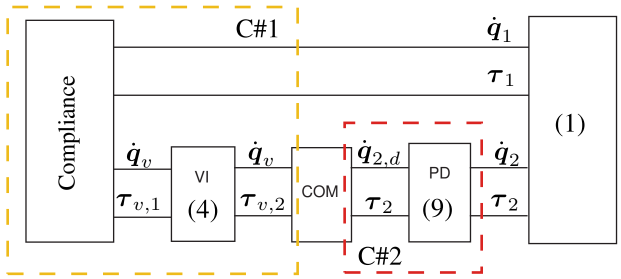


Fig. 4. System interconnection for the design of the Cartesian compliance control law (VI ... virtual inertia, COM ... wave communication).

In the following the steady state behavior of the wave transmission will be of particular importance. When a constant external torque $\boldsymbol{\tau}_{v,2} = \boldsymbol{\tau}_2$ acts, the steady state difference between the virtual and the physical coordinates must be equal to $\boldsymbol{q}_v - \boldsymbol{q}_2 = (\frac{T_d}{b}\boldsymbol{I} + \boldsymbol{K}^{-1})\boldsymbol{\tau}_2$ according to the interconnection of the passivated communication and the PD controller, which corresponds to an overall effective stiffness of $\boldsymbol{K}_c := (\frac{T_d}{b}\boldsymbol{I} + \boldsymbol{K}^{-1})^{-1}$. For small time delays the system therefore resembles a robot with flexible joints $\boldsymbol{q}_2$, in which $\boldsymbol{M}_v$ corresponds to the matrix of motor inertias and $(\frac{T_d}{b}\boldsymbol{I} + \boldsymbol{K}^{-1})^{-1}$ corresponds to the joint stiffness. This motivates the application of the passivity based control approach developed in [15], [16] for the implementation of the Cartesian compliance. It should be mentioned in advance that the passivity property of the resulting controller will not rely on this analogy to a flexible joint robot. While the analogy can give an intuitive physical interpretation for the control law developed in this section, it will be less useful for the discussion of a modified control law in Section VI.
The goal of the compliance control is to achieve a given stiffness and damping behavior. In the following we aim at a controller structure similar to (3) consisting of gravity compensation as well as implementation of stiffness and damping, but without requiring feedback of the delayed (and thus noncollocated) variable $\boldsymbol{q}_2$. For the gravity compensation we use the design presented in [15] with regard to the above mentioned analogy to robots with flexible joints.

### A. Gravity compensation

The basic idea for the gravity compensation in [15] was to use the steady state equation of the free (i.e. for $\boldsymbol{\tau}_{ext} = \boldsymbol{0}$) dynamics in order to compute a "quasi-static" estimate of the underactuated coordinates. In the present case the free steady state conditions follow from (1) and (4) and are given by

$$\boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{\tau}_c = \begin{pmatrix} \boldsymbol{\tau}_1 \\ \boldsymbol{\tau}_2 \end{pmatrix} , \tag{14}$$

$$\boldsymbol{\tau}_{v,1} = \boldsymbol{\tau}_{v,2} . \tag{15}$$

By using the static condition $\boldsymbol{\tau}_{v,2} = \boldsymbol{\tau}_2 = \boldsymbol{K}_c(\boldsymbol{q}_v - \boldsymbol{q}_2)$ from the basic wave teleoperator, with $\boldsymbol{K}_c = (\frac{T_d}{b}\boldsymbol{I} + \boldsymbol{K}^{-1})^{-1}$ as the effective steady state stiffness, one gets

$$\boldsymbol{g}_1(\boldsymbol{q}_1, \boldsymbol{q}_2) = \boldsymbol{\tau}_1 , \tag{16}$$

$$\boldsymbol{g}_2(\boldsymbol{q}_1, \boldsymbol{q}_2) = \boldsymbol{\tau}_2 = \boldsymbol{K}_c(\boldsymbol{q}_v - \boldsymbol{q}_2) , \tag{17}$$

