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Bearing-Only Mapping by Sequential Triangulation and Multi-dimensional Scaling

Takehisa Yairi and Hirofumi Kanazaki

Abstract— **In this paper, we introduce an alternative solution to the *Bearing-only Mapping* problem in which a mobile robot builds a map of features (landmarks) using only relative bearing measurements to them and odometry information. Our approach named BOM-STMDS (Bearing-Only Mapping by Sequential Triangulation and Multi-Dimensional Scaling) first tries to estimate *relative distances* among the features, then finds two-dimensional coordinates of all features by using multi-dimensional scaling (MDS) and its enhancements. BOM-STMDS is different from the conventional BOSLAM methods based on Bayesian filtering in that robot self-localization is not mandatory. Another remarkable property is that BOM-STMDS is able to utilize prior information about relative distances among features efficiently. In the experiment, the performance of BOM-STMDS is shown to be competitive with a conventional EKF-based BOSLAM method.**

I. Introduction

In recent years, a number of studies(e.g.[5], [4], [2]) have been made on Bearing-only SLAM (BOSLAM), which is a problem of estimating both positions of features (landmarks) and robot itself simultaneously, using only relative bearing measurements to the features and odometry information of the robot's motion. A conventional approach to BOSLAM can be described as follows. First, prepare motion and measurement models which contain robot's pose and positions of all features as the state variables, and relative bearing measurements to the features as the observation variables. Then, apply some Bayesian filtering method such as Extended Kalman Filter (EKF) to update sequentially the estimates of the state variables.

On the other hand, in this paper, we propose an alternative solution to the bearing-only mapping (and also BOSLAM), which is named Bearing-Only Mapping by Sequential Triangulation and Multi-Dimensional Scaling (BOM-STMDS). In BOM-STMDS, the robot first attempts to estimate the relative distances among the features sequentially using the bearing measurements and odometry information at each time step, then reconstructs the coordinates of the features by applying enhanced versions of multi-dimensional scaling (MDS) to the matrix of inter-feature distances. This two-step estimation procedure is significantly different from the ordinary SLAM methods which estimate the feature locations and robot pose directly from the measurements. The most remarkable feature of BOM-STMDS is that robot self-localization is not mandatory (but optional), whereas mapping and localization are inseparable in the conventional SLAM methods.

Authors are with Research Center for Advanced Science and Technology, University of Tokyo, 4-6-1 Komaba, Meguro-ku, Tokyo, Japan {yairi,kanazaki}@space.rcast.u-tokyo.ac.jp

The motivation for considering yet another bearing-only mapping method other than the traditional SLAM techniques mainly comes from our observation about the robot map building in practice that some relative distances among features are often known in advance and desire to make the best use of such prior information. For example, imagine a situation where a robot is about to build a map of a room containing several pieces of furniture whose sizes and shapes are known but their locations and orientations are not. If we applied the conventional SLAM method based on Bayesian filtering, we would have to prepare special motion and measurement models containing not only the center locations but also the orientations of furniture pieces as the state variables, or otherwise would have to impose geometric constraints on the locations of features (such as corners of a desk) and solve the constrained optimal estimation problem. Either of them requires a very complicated modification to the basic SLAM procedure. In contrast, BOM-STMDS can utilize such prior knowledge easily and consistently by transforming it into the information about relative distances among specific features.

The rest of this paper is organized as follows. In section II, we will briefly review the related work on mobile robot map building including BOSLAM. In section III, we will describe the principles of the proposed method, such as sequential triangulation, updates of inter-feature distance matrix, and reconstruction of feature coordinates by multi-dimensional scaling. In section IV, we will demonstrate how the proposed method actually works by a simulation study. Finally, in section V, we will conclude with a summary and future work.

II. Related Work

A. SLAM

In the last decade, SLAM (simultaneous localization and mapping) based on the probabilistic modeling (or state space models) and Bayesian filtering has been the mainstream of mobile robot map building research[12]. In the general framework of SLAM, motion and measurement models containing robot's pose and features' positions as state variables are prepared beforehand. Then, those state variables are sequentially estimated from obtained measurements by applying Bayesian inference techniques to the models. As is widely known, several approaches have been developed for solving this SLAM problem, including EKF[8], alternate estimation by EM algorithm [13], and Rao-Blackwellized particle filter[10].

