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Two Distributed Algorithms for Heterogeneous Sensor Network Deployment Towards Maximum Coverage[†]

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***Abstract* – Autonomous deployment of mobile agents for coverage enhancement is an important issue in wireless sensor networks. The major challenge lies in the requirement of efficient distributed and localized computing. In addition, managing the coverage of heterogeneous sensing model is complicated due to the diversity of sensing ranges and the irregularity of coverage holes. This paper presents two distributed algorithms for maximizing the sensing coverage in heterogeneous sensor networks. The first algorithm is based on a circle packing technique. We prove the uniqueness of a circle packing up to a given triangulation and boundary conditions, thus the designated coverage layout can be achieved by controlling the boundary conditions. In the second algorithm, we give a formulation of virtual forces among sensor nodes to reduce redundant overlaps and avoid coverage holes. We prove that these virtual forces always give a quasioptimal local coverage. This method is applicable for deployment of sensor nodes in, not only an open field, but also any bounded field of interest and/or in the presence of obstacles. Numerical simulations are showed and these examples verify that the proposed algorithms always yield sensor deployments of wide coverage and collision free motions among sensor nodes. The proposed strategies utilize only the local information about a sensor node and its neighbors, thus providing distributed, efficient and scalable solutions to the deployment problem.**

Index Terms – Circle packing, deployment, mobile sensors, sensing coverage, wireless sensor network.

I. INTRODUCTION

Wireless sensor network has a lot of remarkable merits such as high sensing fidelity, on-board process, fault tolerance, rapid deployment and low cost. These features enable a wide range of practical applications for sensor networks. Each sensor node in a mobile sensor network is capable in communication, environmental sensing, data storage and processing and locomotion. Mobility enables a number of important functionality in sensor networks such as coverage maximization, adaptive sampling, network repair, localization and energy harvesting.

There is a significant necessity of distributed algorithms for autonomous deployment of active sensor networks. The sensor nodes are ready to self-deploy in a dynamic and inaccessible environment. Considerable efforts have been put on the deployment problem of sensor networks. [1] presented a potential-field-based approach to spread sensor nodes in a target environment. However, it does not consider some crucial problems like connectivity maintenance and topology control. The potential-field-based algorithm and the virtual force algorithm (VFA) presented in [2] work in a similar fashion. They increase sensor coverage by considering the virtual attractive and repulsive forces exerted on each sensor node by neighbor nodes and/or obstacles (if any). VFA assumes all sensor nodes are able to communicate with their cluster head which is responsible for calculating sensor movement and the target location. Moreover, these works only consider homogeneous sensing models, for which sensors need to have an identical sensing capability.

Heterogeneous sensor network allows the coexistence of sensors of different genres and ranges on a common platform. The sensing areas of nodes are modeled as circles of different radii. The deployment problem is intuitively transformed as a problem of placing a number of circles of different sizes over an open field or prescribed field of interest. A deployment algorithm based on the circle packing techniques is proposed in [4]. We adopt the Delaunay triangulation to model the network topology. Then, we use a pruning technique to enlarge the boundary of the mesh so that the coverage size would not be constrained due to the convex layout of the original mesh. The radius of each interior sensing circle is adjusted so that the packing condition is satisfied. Then, the geometric realization of the circle packing result is done by fixing one node and one of its neighbors via overlap packing. We proved that a global scaling on a circle packing can always vanish interstices of any triple which represent coverage holes.

In this paper, two distributed and autonomous deployment algorithms for heterogeneous sensor networks are presented. The network topology is constructed as a triangulation. The first deployment algorithm is an enhanced version of the circle packing based method presented in [4]. We prove the uniqueness of circle packing and its dependence on the boundary conditions.

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Consequently, the designated coverage layout can be effectively achieved by controlling the boundary conditions. The second algorithm is a virtual force based algorithm, thus it can deal with obstacles and field of interest. It can be applied to both fixed and adjustable range sensor nodes. We propose an effective method inspired from the angle sum in circle packing to define the virtual forces. It aims at eliminating coverage holes and reducing coverage overlaps (turn out enlarging overall coverage size) among the sensing circles. The proposed approaches can effectively deploy the sensors to give a wide coverage, eliminate coverage holes, reduce coverage overlaps and avoid obstacles. As both algorithms utilize only the local information about a sensor node and its neighbors, they provide a distributed, efficient and scalable solution to the deployment problem.

