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Practical Robust Control for Flexible Joint Robot Manipulators

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***Abstract*— In this paper we proposed a practical robust controller which has simple structure, more easy tuning factor, and control forms having direct relation with control performance. This robust control is designed using the motor side dynamics directly. The design procedure consists of three parts. A model based computed torque control part to decrease stead-state errors, a feedback based control part to increase control accuracy, and robust control part to maintain the tracking performance using the nonlinear H-infinity control. The designed robust control is applied to a 6 DOF robot manipulator with joint flexibilities. The proposed robust controller has better tracking performance and advantage in its application.**

***Index Terms*— Industrial Robot, Joint Flexibility, Robust Control, H-infinity Control and Model Based Control.**

I. INTRODUCTION

To manufacture high quality products, we need high accuracy robot manipulator. However, industrial robots are complex structures with many variable parts such as electronics cables and tool kits, also there are nonlinear friction forces, variable viscose values and stiffness coefficients. In other words, an industrial robot has many uncertainties such as unmodelled dynamics, parameter variations and external disturbances. For the reason, model based controls have a limits of tracking performance. Therefore, robust control must be considered.

A recursive design is applied to the design of a stabilizing controller for a class of nonlinear systems. Every system in the class is a series connection of a finite number of nonlinear subsystems which are individually stabilizable. Interesting progress in the recursive design has been achieved in adaptive control of feedback linearizable systems. If the linearized system is linear with respect to the parameters, the recursive design can be used to develop an adaptive control [1]. However this design is not suitable to multi links industrial robot manipulator.

Since many systems inherently have uncertainties such as parameter variations, external disturbances, and unmodelled dynamics, robust control can be considered in the recursive design. To design robust controllers, it is usual to use Lyapunov's second method, as proceeded in the existing results [2,3]. However, a difficulty of using Lyapunov's second method is that a Lyapunov function for control design is required.

Another robust control, which has attracted attention of many researchers, is H_∞ control. Although the nonlinear H_∞ control is derived by the L_2-gain analysis based on the concept of energy dissipation [4,5], its applications are not easy to implement due to the difficulty of obtaining of solution to Hamilton Jacobi inequality (HJ inequality). The H_∞ control problem in nonlinear systems reduces to the solution to HJ inequality. Many methods have been proposed in recent papers [6,7,8,9].

In recursive design of the robust control for robot manipulators with joint flexibilities, a fictitious control is designed as if the link dynamics had independent control. As the robust control, the nonlinear H_∞ control is used. The solution to the HJ inequality can be obtained through a more tractable nonlinear matrix inequality (NLMI) method due to the fact that the matrices forming the NLMI are bounded [9,10]. The control for the joint dynamics, the second subsystem, is designed recursively to satisfy the stability and robustness of the overall system by Lyapunov's second method. Finally, the saturation-type control input of a recursive robust control becomes the function of angular velocity error and bound function denoted the preceding inequality [11,12,13]. Thus, the designer must chose between robust range and tracking accuracy.

In this paper we proposed a practical robust controller which has simple structure, more easy tuning factor, and control forms having direct relation with control performance. Directly we design a robust control using the motor side dynamics. The design procedure consists of three parts. A model based computed torque control part to decrease steady state error, a feedback based control part to increase control accuracy, and a robust control part to maintain the tracking performance using the nonlinear H-infinity control. And we proposed a more practical robust control using only motor side feedback information. The designed control is applied to a 6 DOF robot manipulator with flexible joints. Simulations are performed for this system with inertia and stiffness uncertainties.

This paper is organized as follows. In Sec. II, the dynamics of flexible joint robot manipulator are presented. In Sec. III, robust control and more practical robust control are designed for the system with uncertainties. In Sec. IV, the simulation is presented with a 6 DOF industrial robot. In Sec. V, the conclusions are presented.

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Fig. 1. A 6-DOF industrial robot.

