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From Crystals to Lattice Robots

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***Abstract*— We identify three fundamental properties of lattice robots such as (1) discreteness (2) translational symmetry and (3) composition, and explain the analogy of lattice robots kinematics and crystal symmetry described by space groups. Then we give the possible connectors symmetries and orientations compatible with space groups, and the possible sliding and hinge joints locations and orientations compatible with the displacements in such groups. We present a framework for the design of lattice robots by assembling compatible joints and connectors into a space group and give a 3D example.**

I. Introduction

A modular robotic system (MRS) is composed of multiple building blocks (i.e. mechatronic modules) having docking interfaces to connect them together. Structure and operating modes of such a system depend on the way the modules are connected together. It is possible to reconfigure the MRS topology by adding/removing one or several modules to the system, or by changing the way the modules are connected together. A reconfiguration is a sequence of connections, disconnections, and displacements of modules. Moreover, self-reconfigurable MRS can change their structure by themselves and must be able to control the state of their connectors and move their modules. The advantages of such systems are several. They are rapidly deployable, reusable, versatile and robust. Applications can be locomotion on hazardous terrain for planetary exploration, modular manipulation, self assembly, and others. A detailed review of these systems can be found in [1].

One can distinguish *chain type systems* such as Polybot [2], Conro [3], and *lattice systems* such as I-Cube [4], Telecube [5], Molecule [6], Microunit [7], Stochastic Modular Robots [8], []. In lattice systems, connections and disconnections occurs at discrete coordinates in a virtual lattice at each step of a reconfiguration, this is not the case in chain type systems. Other systems can have both lattice and chain type configurations, such as Atron [9], Molecube [10], M-Tran [11] and Superbot [12]. This paper proposes a framework for the design of lattice systems. It relies on the discretization of the module representation on two levels. In the first one, the symmetries of the connectors are represented by discrete rotation groups and in the second one the discrete configurations which can be produced on the connectors by the MRS actuators are represented by discrete displacements groups. For representing the different kinematical structures of the system we use special discrete displacement groups, the *chiral space groups* provided by the crystallography science. First we explain the analogy of lattice robots and crystal, then we propose a framework for the kinematical design of lattice robots using chiral space groups, third we explain the advantages of this method, before concluding.

II. Analogy between crystals and lattice robots

In the following, mathematical concept of crystallography such as space group, lattice group, Bravais lattice, point group, orbit, Wyckoff position, site symmetry and plane group are used. An introducing of these concepts can be found in [13]. Complete theory is available in [14], complete data about the 230 space groups is available in [14] and [15].

We explain the role of chiral space groups in the design of lattice robot in an example. Fig.2 shows a set of equivalent coordinates (orbit) into the plane group $p4$. We note G the set of displacement of $p4$ and X the orbit. The figure shows also a set of 2 coordinates for black and white connectors with same positions but opposite orientations. The orbit of the black connectors is X, and the orbit of the white connectors is $\bar{X}$. $\bar{X}$ is such that for each $\bar{x}$ in $\bar{X}$ and for each x in X, $\bar{x}$ has the same position than x but opposite orientation. By convention two connectors can be connected only if they have the same position and opposite orientations (see Fig. 1. We may use these sets of positions to construct a

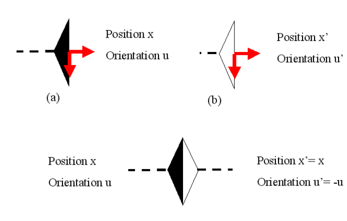

Fig. 1. In (a) the coordinates of a + connector, in (b) the coordinates of a − connector, in (c) the + and − connectors are connected

