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Position and Force Control of the Grasping Function for a Hyperredundant Arm

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Abstract— **The grasping control problem for a hyperredundant manipulator is presented. The dynamic model is derived by using Lagrange equations developed for infinite dimensional systems. The algoritms for the position and force control are proposed. The arm fluid pressure control is inferred and the conditions that ensure the stability of the motion are discussed. Numerical simulations are presented.**

I. INTRODUCTION

A hyperredundant robot is a or hyper-degree-of-freedom manipulator and there has been a rapidly expanding interest in their study and construction lately.

The control of these systems is very complex and a great number of researchers have tried to offer solutions for this difficult problem. In [1] it was analyzed the control by cables or tendons meant to transmit forces to the elements of the arm in order to closely approximate the arm as a truly continuous backbone. In [2], Gravagne analyzed the kinematical model of “hyper-redundant” robots, known as “continuum” robots. Important results were obtained by Chirikjian and Burdick [3]-[5] which laid the foundations for the kinematical theory of hyper-redundant robots. Mochiyama has also investigated the problem of controlling the shape of an HDOF rigid-link robot with two-degree-of-freedom joints using spatial curves [6], [7]. In [8, 9] it is presented the “state of art” of continuum robots, outline their areas of application and introduce some control issues.

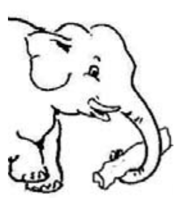

Figure 1. A biological grasping

In other papers [11, 12] several technological solutions for actuators used in hyper-redundant structures are presented and conventional control systems are introduced.

In this paper, the problem of a class of hyperredundant arms with continuum elements that performs the grasping function by coiling is discussed. This function is often met in the animal world as the elephant’s trunk (Figure1), the octopus tentacle or the constrictor snakes. First, the dynamic model of the system is inferred. The difficulties determined by the complexity of the non-linear integral-differential equations, that represent the dynamic model of the system, are avoided by using a very basic energy relationship of this system. Energy-based control laws are introduced for the position control problem. A force control method is proposed by using the ER fluid viscosity control.

The paper is organized as follows: section 2 presents the basic principles of a hyperredundant structure with continuum elements; section 3 studies the dynamic model; section 4 discusses the both problem of grasping by coiling, the position control and force control; section 5 verifies by computer simulation the control laws.

II. BACKGROUND

A. Technological Model

The paper studies a class of hyperredundant arms that can achieve any position and orientation in 3D space, and that can perform a coil function for the grasping (Figure 2). The arm is a high degree of freedom structure or a continuum structure. The general form of the arm is shown in Figure 3. It consists of a number (N) of elements, cylinders made of fiber-reinforced rubber. There are four internal chambers in the cylinder, each of them containing the ER fluid with an individual control circuit. The deformation in each cylinder is controlled by an independent electrohydraulic pressure control system combined with the distributed control of the ER fluid. The cylinder can be bent in any direction by appropriately controlling the pressure in the four chambers. The electrical control of the ER fluid viscosity is obtained by an electrode network distributed on the length of the cylinder.

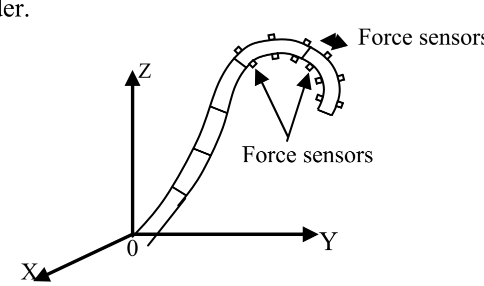

Figure 2. The force sensors distribution

The technological model can be considered as one with a central, highly flexible and elastic backbone. We will assume that the backbone never bends past the “small – strain region”, where an applied stress produced a strain that is recoverable and obeys an approximately linear stress – strain relationship. Also, the system is frictionless and any other damping and friction are neglected.

