2008 IEEE International Conference on
Robotics and Automation
Pasadena, CA, USA, May 19-23, 2008

New Framework for Simultaneous Localization and Mapping: Multi Map SLAM

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Abstract—**The main contribution of this paper arises from the development of a new framework, which has its inspiration in the mechanics of human navigation, for solving the problem of Simultaneous Localization and Mapping (SLAM). The proposed framework has specific relevance to vision based SLAM, in particular, small baseline stereo vision based SLAM and addresses several key issues relevant to the particular sensor domain. Firstly, as observed in the authors' earlier work, the particular sensing device has a highly nonlinear observation model resulting in inconsistent state estimations when standard recursive estimators such as the Extended Kalman Filter (EKF) or the Unscented variants are used. Secondly, vision based approaches tend to have issues related to large feature density, narrow field of view and the potential requirement of maintaining large databases for vision based data association techniques. The proposed Multi Map SLAM solution addresses the filter inconsistency issue by formulating the SLAM problem as a nonlinear batch optimization. Feature management is addressed through a two tier map representation. The two maps have unique attributes assigned to them. The Global Map (GM) is a compact global representation of the robots environment and the Local Map (LM) is exclusively used for low-level navigation between local points in the robot's navigation horizon.**

I. Introduction

THIS exposition follows our previous work [1-3] on the use of small baseline stereo cameras in Simultaneous Localization and Mapping (SLAM). There, a detailed study on sensor behavior and sensor modeling were carried out. It was shown that with the use of small baseline stereo cameras, the non linearity of the observation model manifest within very short ranges leading to inconsistencies in filter estimates. We also showed that a simple linearization as in the EKF cannot handle such nonlinearities requiring a more elegant solution. Here we propose nonlinear batch optimization as a suitable alternative to the standard recursive methods [4-6] used in solving the SLAM problem. We begin the batch formulation similar to [7, 8], however it is then extended into the new Multi Map (MM) framework where techniques from the Variable State Dimension Filter [9, 10] are used in order to realize a consistent and efficient solution to the small base line stereo vision based SLAM problem.

Manuscript received September 14, 2007. This work is supported by the ARC Centre of Excellence program, funded by the Australian Research Council (ARC) and the New South Wales State Government.

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The technique has its inspiration in the way humans move about in society. It has been observed that humans tend to use a few important visual cues such as prominent constructions, roundabouts, etc when navigating from one point of interest to the next and tend to discard most of the information utilized in refining the navigation task such as lane markings, traffic signs, etc in between such points along the path.

We intend to use a similar principle in the development of the novel Multi-Map approach. The new representation consists of a Global Map (GM), analogous to important visual cues. This map enables the SLAM algorithm to be bounded globally and the corresponding map size grows monotonically in dimensionality as the robot's exploration horizon expands. The second map called the Local Map (LM) enables the lower level navigation between two visual cues of the global map, analogous to the lane markings, etc. When a particular segment of the navigation is completed, the local map corresponding to the path segment is marginalized from the robots state vector. Hence, the dimensionality of the local map state vector varies from segment to segment but does not correlate with the growth of the robot's exploration horizon, considerably reducing the computational requirements of the overall SLAM algorithm. This novel representation has the added advantage of maintaining estimator consistency through explicit use of batch optimization techniques in the estimation process.

II. SLAM as an Optimization Problem

SLAM could be posed as a multivariate parameter estimation problem [7, 8] where a set of unknown state variables corresponding to the robot pose and map $(\mathbf{x})$ are estimated via the observations $(\mathbf{z})$ made of the environment through sensors on-board the robot. A general solution to the problem is to obtain the maximum a posteriori (*MAP*) estimate,

$$\begin{aligned} \mathbf{x}^* &\triangleq \underset{\mathbf{x}}{\operatorname{argmax}} \quad p(\mathbf{x} \mid \mathbf{z}) \\ &= \underset{\mathbf{x}}{\operatorname{argmax}} \quad p(\mathbf{z} \mid \mathbf{x})\, p(\mathbf{x}) \end{aligned} \tag{1}$$

where $p(\mathbf{z} \mid \mathbf{x})$ is the observation likelihood and $p(\mathbf{x})$ is the prior. Nonlinear batch optimization techniques generally preserve the entire history of the states to be estimated. The large number of observations used in a single optimization cycle could potentially improve the linearization due to more accurate estimates being available for use in the linearization process.