Recently, Bearing-only SLAM (BOSLAM) has drawn much attention in this field, mainly due to the requirement

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of performing SLAM only with inexpensive sensors such as monocular cameras(e.g.[5], [4], [2]). Although BOSLAM poses several challenges such as landmark (feature) initialization problem because of the insufficient information of single bearing measurement, it is still based on the ordinary SLAM framework above. Actually, Bekris *et.al.*[2] have compared a variety of existing SLAM techniques in BOSLAM setting, and reported that Rao-Blackwelized particle filters are faster and more robust than others.

B. Embedding Approach

The SLAM framework, as seen above, inherently focuses on the spatial relationships between the robot's poses and features' locations. In contrast, there are other approaches to the map building problem focusing on the spatial relationships among the features themselves, rather than the robot-features relationships. In other words, they attempt to construct a global map by merging and embedding pieces of information of local spatial relationships among features which are obtained by observations. Intuitively, this process is similar to putting puzzle pieces together into a picture. For example, *local map alignment* by Lu and Millios[9] and *relaxation* by Duckett *et.al.*[7] are representative map building techniques based on this idea.

On the other hand, the authors have proposed methods of building feature-based maps by applying multi-dimensional scaling (MDS) and related techniques to the inter-feature distance matrices which are estimated from covisibility[16] and similarity of observation history[15]. While these methods also focus on the relationships among features rather than robot-features relationships, they are more distinct from SLAM in that they do not require robot localization.

The proposed method in this paper can be regarded as a special case of the authors' previous studies in which the inter-feature distance matrix is estimated from bearing measurements to the features and robot's odometry information.

III. Proposed Method - BOM-STMDS

In this section, we describe the proposed method named Bearing-only mapping by sequential triangulation and multi-dimensional scaling (BOM-STMDS).

A. Problem Definition and Outline

We hereafter use the following notation.

- F_i, $\boldsymbol{x}_{F_i}$: i-th feature (landmark), and its 2-dimensional coordinates
- R_t, $\boldsymbol{x}_{R_t}$: Robot's position at time t, and its 2-dimensional coordinates
- d_{F_i,F_j} : Distance between F_i and F_j
- $\theta_{i,t}$: Relative bearing measurement to F_i at time t
- l_t : Measurement of distance the robot moves from time $t-1$ to t
- ϕ_t : Measurement of change in heading direction from time $t-1$ to t
- $\boldsymbol{y}_t \equiv [l_t, \phi_t, \theta_{1,t}, \ldots, \theta_{N,t}]^T$: Measurement vector at t.

Figure 1 illustrates an "ideal" relationship among them, assuming there are no measurement errors in l_t, ϕ_t, $\theta_{i,t-1}$, $\theta_{j,t-1}$, $\theta_{i,t}$, and $\theta_{j,t}$. Note that these measurements are subject to errors and sometimes missing in the real environment.

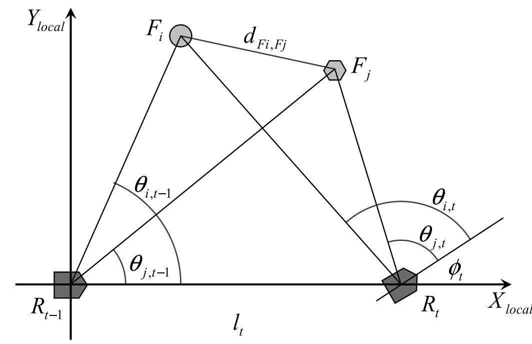


Fig. 1. Ideal relationships among measurements and positions of features and robot

Bearing-only mapping (BOM) is defined as a problem of finding 2-D coordinates of all features $\{\boldsymbol{x}_{F_i}\}_{i=1,\ldots,N}$, for given measurements $\{\boldsymbol{y}_t\}_{t=1,\ldots,T}$ up to time T. In BOSLAM, not only features' positions, but also robot's position at each time step $\{\boldsymbol{x}_{R_t}\}_{t=1,\ldots,T}$ is required to be estimated.

