

# Decentralised Fault Tolerance and Fault Detection of Modular and Reconfigurable Robots with Joint Torque Sensing

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**Abstract**—A decentralised approach to fault tolerant control and fault detection is proposed for modular and reconfigurable robots with joint torque sensing. The proposed fault tolerant control scheme is independent of fault detection, avoiding the chances of delay being introduced by the detection scheme on the fault tolerant control algorithm. Based on a unique joint by joint control approach, the proposed fault tolerant controller for each module neither requires motion states of any other modules, nor the link dynamics. The addition or removal of modules does not affect the control of other joint modules. Uncalibrated torque sensor signals are utilized and actuator performance degradation is considered. Faults are detected and corrective measures are taken at the module level. An observer-based fault detection algorithm is proposed by using a residual generated from the joint velocity estimation and measured joint velocity. Simulation and experimental results have confirmed the effectiveness of the proposed fault tolerant control and fault detection schemes.

## I. INTRODUCTION

With a wide range of applications, especially in aerospace sector, the development of modular and reconfigurable robot (MRR) has been one of the promising research areas in robotics [1]. A recent survey of MRR systems can be found in [2]. The main concept of developing MRRs is based on the use of modular components as building blocks. For this reason, various modules have been proposed for reconfigurable robots. However, most of the reported modules are the traditional mechanical components, i.e., joints and links. While the reported reconfigurable robots may represent excellent mechanical design concepts, the modules of known MRRs are not modular from control systems point of view because of the existence of dynamic coupling among the modules, which is left to be dealt with by the controller.

Dynamic control of manipulators can be performed using joint torque sensing [3] - [5], without the need for modeling of link dynamics. The effectiveness of these approaches depends on accurate joint-torque sensing. Since joint torque sensor gains and offsets are susceptible to changes due to varying temperature and other factors and onboard calibration of joint torque sensors is difficult, it is desirable to estimate torque sensor parameters. In [6] the dynamical equation is parameterized such that torque sensor parameters are included in the overall system parameters to be estimated. This approach in other words would accept uncalibrated torque sensor signals for the controller. In our

previous work [7], a distributed control technique for modular and reconfigurable robots is developed based on joint torque sensing, enabling the joint by joint stabilization of the modular robot and allowing instant adaptation to robot reconfigurations. As there is no coupling effect left on the base joint after the feedback of torque sensor signal, this joint can be stabilized using any control design technique for a single joint, such as decomposition based control scheme [8]. Once the base joint is stabilized independently, the acceleration and velocity of this joint must be bounded and can only cause bounded uncertainty to upper joints. The bounded uncertainty is then compensated at upper modules to achieve stabilization of the succeeding joint. Proceeding similarly, the upper modules are stabilized.

Based on our previous work, the present work is aimed to achieve fault detection and fault tolerance at individual MRR modules, so that potential faults are dealt with at the module level and a faulty module can be repaired or replaced independent of the rest modules. Most approaches of fault tolerant control in robot manipulators are centered on the addition of some form of redundancy. An alternate way of achieving redundancy is by means of analytical relationships among system variables. This form of redundancy termed as analytical redundancy has received significant attention in the past. Several such approaches to fault tolerance of robots have been proposed, such as observer-based approaches [9]-[13], parity based [14] and parameter estimation based methods [15]-[17]. In [9] residuals are generated by comparing the predicted observer outputs with the measured system outputs. However, in these observer-based schemes, the measurement or estimation of acceleration signals are necessary. All these techniques are designed for robot manipulators with fixed configuration, and are not based on distributed control schemes. In [18] an adaptive robot control strategy with consideration of actuator faults is proposed, which incorporates actuator effectiveness factors in a model parameterization with commanded torque as the input.

In this paper decentralised fault tolerant control and fault detection schemes of modular robots are developed based on a joint-by-joint approach. In the proposed fault tolerant control of MRRs, actuator degradation at each joint module is tolerated independently of the other modules and fault detection schemes. For the proposed fault detection that is run in parallel with the fault tolerant control algorithm, a threshold based comparison on joint velocity estimation error is used to indicate the occurrence of a fault at each module. The threshold is determined based on the estimation error bounds obtained during fault free operation of the robot system. A fault is declared when the estimation error

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exceeds this threshold. Faults are detected and corrective measures can be taken at the module level. The addition or removal of a module will not affect the control of other modules, enhancing the modularity and reparability of the overall MRR system. The proposed technique is different from most approaches which detect faults first and then use the information of detected faults in the operation of fault tolerant control. This can cause unknown transients due to the delay in the detection algorithm. The fault tolerant scheme proposed here is independent of the fault detection and does not rely on the fault detection information for the operation of fault tolerant control. This effectively avoids the chances of delay in fault tolerant control due to the detection algorithm. In the proposed fault tolerant control scheme, an adaptation law is used to update actuator effectiveness factors, friction parameters and the torque sensor related parameters. Each of the joint controllers does not require the motion states of other joints or link dynamics, and each joint is controlled independently from other joint modules, making it suitable for control of modular robots.

The paper is organized as follows: Section 2 presents the proposed dynamical model formulation. Fault tolerance scheme is discussed in Section 3, followed by the fault detection scheme in Section 4. The simulations are presented in Section 5 to demonstrate the effectiveness of the fault tolerant and fault detection schemes. In Section 6, the experimental implementation of the proposed strategy on a single module is presented. Finally, in Section 7, some conclusions are drawn.

