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Formation Control Over Delayed Communication Networks

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Abstract— **In this paper we address the problem of formation control of a group of robots that exchange information over a delayed communication network. We consider the Virtual Body Artificial Potential approach for stabilizing a group of robots at a desired formation. We show that it is possible to model the controlled system as a set of elements exchanging energy along a power preserving interconnection structure. We exploit the scattering framework to stabilize the robots in the desired formation independently of any delay in the communication of the information.**

I. Introduction

A central problem in the coordination of multi-robot systems is the formation control. An example is in the framework of mobile sensor networks where it is desirable to control the disposition of the sensors in such a way to maximize the amount of information that they are able to detect. Several approaches for solving the formation control problem have been proposed in the literature [1], [2], [3], [4], [5], [6], [7].

In most of the current approaches for formation control, it is assumed that the information is instantaneously exchanged among the robots and this can be quite restrictive. In fact, in real applications, it is necessary to choose a medium over which the robots exchange information. Packets switched networks are very good candidate to play this role, as noted in [5]. Nevertheless, the use of this kind of medium introduces some problems that need to be addressed. First of all the information is exchanged among the robots with a certain communication delay which tends to destabilize the controlled system. Moreover, some packets can get lost and the communication delay can be variable.

In [7], [8] the virtual body artificial potential (VBAP) approach has been introduced. Artificial potentials are exploited for reproducing the social forces, a concept deduced from biological studies of animal aggregations [9], in a group of mobile robots.The goal of this work is to extend the VBAP approach in order to take into account a possible non negligible delay in the communication among the robots and between the robots and the virtual leaders. We will exploit the scattering framework in the VBAP formation control strategy in order to stabilize a group of robots at a desired formation when the information is exchanged over a delayed communication channel. First of all we will formulate the VBAP strategy within the port-Hamiltonian framework [10], [11] in order to put into evidence the interconnection structure through which energy is exchanged, namely, the communication protocol through which the robots and the elements associated to the artificial potentials exchange the information for obtaining the desired formation. We will then reformulate this communication protocol in terms of scattering variables and we will prove that the new communication strategy allows to stabilize the robots in the desired formation independently of any communication delay.

The paper is organized as follows: in Sec. II we will give some background on the VBAP approach, on port-Hamiltonian systems and on the scattering framework. In Sec. III we will interpret the VBAP control strategy within the port-Hamiltonian framework. In Sec. IV we will exploit the scattering framework for implementing the VBAP formation control over a delayed network. In Sec. V we will provide some simulations to validate the results proposed in the paper and, finally, in Sec. VI we will give some concluding remarks and we will address some future work.

II. Background

A. The VBAP approach for formation control

Consider a group of N agents (that can be mobile robots, UAV, etc.) that we want to take in a desired formation. We will indicate with $x_{a_i} \in \mathbb{R}^3$, the position of the i^{th} agent with respect to an inertial frame. Furthermore, we will indicate with $u_i \in \mathbb{R}^3$ the control input of the i^{th} vehicle. We consider fully actuated, point mass agent models whose dynamics is represented by:

$$\ddot{x}_{a_i}(t) = u_i(t) \qquad i = 1, \ldots, N \tag{1}$$

In the space where the agents are moving, a set of L reference points, called *virtual leaders*, are introduced. The position of the k^{th} virtual leaders is denoted by $x_{l_k} \in \mathbb{R}^3$. In this paper we consider the case in which all the leaders are at rest. This means that we are focusing on the problem of taking all the agents in the desired formation and not on the problem of moving the formation.

Let $x_{a_{ij}} = x_{a_i} - x_{a_j}$ be the relative position of agent i with respect to agent j. Between each pair of agents, an artificial potential $V_I(x_{a_{ij}})$, that we call *interagent potential*, is defined. This potential depends on the distance between the i^{th} and the j^{th} agents and its role is to produce a radial force $f_I(x_{a_{ij}})$ that regulates the distance between the agents. Similarly let $x_{a_i l_k} = x_{a_i} - x_{l_k}$ be the relative position of agent i with respect to leader k. Between each agent and each virtual leader a potential $V_h(x_{a_i l_k})$, that we call *leader potential*, is defined. This potential depends on the distance between the i^{th} agent and the k^{th} leader and its role is to

produce a radial force $f_h(x_{a_i l_k})$ that regulates the distance between the agent and the leader.