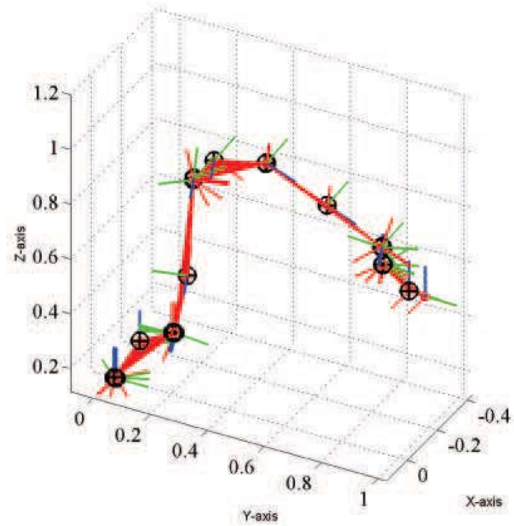

Fig. 2. A SimMechanics model.

II. DYNAMICS OF FLEXIBLE JOINT ROBOT MANIPULATORS

In the flexible manipulator model, the link dynamics is actuated by the spring torque generated by the angular difference between motor and link, and the motor dynamics is actuated by the driving torque.

Consider the dynamics of robot manipulators with joint flexibility. The dynamics are

$$M(x_1)\ddot{x}_1 + C(x_1, \dot{x}_1)\dot{x}_1 + G(x_1) = K(x_2 - x_1) \quad (1)$$

$$J\ddot{x}_2 + K(x_2 - x_1) = \tau \quad (2)$$

where $x_1 \in R^n$ is the link side angle, $x_2 \in R^n$ is motor side angle, $M(x_1)$ is the positive definite symmetric inertia matrix, $C(x_1, \dot{x}_1)$ represents the centripetal and coriolis torque, $G(x_1)$ represents the gravitational torque, J denotes the diagonal inertia matrix of actuator about their principal axes of rotation, and K is the stiffness matrix.

The target model is a heavy payload industrial robot which handles 165kg load as shown in Fig. 1. It is very difficult to model a multi-links serial robot, because of its complex structure in physical relationship. Thus we used MATLAB/SimMechanics toolbox that makes it easy to design a flexible joint mechanical system. Figure 2 is the SimMechanics model of target robot.

III. ROBUST CONTROLLER

The link motion of the robot cannot be directly controlled by the driving torque because of elastic interconnecting mechanism. So usually it is assumed that there is a fictitious control to be used in the position of the motor angle as virtual input for robust stabilization of the link dynamics. And because the fictitious control is not real control, the real control is recursively designed to make the overall system robustly stable. These control method is called back-stepping based robust control.

The finally saturation-type control input of the back-stepping based robust controller becomes the functional equation of angular velocity errors and some bounded values satisfying the inequality equations [13]. In this saturation-type control, the bounded values have an effect on both control accuracy and control robustness. Instead of a only saturation-type control input, we proposed the actual robust control which is composed with a model based computed torque control part, a feedback based control part, and a robust control part.

A. Robust Control Design

Directly we design a robust control using the motor side dynamics. The design procedure consists of three phases. A model based computed torque control part, a feedback based control part, and robust control part to maintain the tracking performance against model uncertainties. The control input, τ is proposed as

$$\tau = \tau_{ct} + \tau_{pd} + \tau_{ro} \quad (3)$$

where τ_{ct} is a computed torque control input, τ_{pd} is a feedback control input, and τ_{ro} is a robust control input to designed with H_∞ theory.

The main target of industrial robots is for robots end point to track the desired trajectory. So we have to derive desired trajectories of motor side angle such as

$$x_{2d} = \hat{K}^{-1}\tau_l + x_{1d} \quad (4)$$

where x_{1d} and x_{2d} are the desired link side angle and the desired motor side angle respectively, and τ_l is link side computed torque control input such as

$$\tau_l = \hat{M}(\ddot{x}_{1d} + K_d\dot{e}_1 + K_p e_1 + K_i \int e_1) + \hat{C}\dot{x}_1 + \hat{G} \quad (5)$$

, $\hat{K}$, $\hat{M}$, $\hat{C}$, $\hat{G}$ are the matrixes with nominal parameter values, and e_1 is link side joint angle error. Being the integral term makes to decrease stead-state error of link-side angle. If there exists parameter uncertainties in link side dynamics, we can drive more reasonable motor side angles, because τ_l has robustness against that.

