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Formation path following control of unicycle-type mobile robots

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Abstract— **This paper presents a control strategy for coordination of multiple mobile robots. A combination of the virtual structure and path following approaches is used to derive the formation architecture. The formation controller is proposed for the kinematic model of two-degrees of freedom unicycle-type mobile robots. The approach is then extended to consider formation controller for its complete dynamic model. The controller is designed in such a way that the path derivative is left as a free input to synchronize the robot's motion. Simulation results with three robots, are included to show the performance of our control system.**

I. INTRODUCTION

During the last years, efforts have been made to give autonomy to single mobile robots by using different sensors, actuators and advanced control algorithms. This was mainly motivated by the necessity to develop complex tasks in an autonomous way, as demanded by service or production applications. In some applications, a valid alternative (or even the mandatory solution) is the use of multiple simple robots which, operating in a coordinated way, can develop complex tasks ([8], [1], [2]). This alternative offers additional advantages, in terms of flexibility in operating the group of robots and failure tolerance due to redundancy in available mobile robots [3]. In the literature, there have been roughly three methods to formation control of multiple robots : leader following, behavioral and virtual structure. Each approach has its own advantages and disadvantages.

In the leader following approach, some vehicles are considered as leaders, whilst the rest of robots in the group act as followers [8], [6]. The leaders track predefined reference trajectories, and the followers track transformed versions of the states of their nearest neighbors according to given schemes. An advantage of the leader-following approach is that it is easy to understand and implement. In addition, the formation can still be maintained even if the leader is perturbed by some disturbances. However, the disadvantage related to this approach is that there is no explicit feedback to the formation, that is, no explicit feedback from the followers to the leader in this case.

The behavioral approach prescribes a set of desired behaviors for each member in the group, and weighs them such that desirable group behavior emerges without an explicit model of the subsystems or the environment. Possible behaviors include trajectory and neighbor tracking, collision and obstacle avoidance, and formation keeping. In [4] the behavioral approach for multi-robot teams is described where formation behaviors are implemented with other navigational behaviors to derive control strategies for goal seeking, collision avoidance and formation maintenance. The advantage is that it is natural to derive control strategies when vehicles have multiple competing objectives, and an explicit feedback is included through communication between neighbors. The disadvantages is that the group behavior cannot be explicitly defined, and it is difficult to analyze the approach mathematically and guarantee the group stability.

In the virtual structure approach, the entire formation is treated as a single, virtual, structure and acts as a single rigid body. The control law for a single vehicle is derived by defining the dynamics of the virtual structure and then translates the motion of the virtual structure into the desired motion for each vehicle ([16], [17], [18]). in [5] virtual structures have been achieved by having all members of the formation tracking assigned nodes which move into desired configuration. A formation feedback has been used to prevent members leaving the group. Each member of the formation tracks a virtual element, while the motion of the elements are governed by a formation function that specifies the desired geometry of the formation. The main advantages of the virtual structure approach is that it is fairly easy to prescribe the coordinated behavior for the group, and the formation can be maintained very well during the manoeuvres, i.e. the virtual structure can evolve as a whole in a given direction with some given orientation and maintain a rigid geometric relationship among multiple vehicles. However, if the formation has to maintain the exact same virtual structure at the times, the potential applications are limited especially when the formation shape is time-varying or needs to be frequently reconfigured.

This paper develops a control law based on virtual structure approach for coordination of a group of N mobile robots. The conventional virtual structure is modified so that the formation shape can change. A formation of a simplified kinematic model of robots is firstly considered to clarify the design philosophy. The proposed technique is then extended to the dynamics of mobile robots with nonholonomic constraints. The conventional virtual structure approach is modified so that the formation shape can vary, i.e. the robots can change their relative positions with respect to the center of the virtual structure during the manoeuvre. The technique of maneuvering design [9] is employed to guarantee convergence of the robots to their locations with a prescribed dynamic. The feedback controller is designed in such a way that the derivative of the path parameter is left

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as an additional control input to synchronize the formation motion.

II. Problem statement

A. Kinematic model

We consider a group of N mobile robots, each of which has the following equations of motion

$$\begin{aligned}\dot{x}_i &= v_i\cos(\theta_i)\\ \dot{y}_i &= v_i\sin(\theta_i)\\ \dot{\theta}_i &= \omega_i\end{aligned} \qquad (1)$$

where, $\eta_i = [x_i, y_i, \theta_i]^\top$ denotes the position and the orientation vector of the i^{th} robot of the group with respect to an inertial coordination frame see Fig 1. v_i and ω_i stand

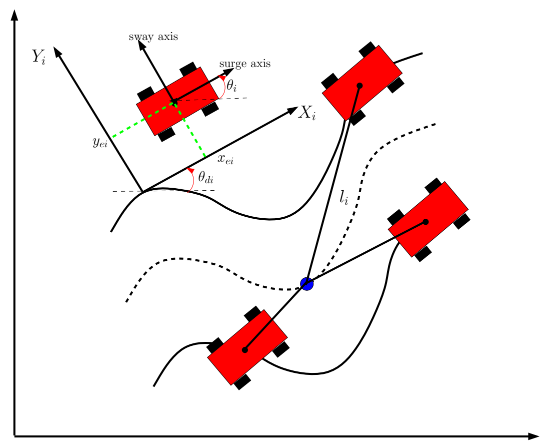

Fig. 1. Formation Setup.