The process of map building by BOM-STMDS is summarized as follows:

1) First, in *Sequential Triangulation* step (III-B), for each pair of features F_i, F_j, an instantaneous estimate of squared distance $\hat{d}^2_{F_i,F_j|t}$ with its variances $\hat{\sigma}^2_{F_i,F_j|t}$ is computed from two successive bearing measurements to the features $\{\theta_{i,t-1}, \theta_{i,t}, \theta_{j,t-1}, \theta_{j,t}\}$, and odometry readings $\{l_t, \phi_t\}$.
2) Next, in *Distance Update* step (III-C), for each pair of features, $\hat{d}^2_{F_i,F_j|t}$ is merged with $\hat{d}^2_{F_i,F_j|1:t-1}$ (filtered estimate up to the previous time step), and a new estimate $\hat{d}^2_{F_i,F_j|1:t}$ is obtained.
3) Finally, in *Coordinates Reconstruction* step (III-D), estimates of feature positions $\{\hat{\boldsymbol{x}}_{F_i|1:t}\}_{i=1,\ldots,N}$ are found by applying multi-dimensional scaling (MDS) to the set of estimated inter-feature squared distances $\{\hat{d}^2_{F_i,F_j|1:t}\}_{i,j=1,\ldots,N}$.

Figure 2 and Table 1 illustrate the outline of BOM-STMDS. In the rest of this section, we will explain these processes in detail.

B. Sequential Triangulation

In Figure 1, consider a local coordinate system whose origin is located at R_{t-1} and x-axis is aligned with the vector from R_{t-1} to R_t. When we assume there are no measurement errors, the location of a feature F_i in this local coordinate system can be represented in terms of two successive bearing measurements $\theta_{i,t-1}$, $\theta_{i,t}$ and odometry readings l_t, ϕ_t as,

$$\boldsymbol{x}^{R_t}_{F_i} = \frac{\sin(\phi_t + \theta_{i,t}) \cdot l_t}{\sin(\phi_t + \theta_{i,t} - \theta_{i,t-1})} \; [\cos\theta_{i,t-1}, \sin\theta_{i,t-1}]^T \quad (1)$$

This is the well-known principle of triangulation. In this paper, we call it *sequential triangulation* because it is performed every time the robot moves to a new position. Using

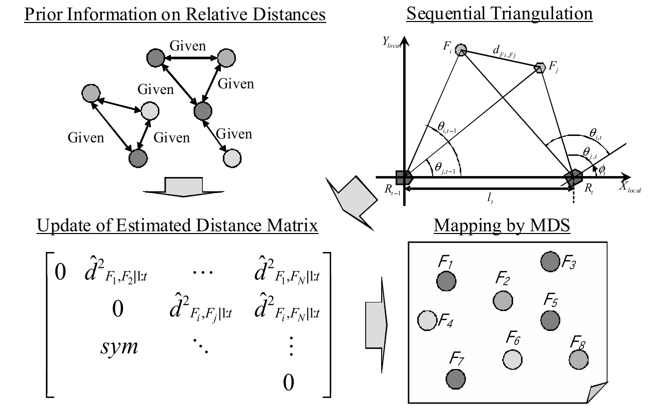


Fig. 2. Sketch of BOM-STMDS

Algorithm 1 Outline of BOM-STMDS

Initialize map and distance matrix
for $t = 1$ **to** T **do**
  Move to next position and obtain odometry information and bearing measurements
  for $i, j =$ every pair of visible features **do**
    Perform sequential triangulation (**III-B**)
    Update estimates of inter-feature distances (**III-C**)
  end for
  Estimate robot-features distances (opt., **III-E**)
  Approximate missing distances if necessary (**III-D.1**)
  Perform classical scaling (**III-D**) or SMACOF (**III-D.2**) to estimate the map (and trajectory)
end for

this equation, distance between two arbitrary features d_{F_i,F_j} can be easily computed. As actual bearing measurements and odometry readings are subject to measurement errors, we denote the instantaneous estimate of the squared distance at time t by $\hat{d^2}_{F_i,F_j|t}$.

