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Continuous Control Law from Unilateral Constraints

Application to Reactive Obstacle Avoidance in Operational Space

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***Abstract*— The control approaches based on tasks, and particularly based on a hierarchy of tasks, enable to build complex behaviors with some nice properties of robustness and portability. However it is difficult to consider directly unilateral constraints in such a framework. Unilateral constraints presents some strong irregularities (in particular at the level of their derivative) that prevents the insertion of unilateral-based tasks at the high-priority level of a hierarchy. In this paper, we present an original method to generalize the hierarchy-based control schemes to take unilateral constraint into account at the top-priority level. We develop our method first at the kinematic level then directly at the dynamic level using the operational space. The method is then validated on a various set of robots by realizing a visual servoing under the constraint of joint limits.**

I. INTRODUCTION

Traditionally, the classical approach to control a robot has been to define the objective directly in the joint space. The control approaches based on the definition of a task objective [14], [8] have been introduced to simplify the control problem. Working directly in a properly-chosen task space produces a more intuitive manner to define the robot objective. It also enables to work directly in the sensor space, which closes more tightly the control loop [4]. Finally, since a same task space is valid for a large set of robot, the control scheme could be easily adapted from one robot structure to another. Moreover, these methods produce directly the kinematic or dynamic model to decouple the motions due to the task to the free remaining motions [9]. A secondary task can then be applied in the space of free motions, and, recursively, a hierarchic set of tasks can be considered [16]. Hierarchy of tasks are more and more popular to build complex behavior on very redundant robot such as humanoid robots [1], [12], [15], [11].

A task is generally defined by an equality of reference such as $\mathbf{e} = 0$ where $\mathbf{e} = \mathbf{s} - \mathbf{s}^*(t)$ is an error to be regulated to 0. The task thus represent a *bilateral constraint*. On the opposite, unilateral constraint are typically represented by an inequality $e_i \leq 0$. Unilateral constraints present an irregularity at the activation point $e_i = 0$. Due to this irregularity, it is impossible to consider an unilateral constraint as a classical (bilateral) task. A first solution is the gradient projection method [9]: the unilateral constraint is embedded in a cost function [7] whose gradient is projected in the space of free motion as a lowest-priority task. However the irregularity still exist and prevents the insertion of the unilateral constraint at any level of the task hierarchy but the lowest-priority one. Indeed, the unilateral constraints have always been considered as a secondary objective to be optimized only when enough degree of freedom (DOF) are available.

We propose here to study a solution to introduce the unilateral constraints at the top-priority level of the hierarchy. Some work have already been proposed to go in this direction. Our work is based on a specific inverse operator introduced in [10] to smooth the irregularity of the unilateral constraint while computing the control law. Using such an operator, we can generalize the notion of hierarchy of task for both bilateral and unilateral constraints at the kinematic (Section III) and at the dynamic levels (Section IV). The tasks used in the experiments are then given in Section V. A set of experiments that validate our approach are discussed in Section VI.

II. INVERSE KINEMATIC CONTROL

We first remind the classical task-based control, valid only for bilateral tasks and show the evidence of discontinuities when considering unilateral constraints in this formulation.

A. Considering only one task

Let $\mathbf{q}$ be the joint position of the robot. The main task is $\mathbf{e}$. The robot is controlled using the joint velocities $\dot{\mathbf{q}}$ (to improve the readability, we will generalize to torque control in Section IV). The Jacobian of the task $\mathbf{e}$ is $\mathbf{J}$ defined by $\dot{\mathbf{e}} = \frac{\partial \mathbf{e}}{\partial \mathbf{q}}\dot{\mathbf{q}} = \mathbf{J}\dot{\mathbf{q}}$. Let n be the number of DOF of the robot ($n = \dim \mathbf{q}$) and m be the size of the task ($m = \dim \mathbf{e}$).

The controller has to regulate $\mathbf{e}$ to $\mathbf{0}$ according to a reference decreasing behavior $\dot{\mathbf{e}}^*$. The joint motion $\dot{\mathbf{q}}$ that realizes $\dot{\mathbf{e}}^*$ is given by the least-square inverse:

$$\dot{\mathbf{q}} = \mathbf{J}^+\dot{\mathbf{e}}^* \tag{1}$$

where $\mathbf{A}^+$ is the least-square inverse of $\mathbf{A}$. By (1), we consider that the Jacobian matrix is perfectly known. If it is not the case, an approximation $\widehat{\mathbf{J}^+}$ has to be used instead of $\mathbf{J}^+$. The control law is stable if $\mathbf{J}\widehat{\mathbf{J}^+} \geq 0$ and asymptotically stable *iff* $\mathbf{J}\widehat{\mathbf{J}^+} > 0$ [14]. In the following, we make the assumption that $\mathbf{J}$ is perfectly known: the control law is thus always stable, and asymptotically stable if $m \leq n$.

In the state of the art, an implicit condition of such control schemes is always the constant rankness of the Jacobian $\mathbf{J}$. Indeed, the pseudo-inverse $\mathbf{J}^+$ is continuous with respect to $\mathbf{J}$ only when the rank is constant. When the rank increases or decreases, the continuity is not ensured, which can result

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in an awkward or even dangerous behavior. This is typically what happens when the robot reaches a singularity.