for the linear and angular velocities respectively. For the group to move in a prescribed formation, each member will require an individual parameterized reference path so that when all paths' parameters are synchronized the member i will be in formation. We can generalize the setup of a single path $\xi(s)$ to n paths by defining the center of a virtual structure that moves along a given reference path $\xi(s_0) = [x_{d0}(s_0), y_{d0}(s_0)]$ with s_0 being a path parameter and create a set of n designation vectors $l_i(x_{d0}(s_i), y_{d0}(s_i))$ relative to the center of the structure. When the center of the virtual structure moves along the path $\xi_0(s_0)$, mobile robot i will then follow the individual desired path given by

$$\xi_i(s_i) = \xi_0(s_0) + \mathbf{R}(\theta_{d0}(s_i))l_i(x_{d0}(s_i), y_{d0}(s_i)) \qquad (2)$$

where $\mathbf{R}(\theta_{d0}(s_i))$ is a rotation matrix from a frame attached to the center of the virtual structure and the earth fixed frame. which is given as

$$\begin{aligned}\mathbf{R}(\theta_{d0}(s_i)) &= \begin{bmatrix}\cos(\theta_0(s_i)) & -\sin(\theta_0(s_i))\\ \sin(\theta_0(s_i)) & \cos(\theta_0(s_i))\end{bmatrix}\\ \theta_{d0}(s_i) &= \arctan\left(\frac{y_{d0}^{s_i}}{x_{d0}^{s_i}}\right)\end{aligned} \qquad (3)$$

where the partial derivatives notations in [9]-[10] are used, i.e.,

$$y_{d0}^{s_i} = \frac{\partial y_{d0}(s_i)}{\partial s_i}, \text{ and } x_{d0}^{s_i} = \frac{\partial x_{d0}(s_i)}{\partial s_i}$$

B. Control objective

Before going onward the design, we first give the following assumption :

Assumption 1:

1) We assume that each mobile robot i of the formation broadcasts its state and reference trajectory to the rest of the robot in the group. Moreover it can receive states and reference trajectory from the other robot of the group.
2) The desired geometric path is regularly parameterized i.e. there exist strictly positive constants ε_{1i} and ε_{2i}, $1 \le i \le N$ such that

$$(x_{di}^{s_i})^2(s) + (y_{di}^{s_i})^2(s) \ge \varepsilon_{1i}, \quad \dot{s}_0 \ge \varepsilon_{2i} \qquad (4)$$

Remark 1: The last inequality of equation (4), means that the center of the virtual structure moves forward.

The overall control objective is to solve the Formation Problem for N mobile robots which consists of two parts [9] : The geometric task which ensures that the individual mobile robot converges to and stay at its designation in the formation. The dynamic task ensures that the formation maintains a speed along the path according to the given speed assignment. The formation control objective boils down to design controller v_i and ω_i such that

$$\begin{aligned}\lim_{t\to\infty}\|\eta_i(t) - \eta_{di}(t)\| &= 0\\ \lim_{t\to\infty}|s_i(t) - s_0(t)| &= 0\end{aligned} \qquad (5)$$

where η_{id} denotes the i^{th} reference path to be tracked by robot i.

III. Path following error dynamic

The problem we consider here is the path following for each mobile robot in the formation ; that is we wish to find control law v_i and ω_i such that the robot follows a reference point in the path with position $\eta_{di} = [x_{di}, y_{di}, \theta_{di}]^\top$ and inputs v_{di} and ω_{di}. The path error is therefore interpreted in a frame attached to the reference path $\xi(s_i)$. Following [11] we define the error coordinates

$$\begin{bmatrix}x_{ei}\\ y_{ei}\\ \psi_{ei}\end{bmatrix} = \begin{bmatrix}\cos(\theta_i) & \sin(\theta_i) & 0\\ -\sin(\theta_i) & \cos(\theta_i) & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x_i - x_{di}\\ y_i - y_{di}\\ \theta_i - \theta_{di}\end{bmatrix} \qquad (6)$$

where θ_{di} is defined as

$$\theta_{di} = \arctan\left(\frac{y_{di}^{s_i}}{x_{di}^{s_i}}\right) \qquad (7)$$

The error dynamics are then

$$\begin{aligned}\dot{x}_{ei} &= v_i - v_{di}\cos(\theta_{ei}) + y_{ei}\omega_i\\ \dot{y}_{ei} &= v_{di}\sin(\theta_{ei}) - x_{ei}\omega_i\\ \dot{\theta}_{ei} &= \omega_i - \omega_{di}\end{aligned} \qquad (8)$$

where we have defined

$$v_{di} = \sqrt{x_{di}^{s_i}(s_i)^2 + y_{di}^{s_i}(s_i)^2}\dot{s}_i := \bar{v}_{di}(s_i)\dot{s}_i \quad (9)$$

$$\omega_{di} = \frac{x_{di}^{s_i}(s_i)y_{di}^{s_i^2}(s_i) - x_{di}^{s_i^2}(s_i)y_{di}^{s_i}(s_i)}{x_{di}^{s_i}(s_i)^2 + y_{di}^{s_i}(s_i)^2}\dot{s}_i := \bar{\omega}_{di}(s_i)\dot{s}_i \quad (10)$$