$$\boldsymbol{\tau}_{v,1} = \boldsymbol{\tau}_2 , \tag{18}$$

where the gravity torque $\boldsymbol{g}(\boldsymbol{q})$ has been split up into the components $\boldsymbol{g}_1(\boldsymbol{q}_1, \boldsymbol{q}_2)$ and $\boldsymbol{g}_2(\boldsymbol{q}_1, \boldsymbol{q}_2)$ which act on $\boldsymbol{q}_1$ and $\boldsymbol{q}_2$, respectively. The goal is now to find a feedback law for $\boldsymbol{\tau}_{v,1}$ which exactly counterbalances the gravity term $\boldsymbol{g}_2(\boldsymbol{q}_1, \boldsymbol{q}_2)$ in steady state without requiring measurement of the (noncollocated) variable $\boldsymbol{q}_2$. Following the design idea presented in [15] we solve (17) for $\boldsymbol{q}_2$ and use the resulting function $\bar{\boldsymbol{q}}_2(\boldsymbol{q}_1, \boldsymbol{q}_v)$ as a quasi-static estimate of $\boldsymbol{q}_2$. This quasi-static estimate can further be applied for constructing a gravity compensation term of the form

$$\bar{\boldsymbol{g}}(\boldsymbol{q}_1, \boldsymbol{q}_v) := \boldsymbol{g}(\boldsymbol{q})|_{\boldsymbol{q}_2 = \bar{\boldsymbol{q}}_2(\boldsymbol{q}_1, \boldsymbol{q}_v)} .$$

*Remark 1: Solution of (17): In order to be able to ensure existence and uniqueness of the solution $\bar{\boldsymbol{q}}_2(\boldsymbol{q}_1, \boldsymbol{q}_v)$ from (17) one needs an additional requirement. Namely, the virtual stiffness $\boldsymbol{K}_c$ must dominate the Hessian $\boldsymbol{H}(\boldsymbol{q}_1, \boldsymbol{q}_2) := \partial \boldsymbol{g}_2(\boldsymbol{q})/\partial \boldsymbol{q}_2$ of the gravity potential with respect to $\boldsymbol{q}_2$, i.e. $||\boldsymbol{K}_c|| > ||\boldsymbol{H}(\boldsymbol{q}_1, \boldsymbol{q}_2)||$. For a moderately large controller gain $\boldsymbol{K}$ and small communication delay[3] $T_d$ this, however, is not a critical assumption. This assumption in particular ensures that the function $\boldsymbol{T}(\boldsymbol{q}_2) := \boldsymbol{q}_v - \boldsymbol{K}_c^{-1}\boldsymbol{g}_2(\boldsymbol{q}_1, \boldsymbol{q}_2)$ is a contraction mapping and thus can be used for computing $\bar{\boldsymbol{q}}_2(\boldsymbol{q}_1, \boldsymbol{q}_v)$ by iteration (see [15] for more details).*
Notice that while (17) stems from a static analysis we use it as a general definition of the quasi-static estimate which is used for the gravity compensation also in the dynamic case. In any case, due to its particular construction it is ensured that $\bar{\boldsymbol{q}}_2(\boldsymbol{q}_1, \boldsymbol{q}_v)$ fulfills the equation

$$\boldsymbol{g}_2(\boldsymbol{q}_1, \bar{\boldsymbol{q}}_2(\boldsymbol{q}_1, \boldsymbol{q}_v)) = \boldsymbol{K}_c(\boldsymbol{q}_v - \bar{\boldsymbol{q}}_2(\boldsymbol{q}_1, \boldsymbol{q}_v)) \tag{19}$$

everywhere, i.e. $\forall \boldsymbol{q}_1 \in \mathbb{R}^1, \boldsymbol{q}_v \in \mathbb{R}^2$. Finally, it should be mentioned that the application of the term $\bar{\boldsymbol{q}}_2(\boldsymbol{q}_1, \boldsymbol{q}_v)$ for gravity compensation in the dynamic case is justified by the fact that it is associated with a lower bounded[4] potential function $V_{\bar{g}}(\boldsymbol{q}_1, \boldsymbol{q}_v)$ which guarantees the passivity. Without going into the detail about its derivation (see [15] for this result) this potential function is written as

$$\begin{aligned} V_{\bar{g}}(\boldsymbol{q}_1, \boldsymbol{q}_v) &= V_g(\boldsymbol{q}_1, \boldsymbol{q}_2(\boldsymbol{q}_1, \boldsymbol{q}_v)) + \\ &\quad \frac{1}{2}(\boldsymbol{q}_v - \bar{\boldsymbol{q}}_2(\boldsymbol{q}_1, \boldsymbol{q}_v))^T \boldsymbol{K}_c(\boldsymbol{q}_v - \bar{\boldsymbol{q}}_2(\boldsymbol{q}_1, \boldsymbol{q}_v)) \end{aligned}$$