For the first time, a model based computed torque control input is designed, which controls the nominal model, as

$$\tau_{ct} = \hat{J}\ddot{x}_{2d} + \tau_l. \tag{6}$$

At the Second part, a feedback based PD control input can be chosen as

$$\tau_{pd} = \hat{J}\Lambda\dot{e}_2 + \hat{K}\dot{e}_2 + \hat{K}\Lambda e_2 \tag{7}$$

where $e_2 = x_{2d} - x_2$, and Λ is a positive constant diagonal matrix. This control part has ability to increase tracking performance and converse quickly. We do not contain integral term in this control part to prevent conversing to desired motor angles. Our target is a accuracy tracking of link side angle.

To use the H_∞ theory, the new state s, which is the modified error for motor side joint tracking, is defined as

$$s = -\dot{e}_2 - \Lambda e_2 = -(\dot{x}_{2d} - \dot{x}_2) - \Lambda(x_{2d} - x_2). \tag{8}$$

If the state elements approach zero at $t \to \infty$, the tracking errors of joints approach zero.

Then, the motor side dynamics transforms to

$$\dot{s} = As + Bw + B\tau_{ro}, \tag{9}$$

where $A = -\hat{J}^{-1}\hat{K}$, $B = \hat{J}^{-1}$, and

$$\begin{aligned} w &= (\hat{J} - J)\ddot{x}_2 + \tau_l - K(x_2 - x_1) \\ &= (\hat{J} - J)\ddot{x}_2 + (\hat{M} - M)\ddot{x}_{1d} + (\hat{C} - C)\dot{x}_1 + (\hat{G} - G) \\ &\quad + M(\ddot{x}_{1d} - \ddot{x}_1) + \hat{M}K_p e_1 + \hat{M}K_d\dot{e}_1 + \hat{M}K_i \int e_1, \end{aligned} \tag{10}$$

which is a disturbance vector caused by model uncertainties. And the matrices $\hat{J}$ and $\hat{K}$ are a constant motor inertia and stiffness matrix, respectively.

The performance index matrix, z is designed such as

$$z = Hs + D\tau_{ro}, \quad H^T D = 0, \quad D^T D > 0 \tag{11}$$

where H and D are the constant matrices of suitable dimensions.

There exists a non-negative function $V(s) = s^T P s \geq 0$. The time-derivative of the non-negative energy storage function is

$$\begin{aligned} \dot{V} &= 2s^T P^T \dot{s} \\ &= 2s^T P^T (As + Bw + B\tau_r) \\ &= s^T (P^T A + A^T P)s + 2s^T P^T (Bw + B\tau_{ro}). \end{aligned} \tag{12}$$

Introducing $\gamma^2\|w\|^2 - \|z\|^2$ into the upper equation,

$$\begin{aligned} \dot{V} &= \gamma^2\|w\|^2 - \|z\|^2 - \gamma^2\|w - \frac{1}{\gamma^2}B^T P s\|^2 \\ &\quad + \|D\tau_{ro} + D^{-T}B^T P s\|^2 + s^T\{P^T A + A^T P \\ &\quad + \frac{1}{\gamma^2}P^T B B^T P - P^T B[D^T D]^{-1}B^T P + H^T H\}s \\ &\quad + 2s^T H^T D\tau_{ro}. \end{aligned} \tag{13}$$