IV. CONTROL DESIGN

In the tracking model (8), y_{ei} could not be directly controlled, and to overcome this difficulty, we employ the backstepping approach. [13].
Define the following variable

$$\dot{\tilde{s}}_i = \dot{s}_i + \varpi_i(t, x_e, y_e, \theta_e) \quad (11)$$

where $x_e = [x_{e1}, x_{e2}, \ldots, x_{en}]^\top$, $y_e = [y_{e1}, y_{e2}, \ldots, y_{en}]^\top$ and $\theta_e = [\theta_{e1}, \theta_{e2}, \ldots, \theta_{en}]^\top$, $\varpi_i(t, x_e, y_e, \theta_e)$ is a strictly positive function that specifies how fast the i^{th} mobile robot should move to maintain the formation since $\dot{s}_i$ is related to the desired forward speed v_{di}.

Step 1: Design a controller to stabilize the x_{ei} dynamic. The variable $\dot{\tilde{s}}_i$ should be left as an extra controller in order to synchronize all the path parameters of each mobile robot in the formation. We therefore choose the following control for x_{ei}

$$v_i = -k_{1i}x_{ei} + \bar{v}_{di}\varpi\cos(\theta_{ei}) \quad (12)$$

in closed loop the x_{ei} dynamic rewrites

$$\dot{x}_{ei} = -k_{1i}x_{ei} - \bar{v}_{di}\dot{\tilde{s}}_i\cos(\theta_{ei}) + y_{ei}\omega_i \quad (13)$$

From (13), it is seen that if $\dot{\tilde{s}}_i$ and y_{ei} are zero, then x_{ei} globally exponentially converges to zero. The next step of the design will focus on designing a controller to stabilize θ_{ei} at the origin.

Step 2: Design a controller for ω_i. Consider the following Lyapunov function

$$V = \frac{1}{2}\sum_{i=1}^{n}(x_{ei}^2 + y_{ei}^2 + \theta_{ei}^2 + \gamma_i(s_i - s_0)^2) \quad (14)$$

where γ_i is a positive constant. The time derivative of (14) along the solutions of (13) and (8) yields

$$\begin{aligned}\dot{V} &= \sum_{i=1}^{n} x_{ei}(-k_{1i}x_{ei} - \bar{v}_{di}\dot{\tilde{s}}_i\cos(\theta_{ei}) + y_{ei}\omega_i) \\ &\quad + \sum_{i=1}^{n} y_{ei}(\bar{v}_{di}(\dot{\tilde{s}}_i + \varpi_i)\sin(\theta_{ei}) - x_{ei}\omega_i) + \sum_{i=1}^{n}\theta_{ei}(\omega_i \\ &\quad -\bar{\omega}_{di}(\dot{\tilde{s}} + \varpi_i)) + \sum_{i=1}^{n}\gamma_i(s_i - s_0)(\dot{\tilde{s}}_i + \varpi_i - \dot{s}_0)\end{aligned} \quad (15)$$

If we select

$$\varpi_i = \dot{s}_0 \quad (16)$$

the derivative of the Lyapunov function V becomes

$$\begin{aligned}\dot{V} &= \sum_{i=1}^{n} -k_{1i}x_{ei}^2 + \theta_{ei}\Big[y_{ei}\bar{v}_{di}\varpi_i\int_0^1\cos(\lambda\theta_{ei})d\lambda \\ &\quad +\omega_i - \bar{\omega}_{di}\varpi_i\Big] + \sum_{i=1}^{n}\Big[-\bar{v}_{di}\cos(\theta_{ei})x_{ei} \\ &\quad +y_{ei}\bar{v}_{di}\sin(\theta_{ei}) - \theta_{ei}\bar{\omega}_{di} + \gamma_i(s_i - s_0)\Big]\dot{\tilde{s}}_i\end{aligned} \quad (17)$$

To make the derivative of the Lyapunov function V negative, we choose the following controllers

$$\begin{aligned}\omega_i &= -k_{2i}\theta_{ei} - y_{ei}\bar{v}_{di}\varpi_i\int_0^1\cos(\lambda\theta_{ei})d\lambda + \bar{\omega}_{di}\varpi_i \\ \dot{\tilde{s}}_i &= -\varepsilon_\omega\tanh\Big(-\bar{v}_{di}\cos(\theta_{ei})x_{ei} + y_{ei}\bar{v}_{di}\sin(\theta_{ei}) \\ &\quad -\theta_{ei}\bar{\omega}_{di} + \gamma_i(s_i - s_0)\Big)\end{aligned} \quad (18)$$

where ε_ω is a positive constant to be selected later. Substituting the equations of (18) into (17) yields

$$\dot{V} = \sum_{i=1}^{n} -k_{1i}x_{ei}^2 - k_{2i}\theta_{ei}^2 - \varepsilon_\omega\phi_i\tanh(\phi_i) \quad (19)$$

where $\phi_i = -\bar{v}_{di}\cos(\theta_{ei})x_{ei} + y_{ei}\bar{v}_{di}\sin(\theta_{ei}) - \theta_{ei}\bar{\omega}_{di} + \gamma_i(s_i - s_0)$. We are able now to choose an appropriate function for $\dot{s}_0$ such that when the tracking errors are large, the center of the virtual structure will wait for the rest of the group however when the errors converge to zero, we impose $\dot{s}_0$ to approach a given positive bounded function $\omega_0(t)$ which means that the center of the virtual structure will move at a desired speed. $\dot{s}_0$ can be chosen as among many others [14]