for which one can verify the desired condition $\bar{\boldsymbol{g}}(\boldsymbol{q}_1, \boldsymbol{q}_v) = \frac{\partial V_{\bar{g}}(\boldsymbol{q}_1, \boldsymbol{q}_v)}{\partial(\boldsymbol{q}_1, \boldsymbol{q}_v)}$ by straight-forward calculation and utilization of the property (19).
From a practical point one can expect that the difference between $\boldsymbol{q}_2$ and its quasi-static estimate $\bar{\boldsymbol{q}}_2(\boldsymbol{q}_1, \boldsymbol{q}_v)$ will be quite small as long as $\boldsymbol{K}_c$ is large, which corresponds to a small time delay $T_d$.

[3] remember $\boldsymbol{K}_c = (\frac{T_d}{b}\boldsymbol{I} + \boldsymbol{K}^{-1})^{-1}$

[4] It is lower bounded if the gravity potential $V_g(\boldsymbol{q})$ is lower bounded which holds for all robots with only rotational joints. For a robot with prismatic joints this holds only within a limited workspace.

## B. Controller formulation

The gravity compensation of the last sub-section is then augmented by a Cartesian stiffness and damping term which is evaluated by simply replacing $\boldsymbol{q}_2$ by $\boldsymbol{q}_v$ in (3). This results in the control law for $\boldsymbol{\tau}_1$ and $\boldsymbol{\tau}_{v,1}$:

$$\begin{aligned}\begin{pmatrix}\boldsymbol{\tau}_1\\ \boldsymbol{\tau}_{v,1}\end{pmatrix} &= \bar{\boldsymbol{g}}(\boldsymbol{q}_1,\boldsymbol{q}_v) - \boldsymbol{D}(\boldsymbol{q}_1,\boldsymbol{q}_v)\begin{pmatrix}\dot{\boldsymbol{q}}_1\\ \dot{\boldsymbol{q}}_v\end{pmatrix} + \\ &\quad \boldsymbol{J}(\boldsymbol{q}_1,\boldsymbol{q}_v)^T\boldsymbol{K}_d(\boldsymbol{r}_d - \boldsymbol{f}(\boldsymbol{q}_1,\boldsymbol{q}_v))\ , \\ \boldsymbol{D}(\boldsymbol{q}_1,\boldsymbol{q}_v) &:= \boldsymbol{J}(\boldsymbol{q}_1,\boldsymbol{q}_v)^T\boldsymbol{D}_d\boldsymbol{J}(\boldsymbol{q}_1,\boldsymbol{q}_v)\end{aligned} \tag{20}$$

Finally, let us summarize the complete control law. On computer C#1 one must implement the virtual dynamics (4), the feedback laws (11) and (20), as well as the computation of the wave signal (12) which is transmitted to computer C#2. On computer C#2 one must implement (10), the PD control law (9), as well as the computation of the wave signal (13) which is transmitted to computer C#1.

Notice that all the controller parts shown in Figure 4 have been designed as passive subsystems. Therefore, also the passivity of their interconnection is assured which guarantees robust interaction with unknown passive environments.

In order to get a good approximation of (3) for small time delays one should choose a small virtual inertia $\boldsymbol{M}_v$ and a high PD gain $\boldsymbol{K}$. The parameters $b$ and $\boldsymbol{B}$ give additional freedom for adjusting the damping. However, one important remark must be made about the resulting Cartesian stiffness. By considering the control law (20) one implements a Cartesian spring with stiffness $\boldsymbol{K}_d$ in series to the joint level spring $\boldsymbol{K}_c$. The achieved Cartesian stiffness $\boldsymbol{K}_{ext}$ as seen from the tip of the robot is locally given by

$$\boldsymbol{K}_{ext} = \left(\boldsymbol{K}_d^{-1} + \boldsymbol{J}(\boldsymbol{q}_1,\boldsymbol{q}_2)\begin{bmatrix}\boldsymbol{0} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{K}_c^{-1}\end{bmatrix}\boldsymbol{J}(\boldsymbol{q}_1,\boldsymbol{q}_2)^T\right)^{-1} \tag{21}$$

instead. However, (21) clearly can be utilized in the design of $\boldsymbol{K}_d$ according to some desired external stiffness $\boldsymbol{K}_{ext}$.

