2008 IEEE International Conference on
Robotics and Automation
Pasadena, CA, USA, May 19-23, 2008

Consensus of Multiple Uncertain Mechanical Systems and Its Application in Cooperative Control of Mobile Robots

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Abstract— **This paper considers the design of control laws for multiple mechanical systems with parameter uncertainty such that the state of each system converges to a point which moves along a desired trajectory. Adaptive cooperative control laws are proposed with the aid of the passivity property of system dynamics and the results for graph theory. As an application, the proposed results are used to solve the cooperative control problem of multiple mobile robots with parameter uncertainty such that multiple mobile robots converge to a desired pattern which moves along a desired trajectory. To show effectiveness of the proposed results, simulation results are presented.**

I. Introduction

Cooperative control of multiple agents has received considerable attention recently due to its challenging features and many applications, e.g., rescue mission, large object moving, troop hunting, formation control, and satellite clustering. Various control strategies have been proposed and are summarized as follows.

Virtual structure for cooperative control of multiple agents is considered in e.g. [1, 2]. Virtual structure methods implement decentralized trajectory-following controllers on each vehicle, but the per-vehicle trajectories are generated in a centralized fashion based on the state of all vehicles. To make the formation robust to internal or external disturbances, [2] introduces feedback of formation errors within the virtual structure. Behavior-based control method is considered in [3, 4]. The cooperative control laws were defined to achieve specific objectives and an arbitration scheme to switch or interpolate between alternative behaviors to implement a mission. In the leader-follower approach [5], one mobile robot is designated as a leader while others are designated as followers. The leader implements a trajectory-following controller to track the desired trajectory. Each follower implements a controller that forces its orientation and offset vector relative to its leader to commanded values, using only its own state and that of its lead vehicle. In [6], the authors discussed the error propagation and stability properties within leader-following based formations. Artificial potentials have been applied to flocking of multi-agents with the aid of other techniques, such as graph theory, virtual bodies/leaders, etc. A framework using artificial potentials and virtual leaders is proposed in [7]. Articles [8] and [9] discuss flocking with the aid of the artificial potentials and virtual agents. Graph theoretical approaches for cooperative control of multiple linear systems have been studied by various authors [10, 11]. In these papers, the structure of the communication network between vehicles was described by Laplacian matrices. Each vehicle was treated as a vertex and the communication links among vehicles were treated as edges. The stability of the whole system was guaranteed by the stability of each modified individual linear system, where the modification to the linear system accounts for the structure of the communication network. However, in these papers, the methods are limited to linear vehicle models. Article [12] considered the stability of multiple agents with nonlinear models in discrete time and time-dependent communication links. Necessary and/or sufficient conditions for the convergence of the state of each individual agent to a consensus vector were presented with the aid of graph theory and convexity.

The consensus problem is closely related to cooperative control and has been widely discussed recently. In [13], cooperative laws were proposed using nearest neighbor rules. In [14], it was shown how to make a group of mobile robots converge to a line or general geometric form by solving the consensus problem. In [15], the consensus problem for networks of dynamic agents with fixed and switching topologies was discussed. Two consensus protocols for networks with and without time-delays were proposed for convergence analysis in different communication cases. In [16], the authors considered the problem of information consensus among multiple agents in the presence of limited and unreliable information exchange with dynamically changing interaction topologies. Updated algorithms were proposed for information consensus in both discrete and continuous cases.

In this paper, we consider the consensus of multiple uncertain nonlinear systems with parameter uncertainty such that they come into a point which moves along a desired trajectory. Since there is parameter uncertainty in the dynamics of each system, the consensus control problem is challenging. To solve this problem, adaptive cooperative controllers are proposed with the aid of results of graph theory and the passivity property of each system. It is shown that our proposed results can make the group of nonlinear systems converge to a desired point which moves along a desired trajectory. As an application of the proposed results, formation control of multiple mobile robots with parameter uncertainty is considered. It is shown that the proposed results can be successfully used to solve the formation control problem. To verify effectiveness of the proposed cooperative control laws, simulation results are included. This paper extends the results for kinematic systems in [17] to dynamic systems.

The rest of the paper is organized as follows. In Section II, we formally state the control problem considered in this paper. In Section III, decentralized control laws are proposed for the defined control problem. In Section IV, applications of the proposed results are included. Section V includes simulation results to show effectiveness of the proposed results. Section VI concludes this paper.

