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Global Continuous Finite-Time Output Feedback Regulation of Robot Manipulators

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Abstract—**A simple continuous output feedback proportional-derivative (PD) plus desired gravity compensation controller is proposed to solve the global finite-time regulation of robot manipulators with position measurements only. The global finite-time convergence is proved by using Lyapunov theory and finite-time stability theory. Simulations preformed on a two degrees-of-freedom (DOF) manipulator demonstrate the expected properties of the proposed approach.**

I. INTRODUCTION

Conventionally, most of the existing results on regulation of robot manipulators is achieved asymptotically [1]-[13]. Asymptotic stability implies that the system trajectories converge to the equilibrium as time goes to infinity. It is now known that finite-time stabilization offers an effective alternative, which yields, in some senses, fast response, high-precision and disturbance rejection properties [14]-[24]. In particular, for robot manipulators, Barambones and Etxebarria [21] formulated a terminal sliding-mode adaptive control scheme for zero trajectory-tracking error in finite time. Gruyitch & Kokosy [22] designed a sliding-mode controller to guarantee the robust global stability and attraction with a finite time. Parra-Vega, Rodriguez-Angeles, and Hirzinger [23] proposed a dynamic sliding controller to implement the perfect tracking defined as the performance of zero tracking errors of position and force in finite time. Yu, Yu, Shirinzadeh, and Man [24] proposed a continuous finite-time tracking controller for robot manipulators by using a new form of terminal sliding modes and showed the faster and high-precision tracking.

While these controllers for robot manipulators are simple, elegant, and intuitively appealing, a major drawback remains for these schemes, i.e. the requirement of measurements of both position and velocity. Velocity measurement increases cost and imposes constraints on the achievable bandwidth. To remove the requirement of the velocity measurements, several control techniques that asymptotic stabilize arbitrary positions of robotic manipulators can be found in the literature [25]-[28]. To the best of our knowledge, the only previous work which targets at the finite-time output feedback regulation is given in [17]. Specially, Hong, Xu, and Huang [17] formulated a nonsmooth PD plus gravity compensation scheme and achieved a local finite-time result, in which a model-based observer is employed to remove the requirement of velocity measurements. As pointed by Gunawardana and Ghorbel [29] and Kasac *et al.* [10], it is often difficult to explicitly characterize the domain of attraction that could be much smaller than the robot workspace. This means that a global result is always more useful for both theoretical analysis and practical implementation.

In this paper the global continuous finite-time output feedback regulation of robot manipulators is addressed, which aims at designing a simple output feedback PD plus gravity compensation control law with the position measurements only, such that the position of the robot can be regulate into a desired position in finite time. Compared with the only finite-time output feedback regulator proposed by Hong *et al.* [17], the advantage of the developed approach is twofold. The simple model-free "dirty derivative" method is used to give the velocity estimation. The global finite-time stability of the closed-loop system is achieved, and thus it is readily implemented.

The reminder of this paper is organized as follows. The robot manipulator model and properties are presented in Section 2. Some preliminaries on the finite-time stability are reviewed in Section 3. Our main results are presented in Section 4, where we formulated a simple continuous output feedback nonsmooth PD plus gravity compensation controller and showed the global finite-time stability by using Lyapunov's direct method and the finite-time stability theory. An illustrative example preformed on a two-DOF robot manipulator is included in Section 5 to show the better performance of the proposed controller. Finally, some concluding remarks are presented in Section 6.

II. ROBOT MANIPULATOR MODEL AND PROPERTIES

In the absence of disturbances, the dynamics of an n-DOF robot manipulator can be written as [1], [30]

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + D\dot{q} + G(q) = \tau \qquad (1)$$

where $q, \dot{q}, \ddot{q} \in \Re^n$ denote the link position, velocity, and acceleration, respectively, $M(q) \in \Re^{n\times n}$ represents the symmetric inertia matrix, $C(q,\dot{q}) \in \Re^{n\times n}$ denotes the centrifugal-Coriolis matrix, $D \in \Re^{n\times n}$ stands for the matrix

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composed of damping friction coefficients for each joint, $G(q) = \partial U(q)/\partial q \in \Re^n$ is a gravitational force, $U(q)$ is the potential energy due to gravitational force, and $\tau \in \Re^n$ denotes the torque input vector.

By recalling the robot manipulators considered, the following properties can be established [1], [9], [30].

Property 1: The matrix D is a diagonal positive definite matrix.