The control of each agent is defined as the sum of the forces induced by the potentials and of a damping term, namely as

$$u_i = -\sum_{i=1}^{N} \frac{\partial V_I(x_{a_{ij}})}{\partial x_{a_{ij}}} - \sum_{k=1}^{L} \frac{\partial V_h(x_{a_i l_k})}{\partial x_{a_i l_k}} - R_i \dot{x}_{a_i} =$$
$$= -\sum_{i=1}^{N} \frac{f_I(x_{a_{ij}})}{\|x_{a_{ij}}\|} x_{a_{ij}} - \sum_{k=1}^{L} \frac{f_h(x_{a_i l_k})}{\|x_{a_i l_k}\|} x_{a_i l_k} - R_i \dot{x}_{a_i} \quad (2)$$

where R_i is a positive definite matrix.

The shape of the potential V_I has to be such to produce a repulsive force when two agents are too close, namely when $\|x_{a_{ij}}\| < d_0$, an attractive force when two agents are far, namely when $\|x_{a_{ij}}\| > d_0$ and a null force when the agents are too far, namely when $\|x_{a_{ij}}\| > d_1 > d_0$; d_1 and d_0 are design parameters. The shape of the potential V_h is similar and it depends on possibly different design parameters h_0 and h_1 which play the same role as d_0 and d_1.

The group of agents is immersed in a potential field which is given by the sum of all the artificial potentials. The minimum of this potential is given by a configuration where all the agents are at a distance d_0 from the neighboring agents and at a distance h_0 from the neighboring leaders. Choosing properly the position of the virtual leaders and the parameters h_0, d_0, h_1 and d_1 it is possible to shape the overall potential in such a way that its minimum corresponds to a desired formation.

Defining the state of the agents group as $x = (x_{a_1}, \ldots, x_{a_N}, \dot{x}_{a_1}, \ldots, \dot{x}_{a_N})^T$ and using as a candidate Lyapunov function

$$V(x) = \frac{1}{2} \sum_{i=1}^{N} (\dot{x}_{a_i}^T \dot{x}_{a_i} + \sum_{j \neq i}^{N} V_I(x_{a_{ij}}) + 2 \sum_{k=1}^{L} V_h(x_{a_i l_k}) \quad (3)$$

it is possible to prove that the configuration $x = (\bar{x}, 0)$, where $\bar{x}$ is the minimum of the sum of the artificial potentials, is asymptotically stable. For further information see [7], [8]

B. Port-Hamiltonian systems and the scattering framework

The port-Hamiltonian modeling framework is the mathematical formalization of the bond-graph strategy for representing physical systems. Loosely speaking, a port-Hamiltonian system is made up of a set of energy processing elements that exchange energy through a power preserving interconnection. More formally, port-Hamiltonian systems are defined on the state manifold of energy variables $\mathcal{X}$ and they are characterized by a lower bounded Hamiltonian energy function $H : \mathcal{X} \mapsto \mathbb{R}$, expressing the amount of stored energy, and by a Dirac structure $\mathcal{D}$, representing the internal energetic interconnections. Using coordinates, in their simplest form, they are represented by the following equations

$$\begin{cases} \dot{x}(t) = (J(x) - R(x)) \frac{\partial H}{\partial x} + g(x)u(t) \\ y(t) = g^T(x) \frac{\partial H}{\partial x} \end{cases} \quad (4)$$

where $x \in \mathbb{R}^n$ is the coordinate vector of the energy variables $J(x)$ is a skew-symmetric matrix representing the Dirac structure, $R(x)$ is a positive semidefinite function representing the energy dissipated by the system, H is the Hamiltonian function, u is the time dependent input vector, $g(x)$ is the input matrix and y is time dependent the conjugated output variable. The dependence on time of the state has been omitted in order to keep the notation simple. The pair of dual power variables (u, y) forms the power port through which the port-Hamiltonian system exchanges energy with the rest of the world. The power provided to the system at the instant t is given by $P(t) = u^T(t)y(t)$. It can be easily seen that the following power balance holds:

$$\dot{H}(t) = y^T(t)u(t) - \frac{\partial^T H}{\partial x} R(x) \frac{\partial H}{\partial x} \leq y^T(t)u(t) \quad (5)$$

