2008 IEEE International Conference on
Robotics and Automation
Pasadena, CA, USA, May 19-23, 2008

Human-friendly Motion Control of a Wheeled Inverted Pendulum by Reduced-order Disturbance Observer

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Abstract— **This paper introduces a novel approach to control a wheeled inverted pendulum when a disturbance is applied by a human. The interaction between a human and a wheeled inverted pendulum involves a pulling or pushing force. This type of action is a severe disturbance for a wheeled inverted pendulum, as the wheeled inverted pendulum tends to maintain its initial position if there is no desired input. Thus, there are many possibilities for the wheeled inverted pendulum to be unstable as a result of interactions with humans.**

To solve this problem, the control algorithm of a wheeled inverted pendulum was designed to move in coordination with the external force of a human. This control algorithm is termed human-friendly motion control. It contains an optimal controller using a full-state feedback and a reduce-order disturbance observer. The disturbance torque from a human was estimated, and the estimated disturbance torque was used to generate a position reference for the human-friendly motion. This control algorithm keeps the wheeled inverted pendulum stable even when a severe disturbance is applied.

I. Introduction

In the past, the humanoid robots used a three or four stable wheel system. In the case of Hadaly-2 [7], two driving wheels and two passive wheels were used as a caster. The mobility of such a stable wheel system was worse than that of a two-wheeled self-balancing system. A two-wheeled system is essentially unstable but can be stabilized independently in the same manner as a human. Therefore, a two-wheel system has mobility that is more natural, which allows it to interact with humans. When a two-wheeled humanoid robot interacts with a human, the stability of the robot can be disturbed by the pulling and pushing forces from the human. The robot can fall down due to these disturbances. A human can even be hurt by the falling robot. To prevent this type of dangerous situation, a novel control algorithm is designed here for human safety issues. This control method is termed the human-friendly motion control [8], [9].

Thus far, a number of studies concerning two-wheeled self-balancing mechanisms have been conducted [1]-[5]. These studies deal with a control method for balancing. However, they did not consider the problem of maintaining the stability after an external force is applied to the mechanism. Another study considered cooperative transportation [6]. An observer was used to estimate disturbance forces and a designed force and position controller was utilized for object transportation after an external force from a human or a robot. However, they assumed that the external force would be applied to the body of the wheeled inverted pendulum at the axle height horizontally. In this case, the stability of the system is not affected from external force as much as when the force is applied to the upper part of the center of gravity (CG) of the pendulum. In such a case, serious stability problems arise.

In this paper, a wheeled inverted pendulum was chosen as a simplified model of a two-wheeled humanoid robot. A case in which an external force from a human was applied to the upper part of a wheeled inverted pendulum is considered. To stabilize the wheeled inverted pendulum even when a severe external force is applied, a human-friendly motion control algorithm was developed. This algorithm allows the pendulum to move naturally coordinated with the external force and maintains a safety environment for the human. For the construction of this algorithm, a position controller for a wheeled inverted pendulum was designed using a LQR method for self-balancing and tracking desired position. In addition, a reduced-order disturbance observer was designed to estimate disturbances by external forces. The estimated external force is used to generate position references. Experiments were conducted to test the human-friendly motion control.