II. Problem Statement

Consider a group of m mechanical systems subjected to nonholonomic constraints. For system j, its motion is defined in the following form [18]

$$M_j(q_{*j})\ddot{q}_{*j} + C_j(q_{*j},\dot{q}_{*j})\dot{q}_{*j} + G_j(q_{*j}) = B_j(q_{*j})\tau_j + J^\top(q_{*j})\lambda_j, \quad (1)$$

$$J(q_{*j})\dot{q}_{*j} = 0 \quad (2)$$

where $q_{*j} = [q_{1j}, \dots, q_{nj}]^\top$ is the state of system j, $M_j(q_{*j})$ is an $n \times n$ bounded positive-definite symmetric matrix, $C_j(q_{*j},\dot{q}_{*j})\dot{q}_{*j}$ presents centripetal and Coriolis force, $G_j(q_{*j})$ is gravitational force, $B_j(q_{*j})$ is an $n \times r$ input transformation matrix, τ_j is a r-vector of control input, $J(q_{*j})$ is a $(n-s-1) \times n$ full rank matrix with $s = n-1-\text{Rank}(J(q_{*j}))$, $2 \le s+1 < n$, $r \ge s+1$. λ_j is an $(n-s-1)$-vector of Lagrange multiplier which expresses the constraint force on system j, and the superscript $\top$ denotes the transpose. In system (1)-(2), the constraint (2) is assumed to be completely nonholonomic for each system [19].

Eqn. (1) has the following two properties for $1 \le j \le m$ [20].

Property 1: Matrix $\dot{M}_j - 2C_j$ is skew-symmetric for a proper definition of C_j.

Property 2: For any differentiable vector $\xi \in R^n$,

$$M_j(q_{*j})\dot{\xi} + C_j(q_{*j},\dot{q}_{*j})\xi + G_j(q_{*j}) = Y_j(q_{*j},\dot{q}_{*j},\xi,\dot{\xi})a_j$$

where a_j is an inertia parameter vector, the regressor matrix $Y_j(q_{*j},\dot{q}_{*j},\xi,\dot{\xi})$ is a function of q_{*j}, $\dot{q}_{*j}$, ξ, and $\dot{\xi}$.

Property 1 is the so-called passivity property of mechanical systems. This property will be applied in the controller design. For each system, we assume that the regressor matrix $Y_j(q_{*j},\dot{q}_{*j},\xi,\dot{\xi})$ is a known function of q_{*j}, $\dot{q}_{*j}$, ξ, and $\dot{\xi}$. But, a_j is assumed to be a unknown constant vector.

The communication between the robots can be described by the edges $\mathcal{E}$ of the digraph $\mathcal{G} = \{\mathcal{V},\mathcal{E}\}$ where the m mobile robots are represented by the m nodes in $\mathcal{V}$ [21]. The existence of an edge $(l,j) \in \mathcal{E}$ means that the state q_{*l} of robot l is available to robot j for control (i.e., unidirectional communication). Bidirectional communication, if it exists, would be represented by the edge (j,l) also being in the digraph $\mathcal{G}$. The symbol $\mathcal{N}_j$ denotes the neighbors of node j and is the set of indices of agents whose state is available to robot j. The information available to robot j for the controller design is the $j-$th robot's own state and the state of each robot l such that $l \in \mathcal{N}_j$. Due to sensor range limitations and bounded communication bandwidth between robots, $\mathcal{N}_j$ may change with time, which means that the edge set $\mathcal{E}$ may be time-varying and consequently the Laplacian matrix L corresponding to $\mathcal{G}$ may be time-varying. In this paper, we make the following assumption.

Assumption 1: The communication between robots are bidirectional and the communication graph $\mathcal{G}$ is strongly connected.

Given a differentiable desired trajectory $q^d(t) = [q_1^d(t), \dots, q_n^d(t)]^\top$ which satisfies

$$J(q^d)\dot{q}^d = 0. \quad (3)$$

The consensus control problem discussed in this article is defined as follows.

Consensus Control Problem: Design a control law τ_j for system j with unknown inertia parameter vector a_j using its own state q_{*j}, the relative state information between system j and system i for $i \in \mathcal{N}_j$, and $q^d(t)$ such that

$$\lim_{t\to\infty}(q_{*j} - q_{*i}) = 0, \quad 1 \le i \ne j \le m \quad (4)$$

$$\lim_{t\to\infty}\left(q^d - \frac{1}{m}\sum_{l=1}^{m} q_{*l}\right) = 0. \quad (5)$$

Remark 1: In the consensus control problem, the control law for system j is designed based on the desired trajectory q^d, the state of system j, and its neighbor's states. Since the inertia parameter vector a_j is unknown, adaptive cooperative control laws will be proposed in this paper.

To solve the consensus control problem, we convert (1)-(2) into a suitable form. Let the vector fields $g_1(q_{*j})$, …, $g_{s+1}(q_{*j})$ form a basis of the null space of $J(q_{*j})$. Then, by (2), there exists an $(s+1)$-vector $u_{*j} = [u_{1j}, \dots, u_{s+1,j}]^\top$ such that

$$\dot{q}_{*j} = g(q_{*j})u_{*j} = g_1(q_{*j})u_{1j} + \dots + g_{s+1}(q_{*j})u_{s+1,j} \quad (6)$$

where $g(q_{*j}) = [g_1(q_{*j}), \dots, g_{s+1}(q_{*j})]^\top \in R^{n\times(s+1)}$. Differentiating both sides of (6) and substituting it into (1) and multiplying both sides by $g^\top(q_{*j})$, we have