Next we consider the estimated variance of $\hat{d^2}_{F_i,F_j|t}$ which is denoted by $\hat{\sigma}^2_{F_i,F_j|t}$. If the covariance matrix of the partial measurement vector $[l_t, \phi_t, \theta_{i,t-1}, \theta_{i,t}, \theta_{j,t-1}, \theta_{j,t}]$ is given as $\Sigma_{i,j,t}$, $\hat{\sigma}^2_{F_i,F_j|t}$ can be approximated to the first order using the Jacobian $J_{i,j,t} \equiv [\frac{\partial(\hat{d^2}_{F_i,F_j|t})}{\partial l_t}, \frac{\partial(\hat{d^2}_{F_i,F_j|t})}{\partial \phi_t}, \ldots]$ as,

$$\hat{\sigma}^2_{F_i,F_j|t} = J_{i,j,t} \Sigma_{i,j,t} J^T_{i,j,t} \tag{2}$$

Intuitively, the variance $\hat{\sigma}^2_{F_i,F_j|t}$ implies the magnitude of estimation error in $\hat{d^2}_{F_i,F_j|t}$.

It should be noted that an estimated (squared) distance between features is independent of the local coordinate system or trajectory of the robot. Thanks to this property, the update process of the distances becomes easy as explained later.

C. Update of Distance Estimates

We assume that the estimation error of a squared distance between features at each time step $e_{i,j|t} = \hat{d^2}_{F_i,F_j|t} - d^2_{F_i,F_j}$ ($t = 1, \ldots, T$) is independent of each other. We denote the estimates of squared distance of a pair of features and its variance estimated taking all measurements up to time t by $\hat{d}^2_{F_i,F_j|1:t}$ and $\hat{\sigma}^2_{F_i,F_j|1:t}$, respectively.

Now consider an instantaneous distance estimate $\hat{d^2}_{F_i,F_j|t+1}$ and its variance $\hat{\sigma}^2_{F_i,F_j|t+1}$ are obtained at the next time step $t+1$ by the sequential triangulation technique described above. Then a reasonable update rule of $\hat{d}^2_{F_i,F_j|1:t+1}$ and $\hat{\sigma}^2_{F_i,F_j|1:t+1}$ are given as below:

$$\hat{\sigma}^2_{F_i,F_j|1:t+1} = (\hat{\sigma}^{-2}_{F_i,F_j|1:t} + \hat{\sigma}^{-2}_{F_i,F_j|t+1})^{-1} \tag{3}$$

$$\begin{aligned} \hat{d^2}_{F_i,F_j|1:t+1} &= \hat{\sigma}^2_{F_i,F_j|1:t+1} \cdot (\hat{\sigma}^{-2}_{F_i,F_j|1:t} \cdot \hat{d^2}_{F_i,F_j|1:t} \\ &+ \hat{\sigma}^{-2}_{F_i,F_j|t+1} \cdot \hat{d^2}_{F_i,F_j|t+1}) \end{aligned} \tag{4}$$

This update rule is optimal in the sense that it minimizes the estimated variance $\hat{\sigma}^2_{F_i,F_j|1:t+1}$.

These equations mean that $\hat{d^2}_{F_i,F_j|1:t+1}$ is the weighted average of $\hat{d^2}_{F_i,F_j|1:t}$ and $\hat{d^2}_{F_i,F_j|t+1}$, where the weights are inverses of corresponding variances. Interestingly, this update rule can be viewed as a reduced form of Kalman filtering for the estimation of the squared distance between two features.

Moreover, when prior information on the relative distances between a specific pair of features is given, it can be utilized directly by setting the initial estimates $\hat{d^2}_{F_i,F_j|0}$ and $\hat{\sigma}^2_{F_i,F_j|0}$ properly. In a later experiment, it is shown that use of such prior knowledge contributes to a significant improvement in the estimation accuracy.