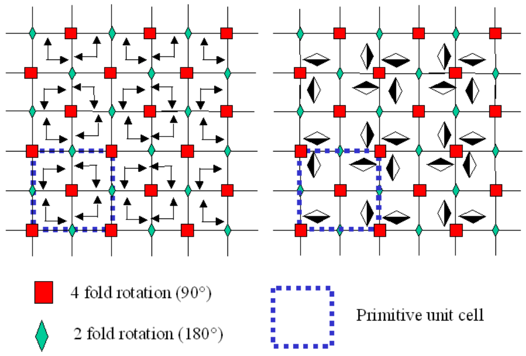

Fig. 2. The frames represent equivalent coordinates of an orbit into the 2-dimensional space group $p4$. The 4-fold and 2-fold rotation axes are represented by squares and rhombus. The unit translations corresponds to the edges of the unit cell in dotted lines.

module with 4 configurations and two connectors shown on Fig.3. To do this we select a white coordinate $\bar{x_0}$ for a white connector, and four black coordinates $x_{0..3}$ for the black connectors. In this example the mechanism can be implemented by two hinge joints: one has its axis aligned with a 4-fold axis of the space groups, and can have two configurations of 0 degree and 90 degrees, the other one has its axis aligned with a 2-fold axis of the space group, and can have two configurations of 0 degree and 180 degrees. The combinations of the configurations of the two hinges yield the 4 configurations of the module. The Fig.3 shows that when several modules are connected in various configurations, the black connectors have their coordinates into X, and the white connectors have their coordinates into $\bar{X}$. We denote T the lattice subgroup of G. The orbit X and $\bar{X}$ have a translation symmetry:

$$\forall x \in X, \forall t \in T, t(x) \in X$$

$$\forall \bar{x} \in \bar{X}, \forall t \in T, t(\bar{x}) \in \bar{X}$$

In this example we have three fundamental properties:

1) Discretness: The connectors of the modules are in discrete positions into a set of coordinate called orbit. Different type of connectors can have different orbits (in the example there are a black orbit and a white orbit).
2) Translation symmetry: The orbits have a discrete translational symmetry.
3) Composition : any set of interconnected modules in any configurations have their connectors into their orbit.

This three properties correspond to the three properties of space groups: a space group (1) has a discrete topology, (2) contains translations, and (3) has a group structure, ie. any composition of transformations is in the group.

These three properties are found in lattice systems such as M-Tran [11], Molecube [10], Molecule [6], I-Cube [4], Telecube [5] and Atron [9]. Moreover, it is not possible to have these properties if the set of positions for connectors are not into orbits of a space group.

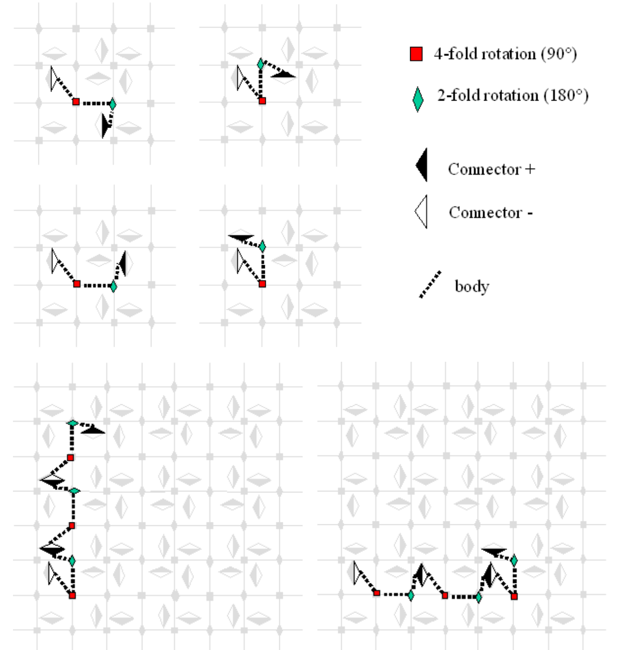

Fig. 3. On the left: a lattice module is built in plane group $p4$ by selecting a set of positions for its connectors. In this example the module has two connectors, one position is chosen for the white connector into the white orbit, and four positions are chosen for the black connector into the black orbit. Therefore the module has 4 configurations. The mechanism is implemented by two hinge joints having their axes coincident with the 4-fold and 2-fold rotation axes of the group. On the right: 3 modules are interconnected in various configurations. In each of these configurations the white connectors have their positions into the white orbit and the black connector have their positions into the black orbit. It is easy to see that for any configuration of interconnected modules the connectors have their positions into their orbit. This is due to the space group symmetry.