The remaining of this paper is organized as follows. Section II presents the preliminaries and assumptions. In Section III, we explain the term *angle sum* and its monotonic properties. We give an overview of the circle packing problem and prove the uniqueness of a packing. In Section IV, we present a virtual force based deployment algorithm. We give the formulation of virtual forces that yield a quasioptimal local coverage of a flower. In Section V, by means of simulation examples, we evaluate the performance of our deployment algorithm. Finally, concluding remarks are given in Section VI.

II. Preliminaries and Assumptions

Suppose we are given a set of n mobile sensor nodes $V=\{v_1,v_2,\cdots,v_n\}$ in the plane. The location of sensor v_i is precisely known and given by *Euclidian coordinates* $\boldsymbol{z_i}$. The *Euclidean distance* between two points v_i and v_j is denoted by d_{ij}, i.e. $d_{ij}=\|\boldsymbol{z_i}-\boldsymbol{z_j}\|$. The sensor nodes v_i have the same *communication range* r_c but different *sensing ranges* r_i, $i=1,\cdots,n$. The communication and sensing areas covered by sensor nodes are modelled as circular discs. The communication graph defined by all wireless nodes V is a *unit-disk graph UDG(V)*, which has an edge v_iv_j if and only if $d_{ij}\le r_c$. We assume this graph is connected when the sensor nodes are initially located at random positions (inside the field of interest if applicable). We further assume that the communication range is longer than the summation of any two of the sensing ranges, i.e. $r_c>r_i+r_j\ \forall i,j$, such that any two sensor nodes are able to communicate while their sensing circles are touching. Each sensor node is capable of broadcasting the identities, positions and sensing ranges of itself and its one-hop neighbors. We assume there is a topology control protocol to construct the network topology as a *connected* and *triangulated* graph, and the degree of every node in the graph is not less than three. The network topology, denoted by *complex K*, is a subgraph of *UDG(V)*. We may employ the localized Delaunay triangulation protocol proposed by [5], while we assume, upon the initial random placement, no holes may exist in the triangulation complex K and every boundary node has at least one neighbor that is an interior node.

III. Alogrithm I: Circle Packing Based Heterogeneous Sensor Network Deployment

This section reviews the circle packing problem and its linkage to the deployment problem. We will then give the proof of uniqueness of circle packing and an enhanced algorithm. A hierarchy of circle packing structure consists of several levels of components, namely *circles*, *triples*, *flowers* and *packings* [6]. In the remaining of this paper, we always refer a circle c_i as the sensing region of a certain sensor node v_i. The coordinates of circle centers refer to sensor node positions $\boldsymbol{z_i}$ and radii refer to the corresponding sensing ranges r_i. The number of petals defines the *degree k* of the central circle. The condition that every circle has such a flower is a local planarity condition that we will enforce on all our packings. We first give an important term, *angle sum,* in circle packing and its monotonic properties. We then define the circle packing problem. We prove that any complex K with fixed boundary radii always has a unique circle packing such that any two circles are tangent whenever their associated nodes form an edge in K. As the computation of a circle packing can be accomplished distributedly, it fits the needs of scalability for sensor networks.

A. Angle Sum

For each triple of radii r_i, r_j and r_k, the Law of Cosines gives the angle α in a corresponding triple of circles, as shown in Fig. 1(a). If we add these individual angles over the k triples involved, we get the angle sum θ for this label at v. Suppose $F_v=\{v;v_1,v_2,\cdots,v_k\}$ is the flower for v in K. Vertex v belongs to m faces, where $m=k$ if v is interior and $m=k-1$ if v is boundary. In a flower having central label r and petal labels $\{r_1,r_2,\cdots,r_k\}$, the angle sum is given by the following summation formula, where $m=k$ and $r_{k+1}=r_1$ if the flower is closed, and $m=k-1$ otherwise:

$$\theta(r;r_1,r_2,\cdots,r_k)=\sum_{j=1}^{m}\cos^{-1}\left[\frac{(r+r_j)^2+(r+r_{j+1})^2-(r_j+r_{j+1})^2}{2(r+r_j)(r+r_{j+1})}\right] \quad (1)$$

Note that the calculation of angle sum does not require any sensor node coordinates, only sensing radii are involved.