Now we define the *estimated squared distance matrix* (ESDM) $\boldsymbol{\Delta}_{F|1:t}$ whose (i,j) element is given by $\hat{d^2}_{F_i,F_j|1:t}$. That is to say,

$$\boldsymbol{\Delta}_{F|1:t} \equiv \begin{bmatrix} 0 & \hat{d^2}_{F_1,F_2|1:t} & \cdots & \hat{d^2}_{F_1,F_N|1:t} \\ \hat{d^2}_{F_1,F_2|1:t} & 0 & \cdots & \hat{d^2}_{F_2,F_N|1:t} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{d^2}_{F_1,F_N|1:t} & \hat{d^2}_{F_2,F_N|1:t} & \cdots & 0 \end{bmatrix} \tag{5}$$

D. Coordinates Reconstruction by Multi-dimensional Scaling

The problem of finding coordinates of "items" in a low dimensional space given a matrix of dissimilarities between them is known as *multi-dimensional scaling* (MDS), and a variety of techniques to solve this problem have been developed[3]. In our case, if the squared distances between all pairs of features are given, in other words, if $\boldsymbol{\Delta}_{F|1:t}$ has neither missing elements nor outliers, *classical scaling* which is the most famous and fundamental MDS technique can be used. Given a dissimilarity matrix (squared distance matrix) $\boldsymbol{D}$, classical scaling finds a coordinate matrix of items $\boldsymbol{X}$ by performing an eigendecomposition of the inner product matrix $\boldsymbol{B}$, which is computed by double centering $\boldsymbol{D}$[3]. An advantage of using classical scaling is that its solution is guaranteed to globally minimize a loss function called *Strain*.

A drawback of classical scaling, on the other hand, is that it is not able to deal with missing elements in the dissimilarity matrix directly. Obviously, this is a serious problem for us, because in the real environment it is very likely that some part of the relative distances between the features $\{\hat{d}^2_{F_i,F_j|t}\}$ are not obtained directly by the sequential triangulation due to various constraints. For example, the distance between two features which are very far from each other or occluded by an obstacle is likely to be missing or too inaccurate.

To overcome this limitation of classical scaling, we propose two approaches – (1) completion of missing or inaccurate elements in the distance matrix by shortest path lengths, and (2) use of another MDS technique called SMACOF. We will explain them in detail.

1) Distance Matrix Correction by Shortest Path Lengths: The idea of the first approach is to apply the ordinary classical scaling to a "repaired" distance matrix $\boldsymbol{\Delta}'_{F|1:t}$ whose missing and deteriorated elements are completed by approximated values using available inter-feature distance information.

More specifically, the approximation of missing distances is performed as follows. First, we consider an undirected graph whose vertices represent the features. In this graph, an edge between two features F_i and F_j is present if the squared distance between them $d^2_{F_i,F_j}$ is properly estimated. Then, we approximate the missing distance between each pair of features by the shortest path length from one to the other on that graph. The shortest paths among features on the graph can be found efficiently by Floyd-Warshall algorithm. We call this technique Distance Matrix Correction by Shortest Path Length (DMC-SPL) in this paper.

DMC-SPL is basically the same technique introduced in ISOMAP[11] which is a non-linear dimensionality reduction method, while the purpose of the latter is not to estimate missing distances, but to approximate geodesic distances on a non-linear manifold in a high-dimensional data space.

Using DMC-SPL with BOM-STMDS has an advantage that the ordinary classical scaling is applicable without modification, once the estimated squared distance matrix is corrected, which means the global optimum solution is always guaranteed. A major drawback of DMC-SPL is that it is an off-line algorithm. Basically, all shortest paths and their lengths need to be recomputed every time a new estimate of distance between a pair of features is obtained by the sequential triangulation.