$$\overline{M}_j(q_{*j})\dot{u}_{*j} + \overline{C}_j(q_{*j},\dot{q}_{*j})u_{*j} + \overline{G}_j(q_{*j}) = \overline{B}_j(q_{*j})\tau_j \quad (7)$$

where we use the fact that $g^\top(q_{*j})J^\top(q_{*j}) = 0$, and

$$\begin{aligned}
\overline{M}_j(q_{*j}) &= g^\top(q_{*j})M_j(q_{*j})g(q_{*j}),\\
\overline{C}_j(q_{*j},\dot{q}_{*j}) &= g^\top(q_{*j})M_j(q_{*j})\dot{g}(q_{*j}) + \\
&\quad g^\top(q_{*j})C_j(q_{*j},\dot{q}_{*j})g(q_{*j}),\\
\overline{G}_j(q_{*j}) &= g^\top(q_{*j})G_j(q_{*j}),\\
\overline{B}_j(q_{*j}) &= g^\top(q_{*j})B_j(q_{*j}).
\end{aligned}$$

Based on Property 1 and Property 2, the following two properties can be easily proved for $1 \le j \le m$.

Property 3: Matrix $\dot{\overline{M}}_j - 2\overline{C}_j$ is skew-symmetric.

Property 4: For any differentiable vector $\xi \in R^{(s+1)\times(s+1)}$

$$\overline{M}_j(q_{*j})\dot{\xi} + \overline{C}_j(q_{*j},\dot{q}_{*j})\xi + \overline{G}_j(q_{*j}) = \overline{Y}_j(q_{*j},\dot{q}_{*j},\xi,\dot{\xi})a_j$$

where $\overline{Y}_j(q_{*j},\dot{q}_{*j},\xi,\dot{\xi}) =$

$$g^\top(q_{*j})Y_j\left(q_{*j},\dot{q}_{*j},g(q_{*j})\xi,\frac{d}{dt}(g(q_{*j})\xi)\right).$$

The reduced system (6)-(7) describes the motion of the original system (1)-(2). Therefore, the consensus control problem can be considered based on the reduced system (6)-(7) instead of system (1)-(2). In order to completely actuate each nonholonomic system, $\overline{B}_j(q)$ is assumed to be a full rank matrix.

System (6)-(7) represents a general nonholonomic dynamic system. For this general system, it is hard to design an effective controller. To simplify the consensus control problem, we assume that $n = 3$ and that eqn. (6) has two inputs and is in the following chained form [19] after suitable state and input transformations.

$$\left\{\begin{array}{lcl} \dot{q}_{1j} & = & u_{1j}, \\ \dot{q}_{2j} & = & u_{2j}, \\ \dot{q}_{3j} & = & u_{1j}q_{2j}. \end{array}\right. \qquad (1 \le j \le m) \tag{8}$$

Remark 2: The necessary and sufficient conditions for the existence of the transformations such that (6) is converted into the chained form (8) have been studied by several authors [19]. If eqn. (6) has three or more inputs and can be converted into one-generator multi-chain form [19], the method developed in this paper also works. Here, we assume $n = 3$ for simplicity.

Since the desired trajectory $q^d(t)$ satisfies the constraint (3), noting the assumptions made above, trajectory q^d satisfies

$$\left\{\begin{array}{lcl} \dot{q}_1^d & = & w_1, \\ \dot{q}_2^d & = & w_2, \\ \dot{q}_3^d & = & w_1 q_2^d \end{array}\right. \tag{9}$$

where w_1 and w_2 are known time-varying functions.

With the above assumption and transformations, the defined consensus control problem with uncertainty is equivalent to finding a control law τ_j for system (7)-(8) with unknown inertia parameter vector a_j using q^d, q_{*j}, and the relative state information between its neighbors such that (4)-(5) are satisfied.

III. Decentralized Controller Design

To facilitate the controller design, the following assumptions are made on the desired trajectory q^d.

Assumption 2: Variable q_2^d is bounded.

Assumption 3: Variable w_1 is bounded and satisfies the following condition:

$$\int_t^{t+T} w_1^2(s)ds \ge \epsilon$$

for some $T > 0$ and $\epsilon > 0$ and for all $t \ge 0$.

Assumptions 2-3 are not stringent. Many $w_1(t)$ satisfy Assumption 3. For example, w_1 may be a nonzero constant, a sine function, etc.

We introduce the following change of states for $1 \le j \le m$

$$\left\{\begin{array}{lcl} z_{1j} & = & q_{1j} - q_1^d \\ z_{2j} & = & q_{2j} - q_2^d + k_3 z_{3j} w_1 \\ z_{3j} & = & q_{3j} - q_3^d \end{array}\right. \tag{10}$$

where constant $k_3 > 0$. Then, we have

$$\left\{\begin{array}{lcl} \dot{z}_{1j} & = & u_{1j} - w_1 \\ \dot{z}_{2j} & = & u_{2j} - w_2 + k_3 z_{3j}\dot{w}_1 - k_3^2 w_1^3 z_{3j} \\ & & +k_3 w_1^2 z_{2j} + (u_{1j} - w_1)k_3 w_1 q_{2j} \\ \dot{z}_{3j} & = & -k_3 w_1^2 z_{3j} + w_1 z_{2j} + (u_{1j} - w_1)q_{2j}. \end{array}\right. \tag{11}$$

Lemma 1: Under Assumption 3, if $\lim_{t\to\infty}(z_{*j} - z_{*i}) = 0$ for $1 \le i \ne j \le m$, then (4) holds. Furthermore, if $\lim_{t\to\infty} z_{*j} = 0$ for $1 \le j \le m$, then (4)-(5) hold.