## II. DYNAMIC MODEL FORMULATION

We consider modular and reconfigurable robots constructed with  $n$  modules, and each module is integrated with a rotary joint with a speed reducer and a torque sensor as illustrated in Fig. 1.

For each module, we assume

- A1. The rotor is symmetric with respect to the rotation axis.
- A2. The joint flexibility is negligible.
- A3. The torque transmission does not fail at the speed reducer, and the inertia between the torque sensor and the speed reducer is negligible.

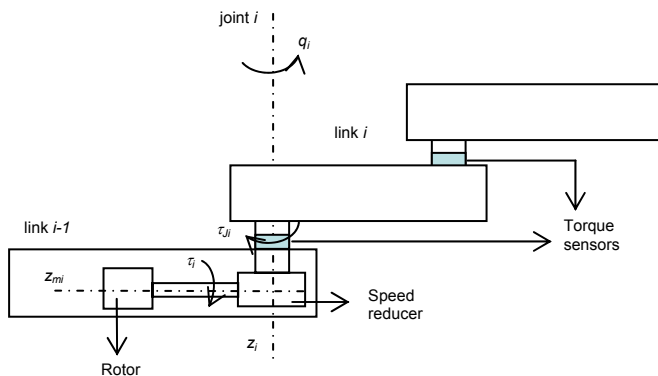


Fig. 1. Schematic diagram of a joint module

For a modular and reconfigurable articulated robot with modules installed in series, each module provides a rotary

joint. The base module is denoted the first module. Modules close to the first module are named lower modules, and modules close to the end-effector are called upper modules.

For the  $i^{th}$  module, we use the following notations:

$I_{mi}$  : rotor moment of inertia about the axis of rotation;

$\gamma_i$  : reduction ratio of the speed reducer ( $\gamma_i \geq 1$ );

$q_i$  : joint angle;

$f_i(q_i, \dot{q}_i)$  : joint friction;

$\tau_{ji}$  : coupling torque at the torque sensor location;

$\tau_i$  : output torque of the rotor;

$z_{mi}$  : unit vector along the axis of rotation of the  $i^{th}$  rotor;

$z_i$  : unit vector along the axis of rotation of joint  $i$ .

Based on the dynamic equations derived in [5], we formulate the dynamic equation of each module as follows.

For the base module,  $i = 1$ ,

$$I_{m1}\gamma_1\ddot{q}_1 + f_1 + \frac{\tau_{J1}}{\gamma_1} = \tau_1 \quad (1)$$

For the second module from the base,  $i = 2$ ,

$$I_{m2}\gamma_2\ddot{q}_2 + f_2 + I_{m2}z_{m2}^T z_1\ddot{q}_1 + \frac{\tau_{J2}}{\gamma_2} = \tau_2 \quad (2)$$

For  $i \geq 3$

$$I_{mi}\gamma_i\ddot{q}_i + f_i(q_i, \dot{q}_i) + I_{mi} \sum_{j=1}^{i-1} z_{mi}^T z_j \ddot{q}_j + I_{mi} \sum_{j=2}^{i-1} \sum_{k=1}^{j-1} z_{mi}^T (z_k \times z_j) \dot{q}_k \dot{q}_j + \frac{\tau_{Ji}}{\gamma_i} = \tau_i \quad (3)$$

The joint friction,  $f_i(q_i, \dot{q}_i)$ , is assumed to be [19],

$$f_i(q_i, \dot{q}_i) = B_i \dot{q}_i + F_q(q_i, \dot{q}_i) + \left( F_{ci} + F_{si} \exp(-F_{\tau i} \dot{q}_i^2) \right) \text{sat}(\dot{q}_i) \quad (4)$$

where  $F_{ci}$  denotes the Coulomb friction related parameter,  $B_i$  denotes the viscous friction coefficient,  $F_{si}$  denotes the static friction related parameter,  $F_{\tau i}$  is a positive parameter related to the Stribeck effect. The saturation function is defined as

$$\text{sat}(\dot{q}_i) = \begin{cases} 1 & \text{for } \dot{q}_i > 0 \\ 0 & \text{for } \dot{q}_i = 0 \\ -1 & \text{for } \dot{q}_i < 0 \end{cases} \quad (5)$$

and  $F_q(q_i, \dot{q}_i)$  denotes the position dependency of friction and other friction modeling errors.

The dynamic equation (3) can be rewritten as

$$I_{mi}\gamma_i\ddot{q}_i + f_i(q_i, \dot{q}_i) + \delta_i + \frac{\tau_{Ji}}{\gamma_i} = \tau_i. \quad (6)$$

The term  $\delta_i$  is constituted of the effects of lower joints on the  $i^{th}$  joint, given by

$$\delta_i = I_{mi} \sum_{j=1}^{i-1} z_{mi}^T z_j \ddot{q}_j + I_{mi} \sum_{j=2}^{i-1} \sum_{k=1}^{j-1} z_{mi}^T (z_k \times z_j) \dot{q}_k \dot{q}_j \quad (7)$$

This term is a source of model uncertainty, which depends on the acceleration and velocities of all the lower  $i-1$  joints. This uncertainty needs to be compensated using a robust controller at the  $i^{\text{th}}$  joint.