B. Evidence of discontinuity

An unilateral constraint can be written $\mathbf{e} < 0$. In that case, the reference behavior $\dot{\mathbf{e}}^*$ is typically set to:

$$\dot{\mathbf{e}}^* = \begin{cases} -\lambda\mathbf{e} & \text{if } \mathbf{e} > 0 \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

Some control laws based on (1) have been proposed [5], [2], [3]. They can be put under the common following form [10]:

$$\dot{\mathbf{q}} = \left(\mathbf{HJ}\right)^+\mathbf{H}\dot{\mathbf{e}}^* \quad (3)$$

where $\mathbf{H} = \text{diag}(h_1, \dots, h_m)$, and $h_i = \begin{cases} 1 & \text{if } e_i > 0 \\ 0 & \text{otherwise} \end{cases}$. The continuity of the control law is then obtained by ensuring that the number of activated feature (*i.e.* the number of non-zero h_i) is large enough to ensure that $\mathbf{HJ}$ has a constant rank. However, such a hypothesis is not admissible in the general case. We can easily check [10] that a large discontinuity of the control law arises each time the rank of $\mathbf{HJ}$ changes (corresponding typically to what happens when the robot comes to a singularity - see Fig. 2 in the experiments). In this article, we will focus on a general solution to solve this discontinuity and ensure a proper behavior of the robot whatever the variation of $\mathbf{H}$.

C. Extension to k tasks

The solution (1) is only one particular solution ensuring $\dot{\mathbf{e}}^*$: it is the solution of least norm. If the rank of $\mathbf{J}$ is smaller than n, a second criterion can be taken into account using the redundancy formalism [14]:

$$\dot{\mathbf{q}} = \mathbf{J}^+\dot{\mathbf{e}}^* + \mathbf{P}\mathbf{z}_{\dot{\mathbf{q}}_2} \quad (4)$$

where $\mathbf{P}$ is the projection operator onto the null space of $\mathbf{J}$ ($\mathbf{P} = \mathbf{I_n} - \mathbf{J}^+\mathbf{J}$), and $\mathbf{z}_{\dot{\mathbf{q}}_2}$ is an arbitrary vector, used to apply a secondary control law. Thanks to $\mathbf{P}$, $\mathbf{z}_{\dot{\mathbf{q}}_2}$ is performed without disturbing $\dot{\mathbf{e}}^*$. Consider now two tasks $\mathbf{e_1}$, $\mathbf{e_2}$. The control law performing $\dot{\mathbf{e_1}}^*$ and if possible $\dot{\mathbf{e_2}}^*$ is [16]:

$$\dot{\mathbf{q}} = \mathbf{J_1}\dot{\mathbf{e_1}}^* + \left(\mathbf{J_2}\mathbf{P_1}\right)^+\left(\dot{\mathbf{e_2}}^* - \mathbf{J_2}\mathbf{J_1}\dot{\mathbf{e_1}}^*\right) \quad (5)$$

This equation can be generalized to perform k tasks, while ensuring a proper hierarchy between them [16]:

$$\dot{\mathbf{q}}_\mathbf{i} = \dot{\mathbf{q}}_\mathbf{i-1} + (\mathbf{J_i}\mathbf{P}^\mathbf{A}_\mathbf{i-1})^+(\dot{\mathbf{e}}_\mathbf{i}^* - \mathbf{J_i}\dot{\mathbf{q}}_\mathbf{i-1}), \quad i = 1..k \quad (6)$$

where $\dot{\mathbf{q}}_\mathbf{0} = 0$ and $\mathbf{P}^\mathbf{A}_\mathbf{i}$ is the projector of $\mathbf{J}^\mathbf{A}_\mathbf{i} = (\mathbf{J_1}, \dots \mathbf{J_i})$. The robot joint velocity realizing all the tasks is $\dot{\mathbf{q}} = \dot{\mathbf{q}}_\mathbf{k}$.

In the following section, we will consider an unilateral-constraint task $\mathbf{e_i}$, and compute a control law that ensure the continuity both at level i and at all the upper levels $i+1 \dots n$.

III. Continuity at the kinematic level

We propose here a new control law whose form is similar to (6) and ensures the continuity for any kind of constraints.

A. Considering only one task

Consider a task function $\mathbf{e}$ (dimension m), its Jacobian $\mathbf{J}$ (dimension $n \times m$ and constant rank r) and its activation matrix $\mathbf{H}$ (diagonal matrix $m \times m$ whose diagonal components $(h_i)_{i=1\dots m}$ are in the interval $[0, 1]$). The continuous inverse of $\mathbf{J}$ activated by $\mathbf{H}$ is defined by [10]:

$$\mathbf{J}^{\mathbf{H}\oplus} = \sum_{\mathcal{P}\in\mathfrak{P}(m)} \Big(\prod_{i\in\mathcal{P}} h_i\Big)\, \mathbf{X}_\mathcal{P} \quad (7)$$

where $\mathfrak{P}(m) = \mathfrak{P}([1..m]) = \Big\{\mathcal{P} | \quad \mathcal{P} \subset [1..m]\Big\}$ are all the subsets composed of the m first integers, and the $\mathbf{X}_\mathcal{P}$ are the coupling matrices of $\mathbf{J}$ defined by:

$$\forall \mathcal{P} \in \mathfrak{P}(m), \mathbf{X}_\mathcal{P} = \mathbf{J}^+_\mathcal{P} - \sum_{\mathcal{Q}\subsetneq\mathcal{P}} \mathbf{X}_\mathcal{Q} \quad (8)$$

where $\mathbf{X}_\varnothing = \mathbf{0}_{n\times m}$ and $\mathbf{J}_\mathcal{P} = \mathbf{H_0}\mathbf{J}$ with $\mathbf{H_0}$ a diagonal matrix whose diagonal components h_i are equal to 1 if $i \in \mathcal{P}$, and to 0 otherwise. This inverse is proved to be continuous with respect to the variation of the activation matrix $\mathbf{H}$ and to be equal to $\left(\mathbf{HJ}\right)^+\mathbf{H}$ when the component of $\mathbf{H}$ are binaries ($h_i = 0$ or $h_i = 1$). This means that the resulting control law is continuous and keeps the same behavior and all the properties of local convergence of the corresponding classical control law. Finally, using this inverse, the control law that applies the task $\mathbf{e}$ upon activation $\mathbf{H}$ is the following [10]:

$$\dot{\mathbf{q}} = \mathbf{J}^{\mathbf{H}\oplus}\dot{\mathbf{e}}^* \quad (9)$$

Using (9) we can verify that each component e_i of $\dot{\mathbf{e}}^*$ is perfectly realized if the corresponding h_i is equal to 1, not taken into account if h_i is zero, partially realized otherwise.