2) SMACOF with Weighted Distance Matrix: Another reasonable way to deal with the missing elements in the estimated squared distance matrix is to *ignore* the missing distances and try to find a set of feature coordinates that satisfy only available inter-feature distances well. This idea can be implemented straightforwardly by defining an appropriate loss function with weight coefficients $\{w_{i,j}\}$ where $w_{i,j}$ is set to 1 if the distance between F_i and F_j is known and reliable or 0 otherwise. The loss function will be minimized in terms of the feature coordinates by means of some optimization techniques such as gradient methods.

In this study, we chose SMACOF algorithm[6] among a number of MDS methods using loss functions of this type, because of its performance and efficiency. In SMACOF, the loss function named *raw stress* as below is locally minimized by iterative *majorization* technique.

$$L_{sma}(\boldsymbol{X}) = \sum_{i<j} w_{i,j}(\hat{d}_{F_i,F_j|1:t} - \|\hat{\boldsymbol{x}}_{F_i} - \hat{\boldsymbol{x}}_{F_j}\|)^2 \quad (6)$$

Compared with DMC-SPL (together with classical scaling), it is easy to modify SMACOF into an on-line algorithm as it is inherently an iterative process. A critical issue for SMACOF, however, is whether a "good" initial solution is given or not, because the optimization process is guaranteed to find only a local minimum.

In the later experiment, we applied SMACOF to the estimated distance matrix with an initial solution which is computed by the classical scaling together with DMC-SPL.

E. Robot Localization

As we have seen so far, BOM-STMDS does not necessarily require the self-localization of the robot. While this is the most remarkable characteristic of BOM-STMDS compared with the conventional BOSLAM methods based on Bayesian filtering, it is possible to enhance our method to estimate the robot positions as well in a natural way.

This modification is achieved by expanding the estimated squared distance matrix of Equation 5 so that it contains the distances among the features and the robot positions in the last p-steps. Specifically, the extended estimated squared distance matrix $\boldsymbol{\Delta}_{E|1:t}$ is defined as,

$$\boldsymbol{\Delta}_{E|1:t} \equiv \begin{bmatrix} \boldsymbol{\Delta}_{F|1:t} & \boldsymbol{\Delta}_{FR|1:t} \\ \boldsymbol{\Delta}_{FR|1:t}{}^T & \boldsymbol{\Delta}_{R|1:t} \end{bmatrix} \quad (7)$$

where, $\boldsymbol{\Delta}_{R|1:t}$ denotes the squared distance matrix among the last p positions the robot have visited which is estimated from the odometry readings, and $\boldsymbol{\Delta}_{FR|1:t}$ denotes the squared distance matrix between the last p robot positions and N features.

Although $\boldsymbol{\Delta}_{FR|1:t}$ is likely to have a number of missing elements, the two techniques mentioned previously are applicable without modification.

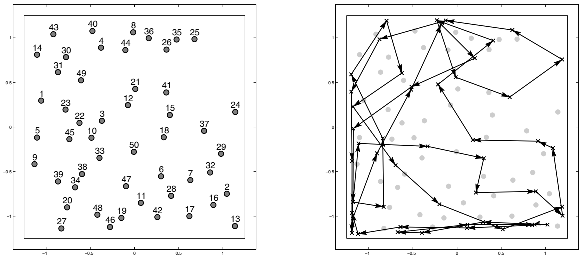
IV. Experiment

We conducted several experiments to evaluate the proposed method - BOM-STMDS in a simulated environment.

A. Experimental Setup and Evaluation Procedure

The simulated environment is a square region whose side length is 2.5[m] containing $N = 50$ randomly placed features. At each time step, the robot moves to the next position according to the randomly chosen l_t and ϕ_t, then obtains a set of relative bearing measurements to recognizable features $\{\theta_{i,t}\}$.