The introduction of the virtual inertia $\boldsymbol{M}_v$ allows to give a simple physical analogy for small delays between the application of the basic wave teleoperator and the physical spring of a flexible joint robot. This was the main design idea followed in the controller design so far. However, the inertia also affects the closed loop dynamics. In the next section it will therefore be shown how this additional virtual inertia can be avoided.

# VI. Elimination of the Virtual Inertia

The idea how to eliminate the virtual inertia from the presented control concept emerges from the fact that the wave-based communication a priori does not assume impedance or admittance causality on either sides of the transmission. In the previous approach (Figure 2) the wave-based communication gives at the power port on the left hand side the torque $\boldsymbol{\tau}_{v,2}$ as an output which is applied to the virtual inertia, while the velocity of the virtual coordinates $\dot{\boldsymbol{q}}_v$ is the corresponding input. In the following we revert this scheme in order to connect the compliance controller from (20) directly to the wave-based communication (see Figure 5).

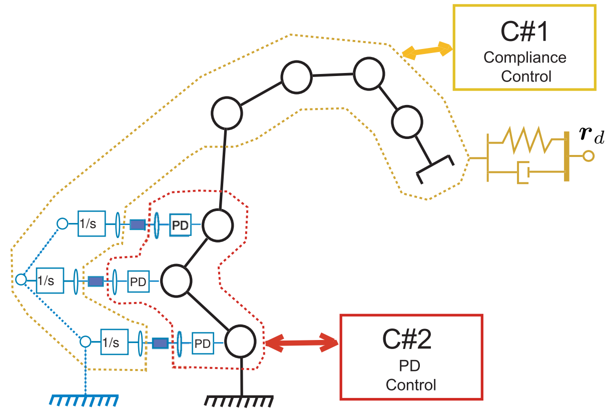


Fig. 5. Modified control approach: By changing the causality for the port on the left hand side of the wave communication the virtual inertia can be avoided.

From (6) applied to the power port on the left hand side of the wave-based communication in Figure 3, i.e. with $\boldsymbol{u}\hat{=}\dot{\boldsymbol{q}}_v$ and $\boldsymbol{F}\hat{=}\boldsymbol{\tau}_{v,2}$, we get

$$b\dot{\boldsymbol{q}}_v = \sqrt{2b}\boldsymbol{v}_v + \boldsymbol{\tau}_{v,2} \tag{22}$$

Furthermore, since we want to eliminate the virtual inertia, we set $\boldsymbol{\tau}_{v,2} = \boldsymbol{\tau}_{v,1}$ (cf. (4)). Using (20) and the abbreviations[5]

$$\begin{aligned}\bar{\boldsymbol{g}}_1 &:= \boldsymbol{g}_1(\boldsymbol{q}_1,\bar{\boldsymbol{q}}_2(\boldsymbol{q}_1,\boldsymbol{q}_v))\ ,\\ \bar{\boldsymbol{g}}_2 &:= \boldsymbol{g}_2(\boldsymbol{q}_1,\bar{\boldsymbol{q}}_2(\boldsymbol{q}_1,\boldsymbol{q}_v))\ ,\\ \bar{\boldsymbol{f}} &:= \boldsymbol{f}(\boldsymbol{q}_1,\boldsymbol{q}_v)\ ,\\ \bar{\boldsymbol{J}}_1 &:= \frac{\partial \boldsymbol{f}(\boldsymbol{q}_1,\boldsymbol{q}_v)}{\partial \boldsymbol{q}_1}\ ,\\ \bar{\boldsymbol{J}}_2 &:= \frac{\partial \boldsymbol{f}(\boldsymbol{q}_1,\boldsymbol{q}_v)}{\partial \boldsymbol{q}_v}\ ,\\ \bar{\boldsymbol{D}}_{11} &:= \bar{\boldsymbol{J}}_1^T\boldsymbol{D}_d\bar{\boldsymbol{J}}_1\ ,\\ \bar{\boldsymbol{D}}_{12} &:= \bar{\boldsymbol{J}}_1^T\boldsymbol{D}_d\bar{\boldsymbol{J}}_2\ ,\\ \bar{\boldsymbol{D}}_{21} &:= \bar{\boldsymbol{J}}_2^T\boldsymbol{D}_d\bar{\boldsymbol{J}}_1\ ,\\ \bar{\boldsymbol{D}}_{22} &:= \bar{\boldsymbol{J}}_2^T\boldsymbol{D}_d\bar{\boldsymbol{J}}_2\ ,\end{aligned}$$