B. Extension to two tasks

The general solution corresponding to (9) can be written:

$$\dot{\mathbf{q}} = \dot{\mathbf{q}}_1 + \mathbf{P}_\oplus\mathbf{z}_{\dot{\mathbf{q}}_2} \quad (10)$$

where $\dot{\mathbf{q}}_1 = \mathbf{J}^{\mathbf{H}\oplus}\dot{\mathbf{e}}^*$ and $\mathbf{P}_\oplus = \mathbf{I} - \mathbf{J}^{\mathbf{H}\oplus}\mathbf{J}$. Consider now two tasks $\left(\mathbf{e_1}, \mathbf{H}\right)$ and $\mathbf{e_2}$. We obtain $\dot{\mathbf{e_2}} = \dot{\mathbf{q}}_1 + \mathbf{J_2}\mathbf{P}_\oplus\mathbf{z}_{\dot{\mathbf{q}}_2}$. We now search the optimal $\mathbf{z}_{\dot{\mathbf{q}}_2}$ that performs $\mathbf{e_2}$. Inverting directly this last equation by analogy to (5) would result in:

$$\dot{\mathbf{q}} = \dot{\mathbf{q}}_1 + \mathbf{P}_\oplus\left(\mathbf{J_2}\mathbf{P}_\oplus\right)^+\left(\dot{\mathbf{e_2}}^* - \mathbf{J_2}\dot{\mathbf{q}}_1\right) \quad (11)$$

However, since $\mathbf{P}_\oplus$ does not have a constant rank, this leads to the same discontinuities than the classical control laws. Instead, we recognize a form similar to (3), with $\mathbf{P}_\oplus$ at the place of the activation matrix. The continuous inverse operator can then be used to solve the discontinuities. However, it is not possible to apply directly this operator since $\mathbf{P}_\oplus$ is not diagonal. We will thus first generalize the inverse operator for such a matrix.

C. Continuous inverse activated by a non diagonal matrix

Let $\mathbf{Q}$ be any matrix $m \times n$, $n \leq m$ and $\mathbf{W}$ be a positive symmetric matrix of size m whose singular value are all between 0 and 1. We note $\mathbf{U}, \sigma$ the eigen-value decomposition of $\mathbf{W}$ and $\mathbf{S} = \text{diag}(\sigma)$. We can write:

$$\left(\mathbf{WQ}\right)^+\mathbf{W} = \left(\mathbf{SQ_u}\right)^+\mathbf{SU} \quad (12)$$

where $\mathbf{Q_u} = \mathbf{U}^\top\mathbf{Q}$. We now recognize the previous form (3), with $\mathbf{Q}$ the matrix to be inverted, and $\mathbf{S}$ a proper diagonal activation matrix. We thus defined the continuous inverse of $\mathbf{Q}$ activated by any positive symmetric matrix whose eigen values are in $[0,1]$ by the following:

$$\mathbf{Q}^{\oplus\mathbf{W}} = \mathbf{Q_u}^{\oplus\mathbf{S}}\mathbf{U} = \sum_{\mathcal{P}\in\mathfrak{P}(m)} \Big(\prod_{i\in\mathcal{P}} \sigma_i\Big)\, \mathbf{X}_{\mathcal{P}}{}^{\mathbf{Q_u}}\mathbf{U} \tag{13}$$

where $\mathbf{X}_{\mathcal{P}}{}^{\mathbf{Q_u}}$ are the coupling matrix of $\mathbf{Q_u}$.

We also define the continuous inverse activated on the right (*i.e.* activation of the columns of $\mathbf{Q}$), noted $\mathbf{Q}^{\mathbf{H}\oplus}$:

$$\mathbf{Q}^{\mathbf{H}\oplus} = \Big(\big(\mathbf{Q}^\top\big)^{\oplus\mathbf{H}}\Big)^\top = \sum_{\mathcal{P}\in\mathfrak{P}(n)} \Big(\prod_{i\in\mathcal{P}} h_i\Big)\, \mathbf{X}_{\mathcal{P}}{}^\top \tag{14}$$

where the $\mathbf{X}_{\mathcal{P}}$ are the coupling matrices of $\mathbf{Q}^\top$. Finally, the continuous inverse of $\mathbf{Q}$ activated on the right by a symmetric matrix $\mathbf{W}$ is defined similarly $\mathbf{Q}^{\mathbf{W}\oplus} = \Big(\big(\mathbf{Q}^\top\big)^{\oplus\mathbf{W}}\Big)^\top$.