The actual joint torques measured using torque sensors are given by

$$\tau_{J_i} = \eta_i \tau_{si} + \mu_i \quad (8)$$

where  $\eta_i$  and  $\mu_i$  are sensor gain and offsets,  $\tau_{si}$  denotes the torque sensor output signal. The dynamic equation (3) can be rewritten as

$$I_{mi} \gamma_i \ddot{q}_i + f_i(q_i, \dot{q}_i) + I_{mi} \sum_{j=1}^{i-1} z_{mi}^T z_j \ddot{q}_j \quad (9)$$

$$+ I_{mi} \sum_{j=2}^{i-1} \sum_{k=1}^{j-1} z_{mi}^T (z_k \times z_j) \dot{q}_k \dot{q}_j + \kappa_i \tau_{si} + l_i = K_{ii} \tau_{ci} \quad (10)$$

$$Y_i(\ddot{q}_i, \dot{q}_i, q_i, \tau_{si}) P_i + \delta_i = K_{ii} \tau_{ci}$$

with  $\kappa_i = \eta_i / \gamma_i$ ,  $l_i = \mu_i / \gamma_i$ , and  $K_{ii}$ ,  $\tau_{ci}$  are the actuator effectiveness factor and commanded torque of  $i^{\text{th}}$  motor. If  $K_{ii} = 1$  the actuator is fault free and  $K_{ii} = 0$  indicates complete failure of the actuator.

$$Y_i(\ddot{q}_i, \dot{q}_i, q_i, \tau_{si}) = \left[ \ddot{q}_i \quad \dot{q}_i \quad \text{sat}(\dot{q}_i) \quad \exp(-F_{\tau i} \dot{q}_i^2) \text{sat}(\dot{q}_i) \quad \tau_{si} \quad 1 \right] \quad (11)$$

$$P_i = \begin{bmatrix} M_i & B_i & F_{ci} & F_{si} & \kappa_i & l_i \end{bmatrix}^T$$

where  $M_i = I_{mi} \gamma_i$ .

From (10) we have for the  $i^{\text{th}}$  module

$$\tau_{ci} = K_{ii}^{-1} Y_i(\ddot{q}_i, \dot{q}_i, q_i, \tau_{si}) P_i + K_{ii}^{-1} \delta_i \quad (12)$$

$$= Y_i(\ddot{q}_i, \dot{q}_i, q_i, \tau_{si}) \theta_i + K_{ii}^{-1} \delta_i$$

with

$$\theta_i = \begin{bmatrix} K_{ii}^{-1} M_i & K_{ii}^{-1} B_i & K_{ii}^{-1} F_{ci} & K_{ii}^{-1} F_{si} & K_{ii}^{-1} \kappa_i & K_{ii}^{-1} l_i \end{bmatrix}^T.$$

Denoting  $\hat{\theta}_i$  as the estimate of the parameter vector  $\theta_i$  and then the uncertainties in joint parameters and actuator effectiveness factors are contained in the parameter estimation inaccuracy  $\tilde{\theta}_i$ , which is given by

$$\tilde{\theta}_i = \hat{\theta}_i - \theta_i \quad (13)$$

The joint by joint stabilization of the modular robot is achieved [7] using the distributed control approach based on joint torque sensing. The following properties are used in the subsequent analysis and design of the control law.

*Property 1:* The acceleration and velocities of the stabilized joints are bounded. Hence the following upper bounds exist:

$$|\delta_i| \leq \rho_{Di} \quad (14)$$

$$\left| K_{ii}^{-1} \delta_i \right| \leq \rho_{Dfi} \quad (15)$$

With higher gear ratios, this upper bounds are significantly lower compared to gravitational torque and hence would not cause large controller gains.

*Property 2:* Since the term  $F_q(q_i, \dot{q}_i)$  is bounded the following upper bound exists

$$\left| F_q(q_i, \dot{q}_i) \right| \leq \rho_f \quad (16)$$

$$\left| K_{ii}^{-1} F_q(q_i, \dot{q}_i) \right| \leq \rho_{fq} \quad (17)$$

From (14) - (17), the following upper bound exists,

$$\left| \delta_i + F_q(q_i, \dot{q}_i) \right| \leq \rho_{Fi} \quad (18)$$

$$\left| K_{ii}^{-1} \delta_i + K_{ii}^{-1} F_q(q_i, \dot{q}_i) \right| \leq \rho \quad (19)$$

### III. FAULT TOLERANT CONTROL

In the design of the proposed fault tolerant control method, we consider actuator fault and assume that both the position and torque sensors are fault free.

For presenting the proposed control law, the following variables are defined first:

$$\begin{aligned} \tilde{q}_i &= q_i - q_{di}, & q_{vi} &= \dot{q}_{di} - \lambda_i \tilde{q}_i \\ q_{ri} &= \dot{\tilde{q}}_i + \lambda_i \tilde{q}_i, & q_{ai} &= \ddot{q}_{di} - 2\lambda_i \dot{\tilde{q}}_i - \lambda_i^2 \tilde{q}_i \end{aligned} \quad (20)$$

where  $\lambda_i$  is a positive constant.