Therefore all lattice systems with these properties can be designed by using displacements of space group and corresponding orbits for the connectors.

Space groups allow to describe the symmetries of crystals by giving the transformations between the atoms of the crystal (considered as infinite at nanoscale). The analogies between crystals and lattice robots are the following:

In a crystal each type of atom has an infinite set of equivalent positions called orbit. In a lattice robot each type of connector has its coordinate into a set of possible positions corresponding to an orbit.

The space group corresponding to a crystal gives the set of transformation between the positions of the atoms of same orbit (for atoms of other orbits the set of transformation is the same). The space group corresponding to a lattice robot gives the set of transformations between the coordinates of connectors of same orbit (for connector of other orbits the set of transformation is the same).

The Wyckoff sites locate and orient the invariants of symmetry operations into the euclidean space, such as rotation axes, reflexion planes or inversion points. These Wyckoff sites give the possible rotation axes of the hinge joint of a lattice robots, and the rotation axes for the symmetries of the connectors.

The differences are: (1) for crystals the symmetries apply to *existing* positions of the atoms while for lattice robots the symmetries apply to the *possible* positions for the connectors, (2) for crystals an atom has only a position while for a lattice robot a connector has a position *and* an orientation, (3) for crystals any isometry can occur while for lattice robots only displacements are involved, (4) for crystals the transformations are symmetries on motionless atoms while for lattice robots the transformations move the connectors.

III. Design of lattice robots

We propose a framework for the preliminary design of lattice modules. The design concerns here only their kinematical structure, and the symmetries of the connectors, the geometric shape of the module is not considered. From a kinematical point of view, a module is defined by a set of connectors and a set of configurations of the connectors. In our framework the displacements between the configurations of the connectors are elements of a *chiral space group*which are space groups *having only displacements*. Moreover, the connectors are also described by symmetries of the chiral space groups. There are 65 types of such groups (see Table I). Since the set of the space groups has a hierarchical structure, it is possible to design all lattice systems in only 2 chiral space groups which contain the symmetries of all the other chiral space groups. These are the groups P622 and P432 (see tables A2 and A3 in [16]).

A. Connectors

1) Connector orbit: In space groups the sets of equivalent positions are called orbits. The connectors have a position but are also oriented. Therefore we consider that the coordinate x of a connector is a a position *and* an orientation. The orbit X of a connector is the set of coordinates having equivalent positions and orientations as in Fig.2.
By convention, two connectors can connect together only if they have same position and opposite orientations. We define by *opposite coordinate* a coordinate $\bar{x}$ which has same position than x but opposite orientation. Likewise, we define by *opposite orbit* the orbit $\bar{X}$ such that for $\forall x \in X, \forall \bar{x} \in \bar{X}$, $\bar{x}$ and x are opposite coordinates. For example, in section II, x and $\bar{x}$ are opposite coordinates and X and $\bar{X}$ are opposite orbits.
In what follows, we will see what are the possible symmetries for the connectors in space groups, and the relation between their positions, orientation and symmetry.

2) Connector type: We consider only connexion plates and not punctual connectors. The set of contact points of a connector may have symmetries. We distinguish two types of connector symmetries: (1) if the connector has a 2-fold tangential rotation symmetry then it is hermaphrodite (see Fig.4) else it is male or female, (2) it