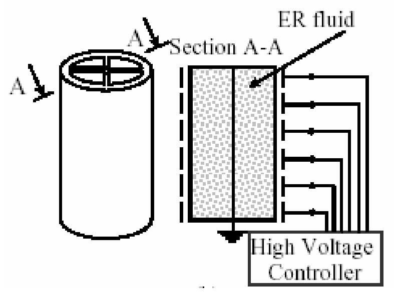

Figure 3. The cylinder structure

The last m elements $(m < N)$ represent the grasping terminal. These elements contain a number of force sensors distributed on the surface of the cylinders. These sensors measure the contact with the load and ensure the distributed force control during the grasping. The sensor network is constituted by a number of impedance devices [11] (see Figure 3) that define the dynamic relationship between the grasping element displacement and the contact force.

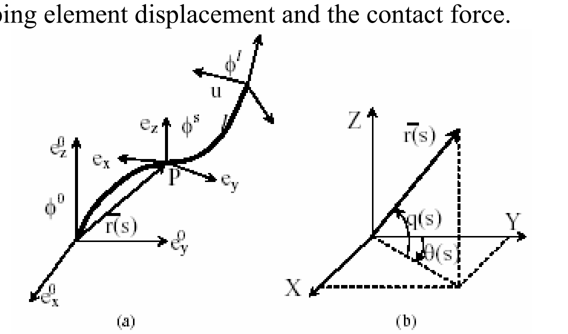

Figure 4. (a) The backbone structure; (b) The backbone parameters

B. *Theoretical model*

The essence of the hyperredundant model is a 3-dimensional backbone curve C that is parametrically described by a vector $\bar{r}(s) \in R^3$ and an associated frame $\phi(s) \in R^{3\times 3}$ whose columns create the frame bases (see Figure 4). The independent parameter s is related to the arc-length from the origin of the curve C, $s \in [0, \quad L]$, where: $L = \sum_{i=1}^{N} l_i$, where l_i represent the length of the elements i of the arm in the initial position.

The position of a point s on curve C is defined by the position vector: $\bar{r} = \bar{r}(s)$, when $s \in [0, \quad L]$. For a dynamic motion, the time variable will be introduced, $\bar{r} = \bar{r}(s,t)$.

We used a parameterization of the curve C based upon two "continuous angles" $\theta(s)$ and $q(s)$ [3-5] (Figure 4).

The position vector on curve C is given by

$$\bar{r}(s,t) = [x(s,t) \quad y(s,t) \quad z(s,t)]^T \tag{1}$$

where

$$x(s,t) = \int_0^s \sin\theta(s',t)\cos q(s',t)ds' \tag{2}$$

$$y(s,t) = \int_0^s \cos\theta(s',t)\cos q(s',t)ds' \tag{3}$$

$$z(s,t) = \int_0^s \sin q(s',t)ds', \ s' \in [0, \quad s] \tag{4}$$

For an element dm, kinetic and gravitational potential energy will be:

$$dT = \frac{1}{2}dm\left(v_x^2 + v_y^2 + v_z^2\right),\ dV = dm \cdot g \cdot z \tag{5}$$

where $dm = \rho ds$, and ρ is the mass density.

The elastic potential energy will be approximated by the bending of the element [10], $V_e = k\frac{d^2}{4}\sum_{i=1}^{N}\left(q_i^2 + \theta_i^2\right)$.

III. DYNAMIC MODEL

In this paper, the manipulator model is considered a distributed parameter system defined on a variable spatial domain $\Omega = [0, \quad L]$ and the spatial coordinate s. The dynamic model is derived by using Lagrange equations developed for infinite dimensional systems,

$$\begin{aligned}
&\rho\int_0^S\int_0^S\Big(\ddot{q}'\left(\sin q'\sin q''\cos(q'-q'')+\cos q'\cos q''\right)- \\
&-\ddot{\theta}'\cos q'\sin q''\sin(\theta''-\theta')+ \\
&+(\dot{q}')^2\left(\cos q'\sin q''\cos(\theta'-\theta'')-\sin q'\cos q''\right)+ \\
&+(\dot{\theta}')^2\cos q'\sin q''\cos(\theta'-\theta'')- \\
&-\dot{q}'\dot{q}''\sin(q''-q')\Big)ds'ds''+\rho g\int_0^S\cos q' ds'+k^* q = F_q
\end{aligned} \tag{6}$$