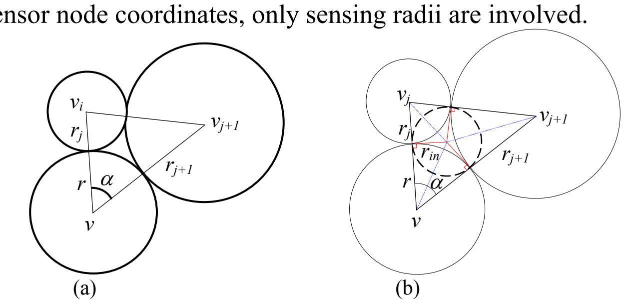


Fig. 1. (a) Angle α defined by the Law of Cosines of a triple. (b) Another derivation of angle α by the constructing the triangle incircle of a triple.

When the petal radii are fixed, a bigger central circle will have a smaller angle sum. On the contrary, if the central circle radius is fixed, the angle sum is strictly increasing with any petal radius. Therefore we come to the following lemma.

LEMMA 1 (*Monotonicity in flower*). *Let $F_v = \{v; v_1, v_2, \cdots v_k\}$ denote a flower with central vertex v and a chain of k petals $\{v_1, v_2, \cdots v_k\}$. Let $\{r; r_1, r_2, \cdots, r_k\}$ be the list of radii of the circles in F_v and write $\theta(v)$ the angle sum at v as a function of $r, r_1, r_2, \cdots, r_k$. Assume $r < \infty$, $\theta(v)$ is strictly decreasing in central circle radius r and strictly increasing in petal radii r_i that are finite. If $r = 0$, $\theta(v) \equiv k\pi$. If $r = \infty$, then $\theta(v) \equiv 0$.*

It is not difficult to prove Lemma 1 by expressing the *inradius* r_{in} (Fig. 1(b)) with the aid of the Heron's formula

$$r_{in} = \sqrt{\frac{r \cdot r_j \cdot r_{j+1}}{r + r_j + r_{j+1}}} \tag{2}$$

to obtain a easier derivative of the angle sum with a new expression. The flower of an interior node can be realized as a *closed* geometric flower if and only if $\theta = 2\pi$. We call $\theta(r; r_1, r_2, \cdots, r_k) = 2\pi$ the *packing condition*.

LEMMA 2. *Let $\theta(r_a; r_1, r_2, \cdots, r_k) := \theta_a$ and $\theta(r_b; r_1', r_2', \cdots, r_k') := \theta_b$ be the angle sums of two flowers F_a and F_b of same number of petals. Suppose $\theta_a = \theta_b$. If $r_i \geq r_i'$ for $i = 1, \cdots, k$ and $r_i > r_i'$ for some i (i.e. r_i and r_i' are not all equal), then $r_a > r_b$.*

Lemma 2 is easily followed from the two monotonicities in Lemma 1. In particular, for closed flowers (i.e. $\theta = 2\pi$), Lemma 2 suggests a resizing mechanism; increasing (decreasing) some petal radii will spontaneously increase (decrease) the central radius in order to maintain the packing condition.

B. Circle Packing Problem

DEFINITION 3 (Circle Packing Problem). Given a complex K and a set of positive numbers correspond to the radii of the boundary nodes, the problem of finding the radii for all interior nodes such that the angle sum at every interior node satisfies the packing condition is called *circle packing problem*.

We call the set of radii imposed for the boundary nodes as *boundary condition* for K. We may also refer boundary condition as the collection of angle sums at the boundary vertices (adjusting the radii will eventually achieve the prescribed angle sums). The set of radii $R = \{r_1, r_2, \cdots\}$ for all vertices, composed of the given radii of boundary nodes and the solution of the set of interior circle radii to the circle packing problem, is called *packing label*. Denote $R(v)$ as the radius of an individual vertex v. $K(R)$ is referred as *labeled complex*. The circle packing P for $K(R)$ is in particular composed of the set of circles of $\{v_i\}$ such that $r_i = R(v_i)$. The importance of the circle packing problem relies on the uniqueness of its solution.