D. Extension to k tasks

The control law (11) can now be re-written using this generalization. The continuous inverse is valid since the eigen values of $\mathbf{P}_\oplus$ are in $[0,1]$. The control law that performs $\mathbf{e_1}$ under activation $\mathbf{H}$ and $\mathbf{e_2}$ if possible is then:

$$\dot{\mathbf{q}} = \mathbf{J_1}^{\oplus\mathbf{H}}\dot{\mathbf{e_1}}^* + \mathbf{J_2}^{\mathbf{P}_\oplus\oplus}\big(\dot{\mathbf{e_2}}^* - \mathbf{J_2}\dot{\mathbf{q}}_1\big) \tag{15}$$

using the notations $\dot{\mathbf{q}}_1$ and $\mathbf{P}_\oplus$ defined in (10). The matrix $\mathbf{P}^\mathbf{2}_\oplus$ corresponding to the projection operator is obtained classically by $\mathbf{P}^\mathbf{2}_\oplus = \mathbf{P}_\oplus - \mathbf{J_2}^{\mathbf{P}_\oplus\oplus}\mathbf{J_2}$. The extension of (15) to k tasks $(\mathbf{e_1}, \mathbf{H})$, $\mathbf{e_2}$ ... $\mathbf{e_k}$ is given by the following:

$$\dot{\mathbf{q}}_\mathbf{i} = \dot{\mathbf{q}}_\mathbf{i-1} + (\mathbf{J_i})^{\mathbf{P}^{\mathbf{i-1}}_\oplus\oplus}(\dot{\mathbf{e}}_\mathbf{i}{}^* - \mathbf{J_i}\dot{\mathbf{q}}_\mathbf{i-1}), \quad i = 1..k \tag{16}$$

with $\dot{\mathbf{q}}_\mathbf{0} = 0$. A similar recursion is used to compute the operators $\mathbf{P}^\mathbf{i}_\oplus = \mathbf{P}^\mathbf{i-1}_\oplus - \mathbf{J_i}^{\mathbf{P}^{\mathbf{i-1}}_\oplus\oplus}\mathbf{J_2}$ and $\mathbf{P}^\mathbf{0}_\oplus = \mathbf{I}$.

All the control law have been written supposing that some local controllers provide a control of the robot in velocity $\dot{\mathbf{q}}$. In the following section, we extend this result to torque control.

IV. Extension to the Operational Space Control

Operational Space Control [8] has been proposed to compute a control law torques as a direct input. Contrary to the inverse-kinematic-based control schemes, operational space control unify at the global level the forces applied by the robot and its displacement in the free space. In this section, we first recall the generic form of the control law in the operational space. We then introduce the continuous inverse account for unilateral constraints in the hierarchy.

A. Classical control law

1) One task: The acceleration $\ddot{\mathbf{q}}$ in free space is defined by $\mathbf{A}\ddot{\mathbf{q}} + \mathbf{g} + \mu = \tau$, where $\mathbf{A}$ is the inertia matrix, $\mathbf{g}$ is the gravity force, μ are the Coriolis forces and τ are the torques applied by the motors on the joints, used as the control input.

Given a task $\mathbf{e}$ with Jacobian $\mathbf{J}$, it is possible to write:

$$\ddot{\mathbf{e}} + \mathbf{b} = \mathbf{J}\mathbf{A}^{-1}\tau \tag{17}$$

with $\mathbf{b} = \mathbf{J}\mathbf{A}^{-1}\big(\mathbf{g} + \mu\big) - \dot{\mathbf{J}}\dot{\mathbf{q}}$. We define $\mathbf{\Omega} = \mathbf{J}\mathbf{A}^{-1}\mathbf{J}^\top$ and $\mathbf{\Lambda} = \mathbf{\Omega}^{-1}$. By multiplying (17) by $\mathbf{\Lambda}$, we obtain:

$$\mathbf{\Lambda}\ddot{\mathbf{e}} + \mathbf{\Lambda}\mathbf{b} = \overline{\mathbf{J}}^\top\tau \tag{18}$$

where $\overline{\mathbf{J}} = \mathbf{A}^{-1}\mathbf{J}^\top\mathbf{\Lambda}$ is the generalized inverse of $\mathbf{J}$ weighted by $\mathbf{A}^{-1}$: $\overline{\mathbf{J}} = \mathbf{J}^{\#\mathbf{A}^{-1}}$. Therefore $\overline{\mathbf{J}}\mathbf{J} = \mathbf{I}$. Multiplying (18) by $\mathbf{J}^\top$, we obtain the control law:

$$\tau = \mathbf{J}^\top\mathbf{\Lambda}\big(\ddot{\mathbf{e}}^* + \mathbf{b}\big) \tag{19}$$

2) Two task: As in the previous section, this control law is only a specific solution: this is the solution of the least acceleration energy [13]. The general solution is [8]:

$$\tau = \mathbf{J}^\top\mathbf{\Lambda}\big(\ddot{\mathbf{e}}^* + \mathbf{b}\big) + \mathbf{N}^\top\mathbf{z}_{\tau_\mathbf{2}} \tag{20}$$

where $\mathbf{z}_{\tau_\mathbf{2}}$ is arbitrary and $\mathbf{N}^\top = \mathbf{I} - \mathbf{J}^\top\overline{\mathbf{J}}^\top$ is the projection operator that ensures the realization of $\ddot{\mathbf{e}}^*$ despite $\mathbf{z}_{\tau_\mathbf{2}}$.

3) k task: The control entry $\mathbf{z}_{\tau_\mathbf{2}}$ can be used to perform a secondary task under the condition $\ddot{\mathbf{e_1}} = \ddot{\mathbf{e_1}}^*$. The control can then be extended by recurrence to k tasks $\mathbf{e_1}$ ... $\mathbf{e_k}$ [15]:

$$\tau_\mathbf{i} = \tau_\mathbf{i-1} + \mathbf{J_i}^\top\mathbf{\Lambda}_{\mathbf{i|i-1}}(\ddot{\mathbf{e}}_\mathbf{i}{}^* + \mathbf{b} - \mathbf{J_i}\mathbf{A}^{-1}\tau_\mathbf{i-1}) \tag{21}$$

with $\tau_\mathbf{0} = \mathbf{0}$, $\mathbf{\Omega}_{\mathbf{i|i-1}} = \mathbf{J_i}\mathbf{A}^{-1}\mathbf{N_{i-1}}^\top\mathbf{J_i}$ and $\mathbf{\Lambda}_{\mathbf{i|i-1}} = \mathbf{\Omega}_{\mathbf{i|i-1}}{}^{-1}$. The control law to be applied on the robot is finally $\tau = \tau_\mathbf{k}$. The projection operator is computed by $\mathbf{N_i}^\top = \mathbf{N_{i-1}} - \mathbf{J_i}^\top\overline{\mathbf{J}_{\mathbf{i|i-1}}}^\top$ where $\overline{\mathbf{J}_{\mathbf{i|i-1}}} = \mathbf{A}^{-1}\mathbf{J_i}^\top\mathbf{\Lambda}_{\mathbf{i|i-1}}$.