$$\begin{aligned}
&\rho\int_0^S\int_0^S\Big(\ddot{q}'\sin q'\cos q''\sin(\theta''-\theta')+ \\
&+\ddot{\theta}'\cos q'\cos q''\cos(\theta''-\theta')- \\
&-(\dot{q}')^2\cos q'\cos q''\sin(\theta''-\theta')+ \\
&+(\dot{\theta}')^2\cos q'\cos q''\sin(\theta''-\theta')- \\
&-\dot{\theta}\dot{q}'\sin q'\cos q''\cos(\theta''-\theta')\Big)ds'ds''+k^*\theta = F_\theta
\end{aligned} \tag{7}$$

where we used the notations: $\dot{q}' = \partial q(s',t)/\partial t$, $\ddot{q}' = \partial^2 q(s',t)/\partial t^2$, $F_q = F_q(s,t)$, $s \in [0, \quad L]$, $s' \in [0, \quad s]$.

The state of this system at any fixed time t is specified by the set $(\omega(t,s), \nu(t,s))$, where $\omega = [\theta \quad q]^T$ represents the generalized coordinates and ν defines the momentum densities. The set of all functions $s \in \Omega$ that ω, ν can take on at any time is state function space $\Gamma(\Omega)$. We will consider that $\Gamma(\Omega) \subset L_2(\Omega)$.

The control forces have the distributed components along the arm, $F_\theta(s,t)$, $F_q(s,t)$, $s \in [0, \quad L]$ that are determined by the lumped torques,

$$F_\theta(s,t) = \sum_{i=1}^{N} \delta(s - il)\tau_{\theta_i}(t) \tag{8}$$

$$F_q(s,t) = \sum_{i=1}^{N} \delta(s - il)\tau_{q_i}(t) \tag{9}$$

where δ is Kronecker delta, $l_1 = l_2 = \ldots = l_N = l$, and

$$\tau_{\theta_i}(t) = \left(p_{\theta_i}^1 - p_{\theta_i}^2\right)S \cdot d/8 \tag{10}$$

$$\tau_{q_i}(t) = \left(p_{q_i}^1 - p_{q_i}^2\right)S \cdot d/8\,,\ i = 1,2,\ldots,N \tag{11}$$

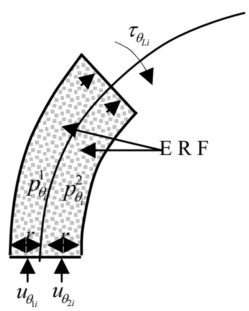

Figure 5. The cylinder driving

In (10), (11), $p_{\theta_i}^1$, $p_{\theta_i}^2$, $p_{q_i}^1$, $p_{q_i}^2$ represent the fluid pressure in the two chamber pairs, θ, q and *S*, *d* are section area and diameter of the cylinder, respectively (Figure 5). The pressure control of the chambers is described by the equations [9]

$$a_{ki}(\theta)\frac{dp_{\theta i}^k}{dt} = u_{\theta ki}\,,\ b_{ki}(q)\frac{dp_{qi}^k}{dt} = u_{qki}\,,\ k = 1,2 \tag{12}$$

where $a_{ki}(\theta)$, $b_{ki}(q)$ are determined by the fluid parameters and the geometry of the chambers and

$$a_{ki}(0) > 0 \tag{13}$$

$$b_{ki}(0) > 0\,,\ k = 1,2\,;\ i = 1,2,\ldots,N\,;\ \theta, q \in \Gamma(\Omega) \tag{14}$$

IV. Control Problem

The control problem of a grasping function by coiling is constituted from two subproblems: the position control of the arm around the object-load and the force control of grasping.

A. Position Control

We consider that the initial state of the system is given by

$$\omega_0 = \omega(0,s) = [\theta_0, \quad q_0]^T \tag{15}$$

$$\nu_0 = \nu(0,s) = [0, \quad 0]^T \tag{16}$$

where

$$\theta_0 = \theta(0,s),\ q_0 = q(0,s),\ s \in [0, \quad L] \tag{17}$$

corresponding to the initial position of the arm defined by the curve C_0

$$C_0 : (\theta_0(s), \quad q_0(s)),\ s \in [0, \quad L] \tag{18}$$

The desired point in $\Gamma(\Omega)$ is represent by a desired position of the arm, the curve C_d that coils the load,