The control law is defined as

$$\tau_{ci} = Y_i(q_{ai}, q_{vi}, q_i, \tau_{si}) \hat{\theta}_i - K_{Di} q_{ri} + U_f \quad (21)$$

and the adaptation law is defined as

$$\dot{\hat{\theta}}_i = -\Gamma_i^{-1} Y_i^T(q_{ai}, q_{vi}, q_i, \tau_{si}) q_{ri} \quad (22)$$

with  $Y_i(q_{ai}, q_{vi}, q_i, \tau_{si}) = \left[ q_{ai} \quad q_{vi} \quad \text{sat}(\dot{q}_i) \quad \exp(-F_{\tau i} \dot{q}_i^2) \text{sat}(\dot{q}_i) \quad \tau_{si} \quad 1 \right]$

and  $\Gamma_i$  is positive definite matrix and  $K_{Di} > 0$ . The control term  $U_f$ , is used to compensate for the term  $\delta_i$  constituted by lower modules and friction term  $F_q(q_i, \dot{q}_i)$  [8][19].

$$U_f = \begin{cases} -\rho \frac{q_{ri}}{|q_{ri}|} & |q_{ri}| > \varepsilon \\ -\rho \frac{q_{ri}}{\varepsilon} & |q_{ri}| \leq \varepsilon \end{cases} \quad (23)$$

where  $\varepsilon$  is a positive control parameter.

From (21) and (13),

$$\begin{aligned} \tau_{ci} &= Y_i(q_{ai}, q_{vi}, q_i, \tau_{si}) \left( \tilde{\theta}_i + \theta_i \right) - K_{Di} q_{ri} \\ &= K_{ii}^{-1} \left( M_i q_{ai} + B_i q_{vi} + F_{ci} \text{sat}(\dot{q}_i) + F_q(q_i, \dot{q}_i) \right) \\ &\quad + F_{si} \exp(-F_{\tau i} \dot{q}_i^2) \text{sat}(\dot{q}_i) + \kappa_i \tau_{si} + l_i + \delta_i \\ &\quad + Y_i(q_{ai}, q_{vi}, q_i, \tau_{si}) \tilde{\theta}_i - K_{Di} q_{ri} + U_f \end{aligned} \quad (24)$$

Combining (6), (12) and (24) we have

$$\begin{aligned} &K_{ii}^{-1} \left( M_i \dot{q}_{ri} + \lambda_i M_i q_{ri} + B_i q_{ri} \right) \\ &= Y_i(q_{ai}, q_{vi}, q_i, \tau_{si}) \tilde{\theta}_i - K_{Di} q_{ri} + U_f + K_{ii}^{-1} F_q(q_i, \dot{q}_i) + K_{ii}^{-1} \delta_i \end{aligned} \quad (25)$$

**Theorem 1:** Given an  $n$ -DOF modular robot, with joint dynamics as given in (1) - (3), actuator faults defined by variations of  $K_{ii}$  in (10) and the uncertainty defined in (13) - (19), then the tracking error of each joint is uniformly ultimately bounded under the control law defined by (21).

**Proof:** A Lyapunov function candidate is defined as

$$V = \frac{1}{2} K_{ii}^{-1} M_i q_{ri}^2 + \frac{1}{2} \tilde{\theta}_i^T \Gamma_i \tilde{\theta}_i \quad (26)$$

Differentiating the above expression yields

$$\dot{V} = K_{ii}^{-1} M_i q_{ri} \dot{q}_{ri} + \tilde{\theta}_i^T \Gamma_i \dot{\tilde{\theta}}_i \quad (27)$$

Since the unknown parameters  $\theta_i$  is constant, we have

$$\dot{\tilde{\theta}}_i = \hat{\dot{\theta}}_i \quad (28)$$

Substituting (25) and (28) into (27) gives

$$\begin{aligned} \dot{V} = & q_{ri} \left( Y_i(q_{ai}, q_{vi}, q_i, \tau_{si}) \tilde{\theta}_i - K_{Di} q_{ri} - K_{ii}^{-1} B_i q_{ri} \right. \\ & \left. - K_{ii}^{-1} \lambda_i M_i q_{ri} + U_f + K_{ii}^{-1} F_q(q_i, \dot{q}_i) + K_{ii}^{-1} \delta_i \right) \\ & - q_{ri} Y_i(q_{ai}, q_{vi}, q_i, \tau_{si}) \Gamma_i^{-1} \Gamma_i \tilde{\theta}_i \\ = & -K_{Di} q_{ri}^2 - K_{ii}^{-1} B_i q_{ri}^2 - K_{ii}^{-1} \lambda_i M_i q_{ri}^2 + q_{ri} U_f \\ & + q_{ri} K_{ii}^{-1} F_q(q_i, \dot{q}_i) + q_{ri} K_{ii}^{-1} \delta_i \end{aligned} \quad (29)$$

If  $|q_{ri}| \geq \varepsilon$ , combining (23) and (29) yields

$$\begin{aligned} \dot{V} = & -K_{Di} q_{ri}^2 - K_{ii}^{-1} B_i q_{ri}^2 - K_{ii}^{-1} \lambda_i M_i q_{ri}^2 \\ & - |q_{ri}| \left( \rho - \left( K_{ii}^{-1} F_q(q_i, \dot{q}_i) + K_{ii}^{-1} \delta_i \right) \frac{q_{ri}}{|q_{ri}|} \right) \\ < & -K_{Di} q_{ri}^2 - K_{ii}^{-1} B_i q_{ri}^2 - K_{ii}^{-1} \lambda_i M_i q_{ri}^2 < 0. \end{aligned} \quad (30)$$