If there exists a matrix P satisfying the following HJ inequality such as

$$\begin{aligned} &P^T A + A^T P + \frac{1}{\gamma^2}P^T B B^T P \\ &-P^T B[D^T D]^{-1}B^T P + H^T H \leq 0 \end{aligned} \tag{14}$$

and control input is designed such as

$$\tau_{ro} = -[D^T D]^{-1}B^T P s. \tag{15}$$

Then the derivative of the storage function satisfies

$$\dot{V} \leq \gamma^2\|w\|^2 - \|z\|^2. \tag{16}$$

To derive the HJ inequality for the robust control input, each matrix term of Eq. 8 is substituted into Eq.14, then

$$\begin{aligned} &-(\hat{J}P^{-T})^{-1}\hat{K} - \hat{K}^T(P^{-1}\hat{J}^T)^{-1} + H^T H \\ &+\frac{1}{\gamma^2}(\hat{J}P^{-T})^{-1}(P^{-1}\hat{J}^T)^{-1} \\ &-(\hat{J}P^{-T})^{-1}[D^T D]^{-1}(P^{-1}\hat{J}^T)^{-1} \leq 0. \end{aligned} \tag{17}$$

By premultiplying and postmultiplying the inequality by the positive definite matrices $\hat{J}P^{-T}$ and $P^{-1}\hat{J}^T$ respectively, the HJ inequality becomes

$$\begin{aligned} &-\hat{K}Q\hat{J}^T - \hat{J}Q^T\hat{K}^T + \hat{J}Q^T H^T H Q\hat{J}^T \\ &+\frac{1}{\gamma^2}I - [D^T D]^{-1} \leq 0. \end{aligned} \tag{18}$$

where $Q = P^{-1}$.

Using the Schur complement, Eq. 18 can be described as a NLMI

$$\begin{bmatrix} W & \hat{J}Q^T H^T \\ HQ\hat{J}^T & -I \end{bmatrix} \leq 0 \tag{19}$$

where $W = -\hat{K}Q\hat{J}^T - \hat{J}Q^T\hat{K}^T + \frac{1}{\gamma^2}I - [D^T D]^{-1}$. Thus we only solve the LMI only one times off-line. To design a practical control, computing time is very important point. In this point, this proposed robust control is of great advantage.

Therefore, the stabilizing robust control becomes

$$\begin{aligned} \tau &= \tau_{ct} + \tau_{pd} + \tau_{ro} \\ &= \hat{J}\ddot{x}_{2d} + \hat{M}(\ddot{x}_{1d} + K_d\dot{e}_1 + K_p e_1 + K_i \int e_1) \\ &+ \hat{C}\dot{x}_1 + \hat{G} + \hat{J}\Lambda\dot{e}_2 + \hat{K}\dot{e}_2 + \hat{K}\Lambda e_2 \\ &- [D^T D]^{-1}\hat{J}^{-T}Ps. \end{aligned} \tag{20}$$

B. More Practical Robust Control Design

In industrial robots, angular sensors are located generally on the motor side only. However, the proposed robust control contains link side feedback information. For more practical robust control, we transformed a model based computed torque control input to feed-forward dynamic terms such as

$$\tau_{ct} = \hat{J}\ddot{x}_{2d} + \tau_{ld} \tag{21}$$

where $\tau_{ld} = \hat{M}(\ddot{x}_{1d}) + \hat{C}\dot{x}_{1d} + \hat{G}$. And, the motor side dynamics transforms to

$$\dot{s} = As + Bw + B\tau_r, \tag{22}$$

where $A = -\hat{J}^{-1}\hat{K}$, $B = \hat{J}^{-1}$, and

$$\begin{aligned} w &= (\hat{J} - J)\ddot{x}_2 + \tau_{ld} - K(x_2 - x_1) \\ &= (\hat{J} - J)\ddot{x}_2 + (\hat{M} - M)\ddot{x}_1 + (\hat{C} - C)\dot{x}_1 + (\hat{G} - G) \\ &+ \hat{M}(\ddot{x}_{1d} - \ddot{x}_1) + \hat{C}(\dot{x}_{1d} - \dot{x}_1). \end{aligned} \tag{23}$$

In this case, we can not remove steady state errors. therefore the bigger bound of uncertainties leads to the more inaccuracy performance. Thus we need observer.

