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Stability of Bilateral Teleoperators with Projection-Based Force Reflection Algorithms

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Abstract— **A general stability result for force-reflecting teleoperator systems with projection-based force reflection algorithms is established. It is shown that the closed-loop system's gain can be assigned arbitrarily by an appropriate choice of certain weighting function of the projection-based force reflection algorithm. In particular, this allows to achieve stability of the force-reflecting teleoperator system in presence of time-varying irregular delays for arbitrarily large force-reflecting gain and arbitrarily low damping and stiffness of the master. The proposed approach solves, to some extent, the trade-off between stability, manoeuvrability, and high force reflection gain in force-reflecting teleoperator system with network-induced communication constraints.**

I. Introduction

Design of high performance teleoperator systems often involves trade-offs between conflicting design objectives. One such a trade-off that arises in force-reflecting teleoperators is between overall stability and high force reflecting gain. Higher force reflecting gain generally implies improved haptic perception of the remote object, however, it also increases the closed-loop gain which leads to instability. The mechanism of such an instability is analyzed in great details in [1]. According to this work, the instability can be explained in terms of so called induced master motion. The induced master motion is an unintentional from the human point of view movement of the master manipulator which is created by the force reflection signal from the slave side. Since the master trajectory is then used as the reference trajectory for the slave manipulator, the induced master motion in turn creates similar reaction of the slave subsystem, etc. Essentially, such an interaction forms a control loop, and the corresponding closed-loop gain is directly proportional to the force feedback gain; as a result, high force feedback gain leads to instability of the teleoperator system.

Some earlier approaches that address the above mentioned trade-off include increasing damping and stiffness of the master manipulator [2], low-pass filtering of the slave reference trajectory [3], low-pass filtering of the reflected force [4]. All these approaches generally lead to different forms of transparency/performance deterioration. The model-based cancelling of the induced master motion from the slave reference trajectory is proposed in [1]. This approach also has some shortcomings, in particular, the fact that the human physiological reaction to the force disturbances normally includes changes in mechanical parameters (in particular, increased stiffness [5]) of the human hand is not taken into account. On the other hand, projection-based force reflection (FR) algorithms were introduced in [6], [7]. These algorithms uses the fact that the human perception of an external force depends critically on the direction and the magnitude of her/his own forces applied to the master manipulator (which in this case simultaneously plays a role of a haptic display). The idea behind the projection-based FR algorithms, therefore, is to alternate the force reflection signal depending on the forces applied by the human operator; it can be done in such a way that the human does not feel this alteration, however, the resulting induced master motion can be eliminated or reduced to an appropriate level. The improvement to stability properties brought in by such force-reflecting algorithms are demonstrated in [6], [7] for some particular force-reflecting teleoperator schemes in presence of time delays.

In this paper, general stability results for force reflecting teleoperator systems with projection-based force reflection algorithms are presented. More precisely, we identify a broad class of force reflection algorithms, and prove stability results for a general force-reflecting teleoperator system with a FR algorithm from this class in the situation where the communication between the master and the slave is subject to time-varying discontinuous possibly unbounded communication delays, occasional packet losses are admitted, and the human force measurement/estimation process is possibly corrupted. To prove these stability results, we utilize a new version of the small gain theorem for multichannel systems with multiple communication delays presented in [8]. The important advantage of the small-gain approach over more traditional one that is based on passivity and scattering/wave variables formalism [9]–[11] is that the small-gain approach does not impose any "phase" restrictions on the subsystems (in particular, on the communication channels between the master and the slave), which makes it highly suitable in dealing with time-varying irregular communication delays (which are typical if the communication is performed over the Internet). On the contrary, the passivity based approach can hardly be extended to the case of irregular communication delays because in this case the passivity can be lost due to signal distortion (although some partial extensions are available, [12]–[14]). However, when applied directly, the small gain approach leads to conservative results; in particular, high damping and stiffness of the master robot are required for the overall stability. One of the most interesting properties

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of the projection-based force reflecting algorithms is that, formally speaking, they allow to disconnect in some sense the closed loop with respect to the force reflection signal (more precisely, the gain between the force reflection signal and the corresponding induced master motion can be made arbitrarily low), which results in significant (theoretically, arbitrary) improvement of the admissible force reflection gain without increasing the damping and the stiffness of the master. Thus, the projection based force reflection algorithms solve, to some extent, the trade-off between stability, manueurability, and high force reflection gain in bilateral teleoperation with communication delay.