$$\begin{aligned}\dot{s}_0 &= \omega_0(t)(1 - \kappa_1 e^{-\kappa_2(t-t_0)})(1 - \kappa_3\tanh(x_e^\top\Gamma_x x_e \\ &\quad +y_e^\top\Gamma_y y_e + \theta_e^\top\Gamma_\theta\theta_e))\end{aligned} \quad (20)$$

where $\omega_0(t)$ is a strictly positive and bounded function. This function specifies how fast the whole group of robots should move. Γ_x, Γ_y and Γ_θ are weighting positive matrices. All the constants κ_1, κ_2 and κ_3 are nonnegative and κ_1 and κ_3 should be less than 1. Now if we choose the constant ε_ω such that

$$\varepsilon_\omega < \omega_0(t)(1 - \kappa_1)(1 - \kappa_3) \quad (21)$$

then we have

$$\begin{aligned}\dot{s}_i &= -\varepsilon_\omega\tanh(\phi_i) + \omega_0(t)(1 - \kappa_1 e^{-\kappa_2(t-t_0)}) \\ &\quad \times(1 - \kappa_3\tanh(x_e^\top\Gamma_x x_e + y_e^\top\Gamma_y y_e + \theta_e^\top\Gamma_\theta\theta_e)) \\ &\geq -\varepsilon_\omega + \omega_0(t)(1 - \kappa_1)(1 - \kappa_3) > 0\end{aligned} \quad (22)$$

From the above control design, we have the closed loop system

$$
\begin{aligned}
\dot{x}_{ei} &= -k_{1i}x_{ei} - \bar{v}_{di}\dot{\tilde{s}}_i\cos(\theta_{ei}) + y_{ei}\omega_i \\
\dot{y}_{ei} &= v_{di}\sin(\theta_{ei}) + k_{2i}\theta_{ei}x_{ei} + y_{ei}x_{ei}\bar{v}_{di} \\
&\quad \times\varpi_i\int_0^1 \cos(\lambda\theta_{ei})d\lambda - x_{ei}\bar{\omega}_{di}\varpi_i \qquad (23)\\
\dot{\theta}_{ei} &= -k_{2i}\theta_{ei} - y_{ei}\bar{v}_{di}\varpi_i\int_0^1 \cos(\lambda\theta_{ei})d\lambda - \bar{\omega}_{di}\dot{\tilde{s}}_i \\
\dot{s}_i &= -\varepsilon_\omega\tanh(\phi_i) + \omega_0(t)(1-\kappa_1 e^{-\kappa_2(t-t_0)}) \\
&\quad \times(1-\kappa_3\tanh(x_e^\top\Gamma_x x_e + y_e^\top\Gamma_y y_e + \theta_e^\top\Gamma_\theta\theta_e))
\end{aligned}
$$

We now state the main result of the formation control for kinematic model of unicycle-type mobile robots.

Theorem 1: Under Assumption 1, the control inputs v_i and ω_i given in (12) and (18) and the time derivative of each individual path $\dot{s}_i$ given in (22) for the mobile robot i solve the formation control objective. In particular the closed loop system is forward complete and the position and orientation of the robots track their path reference asymptotically in the sense of (5)

Proof: (*Forward completness*) From (19), we have that $\dot{V} \leq 0$, which implies that

$$V(t) \leq V(t_0), \quad \forall t \geq t_0 \qquad (24)$$

From the definition of V, the right hand side of (24) is bounded by a positive constant depending on the initial conditions. The boundedness of the left hand side of (24) also implies those of $x_{ei}, y_{ei}, \theta_{ei}$ and $s_i - s_0$ for all $t \geq t_0 \geq 0$. The assumed boundedness of $\omega_0(t)$ implies that of the right hand side of the last equation of (23), depends continuously on t through bounded functions. It follows that s_i and therefore s_0 are bounded on the maximal interval of definition $[0, T)$, this excludes finite escape time so $T = +\infty$. This in turn implies by construction that $x_i(t), y_i(t), \theta_i(t), s_i$ and s_0 do not go to infinity in finite time.
(*Stability of* $(x_{ei}, y_{ei}, \theta_{ei})$) From the above argument on the boundedness of $(x_{ei}, y_{ei}, \theta_{ei}, s_i - s_0)$, applying Barbalt's lemma [19] to (19) results in

$$
\begin{aligned}
\lim_{t\to\infty}(x_{ei}, \theta_{ei}) &= 0 \\
\lim_{t\to\infty}\phi_i\tanh(\phi_i) &= 0
\end{aligned}
\qquad (25)
$$

On the other hand, from the second equation of (25), we conclude that

$$
\begin{aligned}
\lim_{t\to\infty}\phi_i &= 0 \\
\lim_{t\to\infty}\dot{\tilde{s}}_i &= 0
\end{aligned}
\qquad (26)
$$