$$\omega_d = [\theta_d, \quad q_d]^T \tag{19}$$

$$\nu_d = [0, \quad 0]^T \tag{20}$$

$$C_d : (\theta_d(s), \quad q_d(s)),\ s \in [0, \quad L] \tag{21}$$

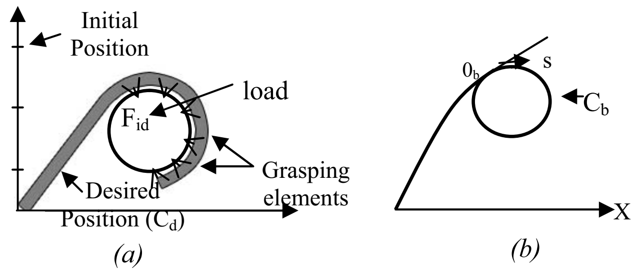

Figure 6. (a) The grasping position; (b) The grasping parameters

In a grasping function by coiling, only the last $n+1-m$ elements $(m < N)$ are used. Let l_g be the active grasping length,

$$l_g = \sum_{i=m}^{n} l_i \tag{22}$$

Let C_b be the curve defines the boundary of the load and we denote by O_b the origin of the coiling function, when O_b is the intersection between the tangent from origin *O* and the curve C_L (Figure 6.b). This curve can be expressed in the coordinates $(\theta, \quad q) \in \Gamma(\Omega)$.

$$C_b : \left(\theta_b\left(s^*\right), \quad q_b\left(s^*\right)\right),\ s^* \in [0, \quad L_b] \tag{23}$$

where L_b is the length of the coiling measured on the boundary C_b and

$$s = L - l_g + s^* \tag{24}$$

We define by $e_p(t)$ the position error

$$e_p(t) = \int_{L-l_g}^{L} \left(\left(\theta(s,t) - \theta_b(s)\right) + \left(q(s,t) - q_b(s)\right)\right)ds \tag{25}$$

It is difficult to measure practically the angles θ, q for all $s \in [0, \quad L]$. These angles can be evaluated or measured at the terminal point of each element. In this case, the relation (25) becomes

$$e_p(t) = \sum_{i=m}^{N} \left(\left(\theta_i(t) - \theta_{bi}\right) + \left(q_i(t) - q_{bi}\right)\right) \tag{26}$$

The error can also be expressed with respect to the global desired position C_d

$$e_p(t)=\sum_{i=1}^{N}\left(\left(\theta_i(t)-\theta_{di}\right)+\left(q_i(t)-q_{di}\right)\right) \quad (27)$$

$$e_p(t)=\sum_{i=1}^{N}\left(e_{\theta i}(t)+e_{qi}(t)\right) \quad (28)$$

The position control of the arm means the motion control from the initial position C_0 to the desired position C_b in order to minimize the error.

Theorem 1. The closed-loop control system of the position (6), (7), (12) is stable if the fluid pressures control law in the chambers of the elements given by:

$$u_{\theta ji}(t)=-a_{ji}(\theta)\left(k_{\theta i}^{j1}\dot{e}_{\theta i}(t)+k_{\theta i}^{j2}\ddot{e}_{\theta i}(t)\right) \quad (29)$$

$$u_{qji}(t)=-b_{ji}(\theta)\left(k_{qi}^{j1}\dot{e}_{qi}(t)+k_{qi}^{j2}\ddot{e}_{qi}(t)\right), \quad (30)$$

where $j=1,2$; $i=1,2,\dots,N$, with initial conditions:

$$p_{\theta i}^1(0)-p_{\theta i}^2(0)=\left(k_{\theta i}^{11}-k_{\theta i}^{21}\right)e_{\theta i}(0) \quad (31)$$

$$p_{qi}^1(0)-p_{qi}^2(0)=\left(k_{qi}^{11}-k_{qi}^{21}\right)e_{qi}(0) \quad (32)$$

$$\dot{e}_{\theta i}(0)=0 \quad (33)$$

$$\dot{e}_{qi}(0)=0,\ i=1,2,\dots,N \quad (34)$$

and the coefficients $k_{\theta i}$, k_{qi}, $k_{\theta i}^{mn}$, k_{qi}^{mn} are positive and verify the conditions

$$k_{\theta i}^{11}>k_{\theta i}^{21};\ k_{\theta i}^{12}>k_{\theta i}^{22} \quad (35)$$

$$k_{qi}^{11}>k_{qi}^{21};\ k_{qi}^{12}>k_{qi}^{22},\ i=1,2,\dots,N \quad (36)$$

Proof. See Appendix 1.