II. System Design

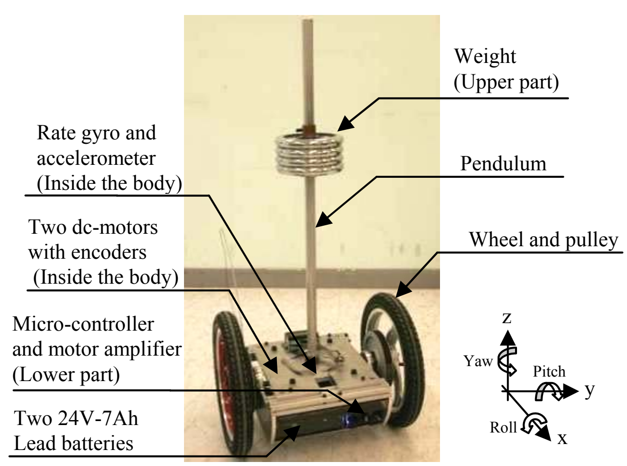

Figure 1. The structure of the wheeled inverted pendulum

Manuscript received September 13, 2007.
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The structure of the wheeled inverted pendulum used in this research is shown in Figure 1. It was assumed that a wheeled inverted pendulum could be used as a service robot in a public area. Accordingly, it was designed to be one meter in height. At the upper part of the pendulum, an adjustable weight was added. This makes testing the various mass and inertia conditions easier. At the lower part of the pendulum, two dc-motors, two encoders, two 24V-7Ah batteries, a micro-controller, a two-channel motor amplifier, a rate gyro and an accelerometer were added. A two-stage pulley-belt system with a 14:1 reduction ratio and a quiet drive without backlash were also added to the device. For control of the motor, a micro-controller and a two-channel motor amplifier were developed. For the micro-controller, a TI TMS320F2808 digital signal controller was used. A custom-made motor amplifier handled two high-power dc-motors (1kW) in a single board despite its relatively small size (100x100x40mm^3). A rate gyro and an accelerometer were used to measure the tilt angle in the pitch direction. These two sensors were combined with complementary filters to provide more accurate tilt information. The accelerometer provides accurate static tilt information when the robot is not accelerating. The rate gyro can be integrated to provide accurate dynamic tilt information, but the integration tends to drift over time. By combining two sensors, it was possible to measure robust inertial information.

III. MODELING

A mathematical model of the developed control system of the wheeled inverted pendulum is described in Figure 2.

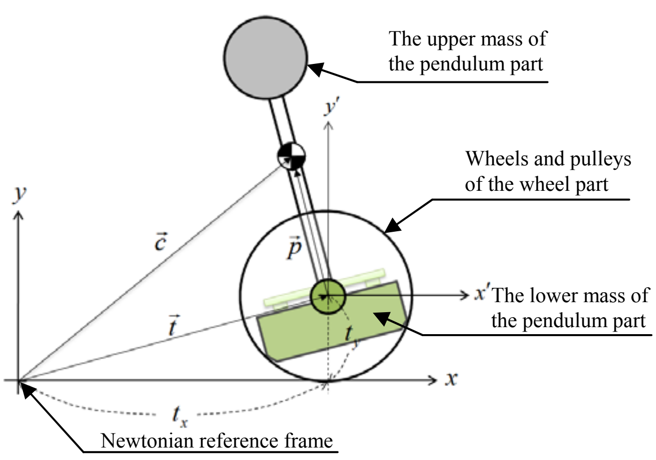
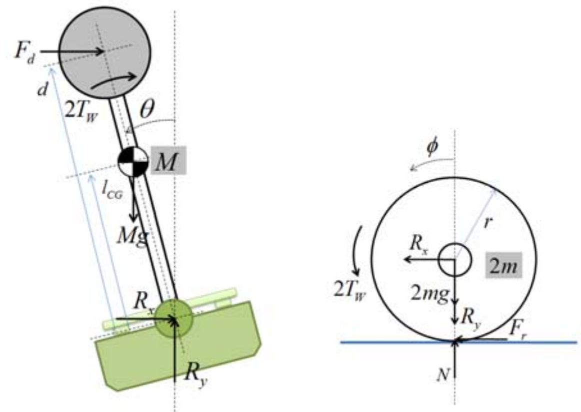


Figure 2. A simplified diagram of the wheeled inverted pendulum

The wheeled inverted pendulum can be decoupled in two different subsystems [1]. The first of these is a pendulum system and the second is a rotation system. The pendulum system is an unstable system and the rotation system is a neutrally stable system. Effort here was focused on the problem of controlling the pendulum system, as it was unstable. Therefore, it was assumed that no yaw motion exists in the pendulum system. Additionally, it was assumed that the wheels of the inverted pendulum are always in contact with the ground and that no slip exists at the contact patches. Therefore, no roll motion exists. Finally, only the longitudinal motion along the x-axis and the pitch motion are considered. The longitudinal movement of the pendulum is characterized by the angular displacement of the wheel ϕ and the pitch motion is characterized by the tilt angle of the pendulum θ.