The scattering framework allows to interpret the power flowing through a power port as the difference of incoming power wave and outgoing power wave rather than as a product of power variables. More formally, given a power port $(e(t), f(t))$, the power flowing through it can be decomposed into an incoming power wave $s^+(t)$ and an outgoing power wave $s^-(t)$ in such a way that

$$e^T(t)f(t) = \frac{1}{2}\|s^+(t)\|^2 - \frac{1}{2}\|s^-(t)\|^2 \quad (6)$$

where $\|\cdot\|$ is the standard Euclidean norm and the pair of scattering variables $(s^+(t), s^-(t))$ is defined as

$$\begin{cases} s^+(t) = \frac{1}{\sqrt{2b}}(e(t) + bf(t)) \\ s^-(t) = \frac{1}{\sqrt{2b}}(e(t) - bf(t)) \end{cases} \quad (7)$$

where $b > 0$ is the impedance of the scattering transformation. The relation in Eq.(7) is bijective and, consequently, a power port can be equivalently represented both as a pair of conjugated power variables or as a pair of scattering variables. For further information see [10], [11].

III. A PORT-HAMILTONIAN INTERPRETATION OF VBAP APPROACH

In this section we will show that a group of agents whose formation is controlled by means of the VBAP strategy can be interpreted as a set of energy processing elements exchanging energy along a power preserving interconnection, that is as a port-Hamiltonian system. First of all, let us consider the main actors in the VBAP approach: the agents, the interagent potentials and leader potentials.

Each agent can be modeled as a kinetic energy storing element, namely as a mass characterized by a certain inertia. Thus, the energy variable describing the state of the agent i is the momentum p_{a_i} and the kinetic energy function associated to agent i is given by $H_{a_i} = \frac{1}{2} M_i^{-1} p_{a_i}^T p_{a_i}$, where M_i is the inertia matrix of the agent. In order to take into account the damping effect introduced by the VBAP strategy for asymptotically stabilizing the formation, we embed a dissipative effect to the dynamics of each agent.

Thus, indicating with (e_{a_i}, f_{a_i}) the power port associated to agent i we have that a generic agent can be described by

$$\begin{cases} \dot{p}_{a_i} = -R_i M_i^{-1} p_i + e_{a_i} \\ f_{a_i} = M_i^{-1} p_i \end{cases} \tag{8}$$

where $p_{a_i} = M_i \dot{x}_{a_i} \in \mathbb{R}^3$ is the momentum of the agent and R_i is a symmetric positive definite matrix representing the damping imposed by the VBAP control strategy on the i^{th} agent. The effort e_{a_i} corresponds to the input u_i in Eq.(1). We can easily see that $e_{a_i}^T f_{a_i} = \dot{H}_{a_i} + p_{a_i}^T M_i^{-T} R_i M^{-1} p_{a_i}$, namely that the power incoming through the power port is either stored in form of kinetic energy or dissipated. Both the interagent potentials and the leader potentials necessary for regulating the relative distance between the agents and between the agents and the leaders can be modeled as a set of potential energy storing elements which can be interpreted as virtual springs. The energy variables representing the state of the virtual spring between agent i and agent j is given by $x_{a_{ij}} \in \mathbb{R}^3$, namely by the relative position of agent i with respect to agent j. The energy variable representing the state of the virtual spring between agent i and leader k is given by $x_{a_i l_k} \in \mathbb{R}^3$, namely by the relative position of agent i with respect to leader k. The potential energy function representing the amount of energy stored in a given configuration for the elements associated to the interagent potentials and to the leader potentials are $V_I(\cdot)$ and $V_h(\cdot)$ respectively. The elements representing the interagent potential between agent i and agent j and to the potential between the agent i and leader k are described by

$$\begin{cases} \dot{x}_{a_{ij}} = f_{a_{ij}} \\ e_{a_{ij}} = \frac{\partial V_I(x_{a_{ij}})}{\partial x_{a_{ij}}} \end{cases} \qquad \begin{cases} \dot{x}_{a_i l_k} = f_{a_i l_k} \\ e_{a_i l_k} = \frac{\partial V_h(x_{a_i l_k})}{\partial x_{a_i l_k}} \end{cases} \tag{9}$$

where $(e_{a_{ij}}, f_{a_{ij}})$ and $(e_{a_i l_k}, f_{a_i l_k})$ are the the power ports associated to the elements representing the interagent potential and the leader respectively. The efforts $e_{a_{ij}}$ and $e_{a_i l_k}$ represent the forces that have to be applied to agent i because of the interagent potential regulating the distance from agent j and of the leader potential regulating the distance from leader k respectively. It follows directly from Eq.(9) that $e_{a_{ij}}^T f_{a_{ij}} = \dot{V}_I$ and $e_{a_i l_k}^T f_{a_i l_k} = \dot{V}_h$, namely the power incoming through the power ports is stored in form of potential energy.