If  $|q_{ri}| < \varepsilon$ ,

$$\begin{aligned} \dot{V} = & -K_{Di} q_{ri}^2 - K_{ii}^{-1} B_i q_{ri}^2 - K_{ii}^{-1} \lambda_i M_i q_{ri}^2 \\ & + q_{ri} \left( -\rho \frac{q_{ri}}{\varepsilon} - K_{ii}^{-1} F_q(q_i, \dot{q}_i) - K_{ii}^{-1} \delta_i \right) \\ \leq & -K_{Di} q_{ri}^2 - K_{ii}^{-1} B_i q_{ri}^2 - K_{ii}^{-1} \lambda_i M_i q_{ri}^2 - \frac{\rho}{\varepsilon} q_{ri}^2 + \rho |q_{ri}| \end{aligned} \quad (31)$$

Since the last four terms of (31) achieves a maximum value at  $|q_{ri}| \leq \varepsilon/2$ , we have

$$\dot{V} \leq -K_{Di} q_{ri}^2 - K_{ii}^{-1} B_i q_{ri}^2 - K_{ii}^{-1} \lambda_i M_i q_{ri}^2 + (\rho \varepsilon/4) \quad (32)$$

The rest of the proof follows straightforwardly Slotine and Li [20].

#### IV. FAULT DETECTION

The proposed fault detection method is an observer based scheme where actual joint velocity is compared against an estimated joint velocity. In the design of a fault detection method, it is assumed that no two faults occur at the same instant, which is a reasonable assumption normally made in reliability engineering. Then actuator fault occurring at each joint can be found using an observer depending on all the other sensor signals.

The dynamical equation of the  $i^{\text{th}}$  joint module of a modular robot is given by

$$\begin{aligned} \tau_i = & I_{mi} \gamma_i \ddot{q}_i + \left( F_c + F_s \exp(-F_\tau \dot{q}_i^2) \right) \text{sat}(\dot{q}_i) + \frac{\tau_{ji}}{\gamma_i} \\ & + F_q(q_i, \dot{q}_i) + B \dot{q}_i + \delta_i \end{aligned} \quad (33)$$

The nominal values of friction model parameters can be estimated through offline techniques. The bound of velocity estimation error can be found during fault free operation of the system. The velocity estimation error is defined as

$$e_i = v_i - \dot{q}_i \quad (34)$$

where  $v_i$  denotes the velocity estimate.

The velocity estimate can be obtained from the dynamical equation of  $i^{\text{th}}$  joint given by (33) and the nonlinear observer proposed in [21] to guarantee error convergence in the presence of uncertainties. The following velocity estimation is obtained by integrating the acceleration term in (33):

$$v_i = \frac{1}{I_{mi} \gamma_i} \left( \int_0^t \tau_i dt - \int_0^t \frac{\tau_{ji}}{\gamma_i} dt - \int_0^t B \dot{q}_i dt \right) - L_i \quad (35)$$

and

$$\dot{v}_i = \frac{1}{I_{mi} \gamma_i} \left( \tau_i - \frac{\tau_{ji}}{\gamma_i} - B \dot{q}_i - \left( F_c + F_s \exp(-F_\tau \dot{q}_i^2) \right) \text{sat}(\dot{q}_i) \right) - \dot{L}_i \quad (36)$$

with the observer  $L_i$  given by

$$L_i = \int_0^t K_1 e_i(\sigma) d\sigma + \int_0^t K_2 \text{sat}(e_i(\sigma)) d\sigma - \int_0^t \frac{U_i}{I_{mi} \gamma_i} d\sigma \quad (37)$$

where

$$U_i = \begin{cases} -\rho_{Fi} \frac{e_i}{|e_i|} & |e_i| \geq \varepsilon_0 \\ -\rho_{Fi} \frac{e_i}{\varepsilon_i} & |e_i| < \varepsilon_0 \end{cases} \quad (38)$$

$K_1 > 0, K_2 > 0$  and  $\varepsilon_0$  is a positive parameter. The term  $U_i$  is used to compensate for the term  $\delta_i$  constituted by the effects of lower  $i-1$  joint modules.

Estimation error  $e_i$  obtained from observed and measured velocity is used as the residual vector for fault detection with a threshold  $\varepsilon_{if}$ , which is a positive value obtained from fault free operation of the system. A fault is declared if  $|e_i| > \varepsilon_{if}$ , i.e., the estimation error exceeds the selected threshold. The thresholds can be set based on the various trials conducted in absence of faults. The time derivative of (34) is given by

$$\dot{e}_i = \dot{v}_i - \ddot{q}_i \quad (39)$$

$$\dot{e}_i = -K_1 e_i(t) - K_2 \text{sat}(e_i(t)) + \left( \frac{U_i}{I_{mi} \gamma_i} + \frac{\delta_i}{I_{mi} \gamma_i} + \frac{F_q(q_i, \dot{q}_i)}{I_{mi} \gamma_i} \right) \quad (40)$$

**Theorem 2:** Given an  $n$ -DOF modular robot, with joint dynamics as given in (33) and an observer defined in (37), then for each joint, the velocity estimation error given by (34) is uniformly ultimately bounded during the fault free operation of the modular robot.

