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The Generation of Environmental Map Based on a NDT Grid Mapping
-Proposal of Convergence Calculation Corresponding to High Resolution Grid-

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Abstract— **The paper presents a new convergence calculation method of the Normal Distributions Transform (NDT) scan matching for high resolution grid map. The proposed method employs Interactive Closest Point(ICP) algorithm to find corresponding point, even if the input scan points are in the outside of the reference grid area. It also enlarges the convergence area by modifying the eigenvalue of normal distribution so that the evaluation value can be driven effectively by all the pairing data for improving convergence speed. In the normal NDT scan matching algorithm, the grid size especially has much effect on the capability of convergence area and speed. Thus, it is difficult to generate the detailed map with small grids. The proposed convergence approach can solve the problem by the phased adjustment of convergence area. Experimental result shows the feasibility of the detailed environmental map generated by the NDT grid mapping.**

I. INTRODUCTION

Simultaneous Localization and Mapping (SLAM) is important technique for discovering unknown environment by the mobile robot. Normal Distributions Transform (NDT) algorithm [1] is one of the scan matching methods for the convergence of measured points. It employs the grid mapping method to storage input data. The NDT scan matching has features of fast convergence speed and memory saving for data administration. When we use the NDT for small robots, the grid size should be small and detailed map is necessary according to the robot size. However, there are problems to apply the NDT to the detailed map using small grid. Since the grid size is small, the grid can capture only a few input points in one scanning, so that there are only a few grids having the normal distribution value. Furthermore, a small error makes the input data be in the outside of the reference grid area. If we have the compact and high resolution sensor, the first problem is solved but second is difficult. In this paper, we propose a new convergence calculation method of the Normal Distributions Transform (NDT) scan matching for high resolution grid map.

For generating high resolution grid map with NDT algorithm, we propose two approaches. First, the Interactive Closest Point (ICP) algorithm is employed to find corresponding point when the input scan points are in the outside of the reference grid area.Secondly, the convergence area adjustment method is proposed to improve the evaluation value of all the pairing data. The convergence area magnification is controlled by using the characteristics that the normal distribution expands as its eigenvalue becomes large. The method is implemented by the phased adjustment of convergence area. The convergence area is expanded according to the average distance between the initial input scanning points and the corresponding reference grids. After the input points come close to the reference grids, the input points are evaluated and adjusted precisely by using the original normal distribution.

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Let M and N be the number of points of the reference scan and the input scan respectively.

II. PREVIOUS WORK

A LRF or a stereo camera is used in many researches to generate an environmental map. There are several kinds of map representation, e.g. raw point data, grid, edge and plane. The map expressed by point data is highly precise, but there is a problem that the data amount increases as the number of measurement grows. On the other hand, the grid expression can suppress the increase of the amount of data [2], but the accuracy of the grid map depends on its grid size. When the capacity of memory is limited, the grid expression is effective.

In map-making, scan matching is necessary to prevent accumulation of errors of odometry and dead reckoning. Iterative Closest Point (ICP) algorithm [3] is the most famous scan matching algorithm. This algorithm estimate the optimal pose by minimizing the sum of squared distance between corresponding points of the two scans. To find the nearest neighboring point of each input point, the calculation lord takes $O(MN)$. From the point of view of the data save, there is a difficulty that the data volume will be increased in proportion to the number of measurements. The major calculation lord of the ICP is searching of a nearest neighboring point. With using k-d tree space quantization algorithm, it takes $O(Nlog(M))$ [4]. Building the k-d tree structure requires additional $NlogN$ time. As another nearest-neighbor search method, there is an Elias method [5] which subdivides the space into grids. Because this method combs every grid sequentially, it is fast to find the reference data when those data are in neighboring grids. However, whether it is faster to use an Elias method or a k-d tree method depends on the shape of the map data. As the recent research of ICP, Ohno uses a modified ICP algorithm and built a real-time 3D map [6].