Property 2: The matrix $M(q)$ is positive definite and bounded by

$$0 < M_m \le \|M(q)\| \le M_M \quad (2)$$

where M_m and M_M are some positive constants.

Property 3: The matrix $C(q,\dot{q})$ is defined using Christoffel symbols, and $\dot{M}(q) - 2C(q,\dot{q})$ is skew-symmetric, i.e.

$$\zeta^T\left(\dot{M}(q) - 2C(q,\dot{q})\right)\zeta = 0, \ \forall \zeta \in \Re^n \quad (3)$$

where $\dot{M}(q)$ is the time derivative of the inertia matrix $M(q)$.

Property 4: The matrix $C(q,\dot{q})$ satisfies the following relationship:

$$C(q,\xi)\upsilon = C(q,\upsilon)\xi, \qquad \forall \xi,\ \upsilon \in \Re^n \quad (4)$$

and is bounded by

$$0 < C_m\|\dot{q}\|^2 \le \|C(q,\dot{q})\dot{q}\| \le C_M\|\dot{q}\|^2, \qquad \forall q, \dot{q} \in \Re^n \quad (5)$$

where C_m and C_M are some positive constants.

Property 5: There exists a positive definite diagonal matrix A such that the following two inequalities, with specified constant $a > 0$, are satisfied simultaneously for any fixed q_d and any q

$$U(q) - U(q_d) - \Delta q^T G(q_d) + \frac{1}{2}\Delta q^T A \Delta q \ge a\|\Delta q\|^2 \quad (6)$$

$$\Delta q^T[G(q) - G(q_d)] + \Delta q^T A \Delta q \ge a\|\Delta q\|^2 \quad (7)$$

where $\Delta q = q - q_d$ denotes the position error, and q and q_d denote the actual and desired coordinates, respectively.

Throughout this paper, we use the notation $\lambda_m(A)$ and $\lambda_M(A)$ to indicate the smallest and largest eigenvalues, respectively, of a symmetric positive definite bounded matrix $A(x)$, for any $x \in \Re^n$. The norm of a vector $x \in \Re^n$ is defined as $\|x\| = \sqrt{x^T x}$ and that of a matrix A is defined as the corresponding induced norm $\|A\| = \sqrt{\lambda_M(A^T A)}$, and I denotes an identity matrix of the appropriate dimension.

III. Preliminaries

We begin the review of the concepts of finite-time stability and stabilization of nonlinear systems following the treatment in [14], [17].

Consider the system

$$\dot{\zeta} = f(\zeta), \quad f(0) = 0, \quad \zeta(0) = \zeta_0, \quad \zeta \in \Re^n \quad (8)$$

with $f: U_0 \to \Re^n$ continuous on an open neighborhood U_0 of the origin. Suppose that system (8) possesses unique solutions in forward time for all initial conditions.

Definition 1: The equilibrium $\zeta = 0$ of system (8) is (locally) finite-time stable if it is Lyapunov stable and finite-time convergent in a neighborhood $U \subset U_0$ of the origin. The finite-time convergence means the existence of a function $T: U \setminus \{0\} \to (0,\infty)$, such that, $\forall\ \zeta_0 \in U \subset \Re^n$, the solution of (8) denoted by $s_t(\zeta_0) = 0$ with ζ_0 as the initial condition is defined, and $s_t(\zeta_0) \in U \setminus \{0\}$ for $t \in [0, T(\zeta_0))$, and $\lim_{t \to T(\zeta_0)} s_t(\zeta_0) = 0$ with $s_t(\zeta_0) = 0$ for $t > T(\zeta_0)$. When $U = \Re^n$, we obtain the concept of global finite-time stability.

Definition 2: Let $(r_1, \ldots, r_n) \in \Re^n$ with $r_i > 0, i = 1, \ldots, n$. Let $V: \Re^n \to \Re$ be a continuous function. V is said to be homogeneous of degree $\sigma > 0$ with respect to $(r_1, \ldots, r_n)$, if, for any given $\varepsilon > 0$,

$$V(\varepsilon^{r_1}\zeta_1, \ldots, \varepsilon^{r_n}\zeta_n) = \varepsilon^{\sigma} V(\zeta), \quad \forall \zeta \in \Re^n \quad (9)$$

Definition 3: Let $f(\zeta) = f(f_1(\zeta), \ldots, f_n(\zeta))^T$ be a continuous vector field. $f(\zeta)$ is said to be homogeneous of degree $\kappa \in \Re$ with respect to $(r_1, \ldots, r_n)$, if, for any given $\varepsilon > 0$,

$$f_i(\varepsilon^{r_1}\zeta_1, \ldots, \varepsilon^{r_n}\zeta_n) = \varepsilon^{\kappa + r_i} f_i(\zeta),\ i = 1, \ldots, n, \forall \zeta \in \Re^n \quad (10)$$

System (8) is said to be homogeneous if $f(\zeta)$ is homogeneous.