The formulation begins by noticing that the observation likelihood in the context of SLAM contains both observations $(\mathbf{z})$ to features as well as odometry measurements $(\mathbf{u})$. These two types of measurements are conditionally independent and could be factorized yielding the following optimization problem,

$$\mathbf{x}^* \triangleq \underset{\mathbf{x}}{\operatorname{argmax}} \quad p(\mathbf{z},\mathbf{u} \mid \mathbf{x}) p(\mathbf{x}) \tag{2}$$

Therefore, the constrained observation model is,

$$p(\mathbf{z}_i, \mathbf{u}_j \mid \mathbf{x}) = p(\mathbf{z}_i \mid \mathbf{x})\, p(\mathbf{u}_j \mid \mathbf{x}) \tag{3}$$

where $\mathbf{x}$ contains both map and the entire set of poses of the robot with $i = 1,\ldots,z \quad j = 1,\ldots,n \quad z,n$ being the number of observations and odometry measurements respectively. Assuming observations to be Gaussian distributed,

$$p(\mathbf{z}_i, \mathbf{u}_j \mid \mathbf{x}) = \eta \exp\left(-\frac{1}{2}(\boldsymbol{\varepsilon}_i^T \mathbf{R}_i^{-1} \boldsymbol{\varepsilon}_i)\right) \exp\left(-\frac{1}{2}(\boldsymbol{\mu}_j^T \mathbf{U}^{-1} \boldsymbol{\mu}_j)\right) \tag{4}$$

where, $\boldsymbol{\varepsilon}_i = (\mathbf{z}_i - \mathbf{h}_i(\mathbf{x}))$ is the innovation of the stereo vision observation and $\mathbf{h}_i(\mathbf{x})$ is the predicted observation to the feature. $\boldsymbol{\mu}_j = \mathbf{u}_j - \mathbf{g}_j(\mathbf{x})$, where $\mathbf{g}_j(\mathbf{x})$ is the predicted odometry measurement. $\mathbf{R}_i$ is the observation covariance (dependent on the observation, see [1]) and $\mathbf{U}$ is the covariance of the odometry measurements. Assuming a uniform prior over the state variables, the cost function ($F(\mathbf{x})$) to minimize in order to solve the SLAM problem is derived by taking the negative of the log of the likelihood function,

$$\begin{aligned} F(\mathbf{x}) &= \frac{1}{2}\sum_{i=1}^{z} \boldsymbol{\varepsilon}_i^T \mathbf{R}_i^{-1} \boldsymbol{\varepsilon}_i + \frac{1}{2}\sum_{j=1}^{n} \mu_j^T \mathbf{U}^{-1} \mu_j \\ &= \frac{1}{2}(\boldsymbol{\varepsilon}^T \mathbf{R}^{-1} \boldsymbol{\varepsilon} + \mu^T \mathbf{U}^{-1} \mu) \end{aligned} \tag{5}$$

Then the new MAP estimate is,

$$\mathbf{x}^* = \underset{\mathbf{x}}{\operatorname{argmin}}(\boldsymbol{\varepsilon}^T \mathbf{R}^{-1} \boldsymbol{\varepsilon} + \mu^T \mathbf{U}^{-1} \mu) \tag{6}$$

This represents the standard least-squares problem generally solved through Gauss-Newton iterations,

$$\mathbf{x}_{k+1} = \mathbf{x}_k + \Delta \mathbf{x}_k \tag{7}$$

In order to minimize the cost function, the algorithm starts at $k = 0$ with an initial estimate of the state vector $\hat{\mathbf{x}}_0$, and proceeds to calculate the approximations to the derivatives. The first derivative,

$$\nabla F(\mathbf{x}) = \mathbf{b} = -\mathbf{J}_h^T \mathbf{R}^{-1} \varepsilon - \mathbf{J}_g^T \mathbf{U}^{-1} \mu \tag{8}$$

and the Hessian matrix,

$$\nabla^2 F(\mathbf{x}) \approx \mathbf{A} = \mathbf{J}_h^T \mathbf{R}^{-1} \mathbf{J}_h + \mathbf{J}_g^T \mathbf{U}^{-1} \mathbf{J}_g \tag{9}$$

Then at the k^{th} iteration,

$$\mathbf{A}_k \Delta \mathbf{x}_k = \mathbf{b}_k \tag{10}$$

and an improved estimate of the states are realized through (7). Iterative sequence continues until residual is minimal.