Thus, all the main actors in the VBAP framework can be modeled as energy storing elements. The dynamics of the state variables and, consequently, the evolution of the agents, depend on the way in which the energy processing elements are interconnected, namely, by the way in which they exchange energy. The VBAP control strategy induces an interconnection structure along which all the elements that we have just defined exchange energy through their power ports. The interconnection structure represents a relationship between the power variables and it expresses the way in which the output variables have to be combined to form the input variables. In other words, it represents a communication protocol that defines how input/output information is exchanged among the energy processing elements. In order to make the notation compact, let us define $f_a = (f_{a_1}, \ldots, f_{a_N})$, $f_{aa} = (f_{a_{12}}, \ldots, f_{a_{N-1N}})$, $f_{al} = (f_{a_1 l_1}, \ldots, f_{a_N l_L})$ and $e_a = (e_{a_1}, \ldots, e_{a_N})$, $e_{aa} = (e_{a_{12}}, \ldots, e_{a_{N-1N}})$, $e_{al} = (e_{a_1 l_1}, \ldots, e_{a_N l_L})$.

Let us consider the interconnection between the agents and the elements representing interagent potentials. Given N agents, there are $N(N-1)/2$ interagent potentials elements, one per each pair of agents. The effort generated by each interagent potential element is applied to the pair of agents that it interconnects with opposite signs, as it happens for two masses connected by a spring. The effort acting on each agent is given by the sum of all the efforts generated by the interagent potential elements that interconnect it with the other agents. A possible interconnection between the agents and the interagent potential elements leads to the following efforts:

$$\begin{aligned} & e_{a_1} = e_{a_{12}} + \cdots + e_{a_{1N}} \\ & e_{a_2} = -e_{a_{12}} + e_{a_{23}} + \cdots + e_{a_{2N}} \\ & \cdots \\ & e_{a_N} = -e_{a_{1N}} - e_{a_{2N}} + \cdots - e_{a_{N-1N}} \end{aligned} \tag{10}$$

The flow to be given as an input to each interagent potential element is given by the difference between the flows of the agents that the elements interconnect. The sign convention chosen for composing the efforts produced by the interagent potential elements to form the efforts to apply to the agents and the power continuity of the interconnection between the agents and the interagent potential elements, induces the sign convention through which composing the flows of the agents in order to form those of the interagent potentials. In summary, the interconnection structure that joins the agents and the interagent potential elements is given by

$$\begin{pmatrix} e_a \\ f_{aa} \end{pmatrix} = \begin{pmatrix} O_1 & J_1 \\ -J_1^T & O_2 \end{pmatrix} \begin{pmatrix} f_a \\ e_{aa} \end{pmatrix} \tag{11}$$

where O_1 and O_2 represent square null matrices of order $3N$ and $3N(N-1)/2$ respectively and J_1 is a $3N \times 3N(N-1)/2$ matrix given by

$$J_1 = \left(\begin{array}{c|c|c|c} \begin{matrix} \text{N-1 columns} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} & \ldots & \mathbf{1} \\ \mathbf{0} & -\mathbf{1} & \mathbf{0} & \ldots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\mathbf{1} & \ldots & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \ldots & -\mathbf{1} \end{matrix} & \begin{matrix} \text{N-2 columns} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \ldots & \mathbf{0} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} & \ldots & \mathbf{1} \\ \mathbf{0} & -\mathbf{1} & \mathbf{0} & \ldots & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \ldots & -\mathbf{1} \end{matrix} & \cdots & \begin{matrix} \mathbf{0} \\ \mathbf{0} \\ \vdots \\ \mathbf{1} \\ -\mathbf{1} \end{matrix} \end{array} \right) \tag{12}$$

where $\mathbf{1}$ and $\mathbf{0}$ represent the 3×3 identity and null matrix respectively.