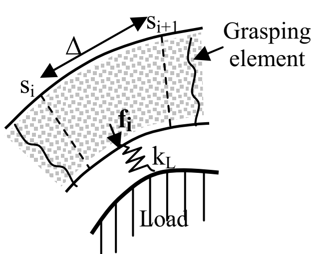

Figure 7. The grasping force

B. Force Control

The grasping by coiling of the continuum terminal elements offers a very good solution in the fore of uncertainty on the geometry of the contact surface. The contact between an element and the load is presented in Figure 7. It is assumed that the grasping is determined by the chambers in θ-plane.

The relation between the fluid pressure and the grasping forces can be inferred for a steady state from [2],

$$\int_0^l k\frac{\partial^2\theta(s)}{\partial s^2}ds+\int_0^l f(s)\widetilde{T}\widetilde{\theta}(s)\int_0^s \widetilde{T}^T\widetilde{\theta}(s)ds= =(p_1-p_2)S\frac{d}{8} \quad (37)$$

where $f(s)$ is the orthogonal force on the curve C_b, $f(s)$ is $F_\theta(s)$ in θ-plane and $F_q(s)$ in q-plane, respectively.

A spatial discretization $s_1, s_2, \dots, s_{l1}$ is introduced and

$$\Delta=s_{i+1}-s_i,\ \theta_i=\theta(s_i),\ i=1,2,\dots,l_1 \quad (38)$$

For small variation $\Delta\theta_i$ around the desired position θ_{id}, in θ-plane, the dynamic model (6) can be approximated by the following discrete model [11],

$$m_i\Delta\ddot{\theta}_i+c_i\Delta\dot{\theta}_i+H_i(\theta_{id}+\Delta\theta_i,\ \theta_{id},\ q_d)- -H(\theta_{id},\ q_d)=d_i(f_i-F_{ei}), \quad (39)$$

where $m_i=\rho S\Delta$, $i=1,2,\dots,l_1$, $H(\theta_{id},\ q_d)$ is a nonlinear function defined on the desired position $(\theta_{id},\ q_d)$,

$$c_i=c_i(\nu,\ \theta_i,\ q_d),\ c_i>0,\ \theta,q\in\Gamma(\Omega) \quad (40)$$

ν is the viscosity of the fluid in the chambers.

$$H_i(\theta_{id}+\Delta\theta_i,\ \theta_{id},\ q_d)-H(\theta_{id},\ q_d)\cong \cong\left.\frac{\partial H_i}{\partial\theta}\right|_{\theta=\theta_d}\Delta\theta_i=h_i(\theta_{id},\ q_d)\cdot\Delta\theta_i \quad (41)$$

and F_{ei} is the external force due to the environment, the load. The equation (39) becomes,

$$m_i\Delta\ddot{\theta}_i+c_i(\nu,\ \theta_i,\ q_d)\Delta\dot{\theta}_i+h_i(\theta_{id},\ q_d)\cdot\Delta\theta_i= =d_i(f_i-F_{ei}) \quad (42)$$

The aim of explicit force control is to exert a desired force F_{id}. If the contact with load is modeled as a linear spring with constant stiffness k_L, the environment force can be modeled as:

$$F_{ei}=k_L\Delta\theta_i \quad (43)$$

The error of the force control may be introduced as

$$e_{fi}=F_{ie}-F_{id} \quad (44)$$

It may be easily shown that the equation (42) becomes

$$\frac{m_i}{k_L}\ddot{e}_{fi}+\frac{c_i}{k_L}\dot{e}_{fi}+\left(\frac{h_i}{k}+d_i\right)e_{fi}=d_if_i-\left(\frac{h_i}{k}+d_i\right)F_{id} \quad (45)$$

Theorem 2. The closed force control system is asymptotic stable if the control law is

$$f_i=\frac{1}{k_Ld_i}\left(\left(h_i+k_Ld_i+m_i\sigma^2\right)e_{fi}-\left(h_i-k_Ld_i\right)F_{id}\right) \quad (46)$$

$$c_i>m_i\sigma \quad (47)$$

Proof. See Appendix 2.