Biber proposed the Normal Distributions Transform (NDT) algorithm as a new scan matching method [1]. This method subdivides space into grids at regular intervals, and in each grid, the distribution of the point data are approximated by the normal distribution. Whereas the ICP

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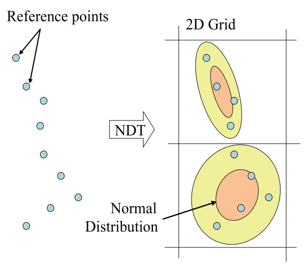


Fig. 1. NDT convertes for reference scan

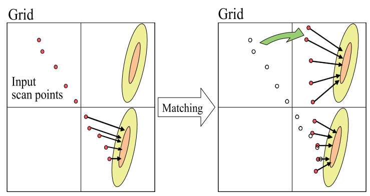


Fig. 2. Flow of NDT scan matching

algorithm matches the points of the two scans, the NDT algorithm calculates evaluation value between the input point and a normal distribution of the reference scan which is in the same grid. It is not necessary for NDT algorithm to find the reference scan for the input scan. The calculation amount is only O(N). Magnusson [7] and Takeuchi [8] extended 2D-NDT to three dimensions. Furthermore, Takeuchi improved performance by multiresolutional method magnifying long-distance grid where the error in the rotation estimate causes greatly wrong movement. In addition to the multi resolution method, Ripperda extended a convergence radius by applying Levenberg-Marquardt method to an iterative calculation [9]. The advantage of the NDT algorithm is that the calculation amount and the data volume are little. These are realized by dividing space, and a matching performance depends on the grid size. In the previous researches, moving range of a robot is set in a grid size. And even if it's small, the grid size is several tens of centimeters. A map becomes rough by only displaying the accumulated grids. Alternatively, the point data saved in each grid are displayed in the NDT. Furthermore, Biber takes another method using point data saved in a grid to upgrade the matching performance.

III. NDT Grid Map

This section describes the NDT scan matching and the grid map.

A. Normal Distributions Transform (NDT)

The NDT algorithm is one of the scan matching methods. In this algorithm, a space is divided into grids, and original point cloud in the grid is converted into one normal distribution which characterizes the distribution of points inside each cell. To evaluate the matching, NDT uses defined score function. Because the calculation is performed only with a grid including both reference scan and input scan, the method can achieve calculation load of O(N).

The NDT process is as follows; First, the point data of each grid are converted into normal distribution, and these are changed into reference scan(Fig.1). Second, when a input scan is obtained, the input scan is matched to the reference scan (Fig.2). For such an algorithm, it is necessary for each grid to store and update an average coordinate and a covariance matrix of reference scan.

Now, we set following variables.

- $p = (p_i)^t_{i=1,2,3} = (t_x, t_y, \theta)^t$: A coordinate transformation parameter between the reference scan and the input scan, where $(t_x, t_y)^t$ discribes the translation and θ is the rotation.
- x_i: A point coordinate of the input scan
- $x_i^{'}$:The input scan data which were transformed by coordinate transformation parameter p
- $q_i, \mathbf{\Sigma_i}$: The covariance matrix and the mean of the corresponding normal distribution to a point $x_i^{'}$

The evaluation function is difined as,

$$E(p) = \sum_{i}^{N-1} exp \frac{-(x_i^{'} - q_i)^t \mathbf{\Sigma}_i^{-1} (x_i^{'} - q_i)}{2}. \tag{1}$$

The expression that input data x_i is converted into $x_i^{'}$ by coordinate transformation parameter p is as follows:

$$x_i^{'} = \begin{pmatrix} x^{'} \\ y^{'} \end{pmatrix} = \begin{pmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \end{pmatrix} = \begin{pmatrix} xcos\theta - ysin\theta + t_x \\ xsin\theta + ycos\theta + t_y \end{pmatrix}. \tag{2}$$

Since the Newton's Algorithm is a nonlinear function minimization algorithm, an optimizing function for NDT scan matching must be;

$$f(p) = -E(X, p). \tag{3}$$

Paramater p is updated by following equation;

$$\Delta p = -H^{-1} g \ , \ p_{new} = p_{old} + \Delta p, \tag{4}$$

where g and H are calculated for each input points by following equations;

$$g = \sum_{i=0}^{N-1} \widetilde{g}_i \quad , \quad H = \sum_{i=0}^{N-1} \widetilde{H}_i. \tag{5}$$