Let us now consider the energetic interconnection between the agents and the leaders. Each leader influences all the agents and, therefore, for each agent there are L energy storing elements yielding the action that each leader exerts on the agent. Thus, in a system with N agents and L leaders, there are NL potential energy storing elements acting on the agents. The effort acting on each agent because of the interaction with the virtual leaders is given by

$$e_{a_i} = -\sum_{k=1}^{L} \frac{\partial V_h(x_{a_i l_k})}{\partial x_{a_i l_k}} = -\sum_{k=1}^{L} e_{a i l_k} \tag{13}$$

The flow the element associated to the action of a leader on the agent i is given by the difference between the velocity

of agent i and the velocity of the leader. Since we are considering the case in which the leaders are not moving, the latter flow is zero. The sign convention chosen for the efforts and the power continuity of the interconnection between the agents and the potential elements induce the sum convention for the flows. Thus, the interconnection structure that joins the agents and the leader potential elements is given by

$$\begin{pmatrix} e_a \\ f_{al} \end{pmatrix} = \begin{pmatrix} O_1 & J_2 \\ -J_2^T & O_3 \end{pmatrix} \begin{pmatrix} f_a \\ e_{al} \end{pmatrix} \tag{14}$$

where O_3 represents a square null matrix of order $3NL$ and J_2 is a $3N \times 3NL$ matrix given by

$$J_2 = \left(\begin{array}{c|c|c|c} \begin{matrix} \text{N columns} \\ -\mathbf{1} & -\mathbf{1} & \dots & \mathbf{1} \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \end{matrix} & \begin{matrix} \text{N columns} \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \\ -\mathbf{1} & -\mathbf{1} & \dots & -\mathbf{1} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \end{matrix} & \cdots & \begin{matrix} \text{N columns} \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots \\ -\mathbf{1} & -\mathbf{1} & \dots & -\mathbf{1} \end{matrix} \end{array}\right) \tag{15}$$

where $\mathbf{1}$ and $\mathbf{0}$ represent the 3×3 identity and null matrix respectively.

Using Eq.(12) and Eq.(15) we have that the interconnection structure along which all the energy processing elements exchange energy is given by

$$\begin{pmatrix} e_a \\ f_{aa} \\ e_{al} \end{pmatrix} = \underbrace{\begin{pmatrix} O_1 & J1 & J2 \\ -J_1^T & O_2 & O_4 \\ -J_2^T & O_4^T & O_3 \end{pmatrix}}_{J} \begin{pmatrix} f_a \\ e_{aa} \\ f_{al} \end{pmatrix} \tag{16}$$

where O_4 denotes the $3N(N-1)/2 \times NL$ null matrix. Agents and elements associated to the artificial potentials exchange efforts and flows, namely energy, along an interconnection structure that is represented by J.

Thanks to the skew-symmetry of J it can be immediately seen that the following balance holds:

$$e_a^T f_a + e_{aa}^T f_{aa} + e_{al}^T f_{al} = 0 \tag{17}$$

which means that energy is neither produced nor dissipated but simply exchanged along the interconnection. This means that J describes a power preserving interconnection that can be represented by a Dirac structure. Using Eq.(8) and Eq.(9) with Eq.(16) we can model a group of agents controlled through the VBAP strategy as a port-Hamiltonian system. In fact we have that

$$\begin{pmatrix} \dot{p}_a \\ \dot{x}_{aa} \\ \dot{x}_{al} \end{pmatrix} = \left[\begin{pmatrix} O_1 & J1 & J2 \\ -J_1^T & O_2 & O_4 \\ -J_2^T & O_4^T & O_3 \end{pmatrix} - \begin{pmatrix} R & O_5 & O_6 \\ O_5^T & O_2 & O_4 \\ O_6^T & O_4^T & O_3 \end{pmatrix} \right] \begin{pmatrix} \frac{\partial H}{\partial p_a} \\ \frac{\partial H}{\partial x_{aa}} \\ \frac{\partial H}{\partial x_{al}} \end{pmatrix} \tag{18}$$

where O_5 and O_6 are null matrices of proper dimensions and $p_a = (p_{a_1}, \dots, p_{a_N})$, $x_{aa} = (x_{a_{12}}, \dots, x_{a_{N-1N}})$, $x_{al} = (x_{a_1 l_1}, \dots, x_{a_N l_L})$ The Hamiltonian function is given by

$$\begin{aligned} H(p_a, x_{aa}, x_{al}) &= H_a(p_a) + H_{aa}(x_{aa}) + H_{la}(x_{la}) = \\ &= \sum_{i=1}^{N} H_{a_i}(p_{a_i}) + \sum_{i=1}^{N} \sum_{j \neq i} V_I(x_{a_{ij}}) + \sum_{i=1}^{N} \sum_{k=1}^{L} V_h(x_{a_i l_k}) \end{aligned} \tag{19}$$