B. Unilateral constraint in the Operational Space

Let us now consider a task $(\mathbf{e}, \mathbf{H})$ with $\mathbf{e}$ the task function and $\mathbf{H}$ the activation matrix. We want to determine the torque entry to perform $\mathbf{e}$ under activation $\mathbf{H}$. Applying directly the classical formalism with jacobian $\mathbf{HJ}$ leads to:

$$\tau = \mathbf{J}^\top\mathbf{H}\big(\mathbf{HJA}^{-1}\mathbf{J}^\top\mathbf{H}\big)^{-1}\mathbf{H}(\ddot{\mathbf{e}}^* - \mathbf{b}) \tag{22}$$

As in the kinematic space, this control law produces some strong discontinuities when the rank of $\mathbf{HJ}$ changes. We recognize the form $\mathbf{J}^\top\mathbf{H}\big(\mathbf{HJA}^{-1}\mathbf{J}^\top\mathbf{H}\big)^{-1}\mathbf{H} = \big(\mathbf{HJA}^{-1}\big)^{\#\mathbf{A}}\mathbf{H}$, the inverse of $\mathbf{HJA}^{-1}$ weighted by the matrix $\mathbf{A}$, whose form is similar to (3). As previously the continuous inverse is introduced to prevent the discontinuity:

$$\tau = \big(\mathbf{JA}^{-1}\big)^{\oplus\mathbf{H},\#\mathbf{A}}(\ddot{\mathbf{e}}^* - \mathbf{b}) \tag{23}$$

where $\big(\mathbf{Q}\big)^{\oplus\mathbf{H},\#\mathbf{A}}$ denotes the generalized continuous inverse of $\mathbf{Q}$ activated by $\mathbf{H}$ and weighted by $\mathbf{A}$, defined by replacing all the classical pseudo-inverse in (7) and (8) by the corresponding generalized inverse weighted by $\mathbf{A}$. By developing the sum (7) in (23) and factorizing by $\mathbf{J}^\top$, we finally obtain the control law on the following form:

$$\tau = \mathbf{J}^\top\mathbf{\Lambda}_\oplus(\ddot{\mathbf{e}}^* - \mathbf{b}) + \mathbf{N}_\oplus{}^\top\mathbf{z}_{\tau_\mathbf{2}} \tag{24}$$

where $\mathbf{\Lambda}_\oplus = \mathbf{\Omega}^{\oplus\mathbf{H}}$, $\mathbf{N}_\oplus{}^\top = \mathbf{I} - \mathbf{J}_\top\overline{\mathbf{J}_\oplus}$, with $\overline{\mathbf{J}_\oplus} = \mathbf{A}^{-1}\mathbf{J}^\top\mathbf{\Lambda}_\oplus$ and $\mathbf{z}_{\tau_\mathbf{2}}$ being any arbitrary torque.

C. Extension to two tasks

This secondary torque τ_2 can be used to perform a secondary task. Let $(\mathbf{e_1}, \mathbf{H})$ and $\mathbf{e_2}$ be two tasks. The equation of motion of the robot constraint by the main task is:

$$\mathbf{A}\ddot{\mathbf{q}} + \mathbf{g} + \mu = \mathbf{J}^\top \mathbf{\Lambda}_\oplus(\ddot{\mathbf{e}}^* - \mathbf{b}) + \mathbf{N}_\oplus{}^\top \tau_2 \qquad (25)$$

By multiplying this last equation by $\mathbf{J_2 A}^{-1}$, we obtain:

$$\ddot{\mathbf{e_2}} + \mathbf{b_2} = \mathbf{J_2 A}^{-1}\tau_1 + \mathbf{J_2 A}^{-1}\mathbf{N}_\oplus{}^\top \tau_2 \qquad (26)$$

where $\mathbf{b_2} = \mathbf{J_2 A}^{-1}(\mathbf{g}+\mu)$ and τ_1 is the control law defined in (24) applied to $\mathbf{e_1}$. It is very tempting to directly inverse (26) to obtain the optimal control τ_2 performing $\mathbf{e_2}$, as it has been done in (21), obtaining the following control law:

$$\tau = \tau_1 + \mathbf{N}_\oplus\big(\mathbf{J_2 A}^{-1}\mathbf{N}_\oplus\big)^{\#\mathbf{A}}\big(\ddot{\mathbf{e_2}}^* - \mathbf{b_2} - \mathbf{J_2 A}^{-1}\tau_1\big) \quad (27)$$