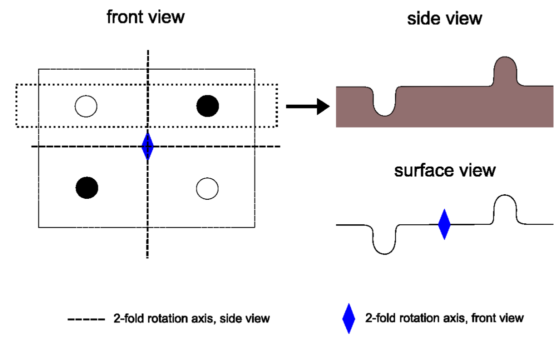

Fig. 4. Connector plate symmetries. The figures show the symmetries of a two fold hermaphrodite connector (type 22). We distinguish tangential and normal symmetries. Dashed line and rhombus represent 2-fold rotation axes in profile and face view. Left : a normal two fold rotation leaves the connector unchanged. Right : a tangential two fold rotation leaves the surface of the connector unchanged.

may have an normal rotation symmetry which allow to connect it with several orientation to another fixed connector, it may be a 2-fold, 3-fold 4-fold or 6-fold rotation. Combining both types of symmetry yields nine types of possible point groups (in Hermann-Mauguin notation): 1, 2, 3, 4, 6, 222, 32, 422, 622. Connectors with point group having only one rotation axis (such as groups 1, 2, 3, 4, 6) have only one symmetry type; else they have both types of symmetry. Nevertheless, this is not sufficient to distinguish all possible connector symmetries because symmetry group 2 can be a tangential 2-fold rotation or a normal 2-fold rotation, therefore a connector type must be added and we get 10 types. We propose another way to denote the connector symmetry using two digits AB, where A is the normal symmetry and can be 1, 2, 3, 4 or 6, and B is 2 if the connector is hermaphrodite, else it is 1 for identity. Moreover connectors without tangential symmetry can have two gender + or -. Therefore we can

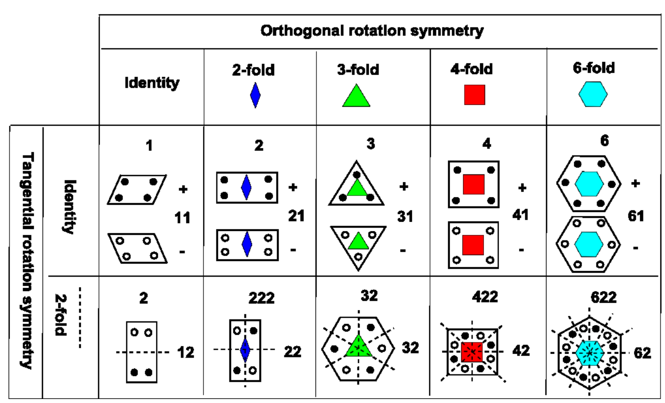

Fig. 5. Connector plates symmetries. The 10 possible symmetries for connector plates compatible with space groups are listed. Dashed line are two-fold tangential rotations. Rhombus, triangle, square and hexagon are, respectively, 2, 3, 4 and 6-fold normal rotation axes. On the top, connectors do not have tangential symmetry, therefore two male/female versions exist for each connector. On the bottom, connectors have 2-fold tangential symmetry (hermaphrodite connectors).

denote A+ and A- connectors of type A1 with gender + and -. The different types of connector are represented in Fig.5.

3) Connector position: A simple way to define the coordinates of a connector is to consider that it is inside a Wyckoff site matching its symmetry. For example a connector having a 4-fold symmetry axis must be on a 4-fold rotation axis of the space group. To define the position of the connector it is only necessary to set the values of its corresponding wyckoff site. The number of parameters of a site depends of its symmetry. Therefore a connector without symmetry (but identity) can be anywhere: three parameters are needed to define its position. If its symmetry is one rotation axis, its site symmetry is a line, one parameter must be set. If it has two or more symmetry axes, its site symmetry is a point, no parameter must be set, only one position is possible. There exists 11 possible point groups for the site symmetry: 1, 2, 3, 4, 6, 222, 23, 32, 422, 432, 622 (see Table I) and 9 possible point groups for the connector symmetry: 1,2,3,4,6,222,32,422,622. The connectors must also be oriented. Below we explain how connectors are oriented in chiral space groups.