Some of the results on finite-time stability of a nonlinear system in [17] that will be used in this paper are summarized by the following two lemmas.

Lemma 1: Consider the following system

$$\dot{\zeta} = f(\zeta) + \hat{f}(\zeta), \quad f(0) = 0, \quad \hat{f}(0) = 0, \quad \zeta \in \Re^n \quad (11)$$

where $f(\zeta)$ is a continuous homogeneous vector field of degree $\kappa < 0$ with respect to $(r_1, \ldots, r_n)$. Assume $\zeta = 0$ is an asymptotically stable equilibrium of the system $\dot{\zeta} = f(\zeta)$. Then $\zeta = 0$ is a locally finite-time stable equilibrium of the system (11) if

$$\lim_{\varepsilon \to 0} \frac{\hat{f}_i(\varepsilon^{r_1}\zeta_1, \ldots, \varepsilon^{r_n}\zeta_n)}{\varepsilon^{\kappa + r_i}} = 0, \quad i = 1, \ldots, n, \forall \zeta \ne 0 \quad (12)$$

Lemma 2: Global asymptotic stability and local finite-time stability of the closed-loop system imply global finite-time stability.

IV. Control Design

A. Control Formulation

To aid the subsequent control design and analysis, we define the vectors $\text{Tanh}(\cdot), \text{Sig}(\cdot)^\alpha \in \Re^n$ and the diagonal matrix $\text{Sech}(\cdot) \in \Re^{n\times n}$ as follows:

$$\text{Tanh}(\xi) = [\tanh(\xi_1), \cdots, \tanh(\xi_n)]^T \tag{13}$$

$$\text{Sig}(\xi)^\alpha = \left[|\xi_1|^\alpha \text{sgn}(\xi_1), \cdots, |\xi_n|^\alpha \text{sgn}(\xi_n)\right]^T \tag{14}$$

$$\text{Sech}(\xi) = \text{diag}(\text{sech}(\xi_1), \cdots, \text{sech}(\xi_n)) \tag{15}$$

where $\xi = [\xi_1, \cdots, \xi_n]^T \in \Re^n$, $0 < \alpha < 1$, $\tanh(\cdot)$ and $\text{sech}(\cdot)$ being the standard hyperbolic tangent and secant functions, respectively, $\text{sgn}(\cdot)$ being the standard signum function, and $\text{diag}(\cdot)$ denotes a diagonal matrix with zeros everywhere except for the main diagonal. Based on the definition of (13)-(15), it can easily be shown that the following expressions hold:

$$\text{Tanh}^T(\xi)\text{Sig}(\xi)^\alpha \geq \text{Tanh}^T(\xi)\text{Tanh}(\xi) \tag{16}$$

$$|\xi_i|^{\alpha+1} \geq \tanh^2(\xi_i) \tag{17}$$

$$\lambda_M(\text{Sech}^2(\xi)) = 1 \tag{18}$$

The proposed output feedback nonsmooth PD plus desired gravity compensation controller is formulated as

$$\tau = G(q_d) - K_p\text{Sig}(\Delta q)^\alpha - K_d\upsilon \tag{19}$$

$$\dot{\upsilon} = -A\upsilon + B\dot{q} \tag{20}$$

where K_p and K_d are positive definite constant diagonal proportional and derivative matrices, respectively, $0 < \alpha < 1$, and A and B are positive definite filter gains.

Substituting (19) into (1), the closed-loop dynamics becomes

$$\begin{aligned} M(q)\ddot{q} + C(q,\dot{q})\dot{q} + D\dot{q} + G(q) - G(q_d) \\ + K_p\text{Sig}(\Delta q)^\alpha + K_d\upsilon = 0 \end{aligned} \tag{21}$$

whose origin $\left[\Delta q^T \ \dot{q}^T \ \upsilon^T\right]^T = 0 \in \Re^{3n}$ is the unique equilibrium.