we get

$$\boldsymbol{\tau}_1 = \bar{\boldsymbol{g}}_1 + \bar{\boldsymbol{J}}_1^T\boldsymbol{K}_d(\boldsymbol{r}_d - \bar{\boldsymbol{f}}) - \bar{\boldsymbol{D}}_{11}\dot{\boldsymbol{q}}_1 - \bar{\boldsymbol{D}}_{12}\dot{\boldsymbol{q}}_v\ , \tag{23}$$

$$\boldsymbol{\tau}_{v,2} = \bar{\boldsymbol{g}}_2 + \bar{\boldsymbol{J}}_2^T\boldsymbol{K}_d(\boldsymbol{r}_d - \bar{\boldsymbol{f}}) - \bar{\boldsymbol{D}}_{21}\dot{\boldsymbol{q}}_1 - \bar{\boldsymbol{D}}_{22}\dot{\boldsymbol{q}}_v\ . \tag{24}$$

From (22) and (24) we finally get

$$\begin{aligned}\dot{\boldsymbol{q}}_v = (b + \bar{\boldsymbol{D}}_{22})^{-1}\ &\Big(\sqrt{2b}\boldsymbol{v}_v + \bar{\boldsymbol{g}}_2 + \\ &\bar{\boldsymbol{J}}_2^T\boldsymbol{K}_d(\boldsymbol{r}_d - \bar{\boldsymbol{f}}) - \bar{\boldsymbol{D}}_{21}\dot{\boldsymbol{q}}_1\Big)\ .\end{aligned} \tag{25}$$

Instead of using the second order integration from (4) we thus integrate (25) and implement the control law (23)-(24) to get $\boldsymbol{\tau}_1$ and $\boldsymbol{\tau}_{v,1} = \boldsymbol{\tau}_{v,2}$. Apart from these changes the controller is the same as the one discussed in the last section.

Notice that the closed loop system still consists of a feedback interconnection of passive sub-systems and thus the overall

[5] suppressing dependence on $\boldsymbol{q}_1$ and $\boldsymbol{q}_v$ for better readability

system will be passive. The only change between the previous controller and the modified one is that the inertia $\boldsymbol{M}_v$ has been eliminated by changing the causality for the port on the left hand side of the wave-based communication.

## VII. SIMULATION

In order to evaluate the proposed method for applying the wave variables concept to the control of a robot with distributed control structure, a simple planar simulation of a three degrees-of-freedom robot is presented. Figure 6 shows a sketch of the simulated model in the starting configuration for the simulations. It is assumed that the computer C#1 is interfaced to the upper two joints while the lower joint is interfaced to a different computer C#2.

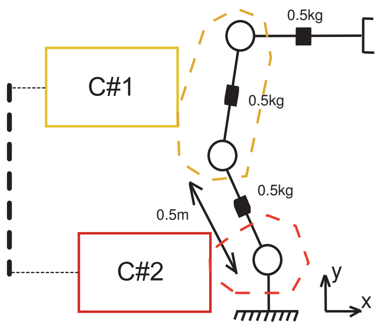


Fig. 6. Simple planar simulation model with 3 degrees-of-freedom. The computer C#1 is interfaced to the upper two joints and the computer C#2 is interfaced to the lower two joints.

As Cartesian coordinates the end-effector position and orientation are chosen. The desired Cartesian stiffness matrix used in the controller $\boldsymbol{K}_d$ has been chosen as a diagonal matrix with values of $1000N/m$ for the translational components and $100Nm/rad$ for the orientation. Notice that $\boldsymbol{K}_d$ is related to the effective stiffness at the end-effector via equation (21). The damping was also chosen as a diagonal matrix with $100Ns/m$ and $10Nms/rad$ for the translational and rotational components, respectively. In the simulations the response for a stepwise command of $10cm$ in $x-$direction is evaluated. All controllers were implemented with a high sampling rate of $0.1ms$ in order to avoid numerical errors in the velocity integration.