In this experiment, we simulated the observation uncertainty by adding Gaussian noises to the ideal measurements of l_t,ϕ_t,$\theta_{i,t}$. The standard deviations of the noises are $\sigma_l =$ 0.03[m], $\sigma_\phi = 3$[deg], and $\sigma_\theta = 3$[deg], respectively. In addition, we assumed that the sensor range is limited and each feature is recognizable only if it is within the distance of 1[m] from the robot position. This means that



(a) An example of ground truth map of features (b) An example of robot's trajectory (50 steps)

Fig. 3. An example of environment and trajectory

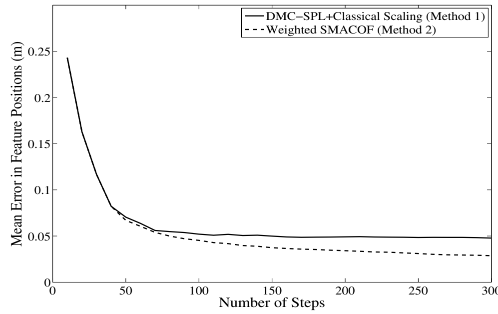


Fig. 4. (Exp.1) Mean errors of estimated feature positions

the estimated squared distance matrix $\mathbf{\Delta}_{F,1:t}$ necessarily contains a number of missing elements.

Figure 3 shows examples of ground truth map of features (left) and robot trajectory (right). We conducted 25 runs by randomly setting the initial robot position 5 times for 5 different layout patterns. Each run consists of 300 steps. Note that the map was update every 10 steps, whereas the distance matrix was updated every step.

Evaluation of an estimated map and trajectory is a little awkward, because the coordinate systems for the ground truth map and estimated map are not necessarily the same. Therefore, we evaluated the accuracy of estimated positions of the features and robots after applying a coordinate transformation of translation, rotation and reflection to the initially obtained map so that it minimizes the sum of squared positional errors of all features.

B. Experiment 1 : Mapping without Localization

The purpose of the first experiment is to examine the performance of BOM-STMDS in its basic form without self-localization. In this experiment, we compared two methods of reconstructing the feature coordinates from the estimated squared distance matrix with missing elements :

[Method 1] : Classical scaling with DMC-SPL (III-D.1) from the beginning to the end.

[Method 2] : Same as [Method 1] before 40 steps, then SMACOF with weights (III-D.2) after 50 steps.

$t = 50$ was chosen for the timing for switching in [Method 2] because all features were observed at least once by that time in all 25 runs.

Figure 4 shows how *mean position errors* (MPEs) change along with the time. MPEs at $t = 300$ in the two cases are 4.78[cm] (Method 1) and 2.87[cm] (Method 2), respectively. It is considered that this difference in accuracy is caused mainly by the difference in the loss functions of classical scaling and SMACOF (See [14] for detail).

C. Experiment 2 : Simultaneous Localization and Mapping

Next, we examined the case of estimating not only feature positions but also robot positions by the proposed methods with the enhancement described in III-E. Specifically, robot positions in the last 6 steps are included in the extended distance matrix $\mathbf{\Delta}_{E,1:t}$. Figure 5 illustrates an example of estimated map and trajectory by the proposed method (Method 2, $t = 100$). We can see both are estimated very accurately.

Moreover, we compared the proposed methods with a conventional BOSLAM method based on Extended Kalman Filter (BOSLAM-EKF). As mentioned earlier, the landmark initialization is a critical issue for BOSLAM-EKF. In this experiment, we employed an initialization method known as *constrained initialization*[1].

Figure 6 shows the estimation errors of *feature* positions in the three cases (i.e., [Method 1], [Method 2], and BOSLAM-EKF). We can see that the BOM-STMDS (especially Method 2) is competitive with BOSLAM-EKF, although the latter outperforms the former in an early stage ($t < 100$).

Figure 7, on the other hand, shows the estimation errors of *self-localization* in the cases. Though BOSLAM-EKF outperforms the proposed methods, we argue that they are achieving reasonable accuracy because the mean estimation errors are bound within 10[cm] (Method 1) and 6[cm] (Method 2), respectively. We consider a main reason why BOM-STMDS was inferior to BOSLAM-EKF in the self-localization accuracy is that BOM-STMDS lacks in a trade-off mechanism between two sources of information – bearing measurements and odometry readings, whereas EKF has that mechanism. For example, BOM-STMDS tends to fail in estimating the robot positions accurately when only a small number of features are observable and the bearing measurements to them contain relatively large errors by chance.