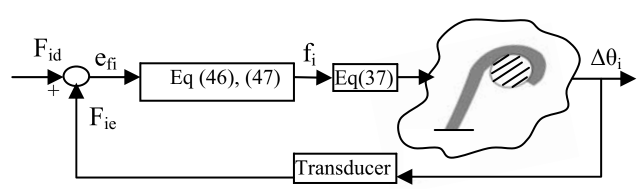

Figure 8. The force control system

In this paper, the force error control may be achieved by using the Direct Sliding Mode Control (DSMC) [12]. This method establishes conditions that force the trajectory along the switching line, directly toward the origin. The block scheme of the force control is presented in Figure 8.

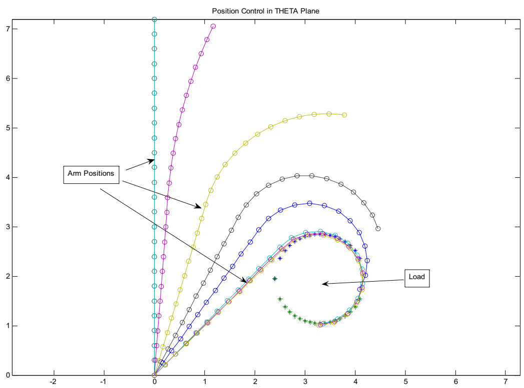

Figure 9. The position control

V. SIMULATION

A hyperredundant manipulator with eight elements is considered. The mechanical parameters are: linear density $\rho = 2.2\, kg/m$ and the length of one element is $l = 0.05m$. The control problem in the θ-plane will be analyzed.

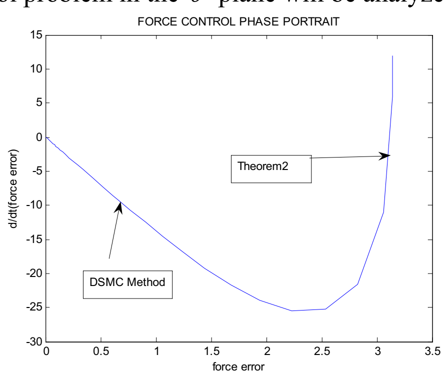

Figure 10. The force control phase portrait

The initial position is the defined by $C_0 : (\theta_0(s) = \pi/2)$ and the grasping function is performed for a circular load defined by $C_b : (x^* - x_0^*)^2 + (y^* - y_0^*)^2 = r^2$, where (x^*, y^*) represent the coordinates in θ-plane. A discretisation for each element with an increment $\Delta = l/3$ is introduced. A control law (29) is used and a MATLAB system is applied. The result is presented in Figure 9.

A force control for the grasping terminals is simulated. The phase portrait of the force error is presented in Figure 10. First, the control (29) is used and then, when the trajectory penetrates the switching line, the viscosity is increased for a damping coefficient $\xi = 1.15$.

VI. CONCLUSION

The paper treats the control problem of a hyperredundant robot with continuum elements that performs the coli function for grasping. The structure of the arm is given by flexible composite materials in conjunction with active-controllable electro-rheological fluids. The dynamic model of the system is inferred by using Lagrange equations developed for infinite dimensional systems.

The grasping problem is divided in two subproblems: the position control and force control. The difficulties determined by the complexity of the non-linear integral-differential equations are avoided by using a very basic energy relationship of this system and energy-based control laws are introduced for the position control problem. The force control is obtained by using the DSMC method in which the evolution of the system on the switching line is controlled by the ER fluid viscosity. Numerical simulations are presented.

Appendix 1

We consider the following Lyapunov function [13],

$$W(t) = T(t) + V(t) + \frac{1}{2}\sum_{i=1}^{N}\left(k_{\theta i} e_{\theta i}^2(t) + k_{qi} e_{qi}^2(t)\right) \quad \text{(A.1.1)}$$

where T, V represent the kinetic and potential energies of the system. $W^*(t)$ is positive definite because the terms that represent the energy T and V are always $T(t) \geq 0$; $V(t) \geq 0$.