and R is the positive definite block diagonal matrices whose diagonal elements are the damping matrices R_i of the agents. The Hamiltonian function represents the total energy of the controlled group of agents, which is the same candidate function used in [7] and, therefore, it has minimum corresponding to the desired configuration of the vehicles. Using the power balance reported in Eq.(5) with Eq.(18) we obtain that

$$\dot{H} = -\frac{\partial^T H}{\partial p_a} R \frac{\partial H}{\partial p_a} \leq 0 \tag{20}$$

which, using LaSalle's invariance principle, proves that the minimum configuration of H is asymptotically stable. This is an alternative way to prove the stability results obtained in [7] using the port-Hamiltonian framework. Nevertheless, the advantage of the port-Hamiltonian representation of the VBAP formation control strategy is the fact that the interconnection structure along which the energy processing elements exchange information is evident and this will be the starting point for extending the VBAP approach over delayed networks.

IV. The effect of communication delay

The interconnection structure reported in Eq.(16) tells how to build the inputs of each element by combining the outputs of the others, namely it provides a communication protocol between the energy processing elements. In case the exchange of information among the energy processing elements takes place over a communication channel characterized by a delay T, we have that the communication protocol is described by:

$$\begin{pmatrix} e_a(t) \\ f_{aa}(t) \\ e_{al}(t) \end{pmatrix} = \begin{pmatrix} O_1 & J1 & J2 \\ -J_1^T & O_2 & O_4 \\ -J_2^T & O_4^T & O_3 \end{pmatrix} \begin{pmatrix} f_a(t-T) \\ e_{aa}(t-T) \\ f_{al}(t-T) \end{pmatrix} \tag{21}$$

The way in which the energy processing elements are interconnected and, in particular, the fact that the interconnection structure is power preserving plays a fundamental role in the formation stabilization process. Because of the communication delay, the balance reported in Eq.(17) doesn't hold anymore and thus, the interconnection structure is no more power preserving. This leads to a production of extra energy associated to the delayed exchange of information which invalidates the inequality reported in Eq.(20) and which prevents from stabilizing the agents in the desired formation. In case of formation control, we have several elements exchanging energy along a power preserving interconnection through their power ports. The main idea for achieving the desired formation control over a delayed network is to represent the power port of each energy processing element in terms of scattering variables. We will find the communication strategy that is equivalent to Eq.(16) in terms of scattering variables and we will see that no regenerative effects due to the communication delay take place and that the agents are stabilized in the desired formation independently of the communication delay.

In the following, for a given power port $(e_i(t), f_i(t))$, where $i \in \{a, aa, al\}$, we will denote with $(s_i^+(t), s_i^-(t)$ the

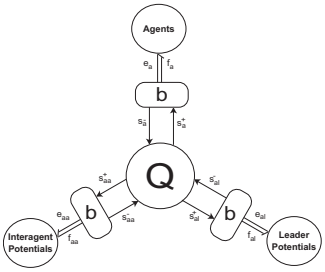

Fig. 1. The overall controlled system

corresponding pair of scattering variables. Using Eq.(7), we can express the efforts and flows in Eq.(16) in terms of scattering variable and, after simple computations we obtain the scattering based expression of the interconnection structure joining the energy processing elements which is given by:

$$\begin{pmatrix} s_a^+(t) \\ s_{aa}^+(t) \\ s_{al}^+(t) \end{pmatrix} = \underbrace{(I_t - J)^{-1}(I_t + J)\mathcal{I}}_{Q} \begin{pmatrix} s_a^-(t) \\ s_{aa}^-(t) \\ s_{al}^-(t) \end{pmatrix} \tag{22}$$

where I_t is the identity matrix of the same size of J and $\mathcal{I} = diag(-I_1, I_2, I_3)$, where I_1, I_2 and I_3 are identity matrices of proper dimensions.

It can be easily proven that Q is orthogonal. In fact we have that the matrix $(I_t - J)^{-1}(I_t + J)$ is orthogonal because it is the Cayley transform of the skew symmetric matrix J. Furthermore, it can be immediately seen that $\mathcal{I}^T = \mathcal{I}^{-1}$ and that therefore $\mathcal{I}$ is orthogonal. Since the product of orthogonal matrices is orthogonal, it follows that Q is orthogonal. The overall controlled system is reported in Fig. 1. Each energy storing element receives a scattering variable from the communication network and it uses it together with its output power variable to compute both the incoming power variable and the outgoing scattering variable to transmit over the network.