For an efficient analysis, a free body diagram of a simplified model is shown in Figure 3. This simplified model can be considered two divided parts: a pendulum part and a wheel part.



(a) The pendulum part (b) The wheel part

Figure 3. Free body diagram of the wheeled inverted pendulum

TABLE I
PARAMETERS OF THE WHEELED INVERTED PENDULUM

Symbol	Quantity
M	The total mass of the pendulum part
l_{CG}	Length between the CG and the wheel axis
d	Length between the point of application of external force F_d and the wheel axis
I_{CG}	Moment of inertia of the pendulum
m	Mass of a wheel and a pulley
r	Radius of a wheel
J	Moment of inertia of a wheel and a pulley
g	The constant of acceleration of gravity
J_m	Moment of inertia of a motor rotor
B_m	Viscous friction coefficient of a motor
K_t	Torque constant of a motor
K_e	Electrical constant of a motor
R_m	The armature resistance of a motor

In the pendulum part, there is an upper mass and a lower mass. The CG of the pendulum is located near the middle of these two masses. The total mass of the pendulum part is characterized as M. It was assumed that external force F_d is applied to the upper mass of the pendulum part as a disturbance from a human. Counter torque from two motors $2T_w$ is applied in the pendulum in the other direction compared to the torque of wheel part. In addition, the reaction forces between the pendulum part and the wheel part R_x, R_y are applied on the axle.

The wheel part contains two wheels and two second-stage pulleys. As there is no yaw motion, two wheels always rotate identically. Therefore, the two wheels and the two pulleys are considered as one body. The friction forces between the wheels and the ground is denoted as F_r.

In an actual control system, the control variable is the voltage of the motor V instead of the motor torque T_w. Therefore, a dynamic equation for the motor and was derived, and that relationship is contained in the equation of motion. Moreover, it is assumed that it is possible to linearize the equations of motion around the operating point ($\theta = 0$). The equations of motion of this model are as follows:

$$\left(I_{CG} + Ml_{CG}^{2} + 2n^2 J_m\right)\ddot{\theta} + \left(Mrl_{CG} - 2n^2 J_m\right)\ddot{\phi} + 2n^2\left(b_m + \frac{K_t K_e}{R_m}\right)\dot{\theta} \quad (1)$$
$$-2n^2\left(b_m + \frac{K_t K_e}{R_m}\right)\dot{\phi} - Ml_{CG}g\ \theta + F_d d = -2n\frac{K_t}{R_m}\cdot V$$

$$\left(Mrl_{CG} - 2n^2 J_m\right)\ddot{\theta} + \left[2J + (M+2m)r^2 + 2n^2 J_m\right]\ddot{\phi} \quad (2)$$
$$-2n^2\left(b_m + \frac{K_t K_e}{R_m}\right)\dot{\theta} + 2n^2\left(b_m + \frac{K_t K_e}{R_m}\right)\dot{\phi} + F_d r = 2n\frac{K_t}{R_m}\cdot V$$

where the parameters are defined in Table I.

IV. Control System Development

In this section, the development of control system is discussed. The equations of motion for the system (1), (2) can be written in state-space form, as follows:

$$\dot{x} = Ax + Bu + W\cdot F_d \quad (3)$$

$$x = \begin{bmatrix}\phi\\ \dot{\phi}\\ \theta\\ \dot{\theta}\end{bmatrix}, A = \begin{bmatrix}0 & 1 & 0 & 0\\ 0 & a_1 & a_2 & a_3\\ 0 & 0 & 0 & 1\\ 0 & a_4 & a_5 & a_6\end{bmatrix}, B = \begin{bmatrix}0\\ b_1\\ 0\\ b_2\end{bmatrix}, W = \begin{bmatrix}0\\ d_1\\ 0\\ d_2\end{bmatrix}, u = V$$

where a_1, a_2, a_3, a_4, a_5, a_6, b_1, b_2, d_1, and d_2 are defined as functions of the parameters of the wheeled inverted pendulum.

With this state-space equation, the following control strategies were designed to maintain the pendulum in equilibrium and to maintain a stable posture even when a disturbance is applied.