**Proof:**

For stability analysis the Lyapunov function candidate is defined as

$$V = \frac{1}{2} e_i^2 \quad (41)$$

Differentiating (41) yields

$$\dot{V} = e_i \dot{e}_i \quad (42)$$

Substituting (40) in (42) gives

$$\begin{aligned} \dot{V} &= e_i \left( -K_1 e_i(t) - K_2 \text{sat}(e_i(t)) + \frac{U_i}{I_{mi} \gamma_i} + \frac{\delta_i}{I_{mi} \gamma_i} + \frac{F_q(q_i, \dot{q}_i)}{I_{mi} \gamma_i} \right) \quad (43) \\ &= -K_1 e_i^2 + \frac{e_i}{I_{mi} \gamma_i} \left( U_i + \delta_i + F_q(q_i, \dot{q}_i) \right) - e_i K_2 \text{sat}(e_i(t)) \end{aligned}$$

Thus if  $|e_i| \geq \varepsilon_0$ , the term  $\dot{V}$  in (43) becomes

$$\begin{aligned} \dot{V} &= -K_1 e_i^2 - e_i K_2 \text{sat}(e_i(t)) + \frac{e_i}{I_{mi} \gamma_i} \left( U_i + \delta_i + F_q(q_i, \dot{q}_i) \right) \\ &= -K_1 e_i^2 - e_i K_2 \text{sat}(e_i(t)) \\ &\quad - \frac{|e_i|}{I_{mi} \gamma_i} \left( \rho_{Fi} - (\delta_i + F_q(q_i, \dot{q}_i)) \frac{e_i}{|e_i|} \right) < 0 \end{aligned}$$

If  $|e_i| < \varepsilon_0$ , then

$$\begin{aligned} \dot{V} &= -K_1 e_i^2 - e_i K_2 \text{sat}(e_i(t)) + \frac{e_i}{I_{mi} \gamma_i} \left( -\rho_{Fi} \frac{e_i}{\varepsilon_0} - (\delta_i + F_q(q_i, \dot{q}_i)) \right) \quad \text{as } \hat{B}_i = 1.2 \text{ Nms/rad}, \hat{F}_{ci} = 1 \text{ Nm}, \hat{F}_{ti} = 80 \text{ s}^2/\text{rad}^2, \hat{F}_{si} = 0.8 \text{ Nm}, \\ &\leq -K_1 e_i^2 - e_i K_2 \text{sat}(e_i(t)) + \frac{1}{I_{mi} \gamma_i} \left( -\rho_{Fi} \frac{e_i^2}{\varepsilon_0} + \rho_{Fi} |e_i| \right) \\ &\leq -K_1 e_i^2 - \frac{\rho_{Fi}}{I_{mi} \gamma_i} \frac{e_i^2}{\varepsilon_0} - \left( K_2 - \frac{\rho_{Fi}}{I_{mi} \gamma_i} \right) |e_i| \end{aligned}$$

The expression  $-\frac{\rho_{Fi}}{I_{mi} \gamma_i} \frac{e_i^2}{\varepsilon_0} - \left( K_2 - \frac{\rho_{Fi}}{I_{mi} \gamma_i} \right) |e_i|$  achieves a maximum of  $(\rho_{Fi} - K_2 I_{mi} \gamma_i)^2 \varepsilon_0 / (4 \rho_{Fi} I_{mi} \gamma_i)$  at  $|e_i| = (\rho_{Fi} - K_2 I_{mi} \gamma_i) \varepsilon_0 / (2 \rho_{Fi})$ . Thus we have,

$$\dot{V} \leq -K_1 e_i^2 + \left( (\rho_{Fi} - K_2 I_{mi} \gamma_i)^2 \varepsilon_0 / (4 \rho_{Fi} I_{mi} \gamma_i) \right).$$

From the above expression it can be concluded that a Lyapunov function can be found if only

$$|e_i| > \sqrt{(\rho_{Fi} - K_2 I_{mi} \gamma_i)^2 \varepsilon_0 / (4 K_1 \rho_{Fi} I_{mi} \gamma_i)}.$$

Define

$$S = \left\{ e_i \in R^1 \mid e_i^2 \leq (\rho_{Fi} - K_2 I_{mi} \gamma_i)^2 \varepsilon_0 / (2 K_1 \rho_{Fi} I_{mi} \gamma_i) \right\}$$

Then on the surface of  $S$ ,  $\partial S$ , we have

$$\dot{V} \leq -(\rho_{Fi} - K_2 I_{mi} \gamma_i)^2 \varepsilon_0 / (4 \rho_{Fi} I_{mi} \gamma_i).$$

Denote  $T$  as the time for the solution trajectory to intersect the surface  $\partial S$ . Then

$$V(e_i(T)) - V(e_i(0)) \leq -\frac{(\rho_{Fi} - K_2 I_{mi} \gamma_i)^2 \varepsilon_0}{(4 \rho_{Fi} I_{mi} \gamma_i)} T.$$

$$\text{Hence, } T \leq \frac{(V(e_i(T)) - V(e_i(0)))(4 \rho_{Fi} I_{mi} \gamma_i)}{(\rho_{Fi} - K_2 I_{mi} \gamma_i)^2 \varepsilon_0}.$$

And the proof is complete.

## V. SIMULATIONS

To study the effectiveness of the proposed fault tolerant and fault detection technique, a 3-DOF serial robot manipulator, working on a horizontal plane, with the following parameters is considered

$$\begin{aligned} F_{\tau i} &= 100 \text{ s}^2/\text{rad}^2, B_i = 1.5 \text{ Nms/rad}, F_{ci} = 3.5 \text{ Nm}, \\ F_{si} &= 1 \text{ Nm}, K_{Di} = 100, \kappa_i = 0.1, l_i = 0, \\ \lambda_i &= 100, \Gamma = 1 \text{ s}^{6 \times 6}, \varepsilon = 0.1, \rho = 2, \rho_F = 1 \end{aligned} \quad (44)$$