From this fact, we also conclude that

$$\lim_{t\to\infty}(s_i - s_0) = 0 \qquad (27)$$

which satisfies the second objective of (5). To satisfy the first objective of (5), we have to show that y_{ei} also converges to zero as t goes to infinity. In closed loop the θ_{ei} dynamic rewrites as follows

$$\dot{\theta}_{ei} = -k_{2i}\theta_{ei} - y_{ei}\bar{v}_{di}\varpi_i\int_0^1 \cos(\lambda\theta_{ei})d\lambda - \bar{\omega}_{di}\dot{\tilde{s}}_i \qquad (28)$$

Let $f(t) = -y_{ei}\bar{v}_{di}\varpi_i\int_0^1\cos(\lambda\theta_{ei})d\lambda - \bar{\omega}_{di}\dot{\tilde{s}}_i$, a direct application of lemma 2 in [13](see appendix), it follows that

$$\lim_{t\to\infty} y_{ei}\bar{v}_{di}\varpi_i\int_0^1 \cos(\lambda\theta_{ei})d\lambda = 0$$

since $\lim_{t\to\infty}\int_0^1\cos(\lambda\theta_{ei})d\lambda = 1$ then $\lim_{t\to\infty} y_{ei} = 0$ which concludes about the first objective of (5). ■

V. Extension of Formation Control to Dynamic Model of the Mobile Robot

In this section, we study the augmented system (8) appended with a dynamic [12].

$$
\begin{aligned}
\dot{x}_{ei} &= v_i - v_{di}\cos(\theta_{ei}) + y_{ei}\omega_i \\
\dot{y}_{ei} &= v_{di}\sin(\theta_{ei}) - x_{ei}\omega_i \\
\dot{\theta}_{ei} &= \omega_i - \omega_{di} \\
\overline{M}_i\dot{\nu}_i &= -\overline{C}_i(\omega_i)\nu_i - \overline{D}_i\nu_i + \overline{B}_i\tau_i
\end{aligned}
\qquad (29)
$$

where $\nu_i = [v_i, \omega_i]^\top, \tau_i = [\tau_{1i}, \tau_{2i}]^\top$ being the control vector torque applied to the wheels of the robot i. The modified mass matrix, Coriolis and Damping matrices are given by

$$
\begin{aligned}
\overline{M}_i &= B_i^{-1}M_iB_i, \quad \overline{C}_i(\omega_i) = B_i^{-1}C_i(\omega_i)B_i \\
\overline{D}_i &= B_i^{-1}D_iB_i
\end{aligned}
$$

where B_i is an invertible matrix given by $B_i = \begin{bmatrix} 1 & b_i \\ 1 & -b_i \end{bmatrix}$ and $M = \begin{bmatrix} m_{11i} & m_{12i} \\ m_{12i} & m_{11i} \end{bmatrix}, C_i(\omega_i) = \begin{bmatrix} 0 & c_i\omega_i \\ -c_i\omega_i & 0 \end{bmatrix}, D_i = \begin{bmatrix} d_{11i} & 0 \\ 0 & d_{22i} \end{bmatrix}$ with $b_i, m_{11i}, m_{22i}, d_{11i}, d_{22i}$ and c_i are the dynamic parameters of each group which are not necessarily know. Definitions of these parameters are given in [12]. In this section we are intended to show that the formation control developed for the kinematic model can also be obtained for the complete dynamic of the unicycle-type mobile robots. The control objective is thereby similar to that of (5) and consists of finding the controller τ_i that satisfies all conditions of (5).

Introduce the following new variables

$$\nu_{ei} = \nu_i - \alpha_{\nu_i}, \quad \Rightarrow \quad v_{ei} = v_i - \alpha_{v_i}, \quad \omega_{ei} = \omega_i - \alpha_{\omega_i} \qquad (30)$$

where α_{v_i} and α_{ω_i} are defined as in (12) and (18) respectively. Following the notation in section IV, in the new coordinates $(x_{ei}, y_{ei}, \theta_{ei}, v_{ei}, \omega_{ei})$ the system (29) is transformed into

$$
\begin{aligned}
\dot{x}_{ei} &= -k_{1i}x_{ei} - \bar{v}_{di}\dot{\hat{s}}_i\cos(\theta_{ei}) + y_{ei}\omega_i + v_{ei} \\
\dot{y}_{ei} &= v_{di}\sin(\theta_{ei}) + k_{2i}\theta_{ei}x_{ei} + y_{ei}x_{ei}\bar{v}_{di} \\
&\quad \times\varpi_i\int_0^1\cos(\lambda\theta_{ei})d\lambda - x_{ei}\bar{\omega}_{di}\varpi_i - x_{ei}\omega_{ei} \\
\dot{\theta}_{ei} &= -k_{2i}\theta_{ei} - y_{ei}\bar{v}_{di}\varpi_i\int_0^1\cos(\lambda\theta_{ei})d\lambda - \bar{\omega}_{di}\dot{\hat{s}}_i \\
&\quad +\omega_{ei} \qquad (31) \\
\overline{M}\nu_{ei} &= -\overline{C}_i(\omega_i)\nu_i - \overline{D}_i\nu_i - \overline{M}_i[\dot{\alpha}_{v_i}, \dot{\alpha}_{\omega_i}]^\top + \overline{B}_i\tau_i \\
&= -\overline{D}_i\nu_{ei} + \Phi_i\Theta_i + \overline{B}_i\tau_i
\end{aligned}
$$

where

$$
\begin{aligned}
\Phi_i &= \begin{bmatrix} \omega_i^2 & -\alpha_{v_i} & -\dot{\alpha}_{v_i} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\omega_i v_i & -\alpha_{\omega_i} & -\dot{\alpha}_{\omega_i} \end{bmatrix} \\
\Theta_i &= \Big[b_i c_i \quad d_{11i} \quad m_{11i} + m_{12i} \quad \frac{c_i}{b_i} \\
&\qquad d_{22i} \quad m_{11i} - m_{12i}\Big]^\top \qquad (32)
\end{aligned}
$$