4) Connector orientation: At a position the connector may have several possible orientations but some orientations can be equivalent because of the normal symmetry of the connector. Since a connector is invariant by its normal symmetry point group, the number of possible orientations of the connector at a position is equal to the order of the point group of its Wyckoff position divided by the order of its normal point group. For instance a connector of type 42 on a Wyckoff position c in P432 has 4 rotations in its normal symmetry point group and the Wyckoff position c point group is 422 and has 8 elements; therefore the connector has $8/4 = 2$ possible orientations at this position.

a) Connector without symmetry: The connector has type 11, its coordinate has three free parameters which must be set by three constants (for instance in the first entry of table A2 in [16]). Its orientation depends only on its position.

b) Connector with a normal symmetry only: Such a connector has type 21, 31, 41 or 61. The connector is on a line and its Wyckoff site has one free parameter. To locate the connector along the line, one parameter must be set. The connector normal symmetry axis is aligned with the line of its Wyckoff site. When it rotates along its normal symmetry it is invariant if the rotation is an element of its normal symmetry point group. For example a connector with a 4-fold normal axis is invariant when it rotates with 90 degrees along its normal axis. A connector with a 2-fold normal axis has its orientation changed when it rotates 90 degrees along its normal axis, but it is invariant by a 180 degrees rotation. Thus a 4-fold connector has only one possible orientation on a 4-fold axis, and a 2-fold axis has two possible orientations on a 4-fold axis.

c) Hermaphrodite connector without normal symmetry: Only connectors with type 12 has this symmetry. The connector is on a line and its Wyckoff site has one free parameter. To locate the connector along a line, one parameter must be set. The connector tangential axis is aligned with the line of its Wyckoff site. Moreover, the Wyckoff site must have a 2-fold, 4-fold or 6-fold rotation symmetry because a 3-fold axis is not compatible with the 2-fold rotation of the tangential axis.

d) Hermaphrodite connector with normal symmetry: This concern connectors of type 22, 32, 42 and 62. This is possible only on Wyckoff sites where several rotation axes intersect. The Wyckoff site no free paramter. The connector is at a point of the Wyckoff site. The two symmetry axes of the connector are orthogonal and the axis of its tangential symmetry can not be a 3-fold axis. Therefore the Wyckoff site must have an axis different to a 3-fold rotation axis, and another rotation axis orthogonal to it. The connector has its tangential axis aligned with a non 3-fold axis of the Wyckoff site, and its normal axis aligned with another axis of the Wyckoff site, orthogonal to the first. When the connector rotates along its normal symmetry axis it is invariant if the rotation is an element of its normal symmetry point group.

B. Joints

In our framework, mechanical parts have to produce displacements which are elements of the space group G in which the system is designed. To implement such mobilities any suitable mechanism can be used. Nevertheless, the rotations of G have their axes on Wyckoff sites, therefore, to produce a rotation of G it is convenient to use hinge a joint whose axis is aligned with its corresponding Wyckoff site. For instance in the space group P432, a 120 degrees rotation is in a Wyckoff site having 3-fold axis, with Wyckoff letter g (see table A2 in [16]), for example on an line (x, x, x). Therefore it can be implemented by using a hinge joint with its axis aligned with (x, x, x). Combinations of rotations of G can be implemented by using several hinge joints with their axes aligned with their corresponding Wyckoff site (as in Fig.3). When the rotations have coincident axes, the Wyckoff site is a the point where the axes intersect. To produce such rotations it is possible to use universal joints or ball joints. Translations of G can be implemented by sliding joints and can have their axis anywhere. The tables A2 and A3 in [16] give the lists of the possible joints at the different Wyckoff sites in P432 and P622. The screw axes are not given by Wyckoff sites but they are represented in [14] by geometric elements representation.