The paper is organized as follows. The projection-based approach to force reflection in bilateral teleoperation is presented in section II. In section III, the general stability results for teleoperator systems with projection-based force reflection algorithms are presented together with all the necessary preliminaries. Simulation results are discussed in section IV and, finally, some conclusions are given in section V.

Notation. $\mathbb{R}^+ := [0, +\infty)$. A function $\alpha\colon \mathbb{R}^+ \to \mathbb{R}^+$ belongs to class $\mathcal{G}$ ($\alpha \in \mathcal{G}$) if and only if $\alpha(0) = 0$ and is nondecreasing. A function α belongs to class $\mathcal{K}$ ($\alpha \in \mathcal{K}$) if and only if $\alpha \in \mathcal{G}$ and strictly increasing. Also, $\alpha \in \mathcal{K}_\infty$ if and only if $\alpha \in \mathcal{K}$ and unbounded ($\lim_{s\to+\infty} \alpha(s) = +\infty$). Finally, $\beta\colon \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}^+$ belongs to class $\mathcal{KL}$ if and only if $\beta(a, b)$ is a $\mathcal{K}_\infty$-class function in a, strictly decreasing in b, and $\lim_{b\to+\infty} \beta(a, b) = 0$ for each $a \in \mathbb{R}^+$.

II. Projection-based force reflection algorithms

The idea behind the projection-based force reflection algorithm is based on the following simple observation: the human operator feels the external force if and only if she/he pushes against it. Moreover, the amount of force that is felt by the human operator is exactly equal to the amount of force exerted by a human hand against the external force. On the other hand, the induced motion of the master manipulator is created where the force-reflecting term is not compensated by the human counter-force; in this situation, however, the force reflection is useless from the point of view of human perception, since the human does not feel it. This observation leads to the following idea: the force reflection term may be altered depending on the forces applied by the human operator. More precisely, since the human operator feels the forces that are directed against her/his own, and the magnitude of the forces felt is not greater than the magnitude of the human forces applied to the master, this implies that all external forces outside these direction and magnitude constraints can be, roughly speaking, filtered out without any impairment of the human force perception. Through such an alternation, however, the potentially harmful induced master motion may be eliminated or, at least, limited to an appropriate level.

Below, we address a force reflection scheme where the force signal applied to the motors of the master $\hat{f}_r$ is described by the following formula

$$\hat{f}_r := \alpha\left(\hat{f}_{env}\right) + [\mathbb{I} - \alpha]\left(\hat{\phi}_{env}\right). \quad (1)$$

Here, $\hat{f}_{env}$ is the force signal that is arrived directly from the slave subsystem, $\hat{\phi}_{env}$ is the signal generated by the projection-based force reflection algorithm described below, and $\alpha \in \mathcal{G}$ is the corresponding weighting function; the last should be chosen to satisfy $[\mathbb{I} - \alpha] \in \mathcal{G}$, where $\mathbb{I} : \mathbb{R}^+ \to \mathbb{R}^+$ is the identity function, $\mathbb{I}(r) = r$ for all $r \geq 0$. The signal $\hat{\phi}_{env}$ can be obtained using the force reflection algorithm, as follows

$$\hat{\phi}_{env} := \operatorname*{Sat}_{[0,1]}\left\{\frac{\hat{f}_{env}^T \bar{f}_h}{\max\{|\bar{f}_h|^2, \epsilon_1\}}\right\}\bar{f}_h, \quad (2)$$

where $\bar{f}_h$ is a measurement/estimate of the human force applied to the master manipulator, $\epsilon_1 > 0$ is a sufficiently small constant, and $\operatorname*{Sat}_{[a,b]}\{x\} := \min\{\max\{a, x\}, b\}$.