By using (5), the derivative of this function will be

$$\dot{W}(t) = \int_0^l \left(F_\theta(s,t)\dot{\theta}(s,t) + F_q(s,t)\dot{q}(s,t)\right) ds + \\ + \sum_{i=1}^{N}\left(k_{\theta i} e_{\theta i}(t)\dot{e}_{\theta i}(t) + k_{qi} e_{qi}(t)\dot{e}_{qi}(t)\right) \quad \text{(A.1.2)}$$

From (8)-(11), the relation (A.1.2.) can be rewritten as

$$\dot{W}(t) = \frac{Sd}{8}\sum_{i=1}^{N}\begin{pmatrix}\left(p_{\theta i}^1(t) - p_{\theta i}^2(t)\right)\dot{\theta}_i(t) + \\ + \left(p_{qi}^1(t) - p_{qi}^2(t)\right)\dot{q}_i(t)\end{pmatrix} + \\ + \sum_{i=1}^{N}\left(k_{\theta i} e_{\theta i}(t)\dot{e}_{\theta i}(t) + k_{qi} e_{qi}(t)\dot{e}_{qi}(t)\right) \quad \text{(A.1.3)}$$

The control law (29), (30) with the initial conditions (31)-(34) determines the pressures of fluid in the chambers,

$$p_{\theta i}^j(t) = -\left(k_{\theta i}^{j1} e_{\theta i}(t) + k_{\theta i}^{j2}\dot{e}_{\theta i}(t)\right),\ j = 1,2 \quad \text{(A.1.4)}$$

$$p_{qi}^j(t) = -\left(k_{qi}^{j1} e_{qi}(t) + k_{qi}^{j2}\dot{e}_{qi}(t)\right),\ j = 1,2 \quad \text{(A.1.5)}$$

Substituting these solutions in (A.1.3) we obtain

$$\dot{W}(t) = -\frac{Sd}{8}\sum_{i=1}^{N}\left(\left(k_{\theta i}^{11} - k_{\theta i}^{21}\right)e_{\theta i}(t)\dot{e}_{\theta i}(t)\right) - \\ -\frac{Sd}{8}\sum_{i=1}^{N}\left(\left(k_{qi}^{11} - k_{qi}^{21}\right)e_{qi}(t)\dot{e}_{qi}(t)\right) - \\ -\frac{Sd}{8}\sum_{i=1}^{N}\left(\left(k_{\theta i}^{12} - k_{\theta i}^{22}\right)\dot{e}_{\theta i}^2(t) + \left(k_{qi}^{11} - k_{qi}^{21}\right)\dot{e}_{qi}^2(t)\right) + \\ + \sum_{i=1}^{N}\left(k_{\theta i} e_{\theta i}(t)\dot{e}_{\theta i}(t) + k_{qi} e_{qi}(t)\dot{e}_{qi}(t)\right) \quad \text{(A.1.6)}$$

If $k_{\theta i} = \frac{Sd}{8}\left(k_{\theta i}^{11} - k_{\theta i}^{21}\right)$, $k_{qi} = \frac{Sd}{8}\left(k_{qi}^{11} - k_{qi}^{21}\right)$ and the conditions (35), (36) are verified, the function $\dot{W}(t)$ becomes:

$$\dot{W}(t) = -\frac{Sd}{8}\sum_{i=1}^{N}\left(\left(k_{\theta i}^{12} - k_{\theta i}^{22}\right)\dot{e}_{\theta i}^{2}(t) + \left(k_{qi}^{11} - k_{qi}^{21}\right)\dot{e}_{qi}^{2}(t)\right) \le 0 \quad \text{(A.1.7)}$$

Q.E.D.

Appendix 2

An error measure of the control system is introduced by the parameter

$$\lambda = \dot{e} + \sigma e \quad \text{(A.2.1)}$$

and $\dot{\lambda} = \ddot{e} + \sigma\dot{e}$.