Therefore, the stabilizing robust control becomes

$$\begin{aligned} \tau &= \tau_{ct} + \tau_{pd} + \tau_{ro} \\ &= \hat{J}\ddot{x}_{2d} + \hat{M}(\ddot{x}_{1d}) + \hat{C}\dot{x}_{1d} + \hat{G} \\ &+ \hat{J}\Lambda\dot{e}_2 + \hat{K}\dot{e}_2 + \hat{K}\Lambda e_2 - [D^T D]^{-1}\hat{J}^{-T}Ps \end{aligned} \tag{24}$$

This control input do not need link side feedback information, so we can used it without sensor addition.

IV. SIMULATIONS

The robust performance of the proposed robust control for the 6 DOF robot manipulators is verified through simulation against inertia and stiffness uncertainties. For estimating the performance of a proposed controller, we use a rectangular trajectory in the 3 dimensional spaces.

The NLMI is solved off-line because motor inertia and stiffness matrix is constant. Thus we only solve the LMI only one times off-line. The performance level can be determined by parameter γ and weighting matrix H, and the control input energy can be adjusted by using matrix D.

The end position errors are shown in Fig. 3. It has very small size error excepting four corners, starting and stoping times. In these phases, acceleration and deceleration are so high, so this situations are occupied. Figure 4 show the joint angle errors. In starting stage, the noisy signal is caused by initial angular velocity errors.

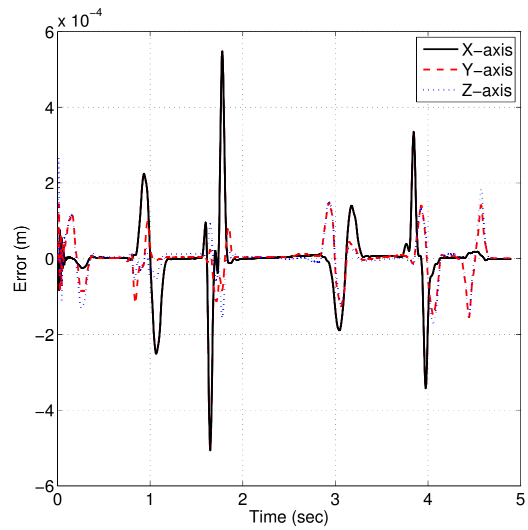

Fig. 3. Position errors of robot end point.

Figure 5 shows the input torque which is the sum of a computed torque control input, a feedback control input and a robust control input.

Figures 6 and 7 show the end position of 6 DOF robot manipulator with model uncertainties of 30%. The proposed robust controller has robustness to the inertia and stiffness uncertainties. 'DT' means a desired trajectory. Therefore, its error is larger. As a result, the proposed robust control has robustness against parameter uncertainty.

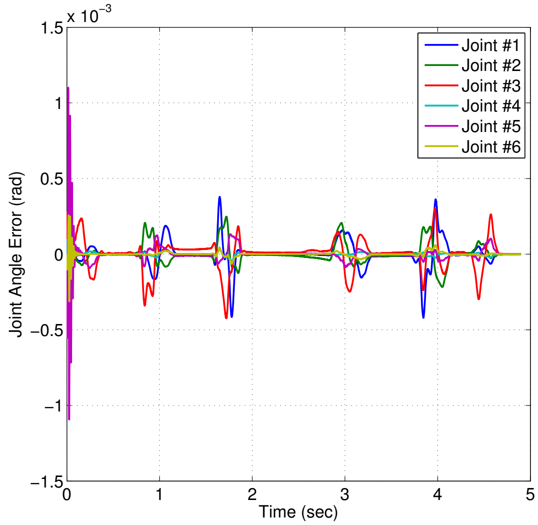

Fig. 4. Joint angle errors.