B. Stability Analysis

Theorem 1: With the proposed output feedback PD plus desired gravity compensation controller (19) and (20), the closed-loop system (21) is globally finite-time stable, provided that the control gains are chosen as follows:

$$D > \lambda_0\left(\sqrt{n}C_M + M_M\right)I \tag{22}$$

$$2K_dAB^{-1} > \lambda_0\lambda_M(K_d) \tag{23}$$

$$K_p > 2(\alpha+1)\lambda_0^2 M_M I \tag{24}$$

$$\begin{aligned} U(q) - U(q_d) - \Delta q^T G(q_d) + \frac{1}{2}\frac{1}{\alpha+1}\sum_{i=1}^n k_{pi}|\Delta q_i|^{\alpha+1} \\ > a\|\text{Tanh}(\Delta q)\|^2 \end{aligned} \tag{25}$$

$$\begin{aligned} \text{Tanh}^T(\Delta q)(G(q) - G(q_d)) + \text{Tanh}^T(\Delta q)K_p\text{Sig}(\Delta q)^\alpha \\ > \left(a + \frac{1}{2}\lambda_M(K_d)\right)\|\text{Tanh}(\Delta q)\|^2 \end{aligned} \tag{26}$$

where k_{pi} denotes the ith diagonal elements of matrix K_p, and λ_0 and a are small positive constants.

Note that the inequalities (25) and (26) correspond to inequalities (6) and (7) of Property 5, respectively, and the existence of such a matrix K_p is confirmed by the same argument given in proposing (6) and (7), and (16) [1], [28].

Proof: The proof proceeds in the following two steps. First, the global asymptotic stability is proved based on Lyapunov's direct method and LaSalle's invariance principle. Second the local finite-time stability is shown using Lemma 1 and Lemma 2 is involved to guarantee the global finite-time stability.

1) Global asymptotic stability: To this end, we propose the following Lyapunov-like function candidate

$$\begin{aligned} V = {} & \frac{1}{2}\dot{q}^T M(q)\dot{q} + \lambda_0 \text{Tanh}^T(\Delta q)M(q)\dot{q} + U(q) \\ & - U(q_d) - \Delta q^T G(q_d) + \frac{1}{\alpha+1}\sum_{i=1}^n k_{pi}|\Delta q_i|^{\alpha+1} \\ & + \frac{1}{2}\upsilon K_d B^{-1}\upsilon + \lambda_0\sum_{i=1}^n d_i \ln(\cosh(\Delta q_i)) \end{aligned} \tag{27}$$

where d_i denotes the ith diagonal elements of matrix D.

We first consider the following

$$\begin{aligned} & \frac{1}{4}\dot{q}^T M(q)\dot{q} + \frac{1}{2(\alpha+1)}\sum_{i=1}^n k_{pi}|\Delta q_i|^{\alpha+1} + \lambda_0\text{Tanh}^T(\Delta q)M(q)\dot{q} \\ & = \frac{1}{4}(\dot{q} + 2\lambda_0\text{Tanh}(\Delta q))^T M(q)(\dot{q} + 2\lambda_0\text{Tanh}(\Delta q)) \\ & \quad - \lambda_0^2\text{Tanh}^T(\Delta q)M(q)\text{Tanh}(\Delta q) + \frac{1}{2(\alpha+1)}\sum_{i=1}^n k_{pi}|\Delta q_i|^{\alpha+1} \\ & \geq \frac{1}{2(\alpha+1)}\sum_{i=1}^n k_{pi}|\Delta q_i|^{\alpha+1} - \lambda_0^2\text{Tanh}^T(\Delta q)M(q)\text{Tanh}(\Delta q) \\ & \geq \sum_{i=1}^n\left[\frac{k_{pi}}{2(\alpha+1)} - \lambda_0^2 M_M\right]\tanh^2(\Delta q_i) \end{aligned} \tag{28}$$

where (2) of Property 2 and (17) have been used.