THEOREM 4 (*Uniqueness of Circle Packing*). *Given any assignment to the radii of boundary vertices of K, there exists a unique circle packing for K. The circle packing problem always yields a unique solution.*

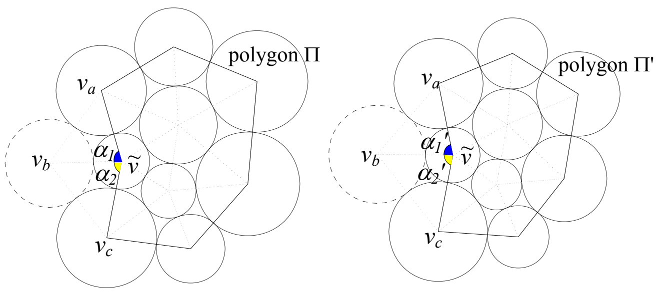
PROOF. We will do the proof by Mathematical Induction (M.I.). Consider the following statement $S(n)$ for all natural number n: for any complex K_n with n interior nodes and a set of fixed boundary node radii, there exists a unique circle packing P_n for K_n.

For $n=1$, complex K_1 is simply a flower as there is only one interior node. The packing condition for a flower to be closed is that the angle sum of the only interior node equals 2π. According to Lemma 1, angle sum θ is strictly decreasing in r. Thus, there exists a unique solution of r for $\theta = 2\pi$. Therefore, $S(1)$ is true.

Assume $S(m)$ is true, that is, there exists a unique circle packing P_m for any complex K_m with m interior nodes and a set of fixed boundary node radii. According to Lemma 2, if any boundary circle radius increases, the radius of the interior circle immediately adjacent to that boundary circle will increase in order to maintain the packing condition. Consequently, like the domino effect, all the interior circle radii will increase simultaneously.

Now we need to prove that $S(m+1)$ is also true, that is for any complex K_{m+1} with $m+1$ interior nodes and a set of fixed boundary node radii, there exists a unique circle packing P_{m+1}. We will do it in a proof by contradiction. Assume to the contrary that there exist two different circle packings P_{m+1} and P_{m+1}' for complex K_{m+1} and they share the same boundary condition. Suppose v_b is a boundary node and connected to one interior node $\tilde{v}$ (Fig. 2). Without loss of generality, we assume $R(\tilde{s}|P_{m+1}) > R(\tilde{s}|P_{m+1}')$, that is the radius label of node $\tilde{v}$ from packing P_{m+1} is greater than that from packing P_{m+1}'. Consider a complex $K_m \subset K_{m+1}$ containing exactly the same contents of complex K_{m+1} except that the node $\tilde{v}$ becomes a boundary node of K_m and the boundary node v_b of K_{m+1} is *absent* in K_m. K_m is literally the clone of complex K_{m+1} with boundary node v_b removed. Consider two particular boundary conditions for K_m: $R(\tilde{v}) = R(\tilde{v}|P_{m+1})$ and $R(\tilde{v}) = R(\tilde{v}|P_{m+1}')$, while all other boundary nodes of K_m, except $\tilde{v}$, have fixed radii which are correspondingly equal to that imposed for K_{m+1}. Since $S(m)$ is true, there exists two distinct circle packings P_m and P_m' for complex K_m under the two boundary conditions respectively. Moreover, $R(\tilde{v}|P_m) > R(\tilde{v}|P_m')$ implies that $R(v|P_m) > R(v|P_m')$ for all interior node v of K_m. Suppose K_m contains N boundary nodes. Then, two N-sided polygons Π and Π' (Fig. 2) are formed by connecting all the centres of the chains of N boundary circles in P_m and P_m' respectively. Each interior angle of the polygons is exactly the angle sum at the corresponding boundary node. Since all the interior circles in P_m is greater than the corresponding circle in P_m', at every boundary vertex except $\tilde{v}$, the interior angle of polygon Π is strictly greater than that of polygon Π'. However, the sum of interior angles of any N-sided polygon is constant (i.e. $(N-2)\pi$), thus the interior angle at $\tilde{v}$ of polygon Π is strictly smaller than that of polygon Π'.