Consider now that a delay is present in the communication between the elements joined through the interconnection. We have that Eq.(22) becomes

$$\begin{pmatrix} s_a^+(t) \\ s_{aa}^+(t) \\ s_{al}^+(t) \end{pmatrix} = Q \begin{pmatrix} s_a^-(t-T) \\ s_{aa}^-(t-T) \\ s_{al}^-(t-T) \end{pmatrix} \tag{23}$$

The power flowing through the interconnection structure is given by

$$\begin{aligned} P(t) = &\frac{1}{2}\|s_a^-(t)\|^2 + \frac{1}{2}\|s_{aa}^-(t)\|^2 + \frac{1}{2}\|s_{al}^-(t)\|^2 - \\ &- \frac{1}{2}\|s_a^+(t)\|^2 - \frac{1}{2}\|s_{aa}^+(t)\|^2 - \frac{1}{2}\|s_{al}^+(t)\|^2 = \\ &\frac{1}{2}\begin{pmatrix} s_a^-(t) & s_{aa}^-(t) & s_{al}^-(t) \end{pmatrix} \begin{pmatrix} s_a^-(t) \\ s_{aa}^-(t) \\ s_{al}^-(t) \end{pmatrix} - \\ &- \frac{1}{2}\begin{pmatrix} s_a^+(t) & s_{aa}^+(t) & s_{al}^+(t) \end{pmatrix} \begin{pmatrix} s_a^+(t) \\ s_{aa}^+(t) \\ s_{al}^+(t) \end{pmatrix} \end{aligned} \tag{24}$$

using Eq.(23) with Eq.(24), we have that

$$\begin{aligned} P(t) = &\frac{1}{2}\begin{pmatrix} s_a^-(t) & s_{aa}^-(t) & s_{al}^-(t) \end{pmatrix} \begin{pmatrix} s_a^-(t) \\ s_{aa}^-(t) \\ s_{al}^-(t) \end{pmatrix} - \\ &- \frac{1}{2}\begin{pmatrix} s_a^-(t-T) & s_{aa}^-(t-T) & s_{al}^-(t-T) \end{pmatrix} Q^T Q \begin{pmatrix} s_a^-(t-T) \\ s_{aa}^-(t-T) \\ s_{al}^-(t-T) \end{pmatrix} \end{aligned} \tag{25}$$

Since Q is orthogonal, we have that

$$\begin{aligned} &P(t) = \frac{1}{2}\|s_a^-(t)\|^2 + \frac{1}{2}\|s_{aa}^-(t)\|^2 + \frac{1}{2}\|s_{al}^-(t)\|^2 - \\ &- \frac{1}{2}\|s_a^-(t-T)\|^2 - \frac{1}{2}\|s_{aa}^-(t-T)\|^2 - \frac{1}{2}\|s_{al}^-(t-T)\|^2 = \\ &= \frac{d}{dt}\int_{t-T}^{t}(\frac{1}{2}\|s_a^-(\tau)\|^2 + \frac{1}{2}\|s_{aa}^-(\tau)\|^2 + \frac{1}{2}\|s_{al}^-(\tau)\|^2)d\tau = \\ &= \dot{H}_{ch}(t) \end{aligned} \tag{26}$$

All the power injected into the interconnection structure is simply stored and, therefore, using the scattering variable, the delayed network used by the agents for exchanging information is characterized by a lossless behavior. Thus, we have that the power exchanged by the actors of the VBAP approach is stored in the communication channel until it is not delivered and that the delay doesn't introduce any regenerative effects in the interconnection.

We are now ready to prove that the agents are stabilized in the desired formation. Consider as a candidate Lyapunov function the total energy of the controlled system namely

$$\begin{aligned} \mathcal{H} &= H(p_a, x_{aa}, x_{la}) + H_{ch}(t) = \\ &= H_a(p_a) + H_{aa}(x_{aa}) + H_{la}(x_{la}) + H_{ch}(t) \end{aligned} \tag{27}$$

From the definition of H_{ch}, we can see that minimum configuration of $\mathcal{H}$ is the same as that the total energy of the non delayed system defined in Eq.(19). Thus, if we prove that the minimum of $\mathcal{H}$ is asymptotically stable, we ensure that agents are controlled in the desired formation independently of the communication delay.