III. MULTI MAP SLAM

From a practical perspective MM framework utilize two different techniques for image registration corresponding to the two map representation. Features corresponding to the high level global map rely on visually salient features that can be recognized using high dimensional descriptors that are scale and affine invariant. The descriptive nature of these features provides the necessary loop closure information. In this work, we have chosen SURF [11]. SURF based Global Map (GM), is sparse and the features could only be observed at certain points of interest (PoI) in the robots trajectory. The low level local map (LM) is used for navigating the robot between the points of interest. The features in the LM need to be tracked between consecutive frames for data association. Thus, the tracker needs to be simple and fast without requiring complex descriptors for feature correspondences. Although there are a number of trackers proposed in the literature, we have chosen the KLT [12, 13] due to its simplicity and speed.

IV. BATCH OPTIMIZATION WITH MULTIPLE MAPS

Let's consider the simple navigation scenario depicted in Fig.1. The robot starts at the origin and completes a small loop by returning to the origin after a temporal lapse. In this particular example, a small cluster of GM features are located near the origin of the robots trajectory marked GM-node 1. There is only a unique PoI at the origin of the reference frame. Once the robot completes the journey, it sees the GM features again and realizes the loop closure. Until the robot reaches the end of the loop, it uses the features from the LM for achieving bounded estimates of its states. The algorithm that extends the batch optimization to accommodate the two maps is described next.

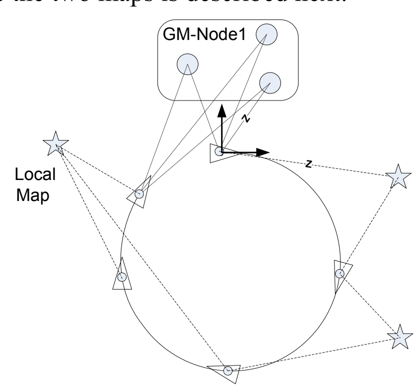


Fig. 1 Simplest form of the MM approach with a single node of Global Map. GM-Node1 provides Loop Closure information

The first extension to the solution presented in previous section occurs when the state vector is extended to accommodate the GM features,

$$\mathbf{x} = \left[\mathbf{x}_r, \mathbf{x}_{LM}, \mathbf{x}_{GM} \right]^T \tag{11}$$

The state vector contains two independent maps. The LM ($\mathbf{x}_{LM}$) contributes to the frame-to-frame optimization whilst the GM ($\mathbf{x}_{GM}$) contributes to the loop closure. The initialization of batch optimization is provided by a local EKF, which only operate between two consecutive PoIs. After carrying out extensive simulation and experimentations, we concluded that EKF can provide a reasonable initialization for the optimization algorithm. Therefore,

$$\hat{\mathbf{x}}_0 = \left[\hat{\mathbf{x}}_r^{EKF}, \hat{\mathbf{x}}_{LM}^{EKF}, \hat{\mathbf{x}}_{GM,0} \right]^T \tag{12}$$

Since the Global Map features need to be integrated in the optimization algorithm, an initial estimate of the GM states ($\hat{\mathbf{x}}_{GM,0}$) is required. The GM feature observations with the EKF robot pose estimates ($\hat{\mathbf{x}}_r^{EKF}$) is used to initialize the map. Once the state and the initial estimates are defined the minimization proceeds as discussed in the previous section.

V. Generalized Multi Map Smoother

In this section, we describe the generalized multi map filter for extended arbitrary navigation. Fig. 2 shows a simplified navigation scenario, which could be used to appreciate the Multi Map Smoothing (MMS) algorithm. Since the robot is not returning to its origin, the GM-node1 features can not be used in closing this larger loop causing EKF based initialization to deviate significantly. This leads to inconsistency in the estimates of the MMF. Thus, PoIs are required at regular intervals of the robot path in order to,

1.) maintain the consistency of the MMF estimates by executing the MMF at shorter intervals so the accuracies of the initial estimates of consecutive optimization cycles are increased.

2.) anticipate loop closures at locations other than the origin.

The efficiency of the MMF algorithm is improved by marginalizing LM features at appropriate intervals. Techniques based on the VSDF [9, 10] algorithm are used to maintain the consistency of the estimates while features are marginalized. The KLT based feature tracker is not capable of associating data with significant temporal laps. This unique character of the LM features results in marginalization exclusive of information loss.