Therefore, the angle sum at $\tilde{v}$ in P_m is strictly smaller than that in P_m', i.e. $\theta(\tilde{v}|P_m) < \theta(\tilde{v}|P_m')$. Let v_a and v_c be the boundary nodes adjacent to v_b. Notice that v_a, v_b and v_c are petals of flower $F_{\tilde{s}}$ in K_{m+1}. Denote α_1, α_2, α_1' and α_2' as the angle α of triples as shown in Fig. 2. According to the proof of Lemma 1, angle α is strictly decreasing in the central circle radius. Since $R(\tilde{v}|P_m) > R(\tilde{v}|P_m')$, we have $\alpha_1 < \alpha_1'$ and $\alpha_2 < \alpha_2'$. However, $\theta(\tilde{v}|P_m) = 2\pi - \alpha_1 - \alpha_2$ and $\theta(\tilde{v}|P_m') = 2\pi - \alpha_1' - \alpha_2'$. Thus, $\theta(\tilde{v}|P_m) > \theta(\tilde{v}|P_m')$. This contradicts to the above result. Consequently, we conclude that $S(m+1)$ is true; there exists a unique circle packing P_{m+1} for complex K_{m+1} with $m+1$ interior nodes and a set of fixed boundary radii. By M.I., $S(n)$ is true for all natural number n, there exists a unique circle packing P for complex K with a specified boundary condition to achieve packing condition at every interior node. ■



(a) Circle Packing P_m (b) Circle Packing P_m'

Fig. 2. Polygons Π and Π' from two different circle packings P_m and P_m', $radius(\tilde{v}|P_m) > radius(\tilde{v}P_m')$.

THEOREM 5 (*Maximal Circle Packing*). *Given any complex K. Assume each boundary node has an upper bound for its radius value while the radii of interior nodes are unlimited. If all boundary radii are assigned to the maximum values, then the total area of circles of the circle packing of K is maximized.*

PROOF. By Lemma 2, the radius of any interior node (as the central) with a boundary neighbor (as a petal) is strictly increasing in the boundary radius for fixed angle sum. Consequently, as we are moving from the exterior circles to the interior of the packing, every interior radius increases monotonically in its petal radii which are also increasing in order to maintain the packing condition. Thus, every circle has its radius maximized when all every boundary radii are assigned to their maximum values, and hence, the total area of circles reaches the maximum. ■

IV. ALGORITHM TWO: VIRTUAL FORCE BASED HETEROGENEOUS SENSOR NETWORK DEPLOYMENT

Algorithm 2 is another distributed autonomous deployment algorithm for heterogeneous sensor networks. It is a virtual force based algorithm, thus it can deal with obstacles and field of interest. It can be applied to both fixed and adjustable range sensor nodes. It is inspired by the angle sum θ discussed in Section 4. Consider a flower of fixed size central and petal circles, no radius can be adjusted. With the monotonicity described in Lemma 1, we conclude that the central radius is too small if the angle sum is greater than 2π; the petal circles should be brought closer to the central circle in order to eliminate the coverage holes. On the contrary, if the angle sum is smaller than 2π, the central radius is too large; the petal circles should be driven further away from the central circle in order to reduce the coverage overlaps.

On condition when sensor nodes are not capable of adjusting their sensing ranges, or if we always need to fully utilize the sensing capability, *Algorithm 1* is not applicable because we do not resize any interior sensing circles. There is no perfect and rigid circle packing obtainable, as for a set of fixed radii of the central and the petal circles of a flower, the packing condition of $\theta = 2\pi$ is generally not achieved. In such cases, the deployment problem becomes the placement of a set of fixed size circles to give a hole-free and maximized covered area. In this section, we formulate the desired distance between the central node and each of the petal nodes in a flower. The idea is to locate the petal circles about the central circle such that sensing coverage is maximized while no coverage hole exists. We prove that the formulation always give a *quasi-optimal flower coverage* subjective to the central node of a flower. Then, we list the procedures of *Algorithm 2* at the end of this section.