The algorithm (2) can be given the following explanation. Assuming $\left|\bar{f}_h\right|^2 \geq \epsilon_1$ and $0 \leq \hat{f}_{env}^T \bar{f}_h / \left|\bar{f}_h\right|^2 \leq 1$ (*i.e.*, the saturation in (2) is not achieved), it is easy to see that $\hat{\phi}_{env}$ is the projection of $\hat{f}_{env}$ onto the direction of $\bar{f}_h$. By placing the lower saturation limit at 0, one guarantees that $-\hat{\phi}_{env}$ and $\bar{f}_h$ are directed against each other; on the other hand, the upper saturation limit at 1 implies that $|\hat{\phi}_{env}|$ does not exceed $|\bar{f}_h|$ (*i.e.*, $|\hat{\phi}_{env}| \leq |\bar{f}_h|$). Finally, sufficiently small $\epsilon_1 > 0$ removes the singularity at $\bar{f}_h = 0$. Thus, the algorithm (2) calculates the component of the environmental force that is directed against the human force, and makes its magnitude bounded by the magnitude of the human force. Therefore, according to the considerations presented above, substitution of $\hat{\phi}_{env}$ for $\hat{f}_{env}$ in the force reflection scheme would not make a difference in terms of the human force perception; however, contrary to the latter, the former does not generate the induced master motion.

The above presented algorithm calculates $\hat{\phi}_{env}$ as the projection of $\hat{f}_{env}$ onto the subspace spanned by the human force estimate $\bar{f}_h$; as a result, $\hat{\phi}_{env}$ is always collinear to $\bar{f}_h$. It is possible to construct another similar force reflection algorithm, where the resulting vector $\hat{\phi}_{env}$ would preserve the direction of the environmental force $\hat{f}_{env}$, however, its magnitude would depend on the magnitude of the projection of $\bar{f}_h$ onto the subspace spanned by $\hat{f}_{env}$. Such an algorithm is described by the formula

$$\hat{\phi}_{env} := \operatorname*{Sat}_{[0,1]}\left\{\frac{\hat{f}_{env}^T \bar{f}_h}{\max\{|\hat{f}_{env}|^2, \epsilon_1\}}\right\}\hat{f}_{env}. \quad (3)$$

Note that algorithms (2) and (3) give the same result if $\hat{f}_{env}$ is collinear to $\bar{f}_h$. It is also possible to use any convex combination of the algorithms (2) and (3).

Remark 1. It can be easily checked that $\hat{\phi}_{env}$ generated by any of the algorithms (2), (3), satisfies the inequality

$$\left|\bar{f}_h - \hat{\phi}_{env}\right| \leq \left|\bar{f}_h\right|. \quad (4)$$

This property will be utilized below, where we formulate and prove stability results for systems with force-reflecting algorithms that satisfy an inequality more general that (4). •

III. Stability results

In this section, we address stability properties of bilateral teleoperator system with force reflection algorithms described above. More precisely, we define a class of force reflection algorithms that, in particular, includes algorithms (2) and (3), and prove general stability result for teleoperator system with any force reflection algorithm from that class.

We assume that the closed-loop "master manipulator plus local master controller" subsystem is described as a general nonlinear system of the form

$$\begin{aligned} \dot{x}_m &= F_m\left(x_m, u_m\right), \\ y_m &= G_m\left(x_m, u_m\right), \end{aligned} \tag{5}$$

where x_m is the state of the master subsystem, and u_m is the master input. We impose general regularity assumptions on F_m and the output map G_m; namely, it is assumed that both $F_m(\cdot,\cdot)$, $G_m(\cdot,\cdot)$ are locally Lipschitz in their arguments. The input of the master subsystem is the external force input

$$u_m = f_h - f_r, \tag{6}$$

where f_h is the force/torque applied by the human operator, and f_r is the force/torque reflection signal. The output y_m of the master subsystem contains an arbitrary set of signals that are to be transmitted to the slave side, these may consist of master positions, velocities, forces/torques, as well as arbitrary combinations of them. During the transmission, the output y_m is subject to time-varying communication delay $\tau_f : \mathbb{R}^+ \to \mathbb{R}^+$; the transmitted version of the master output is then applied to the input of the slave subsystem, as follows

$$u_s(t) := y_m\left(t - \tau_f(t)\right). \tag{7}$$

The closed loop "slave plus environment plus local slave controller" subsystem is also described as a nonlinear system of the form similar to (5), *i.e.*,

$$\begin{aligned} \dot{x}_s &= F_s\left(x_s, u_s\right), \\ y_s &= G_s\left(x_s, u_s\right), \end{aligned} \tag{8}$$