A. Estimation Process

The algorithm could be outlined as follows. The first PoI is created at the origin (GM-Node 1 in Fig. 2) where the robot initiates its navigation as in the case of previous example. Following PoIs are scheduled heuristically considering the anticipated navigation pattern and knowledge of the environment in which the robot operates. Thus in Fig. 2, PoIs are placed after each turn of the robot. As described earlier PoIs are marked with a set of view invariant features described by high dimensional descriptors (GM-node 1,2) thus enabling detection of a Loop Closure. Once a PoI is reached, algorithm sets out to execute the MMS utilizing the LM and GM information currently available (for instance at LC-1 the MMS has GM-Node 1,2 and LM-2 features in the augmented map state vector). This is similar to the previous scenario. VSDF techniques are used to marginalize the features from the state vector that do not contribute to the current execution of the MMF. The VSDF provides a means to preserve the 'contributions' to the estimator from the observations made to these marginalized features by linearizing these measurements and incorporating them into a Gaussian prior within the batch optimization paradigm.

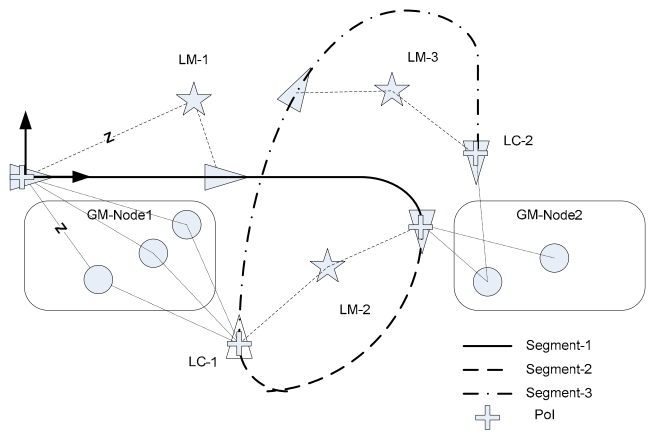


Fig. 2 A simplified illustration of the Generalized Multi Map approach. The robot starting from the origin takes a slightly complicated route. Apart from the origin, PoIs are created when the robot makes a 'sharp' turn. The first Loop Closure (LC-1) occurs when the features observed at the first PoI are observed again from the third PoI. A second Loop Closure (LC-2) occurs when some of the features observed at second PoI are re-observed from 4th PoI.

The MMS is essentially a recursive smoother with dynamically varying state dimension. The algorithm could be summarized as consisting of following components. Assuming that the robot is at the p^{th} PoI, ($p>2$) the current LM contains only a partial set of the LM features that are observed from the PoI at *p-1*. Thus the new cost function to minimize contains a prior reflecting the linearized control input data as well as the non linear measurement data,

$$F(\mathbf{x}) = (\hat{\mathbf{x}}_0 - \mathbf{x})^T \mathbf{A_0} (\hat{\mathbf{x}}_0 - \mathbf{x}) + (\mathbf{d} - f(\mathbf{x}))^T \mathbf{Q}^{-1} (\mathbf{d} - f(\mathbf{x})) + (\mathbf{z} - h(\mathbf{x}))^T \mathbf{R}^{-1} (\mathbf{z} - h(\mathbf{x})) \tag{13}$$

Also it now requires augmenting the state vector with the new LM features, its corresponding observations, robot poses encompassing the current LM and any newly initialized GM features. Thus the new state change from the previous PoI,

$$\mathbf{x}=\begin{bmatrix}\mathbf{x}_r(p-1)\\ \mathbf{x}_{LM}(p-1)\\ \mathbf{x}_{GM}(p-1)\end{bmatrix}\rightarrow\quad \mathbf{x}=\begin{bmatrix}\mathbf{x}_r(p)\\ \mathbf{x}_{LM}(p)\\ \mathbf{x}_{GM}(p-1)\\ \mathbf{x}_{GM}(p)\end{bmatrix} \quad (14)$$

where ($\mathbf{x}_r(p)$) represents the limited pose history attributed to the p^{th} LM and ($\mathbf{x}_{LM}(p)$) contains the LM features belonging to the p^{th} LM. $\mathbf{x}_{GM}(p-1)$ contains GM features that are already initialized during previous PoIs beginning from the origin and ($\mathbf{x}_{GM}(p)$) are the newly observed GM features yet to be estimated. Thus it is clear that the only component of this dynamic state vector that contributes to monotonic increase in the dimension is the observation of new GM features. Contribution from the LM features to the dimensionality of the state vector varies cyclically with the passing of each PoI.