As a reference for the controllers from Section V and Section VI the control law (3) has been simulated for a system without communication delays. The corresponding step response of the Cartesian error is shown in Figure 7. Since the compliance control law (3) implements stiffness and damping without inertia decoupling one also sees some transient error for the translation in $y-$ direction and for the orientation.

Then, a simulation of the same control law but with a communication delay of $T_d = 6ms$ has been performed. For this communication delay the original control law (3) led to an unstable behavior as can be seen in Figure 3.

Next, the control law described in the sections IV and V is evaluated for the same situation. The wave impedance was

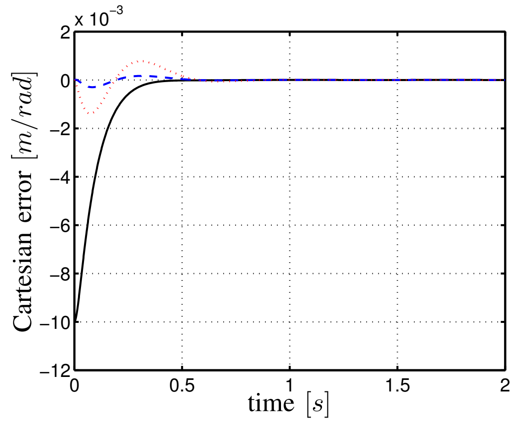


Fig. 7. Cartesian error of the reference controller (3) for a system without communication delay. The solid black and dashed blue line show the translational error in $x-$ and $y-$direction $[m]$, while the dotted red line shows the orientation error $[rad]$.

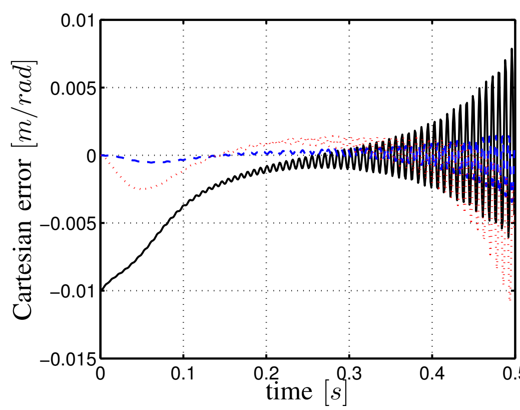


Fig. 8. Cartesian error of the reference controller (3) for a system with a communication delay of $T_d = 6ms$. The simulation was stopped after $0.5s$. The solid black and dashed blue line show the translational error in $x-$ and $y-$direction $[m]$, while the dotted red line shows the orientation error $[rad]$.

chosen as $b = 10Nms/rad$. Therefore, the virtual stiffness of the transmission is given by $K_t = b/T_d = 1667Nm/rad$. Notice that by choosing $b$ one has to make a compromise between the desire for a large "stiffness" $K_t$ and a small "inertia" $M_t$ of the wave communication. For the PD controller of the wave teleoperator values of $\boldsymbol{K} = 1000Nm/rad$ and $\boldsymbol{B} = 40Nms/rad$ have been chosen. For this setting two simulations with different values for the virtual inertia have been performed. First, a simulation with a small value of $\boldsymbol{M}_v = 0.1Nms^2/rad$ is shown in Figure 9. One can see that the system with wave communication now converges and does not become unstable. However, the dynamical response is of course not exactly the same as in Figure 7 due to the communication delay.

Next, the effect of the virtual inertia $\boldsymbol{M}_v$ on the closed loop dynamics shall be observed. Therefore, the same simulation was repeated with a large value of $\boldsymbol{M}_v = 1Nms^2/rad$ for the virtual inertia. The result can be seen in Figure 10. A larger virtual inertia considerably influences the closed loop