A. Quasioptimal Flower Coverage

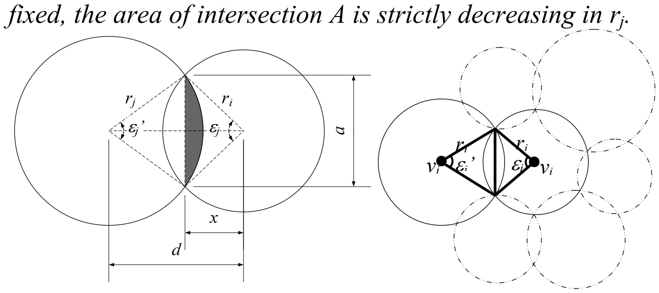
Algorithm 2 is based on virtual forces. Sensor nodes virtually exert attractive and repulse forces to their neighbor nodes. In heterogeneous sensor networks, sensor nodes do not have an identical sensing range. The desired distances among the sensor nodes are to be calculated locally in every sensor node. Since the sensing circles are all in different sizes, we desire a scheme to effectively avoid coverage holes and, at the same time, to maximize overall sensing coverage. This gives rise to a constrained optimization problem, the *optimal flower coverage problem*. It concerns the placement of the petal circles of a flower with respect to the central circle. The circles are no longer tangent among themselves; they are allowed to overlap. The objective is to maximize the total area covered by the circles of a flower, which is interpreted as the total local sensing coverage of a sensor node and its neighbors, under the constraint that no coverage hole may exist among any triples. Here the term "local" refers that the objective concerns the coverage with respect to one sensor node and its neighbors, instead of the coverage of the entire network. As it is a distributed algorithm, it is reasonable and practical for a sensor node to limit the concern in localized coverage. It is conceivable that the maximization problem is dual to the minimization of overlap area among the circles under the same constraints. We look into the expression of overlap area that we attempt to minimize. When two circles c_i and c_j intersect, an asymmetric *lens* composed of two circular segments is formed and it corresponds to the coverage overlaps. Denote d as the distance between the centers of c_i and c_j. The area A of this asymmetric lens is governed by three independent parameters: r_i, r_j and d. We denote ε_j as the central angle of the segment of c_i covered by c_j, ε_j' as the central angle of the segment of c_j covered by c_i, x as the distance between center of c_i to the common chord of c_i and c_j and a as the length of the common chord. Then A can be formulated as a function of r_i, r_j and ε_j:

$$A = \underbrace{\pi r_i^2 \cdot \frac{\varepsilon_j}{2\pi} - \Delta_i}_{\text{area of left circular segment}} + \underbrace{\pi r_j^2 \cdot \frac{\varepsilon_j'}{2\pi} - \Delta_j}_{\text{area of right circular segment}} \tag{3}$$

where $r_i \sin\frac{\varepsilon_j}{2} = r_j \sin\frac{\varepsilon_j'}{2} = \frac{a}{2}$ (i.e. $\varepsilon_j' = 2\sin^{-1}\left(\frac{r_i}{r_j}\sin\frac{\varepsilon_j}{2}\right)$),

$\Delta_i = \frac{1}{2} r_i^2 \sin\varepsilon_j$ and $\Delta_j = \frac{1}{2} a \left(r_j^2 - \frac{a^2}{4}\right)^{\frac{1}{2}}$. Then, we have

LEMMA 6. *Given two intersecting circles c_i and c_j of radii r_i and r_j respectively. Denote ε_j as the central angle of the segment of c_i covered by c_j. If radius r_i and angle ε_j are fixed, the area of intersection A is strictly decreasing in r_j.*



(a) (b)

Fig. 3. (a) Asymmetric lens and (b) quasioptimal flower coverage

The flower coverage maximization problem is equivalent to the minimization of the summation of the overlaps between the central circle and each of its petals. The objective function can be expressed by the summation of areas of asymmetric lenses in (3) and the constraints can be expressed by the inequalities for qualitative test of coverage hole existence in [4]. Note that the overlaps among the petal circles are not considered for simplification, though they can also be expressed by (3). However, as the objective function and the inequality constraints involve numerous trigonometric functions, it is unfeasible to tackle with the exceedingly nonlinear minimization problem for obtaining a global optimal solution. Instead, we are interested in working out a configuration of flower that gives *quasioptimal* (near optimal) coverage and is explicitly expressible in terms of merely sensing ranges.

DEFINITION 7. A *quasioptimal flower coverage* is a configuration of flower such that the coverage holes among any triples (the central node and two adjacent petals) *just vanish*.