$\widetilde{g}_i$ and $\widetilde{H}_i$ are partial differential and second partial differential of optimizing function $f(p)$ with p. (For details, please refer to [1])

When we convert input scan into reference scan, we can reduce computational amount by calculating following equations sequentially, instead of calculating mean q_j and covariance $\mathbf{\Sigma}_j$.

$$m_j = m_{j_{old}} + x \ , \ S_j = S_{j_{old}} + xx \tag{6}$$

$$q_j = \frac{m_j}{M_j} \quad , \quad \mathbf{\Sigma_j} = \frac{S_j - q_j m_j^t}{M_j} \tag{7}$$

Each Grid stores m_j and S_j, and these are calculated by eq. (6) when reference scan need to be updated. A average q_j and a covariance matrix $\mathbf{\Sigma_j}$ are calculated by eq. (7), only when the scan matching is operated.

B. Grid Map

A grid map is one of the representation methods. In this method, space is divided into a lattice of congruent cells. When the distance data from a sensor are saved to the grid map, only the attributes of the data are stored in each grid and coordinate values are destroyed. From such a point, memory consumption does not increase even if a new input scan was stored. In this research, a grid size is 2[cm] and the size of a total grid space is 8[m]. The grid size is equal with the size of robot's foot. Each grid maintains the following elements:

- *DataNum* :The number of data
- m :A summation of the coordinate vectors
- S :A matrix which becomes the covariance matrix

C. Differences between the NDT grid map and the original NDT

The purpose of our research is to produce high resolution grid map. There are some differences between the original NDT and our NDT grid map. In the original NDT, point data are saved and used for map representation, and the grid size is set comparatively large. In other words, the grid is used for an assistance of the matching. On the other hand, in the NDT grid map, grid is used for both map representation and scan matching. Point data obtained from LRF are thrown away when the point data were reflected to reference scan.

IV. The convergence calculation with the high resolution grid

A. Proposal technique

A grid size is a key problem when using the NDT algorithm. If a grid size is small, the map becomes high resolution, but a computational amount increases. On the contrary, if a grid size is big, a computational amount becomes little, but the resolution becomes low.

Even if a alteration of parameter p is small, the small grid leads a problem that combinations of a reference scan and a input scan will be easily changed, and the evaluation value will be unsteadily changed. Furthermore, the original NDT scan matching does not make pairs between the reference scan and the input scan which is in a neighboring grid, and even a little movement puts the input scan out from gravitative area. To solve these problems, we add three following operations to the NDT algorithm.

1) The search of neighboring grids
The NDT algorithm has a merit that it is not necessary for input scan to search for paired reference scan. However, this becomes a problem when making a map with small grids. Therefore we search for nearest neighborhood reference scan from neighboring grids by Elias method.
2) Expansion of normal distribution
When reference scan in neighboring grids is taken into account, but when the distance of the two scans is too long, the conformity degree of the pair becomes small and does not affect evaluation value. As a solution of this problem, we enlarged bases of the normal distribution to give an effectual value for a long-distance pair. Specifically, we enlarged two eigenvalues of the covariance matrix $\mathbf{\Sigma}$ of the eq. (7). Let φ_j be an eigenvector of a eigenvalue λ_j, and let k be an enlargement factor of the eigenvalue, a new covariance matrix $\mathbf{\Sigma_j'}$ is given by following principal component analysis.

$$\mathbf{\Sigma_j'} = V\Lambda' V^t \tag{8}$$

$$V = [\varphi_1 \;\; \varphi_2] \quad , \quad \Lambda' = \begin{bmatrix} k\lambda_1 & 0 \\ 0 & k\lambda_2 \end{bmatrix}$$

3) Adjust updating speed of Newton method
When we use Δp provided by Newton method without modification, input scan moves beyond the grid size frequently and update of the evaluation value changes uncertainly. Therefore we expect the maximum of the evaluation value with magnification of the eigenvalue and the number of pairs, and we adjusted parameter update speed in accordance with a current adaptation situation.