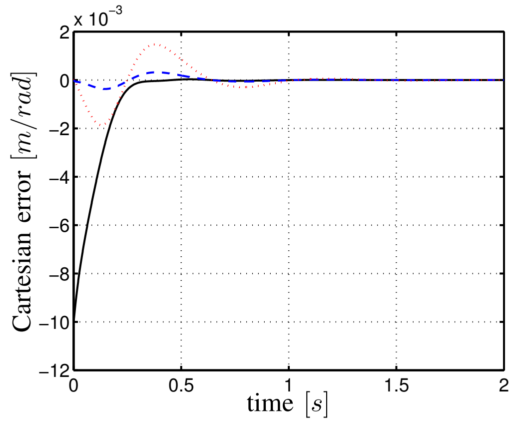


Fig. 9. Cartesian error $\boldsymbol{r}_d - \boldsymbol{f}(\boldsymbol{q}_1, \bar{\boldsymbol{q}}_2(\boldsymbol{q}_1, \boldsymbol{q}_v))$ of the controller (20) from Section V for a system with a communication delay of $T_d = 6ms$ and the virtual inertia set to a small value of $0.1Nms^2/rad$. The solid black and dashed blue line show the translational error in $x-$ and $y-$direction $[m]$, while the dotted red line shows the orientation error $[rad]$.

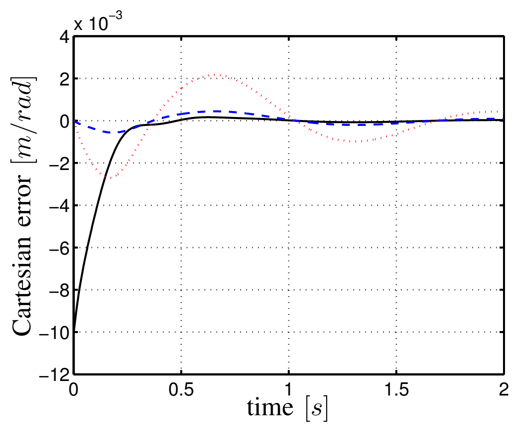


Fig. 10. Cartesian error $\boldsymbol{r}_d - \boldsymbol{f}(\boldsymbol{q}_1, \bar{\boldsymbol{q}}_2(\boldsymbol{q}_1, \boldsymbol{q}_v))$ of the controller (20) from Section V for a system with a communication delay of $T_d = 6ms$ and the virtual inertia set to a large value of $1Nms^2/rad$. The solid black and dashed blue line show the translational error in $x-$ and $y-$direction $[m]$, while the dotted red line shows the orientation error $[rad]$.

dynamics, even if no instability occurs.

This behavior can also be seen by observing the trajectories for the coordinate $\boldsymbol{q}_2$ and the virtual coordinate $\boldsymbol{q}_v$. The simulation results of these signals for the case of small ($\boldsymbol{M}_v = 0.1Nms^2/rad$) and large ($\boldsymbol{M}_v = 1Nms^2/rad$) virtual inertia are shown in Figure 11 and Figure 12, respectively. In these figures one can observe a similar behavior as for the Cartesian error. In addition one can see that in steady state there is a small difference between $\boldsymbol{q}_2$ and $\boldsymbol{q}_v$ according to the virtual stiffness $\boldsymbol{K}_c$ and the gravity load on the coordinates $\boldsymbol{q}_2$.

Finally, the simulation result for the controller from Section VI is shown in Figure 13. The advantage of this controller is that the virtual inertia has been eliminated. One can see that the performance is similar to the controller from Section V with small virtual inertia, cf. Figure 9. This can also be seen by observing the real and virtual coordinates $\boldsymbol{q}_2$ and $\boldsymbol{q}_v$ which are shown in Figure 14.

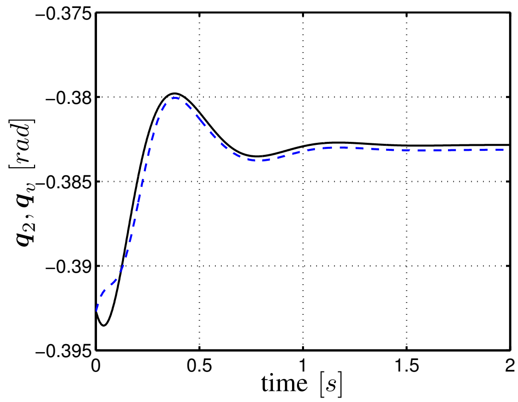


Fig. 11. Real joint angle $\boldsymbol{q}_2$ (black solid) and the virtual joint angle $\boldsymbol{q}_v$ (blue dashed) computed at the computer C#1 for the controller (20) from Section V for a system with a communication delay of $T_d = 6ms$ and the virtual inertia set to a small value of $0.1Nms^2/rad$.