A. Position controller using LQR method

Before the position controller was designed, it was assumed that there was no external force ($F_d = 0$). In order to control the position and posture of the wheeled inverted pendulum, it was necessary to control the rotation of the wheels as a control output V which is the voltage of the wheel.

For the wheeled inverted pendulum used here, the pitch angle of the pendulum θ was measured from a combination of two sensors: a rate gyro and an accelerometer. Also from the rate gyro, the angular velocity of the pitch motion of the pendulum $\dot{\theta}$ was obtained. The angular displacement and angular velocity of the wheel ϕ, $\dot{\phi}$ were measured with the rotary encoder. Therefore, all of the state-variables from the sensor signal were used as feedback. Using this full-state feedback, it was possible to construct control input u, as follows:

$$u = -K\left(x - x_{ref}\right) \quad (4)$$

where x is the state variable vector, x_{ref} is the reference vector, and K is the feedback gain vector K= [K_1 K_2 K_3 K_4].

The feedback gain K was obtained using LQR method, which results in some balance between system errors and control efforts. To use this LQR method, it was necessary to determine three parameters: the performance index R, the state-cost matrix Q, and the weight factors. For simplicity, the performance index matrix was defined as equal to 1. The weighting factors in the state-cost matrix were chosen by trial and error. If the weighting factors of ϕ, $\dot{\phi}$ are increased, the pendulum becomes very sensitive to the disturbance, but more accurate position tracking performance is given. On the other hand, if these factors are decreased, the pendulum is less sensitive to the disturbance but weak to track desired position. In the control strategy, the high weighting factors of ϕ, $\dot{\phi}$ were chosen for better position tracking performance. For the disturbance problem, a position reference to offset disturbance was planned using the observer described in the next section.

B. Reduced-order Disturbance Observer

In this system, it was possible to obtain the state vector, x using the sensor signal. However, it was not possible to measure the external force directly. The purpose of the reduced-order disturbance observer is to make estimations of the external force F_d. The estimated external force will be used in the human-friendly motion control.

The reduced-order observer reduces the order of the estimator by the number of measurable outputs. In this system, the external force is the only unmeasured variable. Therefore, the state vector can be divided into two parts: x_a, which is directly measurable by sensors as x, and x_b, which must be estimated as F_d. By assuming that the derivative of the external force equals zero ($\dot{F}_d = 0$), it was possible to modify the state-space form, as follows:

$$\begin{bmatrix}\dot{x}_a \\ \dot{x}_b\end{bmatrix} = \begin{bmatrix}A_{aa} & A_{ab} \\ A_{ba} & A_{bb}\end{bmatrix}\begin{bmatrix}x_a \\ x_b\end{bmatrix} + \begin{bmatrix}B_a \\ B_b\end{bmatrix}u \tag{5}$$

$$y = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_a \\ x_b\end{bmatrix} \tag{6}$$

where

$$x_a = \begin{bmatrix}\phi & \dot{\phi} & \theta & \dot{\theta}\end{bmatrix}^T, \ x_b = F_d, \ u = V$$

$$A_{aa} = \begin{bmatrix}0 & 1 & 0 & 0 \\ 0 & a_1 & a_2 & a_3 \\ 0 & 0 & 0 & 1 \\ 0 & a_4 & a_5 & a_6\end{bmatrix}, \ A_{ab} = \begin{bmatrix}0 \\ d_1 \\ 0 \\ d_2\end{bmatrix}, \ B_a = \begin{bmatrix}0 \\ b_1 \\ 0 \\ b_2\end{bmatrix}$$

$$A_{ba} = \begin{bmatrix}0 & 0 & 0 & 0\end{bmatrix}, \ A_{bb} = 0, \ B_b = 0$$

With this state-space form, the reduced-order disturbance observer was constructed. The block diagram of the observer is described in Figure 4. A detailed design of the observer [12] is straightforward and is not presented here.