Having defined the state vector and the cost function, Levenberg-Marquardt optimization could be performed by defining the derivatives with respect to the current state vector,

$$\begin{aligned}\mathbf{A}&=\mathbf{A_0}+\mathbf{J}_h^T\mathbf{R}^{-1}\mathbf{J}_h+\mathbf{J}_f^T\mathbf{Q}^{-1}\mathbf{J}_f\\ \mathbf{b}&=\mathbf{b}_0+\mathbf{A_0}(\hat{\mathbf{x}}_0-\hat{\mathbf{x}}^-)+\mathbf{J}_h^T\mathbf{R}^{-1}\boldsymbol{\varepsilon}_x+\mathbf{J}_f^T\mathbf{Q}^{-1}\boldsymbol{\varepsilon}_y\end{aligned} \quad (15)$$

Once optimization has converged LM features belonging to the current segment of the robot path are marginalized, corresponding observations, robot poses and any GM observations related to the removed robot poses are also removed. This smoothing cycle is applied recursively at each new PoI. Thus the consistency of the state estimate is preserved at regular intervals of the robots trajectory, while maintaining an efficient algorithm for arbitrary navigation.

B. Marginalizing LM from the MM

The first requirement in the MMS approach is to remove the LM features from the state once they are used to improve the pose estimates of the relevant potion of the navigation. Consider the state vector in (11), which is the starting point for the Multi Map optimization at a given (p^{th}) PoI ,

$$\mathbf{x}^{MMF}=\left[\mathbf{x}_{r,p},\mathbf{x}_m^{MMF}\right]^T \quad (16)$$

where $\mathbf{x}_m^{MMF}=\left[\mathbf{x}_{LM},\mathbf{x}_{GM}\right]^T$ is the combined multi map at the current PoI. It is required to remove ($\mathbf{x}_{LM}$) from ($\mathbf{x}_m^{MMF}$). This could be achieved by straight forward removal of the corresponding map elements from the overall state vector. Thus,

$$\mathbf{x}^{MMF}=\left[\mathbf{x}_{r,p},\mathbf{x}_{GM}\right]^T \quad (17)$$

However the optimization algorithm now contains a prior inverse covariance term which should also be marginalized accordingly. Assume that $\mathbf{A}_0$ is reordered to reflect the states to be removed ($\mathbf{A}_{LM,*}$) and retained ($\mathbf{A}_{R,*}$) such that,

$$\mathbf{A}_0=\begin{bmatrix}\mathbf{A}_{LM,LM} & \mathbf{A}_{LM,R}\\ \mathbf{A}_{LM,R}^T & \mathbf{A}_{R,R}\end{bmatrix} \quad (18)$$

Then using the matrix inversion lemma, the new prior after marginalization of LM could be calculated as,

$$\mathbf{A}_0\leftarrow\mathbf{A}_{R,R}-\mathbf{A}_{LM,R}^T\mathbf{A}_{LM,LM}^{-1}\mathbf{A}_{LM,R} \quad (19)$$

This technique lends itself to the removal of poses prior to the current PoI as well.

C. Removing Observations Associated with Marginalized Features

As the MM approach extends beyond the first loop closure it becomes necessary to remove observations corresponding to the previous LM features, since they are marginalized as described in the previous section. The technique described in section *A* could be used to linearize such observations. As a consequence the pose estimates belonging to these parts of the journey are no longer as accurate as would be when improved local navigation information was available through the LM. Therefore, these parts of the robot pose should also be marginalized from the state vector. The technique in *A* also lends itself to be used in linearizing any odometry measurements corresponding to marginalized robot poses in the state vector.