C. Building modules

A module can be built by putting together several connectors and joints. In our framework, the joints must produce displacements of the space group G and the connectors must have their types and coordinates compatible

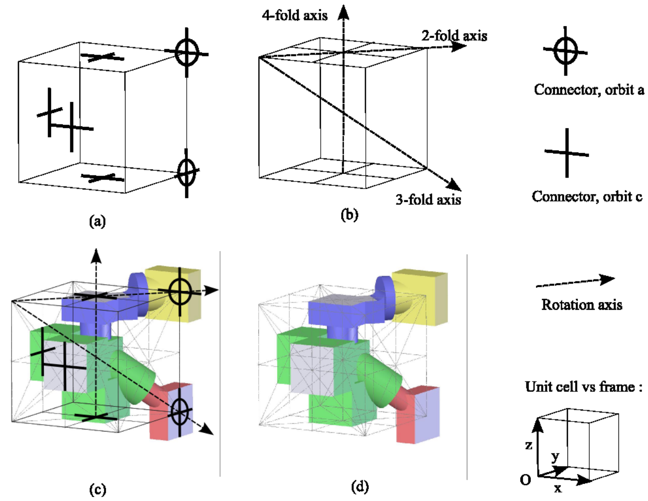

Fig. 6. An example of conception of a lattice module designed in space group P432 within an unique primitive cubic unit cell. It has 1 symmetry type for its connectors, 2 orbits for its connectors, 3 rotations axes, 4 bodies, and 5 connectors. The design process has several stages: in (a) 2 coordinates are selected for connectors with letter a and four coordinates are selected for connectors with letter c, in (b) 3 rotation axes are selected, in (c) 4 bodies are attached to connectors and rotations, in (d) the resulting lattice module.

with the Wyckoff sites of the group G. First a set of connectors and a set of displacement are selected, then a set of bodies is associated to the displacements and connectors. This defines the structure of the module. The following section illustrates the design of a module in the space group P432.

D. Example

We give an example of construction of a module in space group P432, illustrated in Fig.6. In this example the module component are embedded into an unique primitive cubic unit cell, with parameter $a = 1$. The design process can be decomposed in three steps:

In the first step we choose the connector types and coordinates. First we select two orbits X and Y for the connectors: For the orbit X we choose a Wyckoff site with letter a (at the vertices of the unit cell, see table A2 in [16]), for the orbit Y we choose letter c (at the center of the faces of the unit cell). For both orbits we choose connector type 42 which is compatible with sites a and c. Two connectors are selected in orbit X and four connectors are selected in orbit Y. The positions are selected into the entry "coordinates" in table A2 in [16] possibly incremented with lattice translations; we denote $(x, y, z) + (a, b, c)$ the position (x, y, z) incremented with the lattice translation (k_1, k_2, k_3), the resulting position is $(x + k_1, y + k_2, z + k_3)$.
For the two connectors in orbit X we choose (1) position $(0, 0, 0)+(1, 1, 0)$ and orientation $[1, 0, 0]$ and (2) position $(0, 0, 0) + (1, 1, 1)$ and orientation $[0, 1, 0]$. For the four connectors in orbit Y we choose (1) position $(1/2, 1/2, 0)$ and orientation $[0, 0, -1]$, (2) position $(1/2, 0, 1/2)$ and orientation $[0, -1, 0]$, (3) position $(0, 1/2, 1/2)$ and orientation $[-1, 0, 0]$, and (4) position $(1/2, 1/2, 0) + (0, 0, 1)$ and orientation $[0, 0, 1]$.

In the second step we choose a set of displacements. We choose to produce displacements by hinge joints. Three Wyckoff sites compatible with hinge joints are selected for the rotations (see table A2 in [16]): (1) a 4-fold rotation axis into site with letter f with coordinates $(1/2, 1/2, x)$, (2) a 3-fold axis into site with letter g with coordinates $(x, x, -x)$, (3) a-2 fold axis into site with letter i with coordinates $(y, y, 0) + (0, 0, 1)$. Together the three hinges provide $4*3*2 = 24$ displacements of P432.