However, the operator $\mathbf{N}_\oplus$ does not have a constant rank. It is thus necessary to proceed like in II-C, by activated the inverse by the projection operator $\mathbf{N}_\oplus$. The generalization of the continuous inverse is only valid for symmetrical matrices whose singular values are between 0 and 1. This is not the case of $\mathbf{N}_\oplus$, since this matrix is not an orthogonal projection operator. However, we can rewrite the inverse weighted by $\mathbf{A}$ under the following form:

$$\begin{aligned}\mathbf{N}_\oplus\big(\mathbf{J_2 A}^{-1}\mathbf{N}_\oplus\big)^{\#\mathbf{A}} &= \mathbf{N}_\oplus\sqrt{\mathbf{A}}\big(\mathbf{J_2 A}^{-1}\mathbf{N}_\oplus\sqrt{\mathbf{A}}\big)^+ \\ &= \mathbf{N}_\oplus\sqrt{\mathbf{A}}\big(\mathbf{J_2 A}^{-1/2}(\mathbf{A}^{-1/2}\mathbf{N}_\oplus\sqrt{\mathbf{A}})\big)^+\end{aligned} \qquad (28)$$

It is therefore possible to normalize $\mathbf{N}_\oplus$ by setting:

$$\begin{aligned}\mathbf{N}_\oplus^{||}{}^\top &= \mathbf{A}^{-1/2}\mathbf{N}_\oplus{}^\top\sqrt{\mathbf{A}} \\ &= \mathbf{I} - \big(\mathbf{J_1 A}^{-1/2}\big)^{\mathbf{H}\oplus}\big(\mathbf{J_1 A}^{-1/2}\big)\end{aligned} \qquad (29)$$

Using the second part of this equation, it is easy to demonstrate the $\mathbf{N}_\oplus^{||}$ is *normalized* that is to say symmetrical and with proper singular values. The inverse (28) is then finally $\sqrt{\mathbf{A}}\mathbf{N}_\oplus^{||}\big(\mathbf{J_2 A}^{-1/2}\mathbf{N}_\oplus^{||}\big)^+$. We recognize here the form (3), where $\mathbf{J_2 A}^{-1/2}$ is the matrix to be inversed, and $\mathbf{N}_\oplus^{||}$ is the activation matrix. Like in (15), we apply the continuous inverse activated *on the right* by the matrix $\mathbf{N}_\oplus^{||}$. The control law that performs the two task $(\mathbf{e_1}, \mathbf{H})$ and $\mathbf{e_2}$ is then:

$$\tau = \tau_1 + \big(\mathbf{J_2 A}^{-1}\big)^{\mathbf{N}_\oplus^{||}\oplus,\#\mathbf{A}}\big(\ddot{\mathbf{e_2}}^* + \mathbf{b_2} - \mathbf{J_2 A}^{-1}\tau_1\big) \quad (30)$$

D. Extension to k tasks

This control law can easily be extended to a set of task $(\mathbf{e_1}, \mathbf{H})$, ... $\mathbf{e_k}$ by analogy to the developments done in [15]:

$$\tau_i = \tau_{i-1} + \sqrt{\mathbf{A}}\big(\mathbf{J_i A}^{-1/2}\big)^{\mathbf{N}_\oplus^{||}\oplus}(\ddot{\mathbf{e_i}}^* + \mathbf{b_i} - \mathbf{J_i A}^{-1}\tau_{i-1}) \qquad (31)$$

with $\tau_0 = \mathbf{0}$ and $\mathbf{N}_\oplus^{\mathbf{i}}{}^\top = \mathbf{N}_\oplus^{\mathbf{i-1}}{}^\top - \big(\mathbf{J_i A}^{-1}\big)^{\mathbf{N}_\oplus^{||}\oplus,\#\mathbf{A}}\mathbf{J_i A}^{-1}$.

E. Conclusion

The control law (31) generalizes the use of an unilateral constraint for an arbitrary set of tasks in the operational space. The final form is very close to the classical control law (21). Moreover, very simple computations show that the control law (31) is equal to (21) when the singular values of $\mathbf{N}_\oplus^{||}$ are all 0 or 1. Until this point, all the computations have been realized for any set of tasks. In the following section, we introduce the specific tasks used in the experiments.

V. Application to visual servoing

The framework presented above has been experimentally validated by realizing a visual-servoing task while respecting the constraints imposed by the joint limits.

A. Visual servoing task

A visual servoing task [6], [4] is based on an error $\mathbf{e_i} = \mathbf{s_i} - \mathbf{s_i^*}$ where $\mathbf{s_i}$ is the current value of the visual features $\mathbf{e_i}$ and $\mathbf{s_i^*}$ their desired value. The interaction matrix $\mathbf{L_{s_i}}$ related to $\mathbf{s_i}$ is defined so that $\dot{\mathbf{s_i}} = \mathbf{L_{s_i}}\mathbf{v}$, where $\mathbf{v}$ is the instantaneous camera velocity. The task Jacobian $\mathbf{J_i}$ is thus $\mathbf{J_i} = \mathbf{L_{s_i}}\mathbf{M}\mathbf{J_q}$, where $\mathbf{J_q}$ is the robot Jacobian at the camera focal point ($\dot{\mathbf{r}} = \mathbf{J_q}\dot{\mathbf{q}}$) and $\mathbf{M}$ is relates the variation of the camera velocity $\mathbf{v}$ to camera pose parametrization ($\mathbf{v} = \mathbf{M}\dot{\mathbf{r}}$).

The task feature are computed from the image moments. The target is clearly marked by n_p points $P_i = (x_i, y_i)$. The moment $m_{i,j}$ of the image is defined by $m_{i,j} = \sum_{k=1}^{n_p} x_k^i y_k^j$. A task of dimension 3 has been defined to control the position of the target and its dimension in the camera field of view. The two first feature are based on the position of the center of gravity: $(x_g, y_g) = (\frac{m_{10}}{m_{00}}, \frac{m_{01}}{m_{00}})$. The third feature a_Z uses the centered moments of order 2 to control the range between the robot and the target as proposed in [17].