VI. RESULTS

A. Simulation Results

Fig. 3 shows the map estimates from the MMS algorithm for a loop similar to that described in Fig 2. There are two nodes of the GM of which the first node initialized at the beginning of the robots trajectory aids the small loop closure and the second node first initialized during the third leg of the smaller loop aids the final loop closure. An EKF based SLAM was used to generate the initial estimate $\hat{\mathbf{x}}_0$. The estimated error from this filter is shown in Fig. 4(a) which as discussed in [1, 2] produce inconsistent state estimates. Fig. 4(b) shows the error estimate from the new MMS approach. As can be seen from the figure, the estimates are consistent and the two loop closures corresponding to the two nodes of the GM are also visible, indicated by the decreasing uncertainty.

Finally in Fig. 5, the average Normalized Estimation Error Squared (NEES) [14] for 50 trials of the simulation are presented. The NEES is a measure of estimator consistency which could be used when the true locations of the states are known. The estimates are within the 2σ confidence bounds conforming that the MMS is capable of producing consistent state estimates where EKF failed to produce consistent state estimates in comparably smaller sized loops using the particular stereo vision sensor as shown in [1, 2].

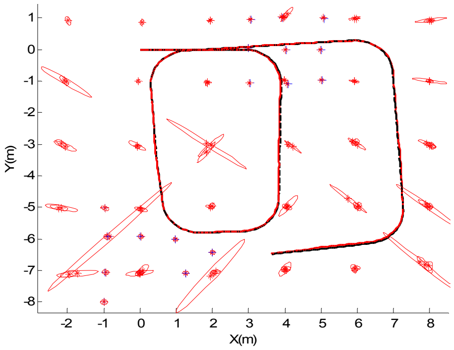
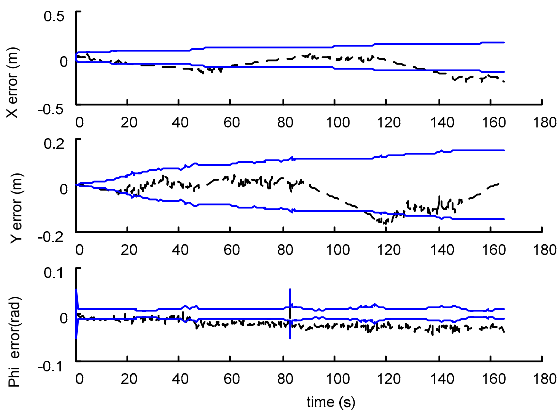
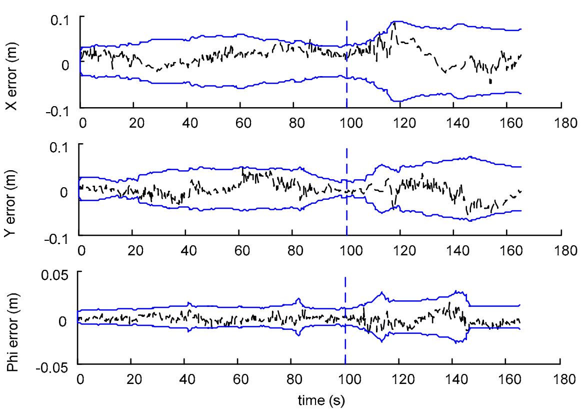


Fig. 3 Map estimate from the MMS. (a) The complete map comprising of the two nodes of GM ('+') and the LM ('*')



(a)



(b)

Fig. 4 Robot pose estimates from (a) EKF and (b) from the MM Smoothing. The first loop closure occurs near the 100sec corresponding to the first node of the GM.

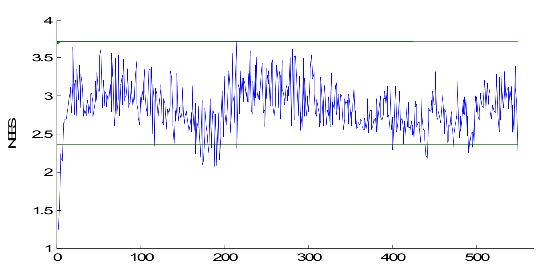


Fig. 5 Average NEES results for MMS

B. *Experimental results*

Below we assess the new framework in a practical indoor application using an Active Media Robotics Pioneer 2 as the robotic platform and videre stereo vision sensor. Fig. 6 shows a map of the environment generated using the pose estimates of the batch algorithm and raw laser scans. The thickness of the walls is a general indication of the error distribution in the robot path estimates. Therefore, quantitatively the algorithm provides sensible estimation of the robot path.