where x_s is a state of the slave+environment interconnection, u_s is the input, and y_s is the output of the slave subsystem. Again, both F_s and G_s are assumed to be locally Lipschitz functions of x_s, u_s. The output y_s of the slave subsystem is again an arbitrary force signal which is to be transmitted to the master subsystem. This force signal may contain information about contact forces due to environment, position errors, velocity errors, or any other signals that depend on the state of the slave+environment subsystem or its inputs. Last but not least, these signals can be multiplied by arbitrary coefficients, which, in particular, corresponds to an arbitrary force reflection gain. The transmission of y_s to the master side is also a subject to time-varying communication delay $\tau_b : \mathbb{R}^+ \to \mathbb{R}^+$, according to the formula

$$\hat{f}_{env}(t) := y_s\left(t - \tau_b(t)\right) \tag{9}$$

Based on $\hat{f}_{env}$, the force reflection signal f_r in (6) is then generated according to the formula (1), where $\hat{\phi}_{env}$ is the outcome of a projection-based force reflection algorithm (such as (2) or (3)).

To formulate our stability results, let us first recall the notion of the input-to-state stability (ISS [15]).

Definition 1. A system of the form $\dot{x} = F\left(x, u\right)$ is said to be input-to-state stable if there exists $\beta \in \mathcal{K}L$, and $\gamma \in \mathcal{K}$ such that $|x(t)| \leq \max\left\{\beta\left(|x(0)|, t\right), \gamma\left(\sup_{s\in[0,t)} |u(s)|\right)\right\}$ holds for all $t \geq 0$. •

Our first assumption is the input-to-state stability of both the master and the slave subsystems.

Assumption 1. Both the master (5) and the slave (8) subsystems are input-to-state stable. •

Remark 2. Due to regularity (local Lipschitzness) assumption imposed on G_m, G_s, the input-to-state stability (ISS) also implies the input-to-output stability (IOS [15]) of both the master and the slave subsystems. Note, however, that neither ISS nor IOS gains are specified in Assumption 1; only the existence of these gains is assumed. As a result, such an assumption allows significant flexibility in design of local controllers for both the master and the slave subsystems; in particular, the ISS property (with possibly high ISS gain) of the master subsystem can be achieved, for example, by using local PD controller with arbitrarily low damping and stiffness coefficients [16]. •

The next assumption, borrowed from [8], is imposed on the communication process between the master and the slave.

Assumption 2. The communication delays $\tau_f, \tau_b : \mathbb{R}^+ \to \mathbb{R}^+$ are Lebesgue measured functions with the following properties:

i) there exist $\tau_* > 0$ and a piecewise continuous function $\tau^*\colon \mathbb{R} \to \mathbb{R}^+$ satisfying $\tau^*(t_2) - \tau^*(t_1) \leq t_2 - t_1$, such that the inequalities $\tau_* \leq \min\{\tau_f(t), \tau_b(t)\} \leq \max\{\tau_f(t), \tau_b(t)\} \leq \tau^*(t)$ hold for all $t \geq 0$;

ii) $t - \max\{\tau_f(t), \tau_b(t)\} \to +\infty$ as $t \to +\infty$. •

Remark 3. Assumption 2 is a technical one; it does not impose any significant restriction on communication process. In particular, it is interesting to note that fulfillment of this assumption does not depend on the characteristics of communication channel (such as bandwidth, packet loss percentage, etc.) On the contrary, this assumption can always be satisfied for any communication channel by implementing standard features such as packet numbering and/or time stamping, unless the communication is totally lost on a semi-infinite time interval. For more details, see [8]. •

Our third assumption describes the class of the force reflecting algorithms. The assumption is a generalization of the property described above in Remark 1.

Assumption 3. There exists $\eta \in \mathcal{K}$ such that $\hat{\phi}_{env}$ in (1) satisfies the inequality

$$\left|\bar{f}_h - \hat{\phi}_{env}\right| \leq \eta\left(\left|\bar{f}_h\right|\right). \bullet \tag{10}$$

Note that, according to Remark 1, both the projection-based force reflection algorithms (2) and (3) satisfy Assumption 3 with $\eta(\cdot) \equiv \mathbb{I}(\cdot)$.

Our fourth assumption is related to the force measurement/estimation process on the master side. In the results presented below, we do not restrict our consideration to the situation where the human force/torque is perfectly measured. On the contrary, we consider the human force measurement/estimation as a process whose accuracy may depend on a values of the human force and its derivatives as well as on disturbance level. More precisely, a sort of input-to-state stability assumption is imposed on the human force measurement/estimation process, as follows.