At this stage, we design the actual control input vector τ_i and updated laws for unknown system parameter vector Θ_i for each robot i. To do so, we consider the following Lyapunov function

$$
V_2 = V + \frac{1}{2}\sum_i^N\left(\nu_{ei}^\top\overline{M}_i\nu_{ei} + \widetilde{\Theta}_i^\top\Gamma_i^{-1}\widetilde{\Theta}_i\right) \qquad (33)
$$

where $\widetilde{\Theta}_i = \Theta_i - \widehat{\Theta}_i$, with $\widehat{\Theta}_i$ being an estimate of Θ_i and Γ_i is a symmetric positive definite matrix. Differentiating both sides of (33) along the solutions of (31) and (17) and choosing the control input τ_i, the path parameter s_i and the updated law $\widehat{\Theta}_i$ as

$$
\begin{aligned}
\tau_i &= B^{-1}\Big(-K_{1i}\nu_{ei} - \Phi_i\widehat{\Theta}_i - \begin{bmatrix} x_{ei} & \theta_{ei} \end{bmatrix}^\top\Big) \\
\dot{\widehat{\Theta}}_i &= \Gamma_i\mathbf{proj}(\Phi_i^\top\nu_{ei}, \widehat{\Theta}_i) \\
\dot{s}_i &= -\varepsilon_\omega\tanh(\phi_i) + \omega_0(t)(1 - \kappa_1 e^{-\kappa_2(t-t_0)}) \\
&\quad \times(1 - \kappa_3\tanh(x_e^\top\Gamma_x x_e + y_e^\top\Gamma_y y_e + \theta_e^\top\Gamma_\theta\theta_e))
\end{aligned} \qquad (34)
$$

Where the operator, **proj**, is the Lipschitz continuous projection [7] algorithm applied in our case as follows :

$$
\begin{aligned}
\mathbf{proj}(\pi, \hat{\mu}) &= \pi, \quad \text{if,} \quad \Xi(\hat{\mu}) \le 0 \\
\mathbf{proj}(\pi, \hat{\mu}) &= \pi, \quad \text{if,} \quad \Xi(\hat{\mu}) \ge 0 \quad \text{and} \quad \Xi_{\hat{\mu}}(\hat{\mu}) \le 0 \\
\mathbf{proj}(\pi, \hat{\mu}) &= (1 - \Xi(\hat{\mu}))\pi, \ \text{if,} \ \Xi(\hat{\mu}) > 0 \ \text{ and } \Xi_{\hat{\mu}}(\hat{\mu}) > 0
\end{aligned}
$$

where $\Xi(\hat{\mu}) = (\hat{\mu}^2 - \mu_M^2)/(k^2 + 2k\mu_M)$, $\Xi_{\hat{\mu}}(\hat{\mu}) = (\frac{\partial\Xi_{\hat{\mu}}(\hat{\mu})}{\partial\hat{\mu}})$, k is an arbitrarily small positive constant, $\hat{\omega}$ is an estimate of μ and $|\mu| \le \mu_M$. The projection algorithm is such that if $\dot{\hat{\mu}} = \mathbf{proj}(\pi, \mu)$ and $\hat{\mu}(t_0) \le \mu_M$ then

a) $\hat{\mu}(t) \le \mu_M + k$, $\forall 0 \le t_0 \le t \le \infty$

b) $\mathbf{proj}(\pi, \hat{\mu})$ is Lipschitz continuous

c) $|\mathbf{proj}(\pi, \hat{\mu})| \le |\pi|$

d) $\tilde{\mu}\mathbf{proj}(\pi, \hat{\mu}) \ge \tilde{\mu}\pi$ with $\tilde{\mu} = \mu - \hat{\mu}$

and K_{1i} is a symmetric positive definite matrix. Using Property d) of the projection algorithm results in

$$
\begin{aligned}
\dot{V}_2 \le & -\sum_{i=1}^n\Big[k_{1i}x_{ei}^2 + k_{2i}\theta_{ei}^2 + \varepsilon_\omega\phi_i\tanh(\phi_i) \\
& + \nu_{ei}^\top(\overline{D}_i + K_{1i})\nu_{ei}\Big]
\end{aligned} \qquad (35)
$$