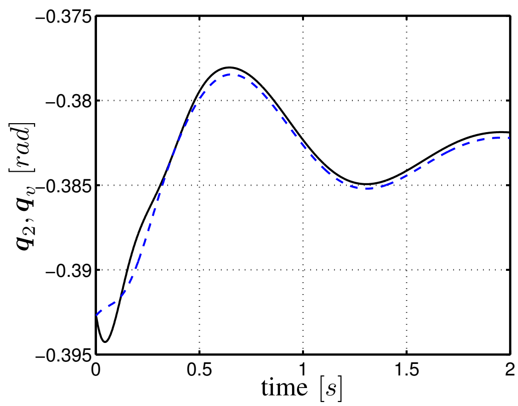


Fig. 12. Real joint angle $\boldsymbol{q}_2$ (black solid) and the virtual joint angle $\boldsymbol{q}_v$ (blue dashed) computed at the computer C#1 for the controller (20) from Section V for a system with a communication delay of $T_d = 6ms$ and the virtual inertia set to a large value of $1Nms^2/rad$.

## VIII. SUMMARY

In this paper the coordinated Cartesian compliance control of a robot system with distributed control architecture was treated. The wave variables concept has been utilized for handling the communication delays between different subsystems. In order to interface the basic wave teleoperator with the Cartesian compliance control law a virtual inertia was introduced. Furthermore, in the implementation of the compliance control law one must take the steady state behavior of the wave teleoperator into account. For this, some techniques developed in the realm of flexible joint robot control have been adopted. In a second step it was shown how the virtual inertia can be avoided such that the compliance control law is directly interfaced to the wave teleoperator. As a consequence the desired dynamics is approximated better and also the parametrization of the controller is simpler. In both approaches the closed loop dynamics consists of several passive sub-systems such that also the overall system is passive. This ensures robust interaction with unknown but passive environments. For this in particular the passivity

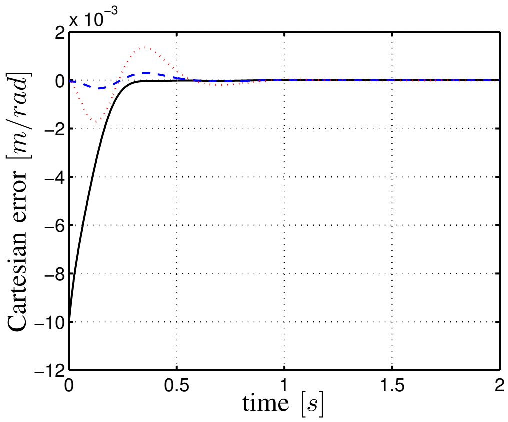


Fig. 13. Cartesian error $(\boldsymbol{r}_d - \bar{\boldsymbol{f}})$ of the controller from Section VI for a system with a communication delay of $T_d = 6ms$. The solid black and dashed blue line show the translational error in $x-$ and $y-$direction $[m]$, while the dotted red line shows the orientation error $[rad]$.

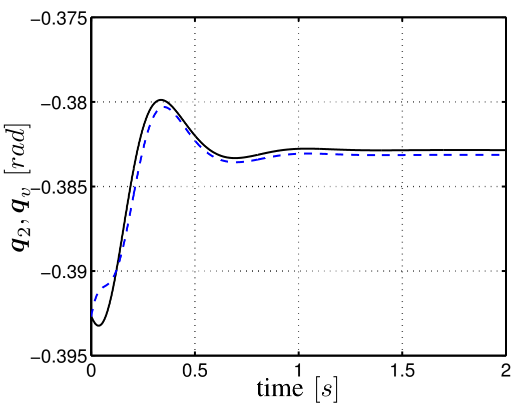


Fig. 14. Real joint angle $\boldsymbol{q}_2$ (black solid) and the virtual joint angle $\boldsymbol{q}_v$ (blue dashed) computed at the computer C#1 for the controller from Section VI for a system with a communication delay of $T_d = 6ms$.

properties of the wave communication and of the compliance control law have been utilized. Finally, some simple planar simulations have been presented which validate the proposed approach.

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