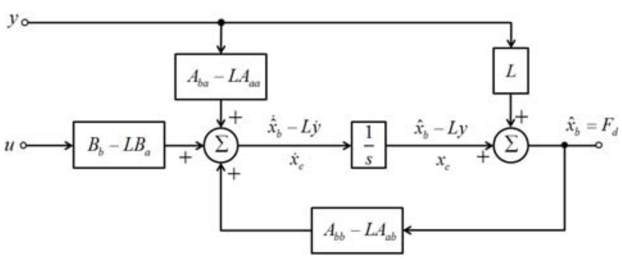


Figure 4. A block diagram of reduced-order disturbance observer

In the above diagram, the observer gain L was obtained using a pole placement method by trial and error by changing the settling time of the observer to balance between a good transient response and a sufficiently low sensor noise. Using this observer, the external force could be estimated.

C. Human-friendly Motion Control

The position controller maintains its original position well against disturbances. If a severe disturbance is applied, it can break its controllable range. To solve this problem, a control algorithm was formulated to generate position reference to prevent falls as a result of an external force. This implies that if the pendulum is pushed or pulled, it moves the way it is commended. But if no external force exists, it maintains its original position. For the generation of the position reference, the following equations were developed:

$$\phi_{ref} = H \cdot \int \left(F_d'\right) dt \tag{7}$$

$$u = -K\left(x - x_{ref} - V \cdot \phi_{ref}\right) \tag{8}$$

where H is a gain obtained experimentally to make the best position reference under various conditions.

No matter how well the observer is designed, it is not possible to estimate the exact external force owing to sensor noise and model mismatch. In some cases, the position reference can be generated by an erroneous estimation, even when an external force is not applied. To prevent this undesired case, the threshold was set so that it ignores estimated external forces that are less than the limit value. By adding the position reference to the feedback system, the pendulum was made coordinate its movement with the external force from a human. A block diagram of the human-friendly motion control system is shown in Figure 5.

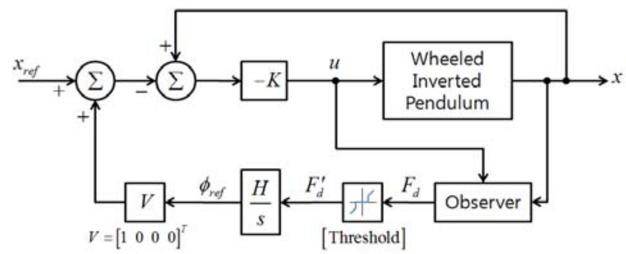


Figure 5. A block diagram of the human-friendly motion control system

V. EXPERIMENTS

A. Estimation of the external force with position control

Using the control law shown in Figure 5, the pendulum was made to stand stably and the external force was estimated. As shown in Figure 6, a static external force was applied. As the external force increased, the pendulum inclined gradually to sustain the force. If the weight was too heavy, the pendulum was not able to maintain a stable range of the pitch angle of $\leq \pm 20^\circ$. Hence, the weight was increased by 0.5kg stepwise within the limit of the pitch angle every 6~7 seconds.

The human-friendly motion control was deactivated for the test of the estimation of the external force in this experiment.

Figure 6. Experiment of estimation of the external force

As the external force is applied horizontally, the estimated external force was converted as the torque using following relationship:

$$T_d = F_d d \cos\theta \tag{9}$$

where d is the height from the axle which is the point of application of the external force and θ is the pitch angle of the pendulum.

B. Experiment of Human-friendly motion control

The purpose of human-friendly motion control is to guarantee human safety when a human pulls or pushes the wheeled inverted pendulum. In Figure 7, a human is shown pushing the upper part of the pendulum. Under a condition in which the human-friendly motion control was deactivated, the pushing force was strong enough to cause the pendulum to topple over. However, the pendulum pushed away stably due to the generated position reference in an integrated manner. Detailed motion data is presented in the next chapter.

Figure 7. The pendulum coordinating with an external force applied by a human

VI. EXPERIMENTAL RESULTS

In Experiment *A*, the estimation of the external force while the pendulum stood stably by the control system was tested. The result of Experiment *A* is presented in Figure 8. It shows the estimated disturbance torque, its mean value, and the actual applied torque.