Assumption 4. Human force (torque) measurement/estimation process satisfies the following estimate

$$\left|f_h(t) - \bar{f}_h(t)\right| \leq \max \left\{ \begin{array}{l} \beta\left(\left|f_h(0) - \bar{f}_h(0)\right|, t\right), \\ \gamma_f^{\{0\}}\Big(\sup_{s\in[0,t)} |f_h(s)|\Big), \ldots, \\ \gamma_f^{\{r\}}\Big(\sup_{s\in[0,t)} |f_h^{\{r\}}(s)|\Big), \\ \gamma_w\Big(\sup_{s\in[0,t)} |w(s)|\Big) \end{array} \right\} \quad (11)$$

for all $t \geq 0$, where f_h is the human force applied to the master, $f_h^{\{i\}}$ is its i-th derivative, w are the external disturbances that affect the measurement process, $\beta_m \in \mathcal{KL}$, $\gamma_f^{\{0\}}, \ldots, ^{\{r\}}, \gamma_w \in \mathcal{K}$. •

Remark 4. Assumption 4 allows to address schemes where the direct input force measurement is performed in presence of sensor noise as well as a wide range of schemes that utilize different input estimation techniques [17]. In particular, the specific form of inequality (11) is motivated by the fact that the accuracy of estimates provided by input observers normally depends on derivatives of input (first derivative in the cases of high-gain and sliding mode input observers, second derivative for "dirty-derivative" filters, etc.) •

Consider now the closed-loop teleoperator system (5)-(9). Since this system involves communication delays (7), (9), it can appropriately be described by a system of functional differential equations (FDEs) rather than ordinary differential equations. To introduce the stability notion that should be satisfied for the closed-loop teleoperator system, consider a system of FDEs of a general form

$$\dot{x} = F\left(x_d, u_d\right), \quad (12)$$

where $x_d(t) = \{x(t-s):\ 0 \leq s \leq t_d(t)\}$, $u_d(t) = \{u(t-s):\ 0 \leq s \leq t_d(t)\}$, and $t_d(\cdot)$ is a nonnegative function defined on $\mathbb{R}^+$. Analogously to Definition 1, the input-to-state stability property for (12) can be defined as follows.

Definition 2. The system (12) is said to be input-to-state stable (with $t_d(0) \geq 0$) if there exists $\beta \in \mathcal{KL}$, and $\gamma \in \mathcal{K}$ such that

$$|x_d(t)| \leq \max\left\{\beta\left(|x_d(0)|, t\right), \gamma\Big(\sup_{s\in[-t_d(0),t)} |u(s)|\Big)\right\}. \bullet$$

Our main stability result can be formulated as follows.

Theorem 1. Consider the closed-loop force reflecting teleoperator system (5)–(9) with force reflection algorithm (1). Suppose Assumptions 1-4 are satisfied. Then there exists $\alpha_* \in \mathcal{K}_\infty$ such that if $\alpha(\cdot) \in \mathcal{G}$ in (1) satisfies $\alpha(s) \leq \alpha_*(s)$ for all $s \geq 0$ then the closed-loop force reflecting teleoperator system with state $\mathbf{x}_d = (x_m^T, x_s^T)_d^T$ and input $\mathbf{u} = (f_h^T, \ldots, (f_h^{\{r\}})^T, w^T)^T$ is input-to-state stable in the sense of Definition 2 with $t_d(0) = \tau^*(0)$. •

Proof of Theorem 1 is based on a version of a small-gain theorem for systems with delays presented in [8]. Essentially, it can be verified that the small-gain stability condition for the system under consideration has a form $\gamma_m \circ \gamma_s \circ \alpha(s) < \mathbb{I}(s)$ for all $s > 0$, where $\gamma_m, \gamma_s \in \mathcal{K}$ are IOS gains of the master and the slave subsystems, respectively, and $\alpha \in \mathcal{G}$ is the weighting function from the formula (1). The above condition can always be met, for example, by choosing the upper bound $\alpha_* \in \mathcal{K}_\infty$ as follows $\alpha_*^{-1}(s) := \gamma_m \circ \gamma_s(s) + s$. All details, however, are omitted due to space limitations. •