From the above control design, we have the closed loop system

$$
\begin{aligned}
\dot{x}_{ei} &= -k_{1i}x_{ei} - \bar{v}_{di}\dot{\hat{s}}_i\cos(\theta_{ei}) + y_{ei}\omega_i + v_{ei} \\
\dot{y}_{ei} &= v_{di}\sin(\theta_{ei}) + k_{2i}\theta_{ei}x_{ei} + y_{ei}x_{ei}\bar{v}_{di} \\
&\quad \times\varpi_i\int_0^1\cos(\lambda\theta_{ei})d\lambda - x_{ei}\bar{\omega}_{di}\varpi_i - x_{ei}\omega_{ei} \\
\dot{\theta}_{ei} &= -k_{2i}\theta_{ei} - y_{ei}\bar{v}_{di}\varpi_i\int_0^1\cos(\lambda\theta_{ei})d\lambda - \bar{\omega}_{di}\dot{\hat{s}}_i \\
&\quad +\omega_{ei} \\
\overline{M}\nu_{ei} &= -(\overline{D}_i + K_{1i})\nu_{ei} + \Phi_i\widetilde{\Theta}_i - \begin{bmatrix} x_{ei} & \theta_{ei} \end{bmatrix}^\top \\
\dot{s}_i &= -\varepsilon_\omega\tanh(\phi_i) + \omega_0(t)(1 - \kappa_1 e^{-\kappa_2(t-t_0)}) \\
&\quad \times(1 - \kappa_3\tanh(x_e^\top\Gamma_x x_e + y_e^\top\Gamma_y y_e + \theta_e^\top\Gamma_\theta\theta_e)) \\
\dot{\widetilde{\Theta}}_i &= -\Gamma_i\mathbf{proj}(\Phi_i^\top\nu_{ei}, \widehat{\Theta}_i)
\end{aligned} \qquad (36)
$$

Theorem 2: Under Assumption 1, the control inputs τ_i and the update law $\dot{\widehat{\Theta}}_i$ given in (34) for the mobile robot i solve the formation control objective. In particular the closed loop system (36) is forward complete and all signals in the closed loop system are Uniformly Ultimately Bounded (UUB).

Proof: Let $X_{ei} = [x_{ei}, y_{ei}, \theta_{ei}]^\top$ and $Z_i = [X_{ei}^\top, \nu_{ei}^\top, \widetilde{\Theta}_i^\top, s_i]^\top$. Considering (35), by subtracting and adding $\frac{1}{2}\sum_{i=1}^N\widetilde{\Theta}_i^\top\Gamma_i^{-1}\widetilde{\Theta}_i$ to its right hand side, we arrive at

$$
\dot{V}_2 \le \delta V_2 + \rho \qquad (37)
$$

where $\delta = \min\big(1, 2\mu_1, \ldots, \mu_N\big)$, $\mu_i = \min\big(k_{1i}, k_{2i}, \varepsilon_\omega\big)$ and $\rho = \frac{1}{2}\sum_{i=1}^N\widetilde{\Theta}_i^\top\Gamma_i^{-1}\widetilde{\Theta}_i$. From (37), it is straightforward to show that

$$
V_2(t) \le V_2(t_0)e^{-\delta(t-t_0)} + \frac{\rho}{\delta} \qquad (38)
$$

This implies that for all t in the maximal interval of definition $[0, T)$

$$
\|Z_i\| \le p_i\|Z_i(t_0)\|e^{-0.5\delta(t-t_0)} + \rho_i \qquad (39)
$$

where p_i is a constant that depends on the elements of Γ_i, and $\rho_i = \sqrt{\frac{\rho}{\delta}}$. Consequently, all the signals in the closed-loop are guaranteed to be UUB.

The assumed boundedness of $\omega_0(t)$ implies that the right hand side of (36) depends continuously on time t through bounded function. With Z_i being bounded it follows that (36) is bounded on the maximal interval of definition. This excludes finite escape times. ■

VI. SIMULATIONS

We carry out a simulation example for the formation of the kinematic model of the robots to illustrate the results, simulation example for the dynamic model is however omitted due to space limitation. The number of robots in the formation group is chosen for simplicity $N = 3$. The initial positions of robots, the initial conditions and the design constants are chosen as follows

$$\begin{aligned} [x_1, y_1, \theta_1]^\top &= [0,0,0]^\top, [x_2, y_2, \theta_2]^\top = [-2,1,0]^\top, \\ [x_3, y_3, \theta_3]^\top &= [-2,-3,0]^\top, \nu_1(0) = \nu_2(0) = [0,0]^\top \\ k_1 &= 150, \quad k_2 = 70 \\ s_i(0) &= 2, \quad \varepsilon_\omega = 0.12, \quad \omega_0 = 5 \\ \kappa_1 &= 0.5, \quad \kappa_2 = 2, \quad \kappa_3 = 0.5 \\ \Gamma_x &= \Gamma_y = \Gamma_\theta = diag(1,1,1) \end{aligned} \tag{40}$$

We run two simulations with two different reference paths for the center of the virtual structure, the first reference path ξ_0 is chosen as $x_d(s) = s, y_d(s) = 0$. The distance from the mobile robots to the center of the virtual structure are therefore as follows

$$\begin{aligned} l_1(x_{d0}(s_1), y_{d0}(s_1)) &= (0,4), \\ l_2(x_{d0}(s_2), y_{d0}(s_2)) &= (3, 8 + 3\tanh(0.5s_1)) \\ l_3(x_{d0}(s_3), y_{d0}(s_3)) &= (3, -8 + 3\tanh(0.5s_1)) \end{aligned}$$