Lemma 1 can be proved by the definition of the variables and some algebra computation. We omit it here.

Next, we design the control laws in two steps. In the first step, we consider u_{*j} as virtual control inputs and design cooperative control laws such that eqns. (4)-(5) are satisfied. In the second step, we design control laws τ_j such that eqns. (4)-(5) are satisfied with the aid of the results in the first step.

Lemma 2: For system (11), under Assumptions 1-3, the control laws $u_{1j} = \eta_{1j}$ and $u_{2j} = \eta_{2j}$ for $1 \le j \le m$ make (4)-(5) hold, where

$$\begin{aligned} \eta_{1j} \;=\; & -\sum_{l\in\mathcal{N}_j} b_{jl}[z_{1j} + k_3 w_1 z_{2j} q_{2j} + z_{3j} q_{2j} - z_{1l} \\ & -k_3 w_1 z_{2l} q_{2l} - z_{3l} q_{2l}] - \mu_j [z_{1j} + k_3 w_1 z_{2j} q_{2j} \\ & +z_{3j} q_{2j}] + w_1 \end{aligned} \tag{12}$$

$$\begin{aligned} \eta_{2j} \;=\; & -\sum_{l\in\mathcal{N}_j} b_{jl}(z_{2j} - z_{2l}) - \mu_j z_{2j} + w_2 - k_3 z_{3j}\dot{w}_1 \\ & -k_3 w_1^2 z_{2j} + k_3^2 w_1^3 z_{3j} \end{aligned} \tag{13}$$

the control parameters $b_{jl}(= b_{lj})$ and k_3 are each positive constants, constants $\mu_j \ge 0$ and $\sum_{j=1}^m \mu_j > 0$.

Proof: Let the positive definite Lyapunov function

$$V = \frac{1}{2}\sum_{j=1}^m \sum_{i=1}^3 z_{ij}^2. \tag{14}$$

Differentiating V along the solutions of the closed-loop systems, we have

$$\begin{aligned} \dot{V} \;=\; & -\sum_{j=1}^m k_3 w_1^2 z_{3j}^2 - z_{2*}^\top L z_{2*} - (z_{1*} + \Delta)^\top L(z_{1*} + \Delta) \\ & -\sum_{j=1}^m \mu_j z_{2j}^2 - \sum_{j=1}^m \mu_j (z_{1j} + \Delta_j)^2 \le 0 \end{aligned}$$

where $z_{1*} = [z_{11}, \ldots, z_{1m}]^\top$, $z_{2*} = [z_{21}, \ldots, z_{2m}]^\top$, $\Delta = [\Delta_1, \ldots, \Delta_m]^\top = [k_3 w_1 z_{21} q_{21} + q_{21} z_{31}, \ldots, k_3 w_1 z_{2m} q_{2m} + q_{2m} z_{3m}]^\top$. Therefore, V is bounded. Hence, z_{ij} are bounded. By Barlalat's Lemma, $\lim_{t\to\infty} \dot{V} = 0$. So

$$\begin{aligned} & \lim_{t\to\infty} w_1^2 z_{3j}^2 = 0, \quad 1 \le j \le m \\ & \lim_{t\to\infty} z_{2*}^\top L z_{2*} = 0, \lim_{t\to\infty} (z_{1*} + \Delta)^\top L(z_{1*} + \Delta) = 0. \\ & \sum_{j=1}^m \mu_j z_{2j}^2 = 0, \quad \sum_{j=1}^m \mu_j (z_{1j} + \Delta_j)^2 = 0. \end{aligned}$$

By Lemma 2 in [22], we have $\lim_{t\to\infty}(z_{2*}(t) - c_2(t)\mathbf{1}) = 0$ and $\lim_{t\to\infty}(z_{1*}(t) + \Delta(t) - c_1(t)\mathbf{1}) = 0$ where c_1 and c_2

are bounded and are defined as

$$c_2 = \frac{1}{m}\sum_{l=1}^{m} z_{2l}, \quad c_1 = \frac{1}{m}\sum_{l=1}^{m}(z_{1l} + \Delta_l).$$

Since at least there is one integer p such that $\mu_p \neq 0$, $\lim_{t\to\infty} z_{2p} = 0$ and $\lim_{t\to\infty}(z_{1p} + \Delta_p) = 0$. Therefore, $\lim_{t\to\infty} z_{2j} = 0$ and $\lim_{t\to\infty}(z_{1j} + \Delta_j) = 0$ for $1 \le j \le m$. Furthermore, we can prove that $\lim_{t\to\infty} z_{3j} = 0$ and $\lim_{t\to\infty} z_{1j} = 0$ for $1 \le j \le n$. By the definitions of the variables, eqns. (4)-(5) hold. ■

With the aid of Lemma 2 and Property 3, we can design the cooperative control laws τ_j such that eqns. (4)-(5) are satisfied.