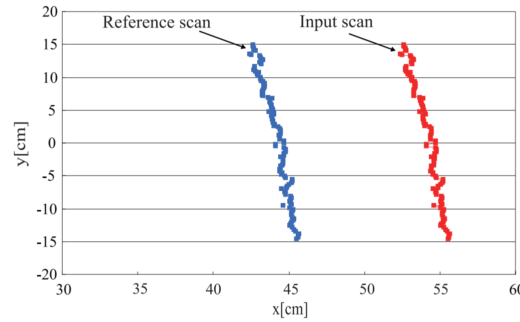
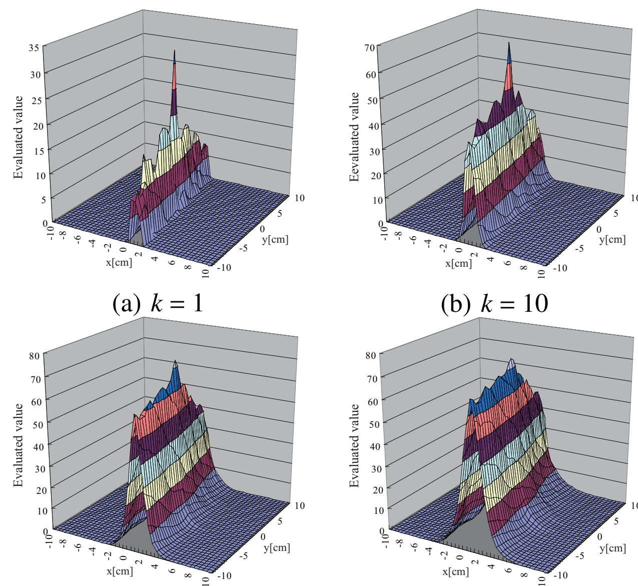


Fig. 3. Sequence of points



(a) $k = 1$ (b) $k = 10$ (c) $k = 45$ (d) $k = 100$

Fig. 4. Behavior of the evaluated value by changing eigenvalue

B. Influence of the eigenvalue expansion

1) Exploratory experiment with sequence of points: Fig.3 is the same point line data consisting of 76 points. We