In the third step we choose 4 bodies S1, S2, S3 and S4 to link the joints and connectors together. As shown in Fig.6, S1 is attached to a connector with orbit X and to the 3-fold hinge. S2 is attached to the 3-fold hinge, to the 4-fold hinge, and to 3 connectors with orbit Y. S3 is attached to the 4-fold hinge, to the 2-fold hinge and to a connector with orbit Y. S4 is attached to the 2-fold hinge and to a connector with orbit X.

This module has connectors with 2 different orbits. Therefore the connectors with orbits X and Y cannot connect together. But the connectors are hermaphrodite (type 42) therefore it is possible to connect a module to another one by using connectors with the same orbit.

E. Constraints on the connectors

The system may have several different modules and different types of connectors in different orbits. It is important that the connectors of the modules can connect together. In our framework, connectors can only connect if they have opposite coordinates (see section III-A.1), and same type. Hermaphrodite connectors are on tangential 2-fold rotation axes, therefore their orbits are equivalent to their opposite orbits, $X = \bar{X}$, the system must be equipped with other connectors of same type and same orbit, in the same module (see Fig.6 and Fig.7b) or in other modules. On the contrary, for non hermaphrodite connectors, connectors with opposite orbit and gender and same type must equip the module (as in Fig. 3) or other modules (see Fig.7a). In example of Fig.6 the connectors of each orbit are hermaphrodite therefore no other connectors are needed in this system.

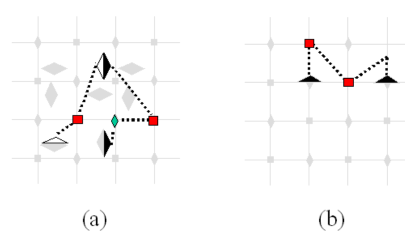

Fig. 7. In (a) a system built in $p4$ with connectors in the same orbit as in Fig.2. The connectors have no symmetry, the system must be equipped with connectors with opposite orientation and gender (type 1+ and 1-). In (b) the connectors are on 2-fold rotation sites, the connectors are hermaphrodite (type 12), one type of connector is sufficient to connect modules together.

IV. Discussion

Based on our framework one can design in P432 and P622 any modular system having the three fundamentals properties seen in section II. Unfortunately it is not possible to represent existing lattice systems in space groups without additional criterions, because there are infinitely many way to do this, due to scaling factors, different possible choices for Wyckoff letters and different possible choices for the space groups. An arbitrary representation would be more confusing than none. Two main applications of our framework can be identified (1) designing lattice robots having joints with discrete configurations, and (2) designing *lattice based* modular robot which can make lattice reconfigurations but may also make chain type reconfiguration if needed because their joints can have non discrete configurations and joint limits compatible with space groups (with rotations at least higher than 60 degrees). Another interesting application is that it helps to design system with *several* different modules types. Every module build in the same space group with compatible types and orbits for the connectors will be compatible together. These modules may also have different actuators with 2-fold, 3-fold or 4-fold joint, or no joint.

V. Conclusions

Thanks to the crystallography theory, we could find out what are all possible discrete displacement groups containing translations. We have identified three fundamental properties that characterize lattice robots. We proposed a framework for the design of the kinematics of all possible lattice robots by using space groups. Moreover, the two displacement spaces P622 and P432 where identified as been sufficient to build all possible systems. We illustrated the method in one example in space groups P432.

References

[1] N. Brener, F. Ben Amar, and P. Bidaud, "Analysis of self-reconfigurable modular systems. a design proposal for multi-modes locomotion," in *Proceedings of the 2004 IEEE International Conference of Robotics and Automation (ICRA)*, Apr. 2004, pp. 996–1001.

[2] D. G. Duff, M. H. Yim, and K. Roufas, "Evolution of polybot: a modular reconfigurable robot," in *Proc. of the Harmonic Drive Int. Symposium and Proc. of COE/Super-Mechano-Systems Workshop*, Nov. 2001.

[3] A. Castano, A. Behar, and P. Will, "The conro modules for reconfigurable robots," *IEEE/ASME Transactions on Mechatronics*, vol. 7(4), pp. 403–409, 2002.