D. Experiment 3 : Utilization of Prior Information

Finally, we examined how the prior information on the relative distances between some pairs of features contributes to the estimation accuracy in BOM-STMDS.

In this experiment, 10% of all the feature pairs (that is $0.1 \times (\frac{50\times 49}{2}) = 123$ pairs) were randomly chosen and their distances were given with the accuracy of $\sigma = 3$[cm] in both x and y axes as prior information in advance. This prior information was used to determine initial distance matrix $\mathbf{\Delta}_{F,0}$.

Figure 8 compares the case where this prior information was incorporated with the case where it was not. The result

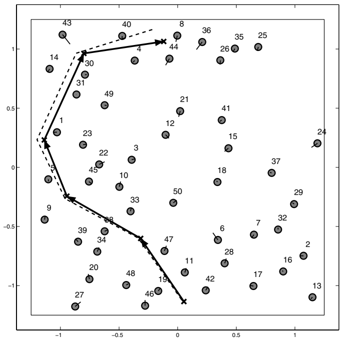


Fig. 5. (Exp.2) Example of estimated map and trajectory ($t = 100$, $p = 6$), Solid and broken lines represent *estimated* and *true* trajectories. Also, the differences between *estimated* and *true* feature positions are emphasized with thin lines.

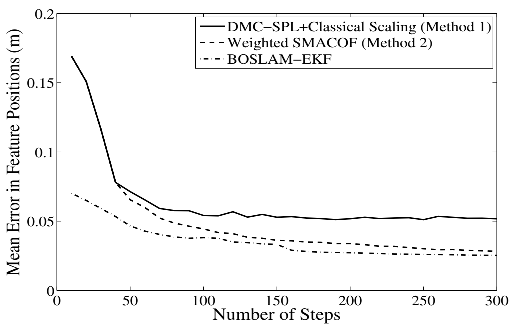


Fig. 6. (Exp.2) Estimation errors in feature positions

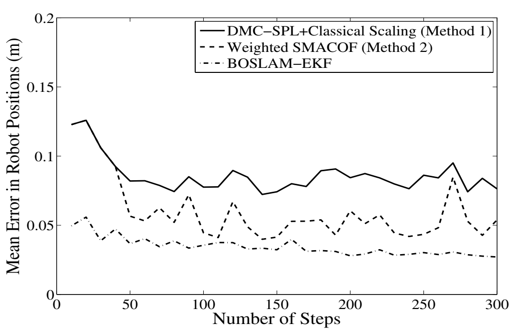


Fig. 7. (Exp.2) Estimation errors in robot positions

demonstrated that using this kind of prior knowledge leads to a significant improvement in the estimation accuracy. Especially, we can see it has the effect of accelerating the mapping in the early stage.

V. CONCLUSION

In this paper, we proposed an alternative approach to the bearing-only mapping problem. The key ideas are to estimate the inter-feature distance matrix using the principle of sequential triangulation, and to use multi-dimensional scaling for reconstructing the coordinates of the features from the distance matrix. While it does not outperform the conventional BOSLAM methods, it has unique properties

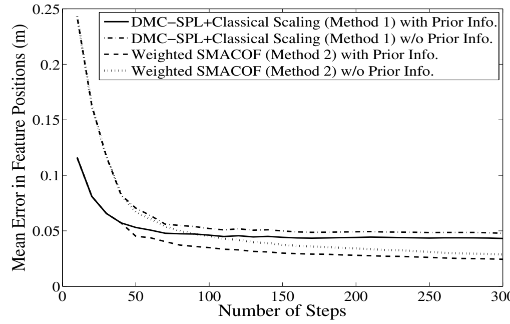


Fig. 8. (Exp.3) Mean errors in estimated feature positions with or without prior information

that prior information of inter-feature distances is efficiently utilized and that the robot self-localization is not necessary.

There are still many interesting issues related to this method, such as data association problem, extension to 3D mapping, and hybridization with the conventional BOSLAM methods.

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