Theorem 1: For system (1)-(2), under Assumptions 1-3, the control laws

$$\tau_j = \tilde{B}_j^{-1}\left(-K\tilde{u}_{*j} + \tilde{Y}_j(q_{*j}, \dot{q}_{*j}, \eta_{*j}, \dot{\eta}_{*j})\hat{a}_j - \Lambda_{*j}\right) \quad (15)$$

and update laws

$$\dot{\hat{a}}_j = -\Gamma_j \tilde{Y}_j^\top(q_{*j}, \dot{q}_{*j}, \eta_{*j}, \dot{\eta}_{*j})\tilde{u}_{*j} \quad (16)$$

for $1 \le j \le m$ make (4)-(5) hold and $\hat{a}_j$ bounded, where symmetric constant matrices $K > 0$ and $\Gamma_j > 0$, $\tilde{u}_{*j} = u_{*j} - \eta_{*j}$, $\eta_{*j} = [\eta_{1j}, \eta_{2j}]^\top$, η_{1j} and η_{2j} are defined in (12)-(13), and

$$\Lambda_{*j} = \begin{bmatrix} z_{1j} + k_3 w_1 z_{2j} q_{2j} + z_{3j} q_{2j} \\ z_{2j} \end{bmatrix}.$$

Proof: Let the nonnegative function

$$V = \frac{1}{2}\sum_{j=1}^{m}(z_{1j}^2 + z_{2j}^2 + z_{3j}^2 + \tilde{u}_{*j}^\top \tilde{M}_j \tilde{u}_{*j} + \tilde{a}_j^\top \Gamma_j^{-1}\tilde{a}_j),$$

differentiating V along the solutions of the closed-loop systems, we have

$$\begin{aligned} \dot{V} = & -(z_{1*} + \Delta)^\top L(z_{1*} + \Delta) - z_{2*}^\top L z_{2*} \\ & -\sum_{j=1}^{m}\mu_j z_{2j}^2 - k_3 w_1^2 z_{3*}^\top z_{3*} - \sum_{j=1}^{m}\mu_j(z_{1j} + \Delta_j)^2 \\ & -\tilde{u}_{*j}^\top K \tilde{u}_{*j} \end{aligned}$$

where we use the fact that $(\dot{\tilde{M}}_j - 2\tilde{C}_j)$ is skew symmetric. Therefore, V is non-increasing. Following the proof of Lemma 2, we can prove that $\lim_{t\to\infty} \tilde{u}_{*j} = 0$ and $\lim_{t\to\infty} z_{ij} = 0$ for $1 \le i \le 3$ and $1 \le j \le m$. By the definitions of the variables, it can be verified that (4)-(5) hold. ■

Remark 3: Cooperative controllers (15)-(16) are decentralized and make the group of systems converge to a point which moves along the desired trajectory. The control law τ_j consists of the relative information between neighbors. The motion of the system is driven by the relative positions and relative velocities among neighbors. The performance of the closed-loop system depends on the connectivity of the communication graph $\mathcal{G}$ [23]. The value $\lambda_2(L)$ (λ_2 is the smallest nonzero eigenvalue of L) affects the convergence rate of z_{1*} and z_{2*}. It depends on the topology of the graph $\mathcal{G}$ and the weights b_{jl}. The estimated parameters $\hat{a}_j$ generally do not converge to their actual values [24]. However, they are bounded. To make the adaptive laws robust to disturbances, robust adaptive techniques may be applied.

IV. Formation Control of Mobile Robots

The proposed results have many applications in the cooperative control of multiple mobile robots. This section presents one application.

Consider a group of m identical wheeled mobile robots which move on a plane. Let $\Xi_{*j} = [x_j, y_j, \theta_j]^\top$ be the state of robot j, where (x_j, y_j) is the coordinate of the middle point of the front wheels of robot j in the fixed coordinate frame O-XY, θ_j is the orientation of robot j with respect to the X-axis of the coordinate frame O-XY, the dynamics of each robot can be described as (1)-(2) with the state Ξ_{*j} instead of q_{*j}, where C_j, G_j, M_j, and B_j are suitable matrices, and

$$J(\Xi_{*j}) = [\sin\theta_j, -\cos\theta_j, 0].$$

Given a desired formation $\mathcal{P}$ described by constant centroid offset vectors (p_{jx}, p_{jy}) $(1 \le j \le m)$ satisfying $\sum_{j=1}^m p_{jx} = 0$ and $\sum_{j=1}^m p_{jy} = 0$ and a desired trajectory $\Xi^d = (x^d, y^d, \theta^d)$ which satisfies

$$\dot{x}^d \sin\theta^d - \dot{y}^d \cos\theta^d = 0. \quad (17)$$

Assume the communication between m robots is described by a graph $\mathcal{G}$, we consider the following problem.