Fig. 7 shows the robot pose error relative to 'ground truth' given by a laser range finder based EKF. The outer (dashed) line is the compounded error estimate $\left(2\sqrt{\sigma_{laser}^2+\sigma_{vision}^2}\right)$. The 2σ error bounds from the batch algorithm as well as from the vision EKF are shown in the graph for comparison. As expected, vision EKF produced optimistic results. Apart from the error in y estimate, both x direction and heading errors appear to be consistently estimated relative to the compounded error bound. It is possible that the off shoot in y direction to be attributed to any accumulated errors in the laser EKF prior to the loop closure.

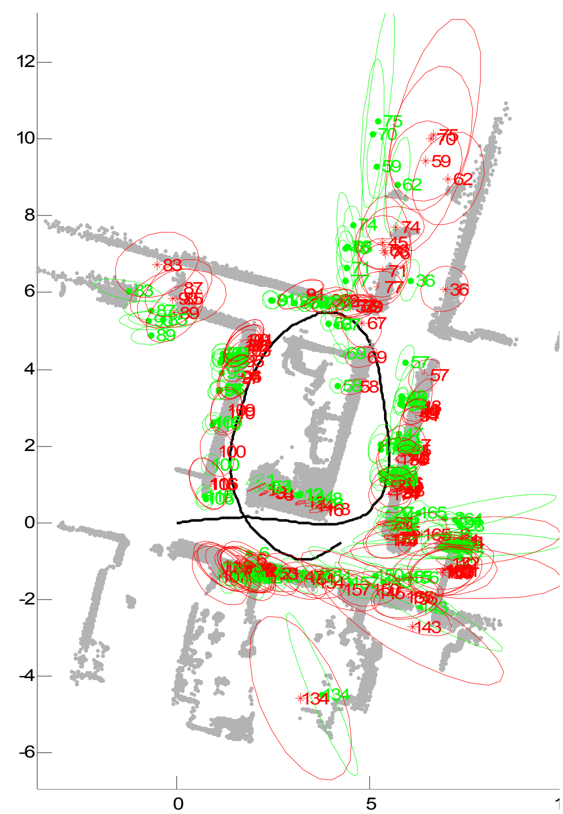


Fig. 6 SLAM results using the batch optimization algorithm. Red '*' indicates map estimates from the batch algorithm. Green '.' indicates the vision EKF map estimates. Green and red ellipses represent the covariance estimates from EKF and batch algorithms respectively.

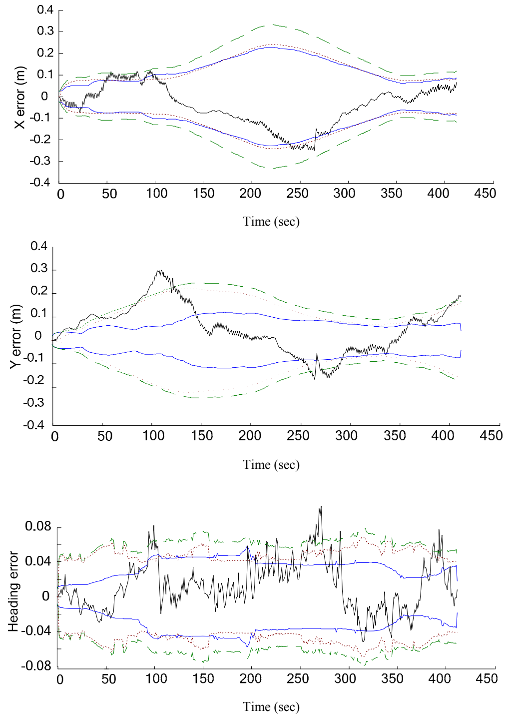


Fig. 7: Comparative robot 2σ bounds (dashed lines indicate the combined estimate of laser and batch algorithm, dotted lines indicate estimates from the batch algorithm and the solid lines are from the vision EKF) and the error in Multi map SLAM robot pose estimation relative to the laser based EKF (ground truth)

VII. Conclusion

This paper presented a novel framework for the small baseline stereo vision based SLAM The unique two tier map representation provides a compact method for representing large environments through a global map. The local map allows for temporary use of large number of features for local navigation. Marginalization of these features maintains the tractability of the developed algorithm within manageable constraints. With high dimensional feature descriptors, the global map provides the loop closure. We have demonstrated the multi map approach in an indoor environment to produce consistent estimations. Further work is being carried out in order to generalize the framework as well as to study the computational aspects of the algorithm.

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