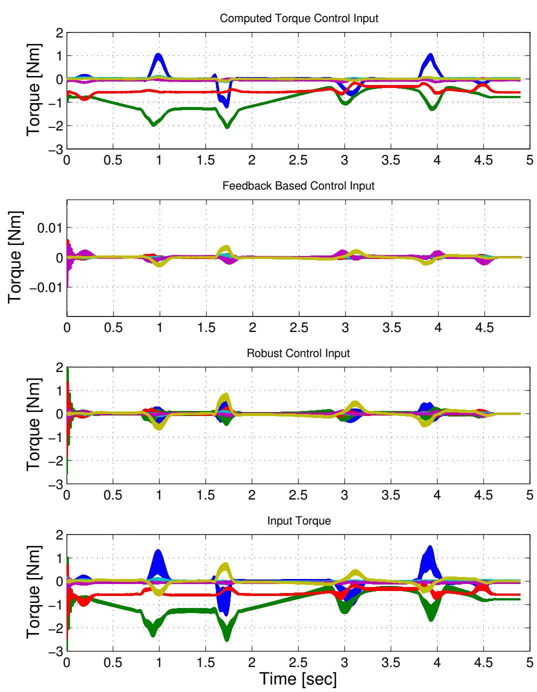

Fig. 5. Input torque.

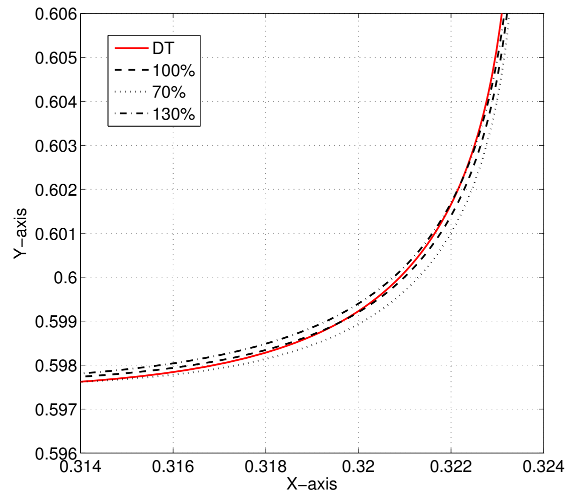

Fig. 6. Position errors under mass uncertainties.

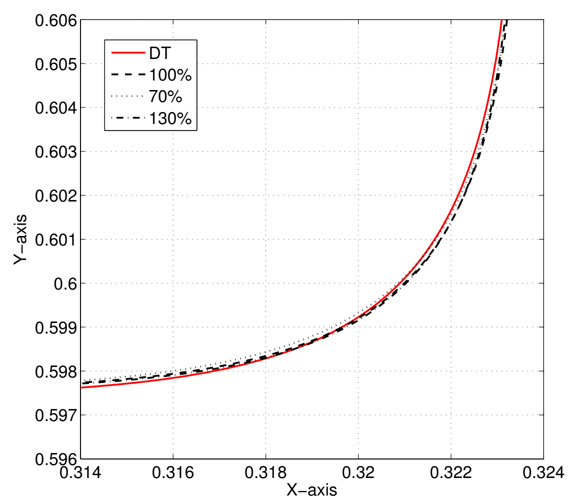

Fig. 7. Position errors under stiffness uncertainties.

V. CONCLUSIONS

A practical robust control was proposed for flexible joint manipulators. It has simple structure, more easy tuning factor, and control forms having direct relation with control performance. Instead of link side dynamics, we design a robust control using the motor side dynamics directly. And the designed controller consists of three phases, a model based computed torque control part, a feedback based control part, and robust control part to maintain the tracking performance against model uncertainties using H-infinity theory. And the proposed robust control has great advantage to computed a algorithm, we only solve the LMI only one times off-line. The effectiveness of the proposed robust control was investigated through simulation on a 6 DOF industrial robot. The designed robust controller has high accuracy performance and robustness against the disturbances and model uncertainties.

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