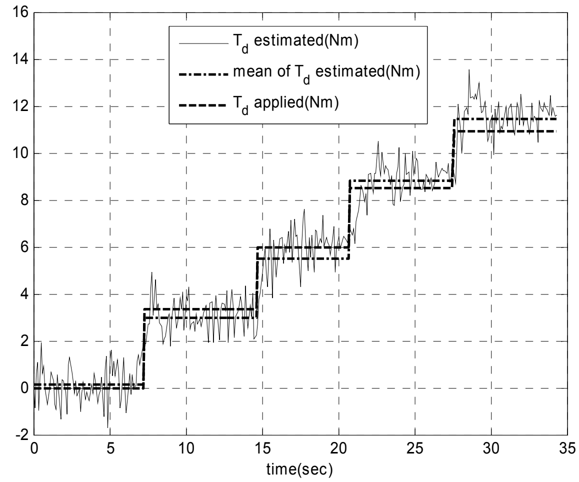


Figure 8. The result of the estimation of the external force

In this figure, it is clear that there is strong vibration in the estimated disturbance torque. However, the average of the estimated value is fairly close to the real value. For a more detailed analysis, the true value, mean value, and the standard deviation of the disturbance torque are shown in Figure 9.

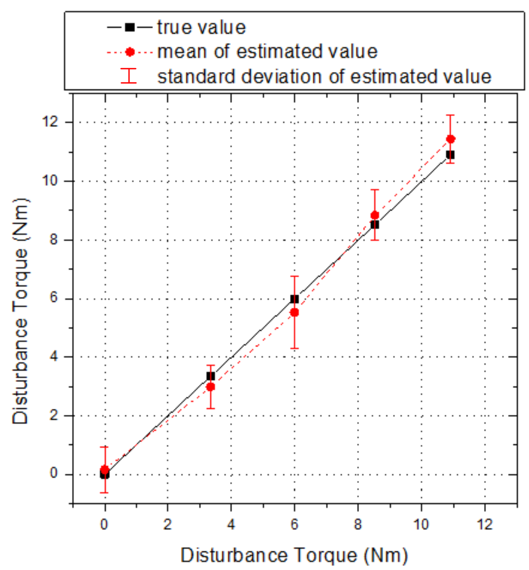


Figure 9. The result of the estimation of the external force

Figure 9 shows that the standard deviation of the estimated value is within the true value, although a vibration exists. Thus, it is confirmed that the external force can be estimated using the reduced-order disturbance observer.

Using this observer, Experiment *B* was conducted. The result of the human-friendly motion control is presented in Figure 10.

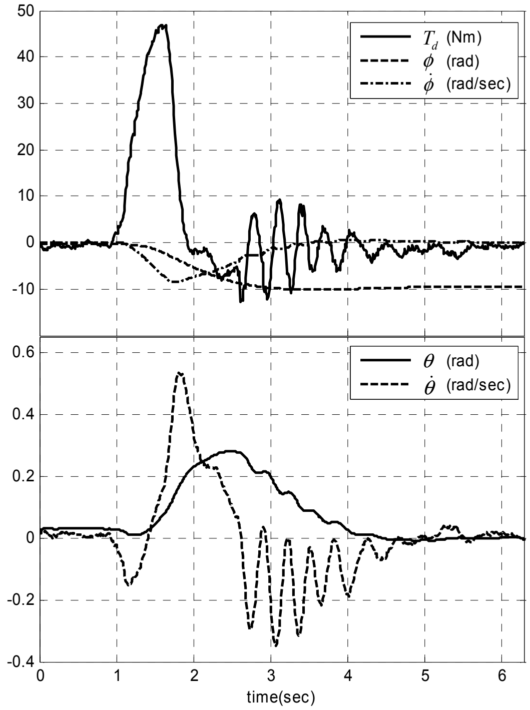


Figure 10. The result of the experiment of human-friendly motion control

After the disturbance had started for approximately 1 second, the angular displacement of the wheel ϕ increased due to the generation of the position reference. It stopped at approximately -10 rad. In the same time, the pitch angle was inclined to its maximum of nearly 0.3 rad ($\simeq 17^\circ$). This value is within the safe tilt angle range of the pendulum. This implies the pendulum offsets the disturbance by moving smoothly in the direction from which the external force is applied. Experiments were conducted regarding the performance of the human-friendly motion control.