When three circles intersect at a point, the coverage hole (interstice) among them just vanishes. It conforms to both fundamental criteria of the original optimization problem: overlap reduction and coverage inexistence. Now, we propose a formulation to define the desired distances between a central sensor node and its neighbors such that the configuration yields a quasioptimal flower coverage. Our idea is to "close" a flower in order to achieve packing condition. Consider a flower of central node v_i of degree k and angle sum θ. When sensing circles c_i is intersecting with sensing circles c_j, $j = 1,2,\cdots,k$, the summation of the area of k asymmetric lenses is literally the coverage overlaps. We adopt the same notation ε_j as the central angle of the segment of c_i covered by c_j for $j = 1,2,\cdots,k$. According to Lemma 6, the area of intersection is strictly decreasing in r_j for fixed r_i and central angle ε_j. In order to reduce these lens areas, and thus, the total coverage overlaps, we define the central angles by

$$\varepsilon_j = \frac{r_j}{\sum_{i=1}^{k} r_i} \cdot 2\pi \tag{4}$$

which means the central angle ε_j is proportional to the neighbor radius r_i. Obviously, (4) implies $\sum_{j=1}^{k} \varepsilon_j = 2\pi$. As shown in Fig. 3(b), when all the interstices are *just* vanished, the summation of central angles equals 2π. Therefore, (4) suggests a configuration of flower to achieve quasioptimal flower coverage. Moreover, the central angles ε_j are assigned in proportion to petal radius r_j, the area of overlaps is reduced due to Lemma 6. The desired distance $\hat{d}$ is:

$$\hat{d}_{ij} = r_i \cos\frac{\varepsilon_j}{2} + r_j \cos\frac{\varepsilon_j'}{2} \tag{5}$$

where

$$\varepsilon_j' = 2\sin^{-1}\left(\frac{r_i}{r_j}\sin\frac{\varepsilon_j}{2}\right). \tag{6}$$

If v_i is a boundary node, the desired distance with its neighbor v_j are simply defined as

$$\hat{d}_{ij} = \frac{\sqrt{3}}{2}(r_i + r_j). \tag{7}$$

B. Procedures of Algorithm 2

Similar to the previous algorithm, the sensor nodes are initially randomly located. If a bounded field of interest is available, the sensor nodes are randomly placed inside the field. In any case, we assume the initial network is connected. A triangulated mesh describing the communication graph is generated upon the initial deployment using the localized Delaunay triangulation in Section II. This algorithm works in an iterative fashion. It increases sensor coverage and avoid collisions by considering the virtual attractive and repulsive forces exerted on each sensor node by neighbor nodes and/or obstacles (if any). In every iteration, each sensor node computes the desired distances $\hat{d}_{ij}$ of its neighbors and exerts virtual forces to them. The virtual force exerts by node v_i on node v_j is

$$\boldsymbol{f_{ij}} = (\hat{d}_{ij} - d_{ij})\frac{\boldsymbol{z_j} - \boldsymbol{z_i}}{d_{ij}}. \tag{8}$$

Each sensor node also scans for field boundaries and obstacles within its sensing range. If any field boundary or obstacle is detected, a virtual repulsive force is applied to the sensor node to drive it directly away to avoid collisions.

We denote $\boldsymbol{f}_i^{OB}$ as the net force arose from obstacles and $\boldsymbol{f}_i^{FoI}$ as the net force arose from the field of interest. Then, each node calculates the resultant force exerted by all its neighbors, obstacles and field of interest, Besides, a reasonable step size ξ should be chosen to avoid sensors from moving too vigorously. Finally, the sensor node v_i self-deploys to a new position expressed by

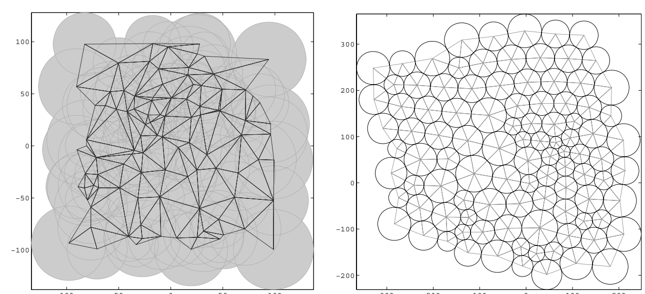
$$\boldsymbol{z}_i{}' = \xi\left(\sum_{j=1}^{k} \boldsymbol{f}_{ji} + \boldsymbol{f}_i^{OB} + \boldsymbol{f}_i^{FoI}\right) + \boldsymbol{z}_i \, . \tag{9}$$