Formation Control with A Desired Trajectory: Design a control law for robot j using Ξ^d, Ξ_{*j}, and the relative information between Ξ_{*j} and Ξ_{*i} for $i \in \mathcal{N}_j$ such that the group of robots come into formation $\mathcal{P}$ and the centroid of the group of robots moves along the desired trajectory, i.e., design control laws for system (1)-(2) such that

$$\lim_{t\to\infty}\left(\begin{bmatrix} x_l - x_j \\ y_l - y_j \end{bmatrix} - \begin{bmatrix} p_{lx} - p_{jx} \\ p_{ly} - p_{jy} \end{bmatrix}\right) = 0, \quad (18)$$

$$\lim_{t\to\infty}(\theta_l - \theta_j) = 0, \quad 1 \le l \neq j \le m \quad (19)$$

$$\lim_{t\to\infty}\left(\frac{1}{m}\sum_{i=1}^{m} x_i - x^d\right) = 0, \quad (20)$$

$$\lim_{t\to\infty}\left(\frac{1}{m}\sum_{i=1}^{m} y_i - y^d\right) = 0. \quad (21)$$

To solve this problem, let

$$g(\Xi_{*j}) = \begin{bmatrix} \cos\theta_j & 0 \\ \sin\theta_j & 0 \\ 0 & 1 \end{bmatrix},$$

the kinematics of each robot are as follows:

$$\dot{x}_j = v_{1j}\cos\theta_j, \quad \dot{y}_j = v_{1j}\sin\theta_j, \quad \dot{\theta}_j = v_{2j}, \quad (22)$$

where v_{1j} and v_{2j} are the velocity and angular rate of robot j, respectively. Similarly, noting (17) we have

$$\dot{x}^d = v_1^d\cos\theta^d, \quad \dot{y}^d = v_1^d\sin\theta^d, \quad \dot{\theta}^d = v_2^d. \quad (23)$$

where $v_1^d = \dot{x}^d\cos\theta^d + \dot{y}^d\sin\theta^d$.

Next, we introduce the state transformation such that (22) is converted into the chained form. Let

$$\begin{cases} q_{1j} = -\theta_j, \\ q_{2j} = (x_j - p_{jx})\cos\theta_j + (y_j - p_{jy})\sin\theta_j \\ q_{3j} = -(x_j - p_{jx})\sin\theta_j + (y_j - p_{jy})\cos\theta_j \end{cases} \quad (24)$$

and

$$u_{1j} = -v_{2j}, \quad u_{2j} = v_{1j} + q_{3j}v_{2j}, \quad (25)$$

we have eqn. (8). The reduced dynamics of each robot is (7). Let

$$\begin{cases} q_1^d = -\theta^d, \\ q_2^d = x^d\cos\theta^d + y^d\sin\theta^d, \\ q_3^d = -x^d\sin\theta^d + y^d\cos\theta^d \\ w_1 = -v_2^d, \\ w_2 = v_1^d + (-x^d\sin\theta^d + y^d\cos\theta^d)v_2^d \end{cases} \quad (26)$$

we have eqn. (9).

Simple calculation derives the following result.

Lemma 3: By the transformations in eqns. (24)-(26), if (4)-(5) hold, then eqns. (18)-(21) are satisfied.

By Lemma 3, the formation control problem with a desired trajectory can be solved by the results obtained in Theorem 1.

V. Simulations

To verify the effectiveness of the proposed results, we present some simulation results. Let $m = 3$ and the initial conditions of the robots be $(-0.2, -20, -0.2)$, $(-3.3, -29.6, 0.3)$, and $(-0.8, -18, 0.2)$. Assume the desired formation $\mathcal{P}$ is a triangular which is defined by $(p_{1x}, p_{1y}) = (-6.6, 0)$, $(p_{2x}, p_{2y}) = (3.3, -5)$, and $(p_{3x}, p_{3y}) = (3.3, 5)$ (Fig. 1). The desired trajectory (x^d, y^d, θ^d) is generated by (23) with $v_1^d = 10m/s$, $v_2^d = 0.5rad/s$. The desired trajectory satisfies Assumptions 2-3. Assume the communication graph $\mathcal{G}$ is shown in Fig. 2. The cooperative controllers can be obtained by Theorem 1. In the simulation, we assume the real inertia parameters $M = 1$ and $I = 1$. We choose the control parameters $b_{ji} = 2$, $k_3 = 10$, $K = 10$, and $\Gamma = 0.1$. Fig. 3 shows the path of the centroid of the three robots and the geometric patterns of the three robots at several times. It is shown that the three robots come into the desired formation and the centroid of the group of robots converges to the desired trajectory. Figs. 4 and 5 show the responses of $\hat{M}_j$ and $\hat{I}_j$. It can be seen that they are bounded.