VII. Conclusion

In this research, it was considered that a stability problem may occur with a wheeled inverted pendulum when it interacts with a human. To address this issue, a human-friendly motion control was constructed. This control strategy allows the pendulum to offset the disturbance from the human safely using an integrated control system.

To realize this control strategy, a position control system was built using full-state feedback to guarantee the stability of the pendulum, and a reduced-order disturbance observer was utilized to estimate the external force. Through the use of the estimated external force, a control algorithm was designed to make proper position references to offset the disturbance.

Experiments showed that the human-friendly motion control was able to estimate the external force while standing safely and that the pendulum can be stabilized even when a severe disturbance is applied.

In the future, the authors aim to adopt the human-friendly motion control to an actual two-wheeled humanoid robot for an evaluation of the control system designed in this research.

References

[1] Felix Gasser, Aldo D'Arrigo, Silvio Colomi, "JOE: A Mobile, Inverted Pendulum," IEEE Transactions on Industrial Electronics, vol. 49, NO. 1, pp 107-114, Feb. 2002

[2] Kaustubh Pathak, Jaume Franch, and Sunil K. Agrawal, "Velocity and Position Control of a Wheeled Inverted Pendulum by Partial Feedback Linearization," IEEE Transactions on Robotics, vol. 21, NO. 3, pp 505-513, June. 2005

[3] Salerno, A. and Angeles, J., "Nonlinear Controllability of Quasiholonomic Mobile Robot," Proc. IEEE ICRA, pp. 1740-1745, April. 2004

[4] Kim, Y.H., Kim, S.H., and Kwak, Y.K., "Dynamic Analysis of a Nonholonomic Two-wheeled Inverted Pendulum Robot," Proc. of the Eighth Int. Symp. on Artificial Life and Robotics (AROB8th, '03). pp. 415-418, Jan. 2003

[5] Nawawi S.W, Ahmad M.N, Osman J.H.S, "Control of Two-wheels Inverted Pendulum Mobile Robot using Full Order Sliding Mode Control," Proc. of Int. Conference on Man-Machine Systems, Sep. 2006

[6] Naoji SHIROMA, Osamu MATSUMOTO, Shuuji KAJITA, Kazuo TANI, "Cooperative Behavior of a Wheeled Inverted Pendulum for Object Transportation," Proc. IROS 96, vol. 2, pp 396-401, Nov. 1996

[7] Toshio MORITA, Koji SHIBUYA, Shigeki SUGANO, "Design and Control of Mobile Manipulation System for Human Symbiotic Humanoid: Hadaly-2," Proc. of the 1998 IEEE ICRA, vol. 2, pp. 1315-1320, May. 1998

[8] Jochen Heinzmann and Alexander Zelinsky, "The Safe Control for Human-Friendly Robots," Proc. of the 1999 IEEE/RSJ, Int. Conference on Intelligent Robots and Systems, vol. 2, pp. 1020-1025, 1999

[9] Sehoon Oh, Naoki Hata, Yoichi Hori, "Proposal of Human-friendly Motion Control: Control Design for Power Assistance Tools and its Application to Wheelchair," IECON 2004. 30th Annual Conference of IEEE, vol. 1, pp. 436-441, Nov. 2004

[10] Chia-Shang Liu, Huei Peng, "Disturbance Observer Based Tracking Control," Journal of Dynamic Systems, Measurement, and Control, vol. 122, June. 2000

[11] Joseph A. Profeta, William G. Vogt, Marin H. Mickle, "Disturbance Estimation and Compensation in Linear Systems," IEEE Transactions on Aerospace and Electronic Systems, vol. 26, NO. 2, March. 1990

[12] Gene F. Franklin, J. David Powell, Abbas Emami-Naeini, "Feedback Control of Dynamic Systems," 4th Edition., Prentice Hall, 2002

[13] Katsuhiko Ogata, "Modern Control Engineering", 4th edition, Prentice Hall, 2001