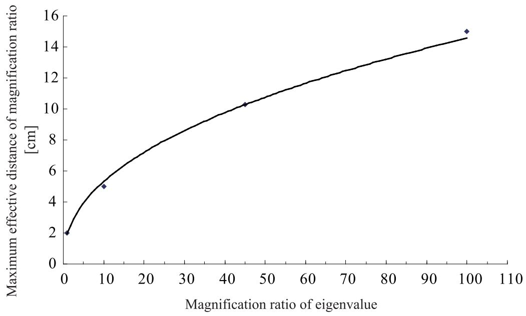


Fig. 5. Relation between magnification ratio k and maximum effective distance

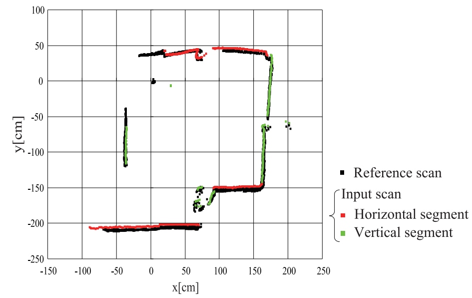


Fig. 6. Reference scan and input scan for analysis

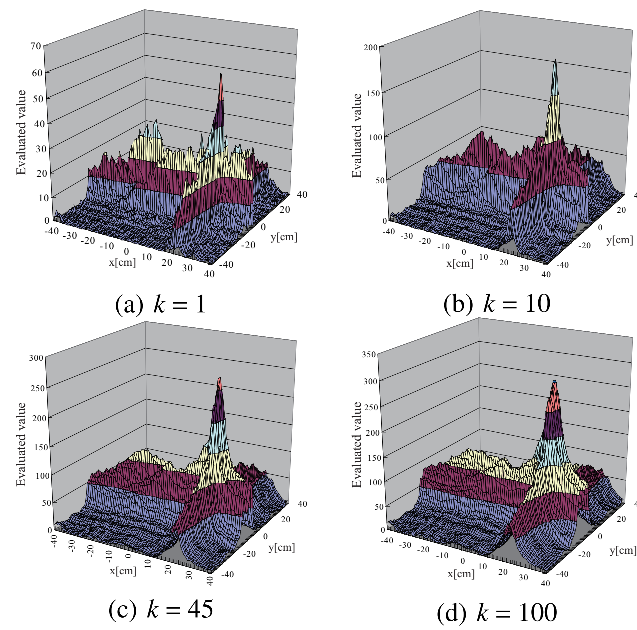


Fig. 7. Relations between the magnification k and the evaluation value of Fig.6

translated the input scan and measured the evaluation value at each location. Magnification ratios are k=1, 10, 45 and 100. And Fig.4 shows the evaluation value of each magnification. The ranges of the translation are ±10[cm] in each direction. This figure shows that the magnifications gently expand slant planes of the evaluation value in a vertical direction. And it also shows that the evaluation value is rugged at lower magnifications and smooth at higher magnifications. In other words, local solutions are decreased by raising magnification ratio. From such a thing, at the beginning, we perform rough matching with higher magnification, and after that, we use a fine adjustment at lower magnification. We defined "gravitative distance" as the maximum distance where input scan can be attracted. Fig.5 shows a relation between the magnification ratio k and the gravitative distance. We got this result by measuring the gravitative distance of the two scans (Fig.3).

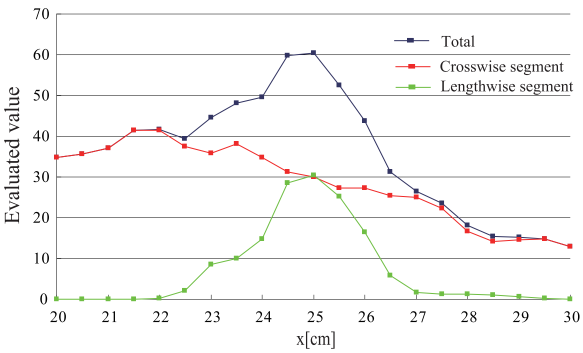
2) Exploratory experiment with map data: In this subsection, we describe an evaluation value of the map data (Fig.6). The figure shows a reference scan (black) and a input scan (red and green), and in this situation, the evaluation value is the maximum and the translation is (x, y)=(25,-5). The reference scan is made of four different reference scans. The red and the green segments are part of the new input scan. Fig.7 is a three-dimensional graph of the evaluation value at each location of each magnification, and the translation range is ±40[cm] in each direction. In Fig.6, because the form of data is divided into a horizontal axis group (red segment) and a vertical axis group (green segment), the shapes of the graphs are like a cross. In addition, the figure shows that the number of local solutions decreases, and a convergence region around the optimum solution is spread. Fig.8 shows changes of the evaluation values of each segment when the input scan translate ±5[cm] in x-axis direction from the maximum position (x, y)=(25,-5). A mountain of the evaluation value becomes smooth across the board, and the evaluation value of a vertical segment is enlarged in particular.

C. The NDT algorithm with the high-resolution grid

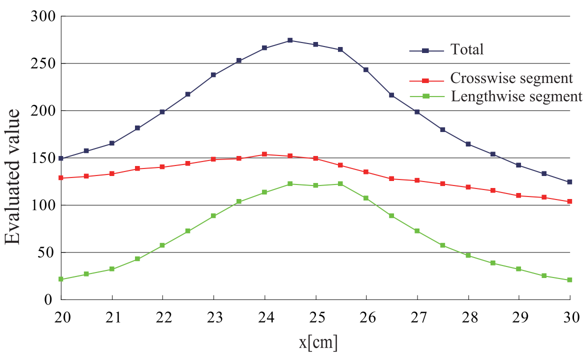
Fig.9 shows a flow of the scan matching by changing magnification of the eigenvalues. In a Rough mode (b), the magnification ratio is raised, and it enables reference scan to attract long-distance input scan. If the distance between the two scans is enough short to attract the input scan, the magnification ratio is lowered and the mode is changed to an Adjustment mode (c). We used an average distance between the reference scan and the input scan as a magnification change condition. This condition judge whether the input scan is attracted to an area where the next eigenvalue magnification ratio is effective. We set a value of the evaluation value for a convergence condition of the last magnification ratio.