The equation of the force error (56) becomes

$$\frac{m_i}{k_L}\dot{\lambda} + \left(\frac{c_i - \sigma m_i}{k_L}\right)\lambda + \left(\frac{h_i}{k_L} + d_i + \frac{m_i}{k_L}\sigma^2\right)e = = d_i f_i + \left(\frac{h_i}{k_L} + d_i\right)F_{id} \quad \text{(A.2.2)}$$

A Lyapunov function of V is introduced

$$V = \frac{1}{2}\lambda^2 \quad \text{(A.2.3)}$$

The derivative of V will be

$$\dot{V} = \dot{\lambda}\lambda \quad \text{(A.2.4)}$$

and substituting the variable $\dot{\lambda}$ from (A.2.2), this relation becomes

$$\dot{V} = -\lambda^2\left(\frac{c_i}{m_i} - \sigma\right) - \lambda\left(\frac{h_i}{k_L} + d_i + \frac{m_i}{k_L}\sigma^2\right)e + + \frac{k_L}{m_i}d_i f_i + \left(\frac{h_i}{m_i} + \frac{k_L d_i}{m_i}\right)F_{id} \quad \text{(A.2.5)}$$

By using the conditions of Theorem 2, (46), (47), this relation will be:

$$\dot{V} = -\lambda^2\left(\frac{c_i}{m_i} - \sigma\right) \quad \text{(A.2.6)}$$

$$V \le 0 \quad \text{(A.2.7)}$$

Q.E.D.

REFERENCES

[1] A. Hemami, *Design of light weight flexible robot arm*, Robots 8 Conference Proceedings, Detroit, USA, June 1984, pp. 1623-1640.

[2] Ian A. Gravagne, Ian D. Walker, *On the kinematics of remotely - actuated continuum robots*, Proc. 2000 IEEE Int. Conf. on Robotics and Automation, San Francisco, April 2000, pp. 2544-2550.

[3] G. S. Chirikjian, J. W. Burdick, *An obstacle avoidance algorithm for hyper-redundant manipulators*, Proc. IEEE Int. Conf. on Robotics and Automation, Cincinnati, Ohio, May 1990, pp. 625 - 631.

[4] G.S.Chirikjian, *A general numerical method for hyper-redundant manipulator inverse kinematics*, Proc. IEEE Int. Conf. Rob Aut, Atlanta, May 1993, pp. 107-112.

[5] G.S. Chirikjian, J. W. Burdick *Kinematically optimal hyperredundant manipulator configurations*, IEEE Trans. Robotics and Automation, vol. 11, no. 6, Dec. 1995, pp. 794 - 798.

[6] H. Mochiyama, H. Kobayashi, *The shape Jacobian of a manipulator with hyper degrees of freedom,* Proc. 1999 IEEE Int. Conf. on Robotics and Automation, Detroit, May 1999, pp. 2837- 2842.

[7] H. Mochiyama, E. Shimeura, H. Kobayashi, *Direct kinematics of manipulators with hyper degrees of freedom and Serret-Frenet formula*, Proc. 1998 IEEE Int. Conf. on Robotics and Automation, Leuven, Belgium, May 1998, pp. 1653-1658.

[8] G. Robinson, J.B.C. Davies, *Continuum robots – a state of the art*, Proc. 1999 IEEE Int. Conf. on Rob and Aut, Detroit, Michigan, May 1999, pp. 2849-2854.

[9] K. Suzumori, S. Iikura, H. Tanaka, *Development of flexible microactuator and its application to robot mechanisms*, IEEE Conf. on Robotics and Automation, Sacramento CA, April 1991, pp 1564 - 1573.

[10] S.K. Singh, D.O. Popa, *An Analysis and Some Fundamental Problems in Adaptive Control of Force*, IEEE Trans. Rob and Aut., Vol. 11, No 6, pp 912-922.

[11] M. Ivanescu, V. Stoian, *A variable structure controller for a tentacle manipulator*, Proc. IEEE Int. Conf. on Robotics and Aut., Nagoya, pp. 3155-3160, 1995.

[12] S.Chiaverini, B.Siciliano, L.Villani, *Force and Position Tracking: Parallel Control with Stiffness Adaptation*, IEEE Contr.Syst, Vol. 18, No 1, pp. 27-33.

[13] S.S.Ge, T.H.Lee, G.Zhu, *Energy-Based Robust Controller Design for Multi-Link Flexible Robots*, Mechatronics, No 7,Vol. 6, pp. 779-798, 1996

[14] M. Ivanescu, *Position dynamic control for a tentacle manipulator*, Proc. IEEE Int. Conf. on Rob. and Aut, Washington, A1-15, May 2002, pp. 1531-1539.