One of the most interesting features of the result presented in the above Theorem 1 is that it does not impose any restrictions on the ISS (IOS) gains of the master and slave subsystem. Instead, given the master and the slave gains, the overall stability can always be achieved by an appropriate choice of the weighting function $\alpha(\cdot) \in \mathcal{G}$ in (1). As one can see from the formula (1), $\alpha(\cdot) \in \mathcal{G}$ and $[\mathbb{I} - \alpha](\cdot) \in \mathcal{G}$ determine the relative weights of the terms $\hat{f}_{env}$ and $\hat{\phi}_{env}$, respectively, in the force reflection signal $\hat{f}_r$. Both these terms provide the haptic feedback to the operator; however, the difference between them is that, contrary to $\hat{f}_{env}$, the term $\hat{\phi}_{env}$ does not create the induced master motion that can potentially destabilize the overall teleoperator system. Thus, the choice of $\alpha(\cdot)$ determines the gain between the force reflection signal and the resulting induced master motion. In particular, if $\alpha(\cdot) \equiv 0$ in (1), the induced master motion is eliminated completely; in this case, the stability is guaranteed simultaneously for all gains as long as both the master and the slave subsystems are stable. Thus, the design of stable force reflecting teleoperator system with communication delay is essentially reduced to the design of two stable subsystems. Theoretically, the choice $\alpha(\cdot) \equiv 0$ disconnects the feedback loop in terms of induced master motion; *i.e.*, the corresponding closed loop gain becomes equal to zero. In particular, this allows to achieve stability for arbitrarily low damping and stiffness of the master manipulator and in presence of arbitrarily high force reflection gain, which has numerous advantages in terms of better operability and transparency. On the other hand, a nonzero $\alpha(\cdot)$ implies that the force reflection term may create some induced motion of the master manipulator; however, the amount of this motion is not sufficient to destroy the overall stability as long as $\alpha(\cdot)$ is small enough. It seems conceivable that certain small enough amount of the induced master motion may be useful in some teleoperation tasks; for example, it may impel the otherwise inactive human operator to apply forces against it. From the simulation results presented below, it can also be noticed that a small amount of the induced master motion may improve force regulation.

IV. Simulations

In this section, we present simulation results that illustrate the improvement to the stability properties of the force reflecting teleoperator system brought in by the projection based force reflecting algorithms. Specifically, we address the question of how the stability of the teleoperator system depends on the force reflection gain, and show that the force reflection gain can be substantially increased in presence of force reflection algorithms (1), (3) without loosing the overall stability (in the case of algorithm (1), (2), very similar results are obtained which are omitted here because of the space constraints). We simulate a force-reflecting master-slave teleoperator system that consists of two identical 2-DOF planar manipulators, master ($i = m$) and slave ($i = s$), whose are described by Euler-Lagrange equations of the form $H_i(q_i)\,\ddot{q}_i + C_i(q_i, \dot{q}_i)\,\dot{q}_i = \tau_i$, where

$$H_i(q) = \begin{bmatrix} (2l_1 \cos q_2 + l_2) l_2 m_2 + l_1^2 (m_1 + m_2) & l_2^2 m_2 + l_1 l_2 m_2 \cos q_2 \\ l_2^2 m_2 + l_1 l_2 m_2 \cos q_2 & l_2^2 m_2 \end{bmatrix},$$

$$C_i(q, \dot{q}) = \begin{bmatrix} -l_1 l_2 m_2 \sin(q_2) \dot{q}_2 & -l_1 l_2 m_2 \sin(q_2)(\dot{q}_1 + \dot{q}_2) \\ l_1 l_2 m_2 \sin(q_2) \dot{q}_1 & 0 \end{bmatrix},$$