[4] C. Unsal and P. Khosla, "Solutions for 3-d selfreconfiguration in a modular robotic system: Implementation and motion planning," in *Sensor Fusion and Decentralized Control in Robotic Systems III*, 2000.

[5] J. W. Suh, S. B. Homans, and M. H. Yim, "Telecubes: Mechanical design of a module for self-reconfigurable robotics," in *IEEE Int. Conf. on Robotics and Automation (ICRA)*, 2002.

[6] K. Kotay and D. Rus, "Locomotion versatility through selfreconfiguration," in *Robotics and Autonomous Systems*, 1999.

[7] E. Yoshida, S. Murata, S. Kokaji, K. Tomita, and H. Kurokawa, "Micro self-reconfigurable modular robot using shape memory alloy," *Journal of Robotics and Mechatronics*, vol. 13 no.2, pp. 212–219, 2001.

[8] N. Napp, S. Burden, and E. Klavins, "The statistical dynamics of programmed self-assembly," in *Proceedings of the 2006 IEEE International Conference of Robotics and Automation*, May 2006, pp. 1469– 1476.

[9] M. W. Jorgensen, E. H. Ostergaard, and H. H. Lund, "Modular atron: Modules for a self-reconfigurable robot," in *Proceedings of 2004 IEEE/RSJ International Conference on Intelligent Robots and Systems*, Sept. 2004.

[10] V. Zykov, E. Mytilinaios, B. Adams, and H. Lipson, "Self-reproducing machines," *Nature*, vol. 435, pp. 212–219, 2005.

[11] S. Murata, E. Yoshida, A. Kamimura, H. Kurokawa, K. Tomita, and S. Kokaji, "M-tran: Self-reconfigurable modular robotic system," in *IEEE/ASME Trans. Mech. Vol. 7, No. 4*, 2002, pp. 431–441.

[12] B. Salemi, M. Moll, and W.-M. Shen, "SUPERBOT: A deployable, multi-functional, and modular self-reconfigurable robotic system," in *Proceedings of the 2006 IEEE/IRSJ International Conference on Intelligent Robots and Systems*, Beijing, China, Oct. 2006.

[13] [Online]. Available: http://www.wikipedia.org

[14] *International Tables for Crystallography, Volume A: Space Group Symmetry*, ser. International Tables for Crystallography. Theo Hahn, 2002.

[15] [Online]. Available: http://www.cryst.ehu.es/

[16] N. Brener, F. Ben Amar, and P. Bidaud, "Designing modular lattice systems with chiral space groups," *The International Journal of Robotics Research*, 2008, to appear.

Appendix
The 65 Chiral Space Groups

The table I lists the 65 chiral space groups types and the 11 point groups types involved in chiral space groups by using the Hermann-Mauguin notation.

TABLE I

The point groups involved in chiral space groups, their order, and their corresponding space groups.

Point group	Order	Chiral space groups
1	1	P1
2	2	P2, $P2_1$, C2,
222	4	P222, $P222_1$, $P2_12_12$, $P2_12_12_1$, $C222_1$, C222, F222, I222, $I2_12_12_1$
4	4	P4, $P4_1$, $P4_2$, $P4_3$, I4, $I4_1$
422	8	P422, $P42_12$, $P4_122$, $P4_12_12$, $P4_222$, $P4_22_12$, $P4_322$, $P4_32_12$, I422, $I4_122$
3	3	P3, $P3_1$, $P3_2$, R3
32	6	P312, P321, $P3_112$, $P3_121$, $P3_212$, $P3_212$, R32
6	6	P6, $P6_1$, $P6_5$, $P6_3$, $P6_2$, $P6_4$
622	12	P622, $P6_122$, $P6_522$, $P6_222$, $P6_422$, $P6_322$
23	12	P23, F23, I23, $P2_13$, $I2_13$
432	24	P432, $P4_232$, F432, $F4_132$, I432, $P4_332$, $P4_132$, $I4_132$