VI. Conclusion

This paper discusses the consensus control problem of multiple uncertain mechanical systems. Adaptive cooperative control laws are proposed with the aid of Lyapunov stability theory and results from graph theory. The proposed results are successfully applied to solve the formation control of multiple mobile robots. Simulation results show effectiveness of the proposed control laws. The obtained results in this paper can be extended to more general nonholonomic systems.

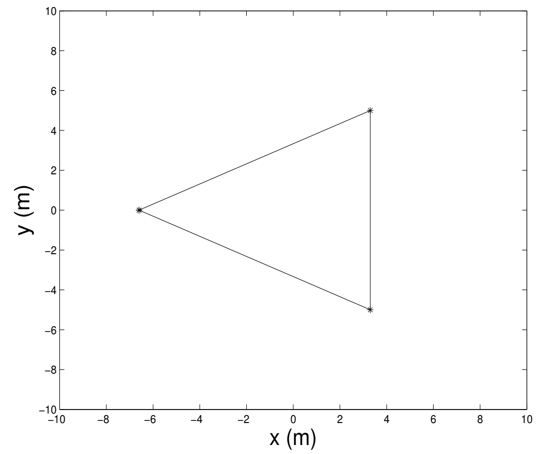

Fig. 1. Desired geometric pattern.

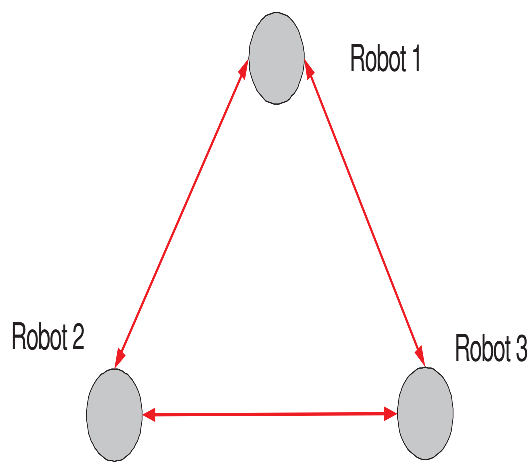

Fig. 2. Communication graph $\mathcal{G}$

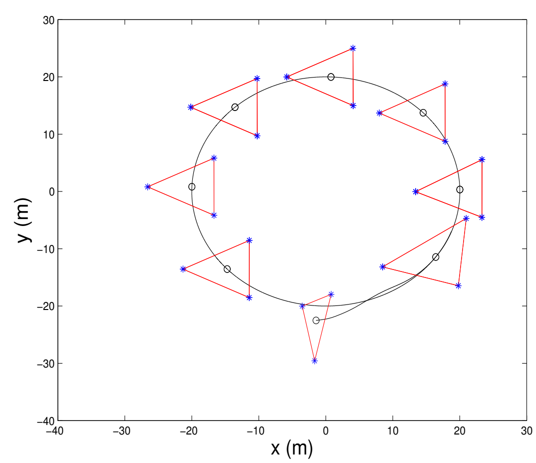

Fig. 3. Paths of the centroid of the three robots and the geometric patterns at several times

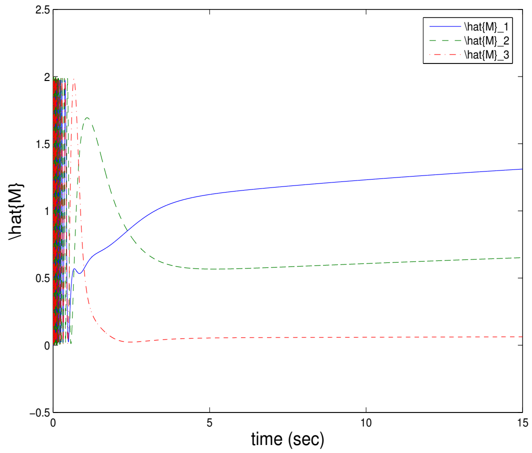

Fig. 4. Response of $\hat{M}_j$ for three robots ($\hat{M}_1$: solid line, $\hat{M}_2$: dashed line, $\hat{M}_3$: dahsdot line)

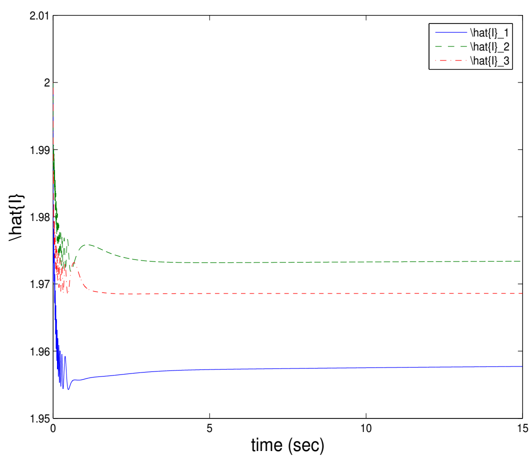

Fig. 5. Response of $\hat{I}_j$ for three robots ($\hat{I}_1$: solid line, $\hat{I}_2$: dashed line, $\hat{I}_3$: dahsdot line)

REFERENCES

[1] M. A. Lewis and K.-H. Tan, "High precision formation control of mobile robots using virtual structures," *Autonomous Robots*, vol. 4, pp. 387–403, 1997.