and the parameters are $m_1 = 10\,kg$, and $m_2 = 5\,kg$, $l_1 = 0.7\,m$, $l_2 = 0.5\,m$. On the master side, $\tau_m = u_m + \mathbb{J}_m^T(q_m, \dot{q}_m)\,f_h - \mathbb{J}_m^T(q_m, \dot{q}_m)\,\hat{f}_r$, where u_m is the local master control law, f_h is the force imposed by the human operator, $\hat{f}_r$ is the force reflection term, and $\mathbb{J}_m(q_m, \dot{q}_m)$ is the (spatial) Jacobian of the master manipulator. The local master controller consists of a damping term $u_m = -D_m \dot{q}_m$, where the damping coefficient $D_m = 50\,N{\cdot}m{\cdot}sec/rad$. The human operator tries to move the end-effector of the master from its initial location at $x_0 = 0$ m, $y_0 = 1.2$ m (which corresponds to $q_1 = \pi/2$, $q_2 = 0$) to the final point $x_{end} = 0.783$ m, $y_{end} = 0.8562$ m, along the straight line between these two points. On the slave side, $\tau_s = u_s - \mathbb{J}_s^T(q_s, \dot{q}_s)\,f_{env}$, where u_s is the local slave control law, f_{env} is the contact force due to environment, and $\mathbb{J}_s$ is the slave Jacobian. The controller has a form $u_s = K_s(\xi_1 - q_s) + D_s(\xi_2 - \dot{q}_s)$, where ξ_1, ξ_2 are estimates of the delayed master position $\hat{q}_m$ and its derivative, respectively, both provided by a dirty-derivative filter. The parameters are $K_s = 500\,N \cdot m/rad$ and $D_s = 500\,N \cdot m \cdot s/rad$. When following the trajectory of the master manipulator, the slave hits an obstacle. The obstacle is a rigid wall which is located at $x = 0.6$ m, and has stiffness coefficient equal to $K_{env} = 10^6\,N/m$; thus, the obstacle can be considered as an extremely rigid one. The contact force due to environment is then transferred to the master side with force reflecting gain $K_f > 0$. Both the forward and the backward communication channels are modelled as one-step delay systems with variable sampling, where the sampling instants are Poisson distributed with mean rate $\lambda = 10\ sec^{-1}$.

First, we simulate the teleoperator system where the force feedback is provided to the master using the standard proportional force-reflection scheme (*i.e.*, $\alpha(\cdot) \equiv \mathbb{I}(\cdot)$ in (1)). Keeping in mind that the slave robot makes a contact with a very stiff environment, it is not surprising that the system with proportional force reflection generally admits a very low force reflection gain without loosing the overall stability. In fact, our simulations show that, under the conditions described above, the border of stability in terms of force reflecting gain lies somewhere between $K_f = 0.4$ and $K_f = 0.7$. Typical plots for $K_f = 0.4$ (stable) and $K_f = 0.7$ (unstable) are shown in figures 1 and 2, respectively. Between these values, the performance of the system decreases gradually.

These results are then compared with the response of the same teleoperator system with the projection-based force reflection algorithm (1), (3). For simplicity, we consider only linear weighting functions $\alpha(\cdot) = \alpha_0 \cdot \mathbb{I}(\cdot)$, $\alpha_0 \in [0, 1]$. It can be seen from the formula (1), that lower values of α_0 imply higher weights of the signal $\hat{\phi}_{env}$, generated by the projection-based algorithm (3), in the overall force reflection term $\hat{f}_r$. According to our theoretical considerations, this results in smaller amount of the induced master motion and, therefore, improved stability properties (in particular, higher force reflection gains can be implemented without loosing the overall stability). This is clearly confirmed by our simulation results. Examples of these results are presented in figures 3, 4, where the responses of the teleoperator system for $K_f = 1$, $\alpha_0 = 0.2$ and $K_f = 10$, $\alpha_0 = 0.01$ are presented, respectively. As one can see from the plots, both these responses are stable.

Overall, our simulations show that, in accordance with theoretical results, the higher weight of the projection-based term $\hat{\phi}_{env}$ in the force reflection signal decreases the amount of induced master motion and, therefore, leads to improved stability characteristics (for example, in terms of higher admissible force reflection gains). On the other hand, it seems like some small enough amount of the induced master motion may have positive influence on the transient response in terms of force regulation . To illustrate this point, consider Fig. 5, (a), (b), where the actual contact force due to environment is shown versus the force signal reflected to the hand of the human operator, for $\alpha = 0.2$ and $\alpha = 0.1$, respectively. In both these cases, the actual contact force and the force reflected converge to each other, however, the convergence is faster for $\alpha = 0.2$. More precisely, it seems like for each set of the task parameters (such as the environmental stiffness, force reflection gain, etc.) there exists some "optimal" value of the weighting coefficient α that results in combination of stable response and fast force convergence. This phenomenon may worth further research.