Using Eq.(8) and Eq.(9) we have that

$$\begin{aligned} &\dot{\mathcal{H}}(t) = \dot{H}_a(t) + \dot{H}_{aa}(t) + \dot{H}_{la}(t) + \dot{H}_{ch}(t) = e_a^T(t)f_a(t) + \\ &+ e_{aa}^T(t)f_{aa}(t) + e_{al}^T(t)f_{al}^T(t) - \frac{\partial^T H_a}{\partial p_a} R \frac{\partial H_a}{\partial p_a} + \dot{H}_{ch}(t) \end{aligned} \tag{28}$$

Using Eq.(6) with Eq.(26) we obtain that

$$\dot{\mathcal{H}}(t) = -\frac{\partial^T H_a}{\partial p_a} R \frac{\partial H_a}{\partial p_a} \tag{29}$$

Thus, recalling the R is a positive definite matrix and using the LaSalle's invariance principle, we have that the minimum configuration of $\mathcal{H}$ are (locally) asymptotically stable. Thus, we have proven the following result

Proposition 1: Using the communication strategy proposed in Eq.(23) the agents are stabilized in the desired formation independently of any constant transmission delay.

Remark 1: Similarly to what happens in bilateral teleoperation, the knowledge of the communication delay is not required for guaranteeing the stabilization of the formation.

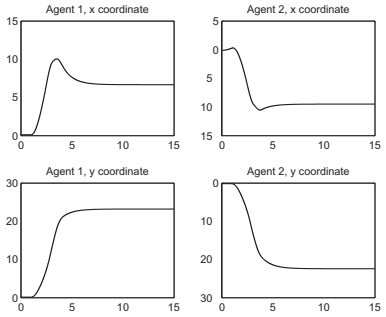

Fig. 2. The positions of the agents when using the VBAP strategy with a communication delay of 1 s.

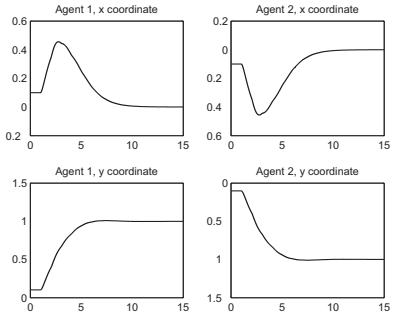

Fig. 3. The positions of the agents when using the VBAP strategy and scattering variables

The transmission of scattering variables guarantees the stability but it causes a bad transient behavior because of the wave reflection phenomenon. This problem is very well known in telerobotics and the techniques developed in the literature can be used in our case. A simple solution, widely used in teleoperation, is to add a feed-through damping action, with damping coefficient equal to the scattering impedance b, to the potential energy storing elements; for more details see [10].

V. Simulations

The aim of this section is to provide some simulations in order to validate the results obtained in the paper. We consider a simple example, treated also in [8], where there are two agents moving on a xy plane. We want to take the agents in the xy coordinates $(0, 1)$ and $(0, -1)$. As reported in [8] this problem can be solved using two leaders set on the x-axis at the configuration $\pm\sqrt{3}/2$.The interagent potential and the leader potential are characterized by the parameters $d_0 = h_0 = 1$ and $d_1 = h_1 = 2$ and the shape of the potentials is the same as in [8]. The initial positions of the agents are $(0.1, 0.1)$ and $(-0.1, -0.1)$. In the first simulation the VBAP control strategy has been implemented with a delay of 1 s. in the exchange of information between the agents and the artificial potential elements. The results are reported in Fig. 2. we can see that the delay induces an unstable behavior. In fact, we have that the forces applied to the agents grow very quickly and this implies that the agents are taken at a relative distance and at a distance from the leaders greater than 2. No force but the damping acts on the agents at this point and, consequently, the agents stop when all the energy they have accumulated is dissipated by the damping. Nevertheless, at steady state, the agents are NOT in the desired formation because of the instability induced by the communication delay.

In the next simulation, we have implemented the communication between the agents and the artificial potential elements using scattering variables (with scattering impedance equal to 1) as described in Sec. IV and we have considered a communication delay of 1 second. The wave reflection problem has been solved by adding a feed-through dissipative action to the artificial potential elements. The results are reported in Fig. 3. We can see that, despite of the communication delay, the agents are taken to the desired formation.

VI. Conclusions and future work

In this paper we have considered the VBAP approach for the stabilization of a formation of a group of point mass agents. We have shown that the controlled group of agents can be modeled as a set of elements exchanging energy over a power preserving interconnection. We have proven that, using the scattering framework, it is possible to keep on stabilizing the agents in the desired formation independently of any communication delay in the exchange of information.

In our future work we aim at extending the proposed framework to the cases in which the communication delay is variable and some information can be lost in the communication, as it often happens when using packet switched networks.

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