Substituting (28) into (27), we have

$$\begin{aligned} V = {} & \frac{1}{2}\dot{q}^T M(q)\dot{q} + \sum_{i=1}^n\left[\frac{k_{pi}}{2(\alpha+1)} - \lambda_0^2 M_M\right]\tanh^2(\Delta q_i) \\ & + U(q) - U(q_d) - \Delta q^T G(q_d) + \frac{1}{2}\upsilon K_d B^{-1}\upsilon \\ & + \frac{1}{2(\alpha+1)}\sum_{i=1}^n k_{pi}|\Delta q_i|^{\alpha+1} + \lambda_0\sum_{i=1}^n d_i\ln(\cosh(\Delta q_i)) \end{aligned} \tag{29}$$

From (29), (24) and (25), we get

$$\begin{aligned} V \geq {} & \frac{1}{4}\dot{q}^T M(q)\dot{q} + \lambda_0\sum_{i=1}^n d_i\ln(\cosh(\Delta q_i)) \\ & + a\|\text{Tanh}(\Delta q)\|^2 + \frac{1}{2}\upsilon K_d B^{-1}\upsilon > 0 \end{aligned} \tag{30}$$

for $\left[\Delta q^T \ \dot{q}^T \ \ \upsilon^T\right]^T \neq 0$.

Hence, we can conclude that V is a positive definite Lyapunov function with respect to $\Delta q, \dot{q}, \upsilon$.

Differentiating V with respect to time, we have

$$\dot V = \frac{1}{2}\dot q^T \dot M(q)\dot q + \dot q^T M(q)\ddot q + \lambda_0(\mathrm{Sech}^2(\Delta q)\Delta\dot q)^T M(q)\dot q + \lambda_0 \mathrm{Tanh}^T(\Delta q)\dot M(q)\dot q + \lambda_0 \mathrm{Tanh}^T(\Delta q)M(q)\ddot q + \dot q^T G(q) - \Delta\dot q^T G(q_d) + \Delta\dot q^T K_p \mathrm{Sig}(\Delta q)^\alpha + \upsilon^T K_d B^{-1}\dot\upsilon + \lambda_0 \mathrm{Tanh}^T(\Delta q) D\Delta\dot q \tag{31}$$

Substituting $M(q)\ddot q$ from (21) and (20) into (31), and using (3) of Property 3, it follows that

$$\begin{aligned}\dot V = &-\upsilon^T K_d A B^{-1}\upsilon - \dot q^T D\dot q - \lambda_0 \mathrm{Tanh}^T(\Delta q) K_d \upsilon \\ &+ \lambda_0\left[\mathrm{Tanh}^T(\Delta q) C^T(q,\dot q)\dot q + (\mathrm{Sech}^2(\Delta q)\Delta\dot q)^T M(q)\dot q\right] \\ &- \lambda_0\left[\mathrm{Tanh}^T(\Delta q)\big(G(q)-G(q_d)\big) + \mathrm{Tanh}^T(\Delta q) K_p \mathrm{Sig}(\Delta q)^\alpha\right]\end{aligned} \tag{32}$$

Using (4) and (5) of Property 4 and (18), the fourth term of the right-hand side of (32) can be upper bounded by

$$\lambda_0\left[\mathrm{Tanh}^T(\Delta q) C^T(q,\dot q)\dot q + (\mathrm{Sech}^2(\Delta q)\Delta\dot q)^T M(q)\dot q\right] \le \lambda_0(\sqrt{n}C_M + M_M)\|\dot q\|^2 \tag{33}$$

Note that the derivation of the first term of (33) we utilized $\|\mathrm{Tanh}(\Delta q)\| \le \sqrt{n}$ according to (13) and $|\tanh(\Delta q_i)| \le 1$.

Substituting (26) and (33) into (32), we have

$$\begin{aligned}\dot V \le &-\upsilon^T K_d A B^{-1}\upsilon - \lambda_0 \mathrm{Tanh}^T(\Delta q) K_d \upsilon + \lambda_0\left(\sqrt{n}C_M + M_M\right)\|\dot q\|^2 \\ &- \dot q^T D\dot q - \lambda_0\left(a + \frac{1}{2}\lambda_M(K_d)\right)\|\mathrm{Tanh}(\Delta q)\|^2 \\ \le &-\frac{1}{2}\upsilon^T\left(2K_d A B^{-1} - \lambda_0\lambda_M(K_d) I\right)\upsilon - a\lambda_0\|\mathrm{Tanh}(\Delta q)\|^2 \\ &- \dot q^T\left[D - \lambda_0\left(\sqrt{n}C_M + M_M\right) I\right]\dot q\end{aligned} \tag{34}$$

where the triangle inequality $a_1 a_2 \le \frac{1}{2}(a_1^2 + a_2^2)$ has been used with $a_1 = \|\mathrm{Tanh}(\Delta q)\|$ and $a_2 = \|\upsilon\|$.