V. Conclusions

Traditionally, the small-gain approach is considered to be not particularly suitable for teleoperation tasks, since it "...would result in conservative design criteria (leading to poor transparency)..." [18]. Indeed, for force reflecting teleoperator systems, the direct application of the small-gain approach results in significant constraints on the force reflection gain, as well as generally requires high damping/stiffness of the master manipulator. In this paper, we show that, using the projection-based force reflection algorithms, the constraints on subsystem's gains can be effectively removed. In particular, this implies that the stability can be achieved for arbitrarily high force reflection gain and arbitrarily low damping/stiffness of the master manipulator. Essentially, the proposed approach reduces the design of stable force reflecting teleoperator system with communication delay to the design of two stable subsystems. It is worth noting that this is achieved by utilizing certain fundamental characteristics of the human force sensing and without paying the price in terms of transparency deterioration. In our opinion, the use of projection-based force reflection algorithms may significantly improve the applicability of the small-gain methods to the design of the force-reflecting teleoperator systems, particularly in presence of network induced communication constraints.

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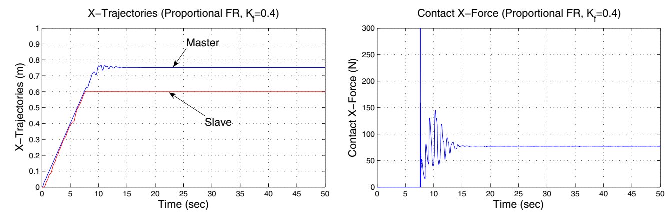
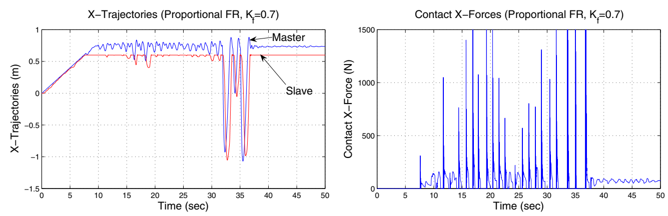


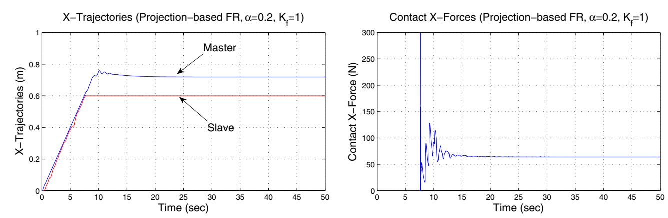
(a) X-Trajectories of the master and the slave (b) X-component of the contact force

Fig. 1. Proportional FR, $K_f = 0.4$



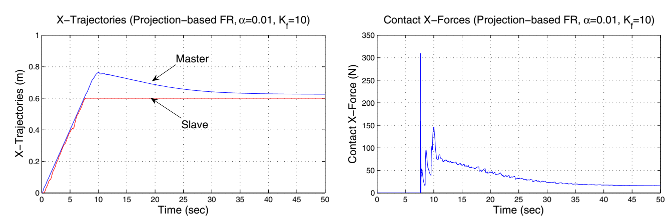
(a) X-Trajectories of the master and the slave (b) X-component of the contact force

Fig. 2. Proportional FR, $K_f = 0.7$



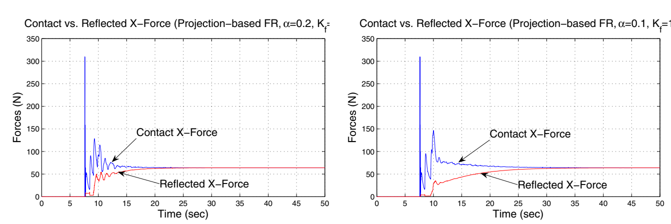
(a) X-Trajectories of the master and the slave (b) X-component of the contact force

Fig. 3. Projection-based FR, $K_f = 1$, $\alpha_0 = 0.2$



(a) X-Trajectories of the master and the slave (b) X-component of the contact force

Fig. 4. Projection-based FR, $K_f = 10$, $\alpha = 0.01$



(a) $\alpha_0 = 0.2$ (b) $\alpha_0 = 0.1$

Fig. 5. Projection-based FR, $K_f = 1$, forces convergence.