The NDT scan matching performs the following steps:

1) Build the NDT of the first input scan, and generate a reference scan. (Fig.1)
2) Get a new input scan and initialize the parameters. (by using odometry data)
3) Transform the input scan with the parameter p.
4) Search the nearest neighborhood reference scan for each input scan point.
5) Estimate the parameter by Newton's method. We change the magnification ratio from high to low se-



(a) $k = 1$



(b) $k = 45$

Fig. 8. A change of the evaluation value of each segment

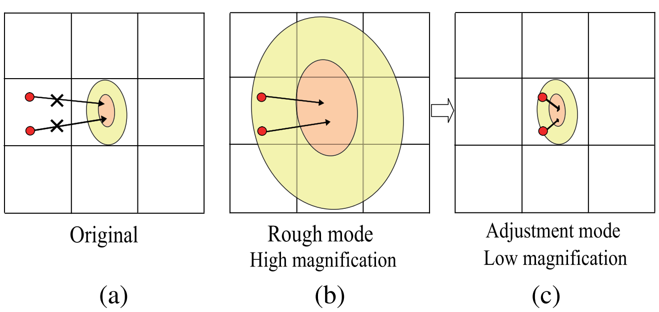


Fig. 9. The matching of long-distance data by extending eigenvalue

quentially.(Fig.9)

6) If the parameter is converged, then go to 7). Otherwise, change k to a higher one, and estimate again.
7) Add transformed input scan to reference scan.
8) Go to 2).

In 6, when an iterative calculation doesn't converge at certain magnification, we change the ratio to a higher one and perform the matching again.

Let r be the number of the magnification ratios to use, computation time will be $O(rN)$ at the maximum. In addition, when there is no reference scan in the grid, the time of the nearest neighborhood search of the ICP is required. The greatest effective range of each magnification showed in Fig.5 is assumed as the search range.

Fig. 10. Experimental environment

V. Experiment

A. Experimental equipments

We used multi-legged robot ASTRERISK[10] as a mobile robot. ASTERISK uses its limbs as an arm and a leg, and each limb has four servomotors. The error of its movement may be occurred by the calibration error of the motor or friction with floor. The range sensor is URG-04LX LRF of Hokuyo Electric Co., Ltd. This sensor measures a 2 dimensional plane, and the range covering ± 120 degrees. It can get 683 points' distances at one scan.

B. Mapping experiment

We made a map with using the robot in an experimental environment such as Fig.10 and Fig.11. The maximum movement is 40[cm] in translation and the maximum rotation is 20 degrees. The measurement was taken 49 times. In this experiment, we used three magnification ratios: $k = 1, 10, 45$. From Fig.5, the maximum gravitative distances of each magnification ratio are 10[cm], 5[cm] and 2[cm]. Newton method is calculated from $k = 45$ to $k = 1$ sequentially. Fig.12(a) is a map using odometry, and NDT scan matching is used from (d) to (b). As the magnification ratio, $k = 1$ is used in (b), $k = 1, 10$ are used in (c), $k = 1, 10, 45$ are used in (d). Note that the points of these figures are not the raw point data. These are 2[cm] grids. And grids including less than 3 points are not displayed because of their lower reliability. From (a) to (d), the average number of points in the grid is increased ((a)4.9 points, (b)6.1 points, (c)7.7 points, (d)8.0 points). The red grids in a map are observation points of ASTERISK. The figure shows that distortions and thickness of the map are reduced by the enlarged eigenvalue.