V. Simulation Examples

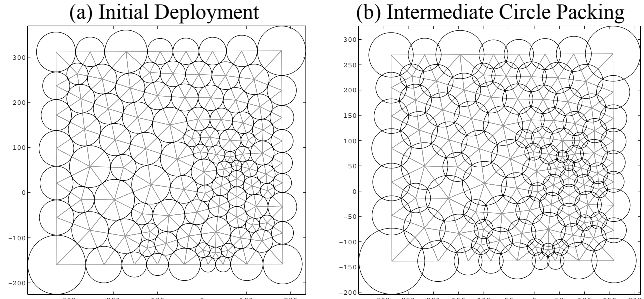
We have implemented the proposed algorithm in Matlab to verify the approach and demonstrate its performance. The simulations performed show that the circle packing approach can always give deployments of large sensing coverage. Two examples are given below.

A. Example 1

In this example, we deploy 100 sensor nodes using *Algorithm 1*. The sensor nodes are initially placed in random positions. A localized triangulated communication graph is constructed upon the initial placements (Fig. 5(a)). Fig. 5(b) shows an intermediate circle packing result while the boundary radii are fixed to certain values. Then, we further adjust the boundary condition by setting appropriate the angle sums at all boundary nodes ($\theta = \pi / 2$ at the four corners and $\theta = \pi$ for the rest). Fig. 5(c) shows a circle packing of rectangular coverage under this boundary condition, and Fig. 5(d) is the final deployment layout under overlap packing with $\eta = \sqrt{3} / 2$.



(a) Initial Deployment (b) Intermediate Circle Packing

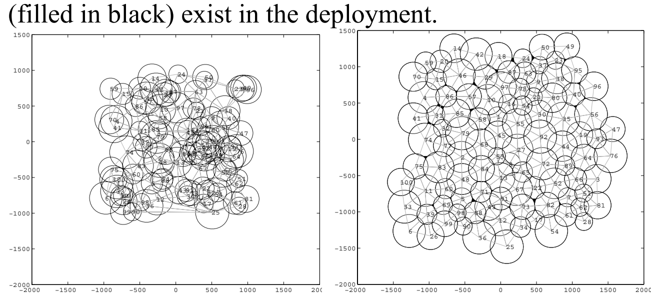


(c) Final Circle Packing (d) Final Deployment by Overlap Packing

Fig. 5. Example 1: A rectangular coverage of 100 sensor nodes using Algorithm 1.

B. Example 2

We demonstrate the effectiveness of *Algorithm 2* in this example. A randomized placement of 100 sensor nodes is show in Fig. 6(a). Each sensor has different sensing ranges. Each node computes the amount of virtual forces exerting on its neighbors by flower closing approach proposed in Section 6. Then, they calculate and move to their new positions by referring the resultant force being exerted. Fig. 6(b) shows the final deployment result, which gives a large coverage and sensible utilization of sensing ranges, by this virtual force algorithm. However, some coverage holes (filled in black) exist in the deployment.



(a) Initial Deployment (d) Final Deployment

Fig. 6. Example 2: Deployment of 100 sensor nodes using Algorithm 2.

VI. Concluding Remarks

We have presented two algorithms to deal with the heterogeneous sensor network deployment problem. The main contributions of this paper lie in two aspects. First, this paper deals with the optimal deterministic coverage problem of heterogeneous sensor networks. We prove that the first algorithm always gives a unique, maximized and hole-free coverage of a given network topology when the sensing ranges of boundary nodes are set to their upper limits and the sensing ranges of interior nodes are unlimited. In the second algorithm, the formulation of the desired distances among the sensor nodes of a flower guarantees the quasioptimal local coverage. Secondly, *Algorithm 1* and *Algorithm 2* are distributed deployment algorithms for mobile sensor network. As both algorithms utilize only the local information about a sensor node and its neighbors, they provide a distributed, efficient and scalable solution to the deployment problem. We have evaluated and demonstrated the performance of our methods with simulation examples. The proposed approaches can effectively deploy the sensors to give a wide coverage, eliminate coverage holes, reduce coverage overlaps and avoid obstacles.

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