[2] W. Ren and R. W. Beard, "Formation feedback control for multiple spacecraft via virtual structures," *IEE Proceedings - Control Theory and Applications*, vol. 151, no. 3, pp. 357–368, 2004.

[3] T. Balch and R. C. Arkin, "Behavior-based formation control for multirobot teams," *IEEE Trans. on Robotics and Automation*, vol. 14, no. 6, pp. 926–939, 1998.

[4] J. Lawton, R. W. Beard, and B. Young, "A decentralized approach to formation maneuvers," *IEEE Trans. on Robotics and Automation*, vol. 19, no. 6, pp. 933–941, 2003.

[5] J. P. Desai, J. P. Ostrowski, and V. Kumar, "Modeling and control of formations of nonholonomic mobile robots," *IEEE Trans. Robot. Automat.*, vol. 17, pp. 905–908, 2001.

[6] H. G. Tanner, G. J. Pappas, and V. Kumar, "Leader-to-formation stability," *IEEE Trans. on Robotics and Automation*, vol. 20, pp. 443–455, 2004.

[7] N. E. Leonard and E. Fiorelli, "Virtual leaders, artificial potentials and coordinated control of groups," *Proc. of the IEEE Conf. on Decision and Control*, pp. 2968–2973, 2001.

[8] R. Olfati-Saber, "Flocking for multi-agent dynamic systems: algorithms and theory," *IEEE Trans. on Auto. Contr.*, vol. 51, no. 3, pp. 401–420, 2006.

[9] H. G. Tanner, A. Jadbabaie, and G. J. Pappas, "Flocking in fixed and switching networks," *IEEE Trans. on Automatic Control*, vol. 52, no. 5, pp. 863–868, 2007.

[10] J. A. Fax and R. M. Murray, "Information flow and cooperative control of vehicle formations," *IEEE Trans. on Auto. Contr.*, vol. 49, pp. 1465–1476, 2004.

[11] G. Lafferriere, A. Williams, J. Caughman, and J. J. P. Veermany, "Decentralized control of vehicle formations," *Systems and Control Letters*, vol. 53, pp. 899–910, 2005.

[12] L. Moreau, "Stability of multiagent systems with time-dependent communication links," *IEEE Trans. on Auto. Contr.*, vol. 50, no. 2, pp. 169–182, 2005.

[13] A. Jadbabaie, J. Lin, and A. S. Morse, "Coordination of groups of mobile autonomous agents using nearest neighbor rules," *IEEE Trans. on Automatic Control*, vol. 48, pp. 988–1001, 2003.

[14] Z. Lin, B. Francis, and M. Maggiore, "Necessary and sufficient graphical conditions for formation control of unicycles," *IEEE Trans. on Auto. Contr.*, vol. 50, no. 1, pp. 121–127, 2005.

[15] R. Olfati-Saber and R. M. Murray, "Consensus problems in networks of agents with switching topology and time-delays," *IEEE Trans. on Auto. Contr.*, vol. 49, pp. 101–115, 2004.

[16] W. Ren and R. W. Beard, "Consensus seeking in multi-agent systems under dynamically changing interaction topologies," *IEEE Trans. on Automatic Control*, vol. 50, no. 5, pp. 655–661, 2005.

[17] W. Dong and J. A. Farrell, "Decentralized cooperative control of multiple nonholonomic systems," *Proc. of Conference of Decision and Control*, 2007.

[18] A. M. Bloch, M. Reyhanoglu, and N. H. McClamroch, "Control and stabilization of nonholonomic dynamic systems," *IEEE Trans. on Automatic Control*, vol. 37, pp. 1746–1757, 1992.

[19] R. M. Murray and S. S. Sastry, "Nonholonomic motion planning: Steering using sinusoids," *IEEE Trans. on Auto. Contr.*, vol. 38, no. 5, pp. 700–716, 1993.

[20] W. Dong and W. L. Xu, "Adaptive tracking control of uncertain nonholonomic dynamic system," *IEEE Trans. on Auto. Contr.*, vol. 43, no. 3, pp. 450–454, 2001.

[21] R. Merris, "Laplacian graph eigenvectors," *Linear Algebra Appl*, vol. 278, pp. 221–236, 1998.

[22] W. Dong and J. A. Farrell, "Decentralized cooperative control of multiple nonholonomic dynamic systems with uncertainty," *submitted to Automatica*, 2007.

[23] M. C. DeGennaro and A. Jadbabaie, "Decentralized control of connectivity for multi-agent systems," *Proc. of IEEE Conference on Decision and Control*, 2006.

[24] W. Dong, W. Huo, S. K. Tso, and W. L. Xu, "Tracking control of dynamic nonholonomic systems and its application to mobile robots with uncertainty," *IEEE Trans. on Robotics and Automation*, vol. 16, no. 6, pp. 870–874, 2000.