B. Joint limits avoidance

The robot lower and upper joint limits for each joint i are denoted $\bar{q}_i^{\min}$ and $\bar{q}_i^{\max}$. The robot configuration $\mathbf{q}$ is acceptable if, for all i, $q_i \in [\bar{q}_i^{\min}, \bar{q}_i^{\max}]$. Given n axes, the joint limits can be expressed by $2 \times n$ unilateral constraints:

$$\forall i = 1..n, \quad q_i > \bar{q}_i^{\min} \text{ and } q_i < \bar{q}_i^{\max} \qquad (32)$$

The corresponding unilateral task can be written $(\mathbf{e_{jl}}, \mathbf{H_{jl}})$, where $\mathbf{e_{jl}} = \mathbf{q^N} = (q_i^N)_{i=1..n}$ is the normalized joint state ($q_i^N = 2\frac{q_i - \bar{q}_i^{\min}}{\bar{q}_i^{\max} - \bar{q}_i^{\min}} - 1 \in [-1, 1]$). The activation matrix $\mathbf{H_{jl}}$ is defined by its diagonal components:

$$h_i = \begin{cases} 1 & \text{if } e_i < -1 \text{ or } e_i > 1 \\ f_\beta(\beta - 1 - e_i) & \text{if } e_i \in [-1, -1+\beta] \\ f_\beta(\beta - 1 + e_i) & \text{if } e_i \in [1-\beta, 1] \\ 0 & \text{otherwise} \end{cases} \qquad (33)$$

where β is the size of the transition interval, used as a gain to tune the robot behavior, and f_β is the transition function, defined to provide a $\mathcal{C}^\infty$ transition from 0 to 1:

$$\forall x \in [0, \beta], \; f_\beta(x) = \frac{1}{2}\left(1 + \tanh\left(\frac{1}{1 - x/\beta} - \frac{\beta}{x}\right)\right) \quad (34)$$

VI. Experiments and results

A. Experiments in simulation

We consider a 7-DOF PA-10 robot in simulation. The task to be executed is a 3-DOF positioning of an embedded camera with respect to a visual target. The joint limits avoidance is ensured by adding the corresponding task at the higher-priority position. We finally add at the lowest-priority

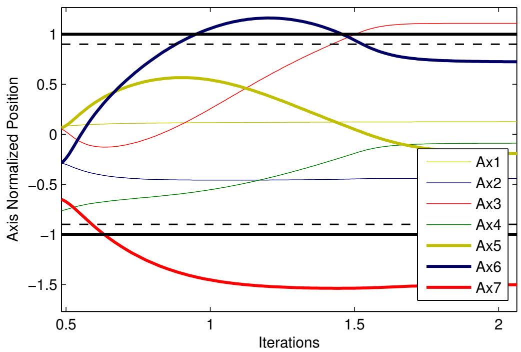

Fig. 1. Positioning the end effector without considering the joint limits.

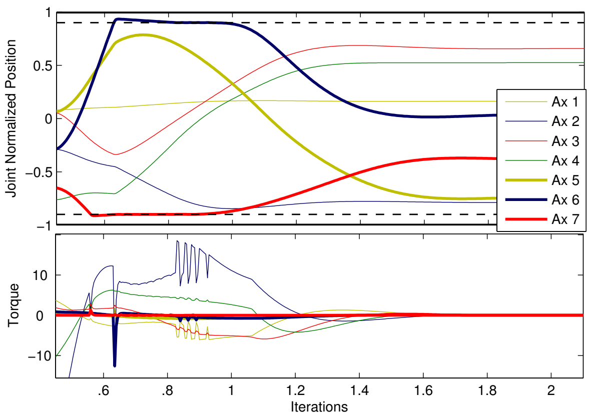

Fig. 2. Positioning the joint limits while ensuring the joint limit avoidance using the non-continuous control law.

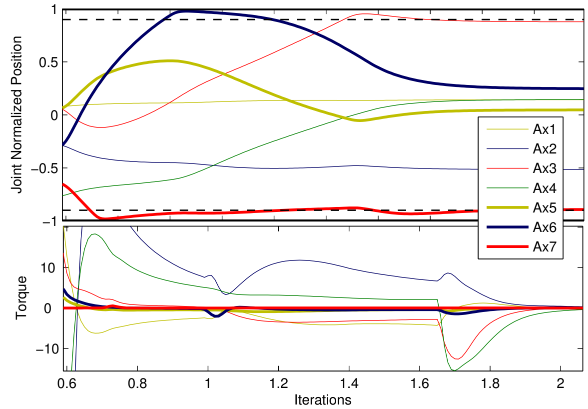

Fig. 3. Positioning the joint limits while ensuring the joint limit avoidance using the continuous control law (31).

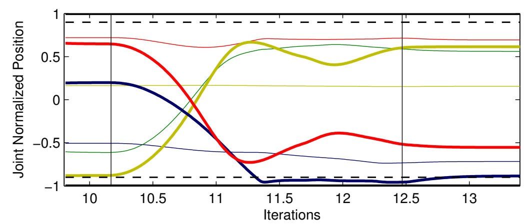

Fig. 4. Motion due to an external force applied on Joint 4, while applying the 3-DOF positioning task and the joint limits. The force started at time t=10.2 and ends at t=12.4

level a posture task (to ensure the stability of the robot, all the DOF have to be constrained).

1) Experiment 1: the target is positioned so that the task is feasible but overcome temporarily the constraint (see Fig. 1). The joint limits are avoided by using directly the pseudo-inverse (see Fig. 2). However, the control law is jerky. The "entering-leaving" oscillation can be clearly observed on the joints 1, 2, 6 and 7. Moreover, the robot cannot practically enter inside the activation buffer but is stuck at the entrance of the buffer, where the oscillations appear (Fig. 2-top). The robot is thus limited in practice to a smaller part of its joint domain. When using the control law (31), the control law is smooth (see Fig. 3). The joint limits are avoided while performing the task. No oscillation appears during any part of the displacement. Moreover, all the joint domain is used.