TABLE I Parameters of the simulated system

	Link 1	Link 2	Link 3
Mass of link (kg)	8	5	4
Length of link (m)	1	1	1
Link inertia (kg-m <sup>2</sup> )	1.0	0.8	0.6
Dist. to centre of mass (m)	0.5	0.5	0.5
Rotor inertia (kg-m <sup>2</sup> )	0.4	0.2	0.1
Gear reduction ratio	10	10	10

The nominal parameters of the friction model are assumed  $\hat{B}_i = 1.2 \text{ Nms/rad}$ ,  $\hat{F}_{ci} = 1 \text{ Nm}$ ,  $\hat{F}_{ti} = 80 \text{ s}^2/\text{rad}^2$ ,  $\hat{F}_{si} = 0.8 \text{ Nm}$ ,  $\hat{\kappa}_i = 0.2$ ,  $\hat{l}_i = 0.5$ . For simplicity, the same friction model and parameters were considered for all the three joints. For simulation, the parameters of the manipulator are chosen as given in Table I. The desired trajectories for each of the three joints are selected as  $q_d = \sin(t) - 0.5 \sin(2t)$  for  $0 \leq t \leq 60 \text{ s}$ .

The actuator effectiveness factor for first joint module was changed from unity to 0.5 at  $t = 7 \text{ s}$  and the corresponding value for third module was changed from unity to 0.3 at  $t = 15 \text{ s}$ . The occurrence of fault is detected by the fault detection algorithm and the fault is tolerated by the control algorithm. The fault tolerant scheme ensures that the tracking error does not diverge even in the presence of actuator faults. This is depicted in Fig. 2. This figure shows the tracking errors for all the three joints, under the above mentioned actuator faults. The variations in actuator effectiveness factors are evident in the appreciable tracking error changes for the first and third joint. But along time the tracking error reduces despite the deviation in actuator effectiveness factor.

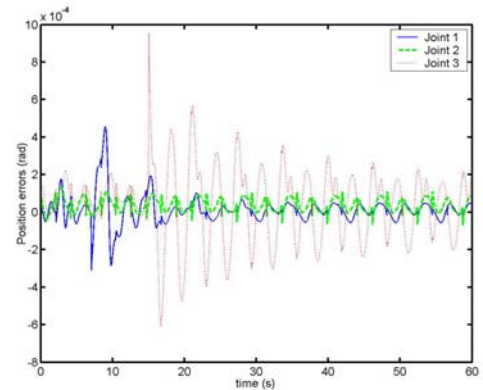


Fig. 2. Tracking errors under actuator faults (at  $t = 7 \text{ s}$  on first joint and  $t = 15 \text{ s}$  on third joint)

For the 3-DOF modular robot, simulations were carried out to study the effectiveness of the fault detection technique. After various trials under absence of faults, the threshold values  $\varepsilon_{if}$  were chosen as 0.01, 0.02 and 0.02 for the first, second and third joints, respectively. The observer gains were chosen as  $K_1 = 20$  and  $K_2 = 10$ .

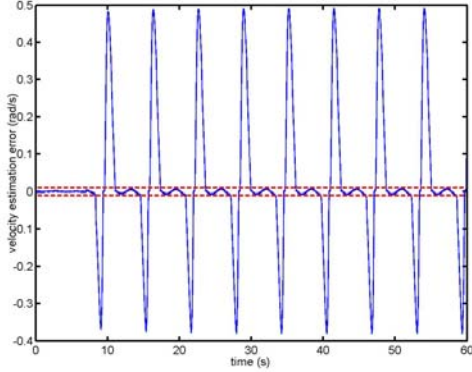


Fig. 3. Velocity estimation error on the first joint with actuator fault at  $t = 7$  s

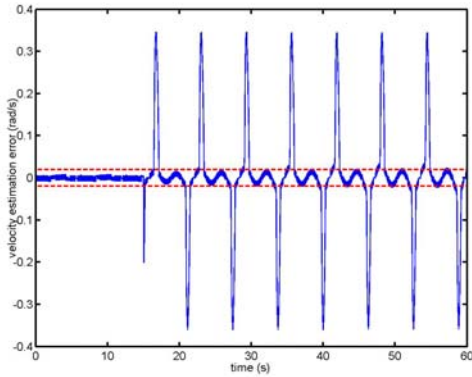


Fig. 4. Velocity estimation error on the third joint with actuator fault at  $t=15$  s

The obtained results are as shown in Fig. 3 and Fig. 4. The plots show the velocity estimation error of first and the third joint module. It is seen that the velocity estimation error exceeds the pre-defined threshold value at the time instant of fault occurrence, at  $t = 7$  s for the first joint and  $t = 15$  s for the third joint.

Depending on the priority of the task under execution and severity of the fault, a decision can be made to immediately abort the process without causing further damage or to finish the task in progress. To give an indication of the severity of fault at each joint, 'health' of each joint needs to be monitored continuously. The information could be used for the maintenance of the system.

## VI. EXPERIMENTS

To demonstrate the effectiveness of the proposed schemes, experimental study has been performed on one module shown in Fig. 5. The robot module consists of a brushless DC motor, a harmonic drive with an integrated torque sensor, an incremental encoder, and is controlled using a DSP based motion controller. For real-time

implementation, the DSP based controller board is programmed in the C language. The reconfiguration of the joint allows rotations on horizontal or vertical plane. In this experiment, the configuration is such that the link moves in a vertical plane. The coupling force is simulated with a payload attached to the manipulator arm. The effect of this change is recorded using the joint torque sensor.