These choices mean that the first robot coincides with the center of the virtual structure which moves on a straight line. The robot's position and orientation are plotted in Fig. 2. The path tracking errors and linear velocities are plotted in Fig. (3-4-5) respectively. It is clear from these figures that each robot in formation asymptotically tracks its own path generated by the center of the virtual structure and the formation is achieved. For the second simulation, the reference path generated for each robot in the formation are sinusoidal, the reference path chosen for the center of the virtual structure is $\xi(s_0) = (s_0, 0)$, this means that the center of the virtual structure is moving along a straight line. The distance from the robots to the center of the virtual structure are as follows

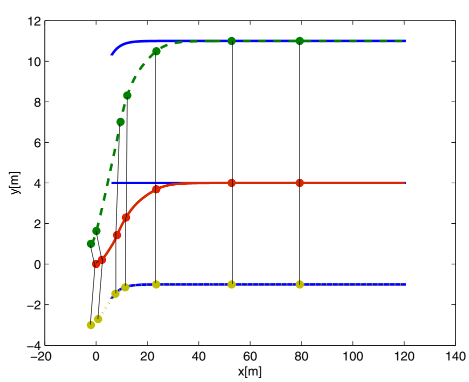

Fig. 2. Robot position in (x, y) plan.

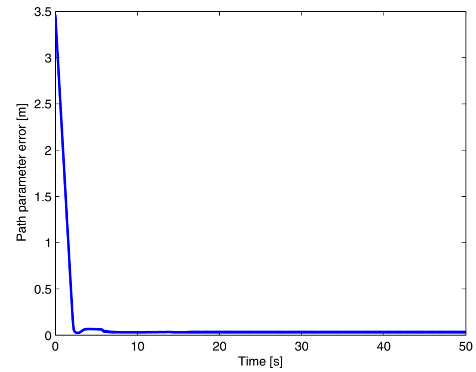

Fig. 3. Path parameter error in the form $\sqrt{\sum_{i=1}^{3}(s_i - s_0)^2}$.

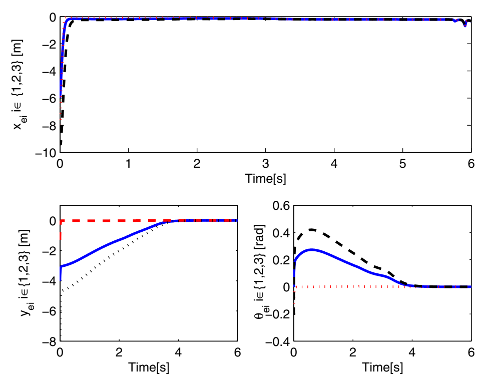

Fig. 4. Tracking errors x_{ei}, y_{ei} and θ_{ei}.

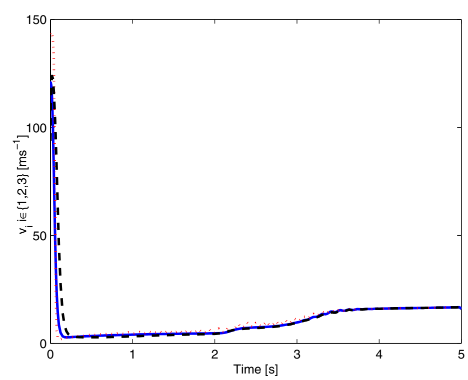

Fig. 5. Time evolution of the forward velocity of each robot.

$$\begin{aligned} l_1(x_{d0}(s_1), y_{d0}(s_1)) &= (0,0), \\ l_2(x_{d0}(s_1), y_{d0}(s_1)) &= (3, 8 + 3\cos(0.5s_1)) \\ l_3(x_{d0}(s_2), y_{d0}(s_2)) &= (3, -8 + 3\sin(0.5s_1)) \end{aligned}$$

The robots' position and orientation is plotted in Fig 6.

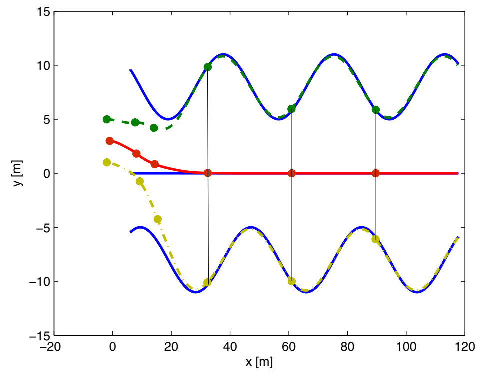

Fig. 6. Robot position in (x, y) plan.

VII. CONCLUSION

This paper has proposed a methodology for formation control of a group of unicycle-type mobile robots represented at a kinematic level and a dynamic level. The approach that has been developed is mainly based on a combination of the virtual structure and path following approaches. The controller is designed in such a way that the derivative of the path parameter is left as an additional control input to synchronize the formation motion. Future work is to design an observer for both kinematic and dynamic model to estimate unavailable signals such as velocities or orientation error tracking.

APPENDIX

Lemma [13]: Consider a scalar system

$$\dot{x} = -kx + f(t)$$

where $k > 0$ and $f(t)$ is a bounded and uniformly continuous function. If, for any initial $t_0 \geq 0$ and any initial condition $x(t_0)$, the solution $x(t)$ is bounded and converges to 0 as $t \to \infty$ then

$$\lim_{t\to\infty} f(t) = 0$$

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