VI. Conclusion

In this paper, we proposed a new convergence calculation method for NDT scan matching using high resolution grid map. The proposed method improves the pairing number for matching and the convergence area. It employs Interactive Closest Point (ICP) algorithm to find corresponding point, and also enlarges the convergence area by modifying the eigenvalue of normal distribution. The proposed convergence

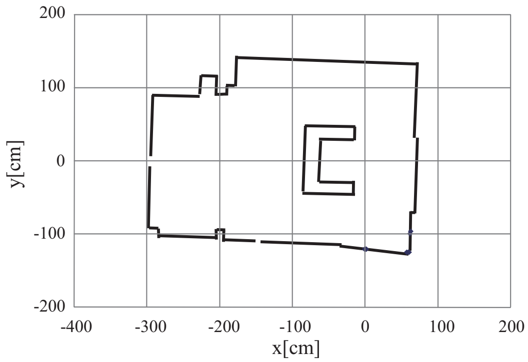


Fig. 11. Sketch of experimental environment

approach can be acheved by the phased adjustment of convergence area. Experimental result shows the feasibility of the detailed environmental map generated by the NDT high resolution grid mapping.

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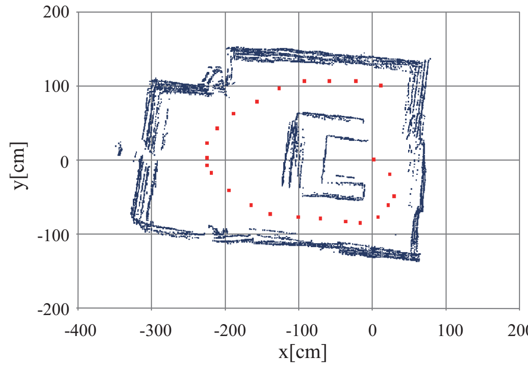
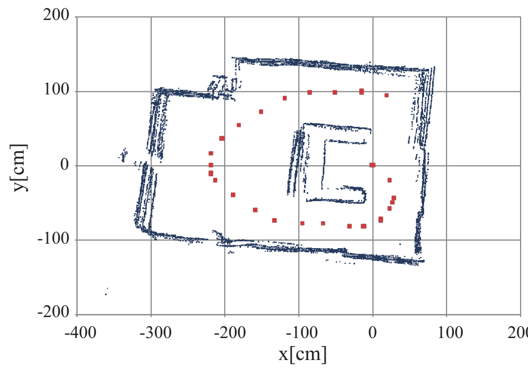
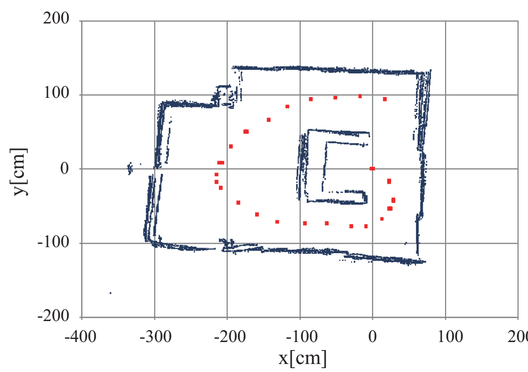


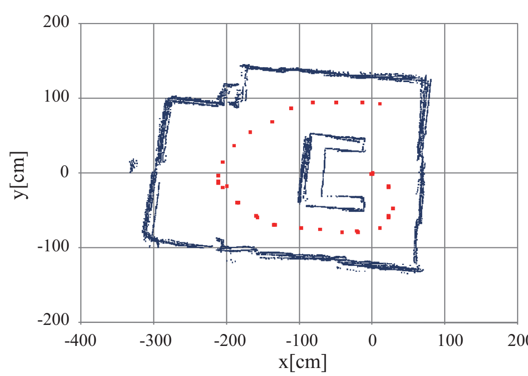
(a)Using odometory



(b)$k = 1$ (original NDT)



(c)$k = 1, 10$



(d)$k = 1, 10, 45$

Fig. 12. Matching result