From (22) and (23), we conclude that $\dot V < 0$. In fact, $\dot V = 0$ means $\mathrm{Tanh}(\Delta q) = 0$, $\dot q = 0$, and $\upsilon = 0$. From the property of hyperbolic tangent function, we have $\Delta q = 0$. Therefore, by Lyapunov's direct method [31], we have the global asymptotic stability about the point $(\Delta q = 0, \dot q = 0, \upsilon = 0)$.

2) Local finite-time stability: Following the idea presented in [17], the local finite-time stability is proved using Lemma 1. To this end, let $x_1 = \Delta q$, $x_2 = \dot x_1 = \dot q$, $x_3 = \upsilon$, and $x = (x_1^T, x_2^T, x_3^T)^T$. The state equation of the closed-loop system is

$$\begin{cases}\dot x_1 = x_2 \\ \dot x_2 = -M^{-1}(x_1+q_d)\big[C(x_1+q_d, x_2)x_2 + Dx_2 \\ \qquad + G(x_1+q_d) - G(q_d) + K_p\mathrm{Sig}(x_1)^\alpha + K_d x_3\big] \\ \dot x_3 = -Ax_3 + Bx_2\end{cases} \tag{35}$$

Clearly, $x = 0$ is the equilibrium of (35). It can be seen that the closed-loop system is not homogeneous. To use Lemma 1, we rewrite (35) as follows:

$$\begin{cases}\dot x_1 = x_2 \\ \dot x_2 = -M^{-1}(q_d)K_p\mathrm{Sig}(x_1)^\alpha + \hat f_2(x) \\ \dot x_3 = Bx_2 + \hat f_3(x)\end{cases} \tag{36}$$

with

$$\hat f_2 = -M^{-1}(x_1+q_d)\big[C(x_1+q_d, x_2)x_2 + G(x_1+q_d) - G(q_d) + Dx_2 + K_d x_3\big] - \widetilde M(x_1, q_d)K_p\mathrm{Sig}(x_1)^\alpha \tag{37}$$

$$\hat f_3 = -Ax_3 \tag{38}$$

and

$$\widetilde M(x_1, q_d) = M^{-1}(x_1+q_d) - M^{-1}(q_d) \tag{39}$$

It can be easily verified that the following system

$$\begin{cases}\dot x_1 = x_2 \\ \dot x_2 = -M^{-1}(q_d)K_p\mathrm{Sig}(x_1)^\alpha \\ \dot x_3 = Bx_2\end{cases} \tag{40}$$

is homogeneous of degree $\kappa = \frac{\alpha-1}{\alpha+1} < 0$ for $0<\alpha<1$ with respect to $(r_{11}, r_{12}, \ldots, r_{1n}, r_{21}, r_{22}, \ldots, r_{2n}, r_{31}, r_{32}, \ldots, r_{3n})$ with $r_{1i} = r_1 = 2/(\alpha+1)$, $r_{2i} = r_2 = 1$, and $r_{3i} = r_3 = r_1$. Note that $f(0) = 0$ and $\hat f(0) = 0$ from (40), and (37) and (38), respectively.

Next we will involve Lemma 1 to show the local finite-time stability of the closed-loop system (35). To this end, first note that, since $M^{-1}(x_1+q_d)$ and $C(x_1+q_d, x_2)$ are smooth and $\kappa<0$ [17], we have

$$\begin{aligned}&\lim_{\varepsilon\to 0} -\left\{\frac{M^{-1}(\varepsilon^{r_1}x_1+q_d)}{\varepsilon^{\kappa+r_2}}\Big[C(\varepsilon^{r_1}x_1+q_d, \varepsilon^{r_2}x_2)\varepsilon^{r_2}x_2 + D\varepsilon^{r_2}x_2 + G(\varepsilon^{r_1}x_1+q_d) - G(q_d) + K_d\varepsilon^{r_3}x_3\Big]\right\} \\ &= -M^{-1}(q_d)\big[C(q_d,0)+D\big]x_2\lim_{\varepsilon\to0}\varepsilon^{-\kappa} - K_d x_3\lim_{\varepsilon\to0}\varepsilon^{r_3-\kappa-r_2} \\ &= 0\end{aligned} \tag{41}$$

Note that the derivation of (41) we have utilized the fact that $r_3 - \kappa - r_2 = \frac{2(1-\alpha)}{\alpha+1} > 0$ for $0<\alpha<1$.