Finally, we apply a force on fourth body of the robot to disturb the control law. Since four DOF are available, the robot can move freely. As shown by Fig. 4, Joint 4 moves freely. Some other joints follow the motion to maintain the task completed. At t=11.4, Joint 7 reaches its limits. The robot thus looses one DOF, which consequently locks Joint 4. The robot is not compliant in the direction of the external force anymore. Joint 7 remains in the buffer while the force is applied on Joint 4. When the force is relaxed, the robot posture changes to take Joint 7 out of the activation buffer.

2) Experiment 2: the target is now positioned so that the required motion is not feasible inside the joint limits. When the robot reaches an equilibrium, we then move the target inside the joint-limit boundary to check that the control law is able to reach the reference position when reachable. We first try to put the positioning task at the top-priority level. As expected, the joint limits are then violated (see Fig. 5). The control law using the pseudo inverse is jerky and could not be applied on a real robot (see Fig. 6). Control law (31) produces a smooth behavior (see Fig. 7). The robot stops at the limit of its joint domain, as close as possible to the desired position. When the target is moved back inside the reachable domain, the robot moves freely away from its limits to reach the desired position.

B. Experiment on the Puma robot

We have then validated our control scheme on a real robot. As in simulation, a camera is attached to the end effector. The task to be accomplished is the same than in simulation. As for the experiment described in 4, an external force is applied on the third joint of the robot to disturb it during the execution of the task. The robot first move freely in the direction of the applied force (Joint 3 moves freely). The wrist (Joint 6) moves to compensate the motion and execute the task. The robot start to resist when Joint 6 reaches its limit. Joint 4 can then be used instead of 6. When both Joints 3 and 4 reaches the activation buffer, the robot resists to the external force.

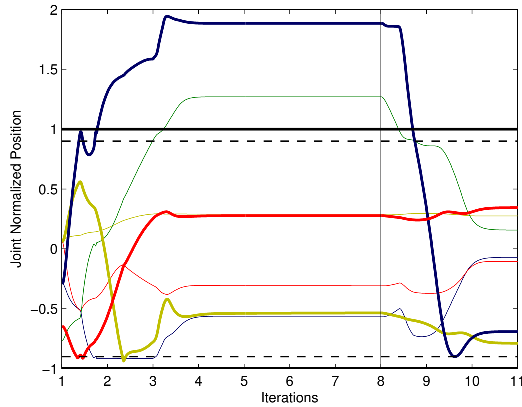

Fig. 5. Avoidance task with low priority. At time t=1, the robot is required to reach a position out of its joint domain. The goal position is set back in the joint domain at $t = 8$.

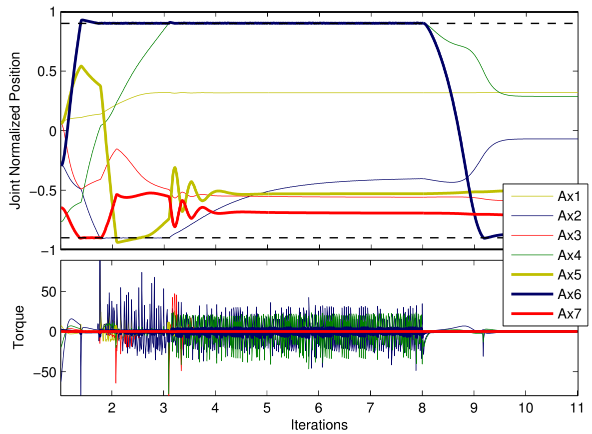

Fig. 6. Oscillation while using the pseudo-inverse.

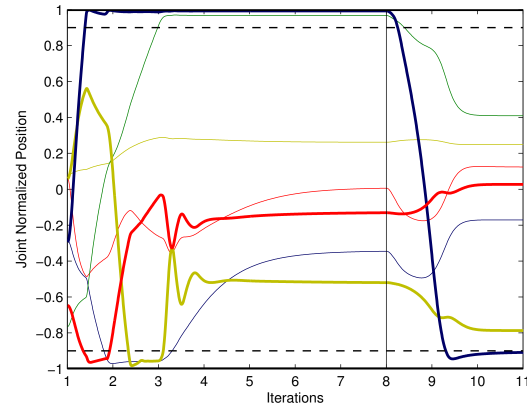

Fig. 7. Joint limits prevent the execution of an non-feasible task: using the continuous inverse

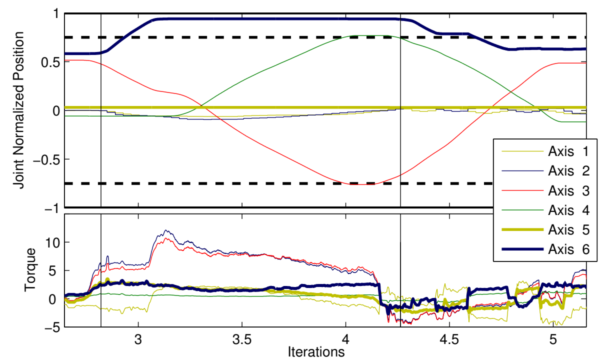

Fig. 8. Experiment on the Puma. An external force is applied to the third joint from time $t = 2.8$ to $t = 4.25$.

VII. Conclusion

In this paper, we have proposed an original scheme to compute a generic control law from a hierarchic set of both unilateral constraints and bilateral tasks. This scheme ensures that the unilateral constraints will be respected whatever the actions of the tasks. The proposed scheme is generic and could be applied for a various set of tasks, constraints and robots. We have demonstrate the validity by applying it on different types of robot with a common visual servoing scheme while insuring the respect of the joint limits.

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