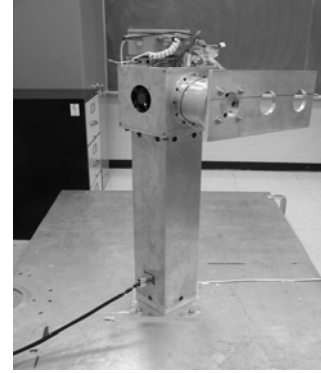


Fig. 5. A modular robot joint system

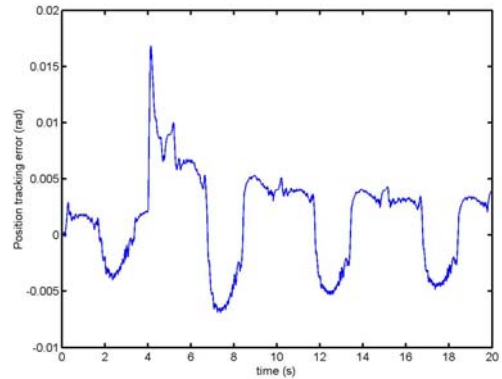


Fig. 6. Joint position error ( fault occurrence at  $t = 4$  s)

The desired trajectory of the joint is selected as

$$q_d = A(\sin(8t/2\pi) - 0.5\sin(16t/2\pi)) \quad \text{for } 0 \leq t \leq 20s.$$

The amplitude of the reference trajectory  $A = 0.2 \text{ rad}$ . The nominal parameters are chosen as  $\hat{\kappa}_i = 0.9$ ,  $\hat{F}_{ci} = 0.2 \text{ Nm}$ ,  $\hat{l}_i = 0.1$ ,  $\hat{F}_{si} = 0.1 \text{ s}^2 / \text{rad}^2$ ,  $\hat{B}_i = 0.002 \text{ Nms} / \text{rad}$ , and  $\hat{F}_{\tau i} = 0.05 \text{ s}^2 / \text{rad}^2$ . The motor rotor inertia is obtained from the data sheets as  $0.00025 \text{ Kg m}^2$  and gear ratio of the harmonic drive is  $\gamma = 101$ . The controller parameters are chosen as  $K_{Di} = 0.05$ ,  $\lambda_i = 40$ ,  $\Gamma = 200 I^{6 \times 6}$ ,  $\rho = 0.4$ ,  $\varepsilon = 0.4$ ,  $\varepsilon_0 = 0.1$  and  $\rho_{Fi} = 0.1$ . The observer gains were chosen as  $K_1 = 30$  and  $K_2 = 20$ .

The actuator fault is introduced by changing the actuator effectiveness factor for the base module from unity to 0.4 at  $t = 4$  s. The obtained results are as shown in Fig. 6 and Fig. 7. The experiments are conducted for two different cases: fault free operation and a fault occurring at  $t = 4$  s. From the fault free operations the threshold value for velocity estimation error was fixed to be 0.05 rad/s.



In Fig. 6, the tracking error occurring due to change in actuator effectiveness factor is shown. The tracking error shows a change at the instance of fault occurrence. It is seen that despite the change in actuator effectiveness factor, the action of the fault tolerant control law ensures the position tracking error does not diverge. This guarantees the performance of the system does not deteriorate due to the actuator fault.

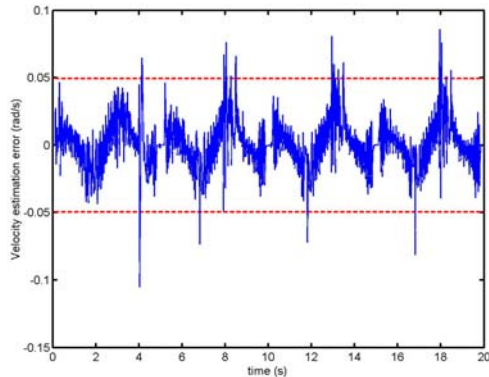


Fig. 7. Velocity estimation error with actuator fault (fault occurrence at  $t = 4$  s)

Though both the changes in payload and actuator effectiveness factor can result in parameter deviations and changes in tracking error, the payload changes are clearly distinguishable when used along with the torque sensor readings. The payload changes are recorded by the changes in torque sensor readings. Thus actuator faults can be easily differentiated from the payload changes.

In Fig. 7, the velocity estimation error occurring due to change in actuator effectiveness factor is shown. The velocity estimation error shows a significant change at the instance of fault occurrence. The actuator fault occurrence at  $t = 4$  s causes the velocity estimation error to exceed the chosen threshold value.

## VII. CONCLUSIONS

An actuator fault tolerant control method and a fault detection scheme for modular and reconfigurable robot with joint torque sensing have been developed and tested in this paper. Fault tolerance and fault detection at each joint module are carried out independently of the other joints, i.e., the controller at each joint does not require motion states of the other modules. This modular approach enables fault tolerant control and fault detection at individual modules, without affecting the control of other joints. Since the fault tolerant control is independent of the fault detection scheme, any delay caused in detection scheme does not affect the fault tolerant control action. The actuator effectiveness factors, torque sensor gains and offsets are incorporated into a parametric dynamical model formulation for each joint module of the robot. Based on this model formulation, a control scheme is designed for compensating the parametric uncertainties including the actuator effectiveness factors, torque sensor gains and offsets. Analysis and simulation

results have confirmed the effectiveness of the proposed fault tolerant control and fault detection schemes, which are further demonstrated with experimental results.

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