Applying the mean value theorem to each entry of $\widetilde M(x_1, q_d)$, it follows that [17]

$$\widetilde M(\varepsilon^{r_1}x_1, q_d) = M^{-1}(\varepsilon^{r_1}x_1+q_d) - M^{-1}(q_d) = o(\varepsilon^{r_1}) \tag{42}$$

As a result, we have

$$\lim_{\varepsilon\to0} -\frac{\widetilde M(\varepsilon^{r_1}x_1, q_d)K_p\mathrm{Sig}(\varepsilon^{r_1}x_1)^\alpha}{\varepsilon^{\kappa+r_2}} = -\lim_{\varepsilon\to0} o(\varepsilon^{r_1-\kappa-r_2}) = -\lim_{\varepsilon\to0} o(\varepsilon^{-2\kappa}) = 0 \tag{43}$$

Thus, for any fixed $x = (x_1^T, x_2^T, x_3^T)^T \in \Re^{3n}$, we get

$$\lim_{\varepsilon\to 0}\frac{\hat{f}_2(\varepsilon^{r_1}x_1, \varepsilon^{r_2}x_2, \varepsilon^{r_3}x_3)}{\varepsilon^{\kappa+r_2}}=0 \tag{44}$$

Similarly, we have

$$\begin{aligned}\lim_{\varepsilon\to 0}\frac{\hat{f}_3(\varepsilon^{r_1}x_1, \varepsilon^{r_2}x_2, \varepsilon^{r_3}x_3)}{\varepsilon^{\kappa+r_3}} &= \lim_{\varepsilon\to 0} -\frac{A\varepsilon^{r_3}x_3}{\varepsilon^{\kappa+r_3}} \\ &= -Ax_3\lim_{\varepsilon\to 0}\varepsilon^{-\kappa}=0\end{aligned} \tag{45}$$

Therefore, according to Lemma 1, we have the local finite-time stability of the closed-loop system.

Finally, by invoking Lemma 2, we get the global finite-time stability. This completes the proof. ■

Remark 1: Condition (22) in Theorem 1 is not excessively restrictive and limitative, due to the fact that the small positive constant λ_0 does not use in the control law formulation. This implies that λ_0 always exists and can be selected as so small.

Remark 2: Although a nonsmooth term appeared in the closed-loop system, it can be conclude that the uniqueness of the solution of the class of robot systems with revolute joints can be guaranteed from the global asymptotic stability proof. However, the uniqueness of the solution for the robot systems with primatic joints remains to be verified.

Remark 3: The above results still hold true when the desired gravity compensation is replaced by the real gravity compensation, i.e. the control law becomes

$$\tau = G(q) - K_p\text{Sig}(\Delta q)^{\alpha} - K_d\upsilon \tag{46}$$

V. Simulations

Simulations on a two-DOF robot manipulator were conducted to illustrate the effectiveness of the proposed simple finite-time output controller. The entries to model the robot manipulator are, respectively [32]

$$\begin{aligned} M &= \begin{bmatrix}\theta_1+2\theta_2\cos(q_2) & \theta_3+\theta_2\cos(q_2)\\ \theta_3+\theta_2\cos(q_2) & \theta_3\end{bmatrix}\\ C &= \begin{bmatrix}-2\theta_2\sin(q_2)\dot{q}_2 & -\theta_2\sin(q_2)\dot{q}_2\\ \theta_2\sin(q_2)\dot{q}_1 & 0\end{bmatrix}\\ G &= \begin{bmatrix}\theta_4\sin(q_1)+\theta_5\sin(q_1+q_2)\\ \theta_5\sin(q_1+q_2)\end{bmatrix}\end{aligned} \tag{47}$$

Furthermore, a Coulomb friction is also considered in the simulations. To keep the notation used for model (1), it is defined $D=\text{diag}(\theta_6,\theta_7)$, and

$$f_c(\dot{q}) = \begin{bmatrix}\theta_8\,\text{sgn}(\dot{q}_1)\\ \theta_9\,\text{sgn}(\dot{q}_2)\end{bmatrix} \tag{48}$$

where the parameters in the simulations are given in SI units and summarized in Table I.

Inserting(47) into (2) and (5), the upper bounds required to determine the parameters of the controller is obtained as

$$\lambda_M(M)=2.533\,\text{kg}\cdot\text{m}^2,\ C_M=0.336\,\text{kg}\cdot\text{m}^2 \tag{49}$$

The final desired positions were $q_d=\left[\frac{\pi}{4},\ \frac{\pi}{2}\right]^T$ (rad). The sampling period was $T=1\,\text{ms}$. All the initial parameters were set as zero. The gains for the proposed nonsmooth PD plus gravity compensation were chosen in accordance with stability conditions (22)-(26) as: $K_p=\text{diag}(250, 100)$, $K_d=\text{diag}(75, 15)$, $A=\text{diag}(130, 80)$, and $B=\text{diag}(110, 60)$, with $\alpha=0.5$ (Finite PD) and $\alpha=1$ (PD), respectively.

TABLE I PARAMETERS OF THE ROBOT MANIPULATOR

Notation	Value	Notation	Value
θ_1	2.351	θ_6	2.288
θ_2	0.084	θ_7	0.175
θ_3	0.102	θ_8	7.170 if $\dot{q}_1>0$ and 8.049 if $\dot{q}_1<0$
θ_4	38.465		
θ_5	1.825	θ_9	1.724

Note that the proposed controller with $\alpha=1$ will be returned to the conventional PD plus desired gravity compensation. Figs. 1 and 2 illustrate the position errors and requested input torques of the proposed output feedback approach with the conventional linear PD plus gravity compensation scheme. It can be seen that the robot targeted at the final desired position correctly, and after a transient due to errors in initial condition, the position errors tend asymptotically to zero. Furthermore, the fast response of the proposed output feedback controller is achieved in comparison with the conventional linear PD plus desired control scheme.

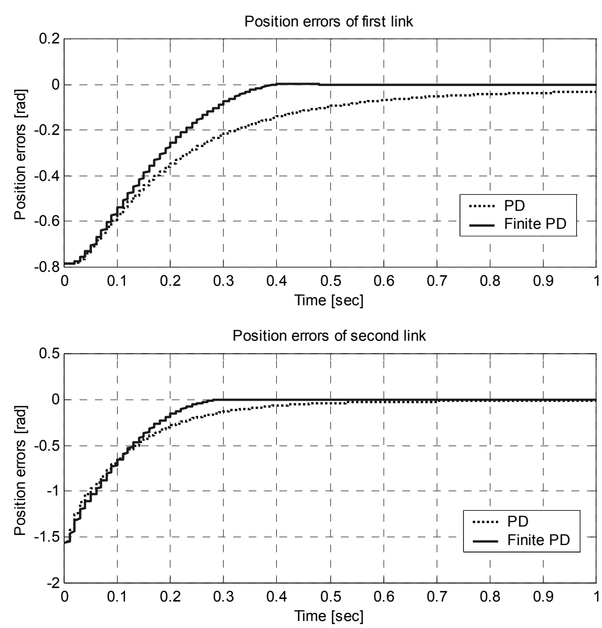

Fig. 1. Position errors.

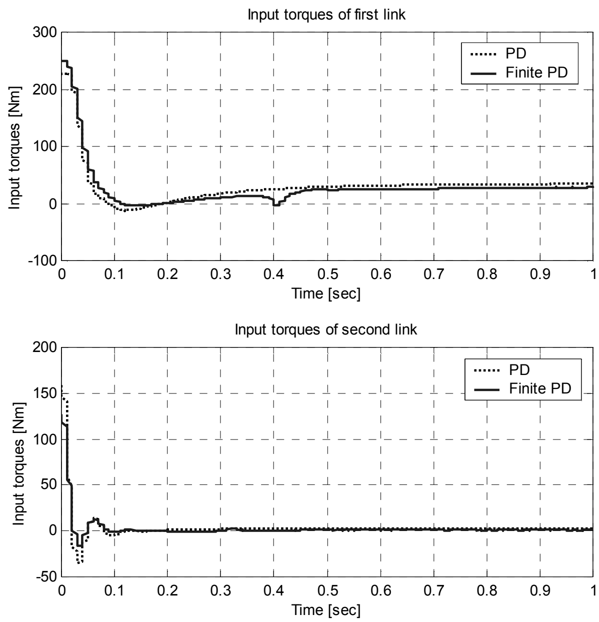

Fig. 2. Input torques.

VI. CONCLUSION

We have proven the global output feedback finite-time regulation of robot manipulators with nonsmooth but continuous PD plus gravity compensation scheme in agreement with Lyapunov's direct method and finite-time stability theory. The developed approach offers an alternative approach for improving the design of the robot regulator, and also solves the global finite-time output feedback control problem for a large class of nonlinear systems with the sole position measurements. The simulations performed on a two-DOF robot manipulator demonstrated the fast response of the proposed controller over the conventional linear PD plus scheme.

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