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Bounded control of a flapping wing micro drone in three dimensions

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Abstract— **This paper presents a bounded control of a flapping Micro Aerial Vehicle (MAV) in three dimensions. First, a simplified model of the flapping MAV is presented aiming to test the control law. The averaging theory shows that for high frequency systems, only the mean aerodynamic forces and torques over a period affect the movement of the body. Therefore, a bounded nonlinear state feedback control, calculated using the averaged model, is applied to the time varying system in order to stabilize it at a desired position in hovering mode. The robustness of the control is tested with respect to the aerodynamic coefficient and to external disturbances.**

I. INTRODUCTION

The flapping flight has been a large domain of exploration in the last few years, aiming to understand and mimic the ingenious strategies developed by animals during the navigation in three dimensions. The big interest in insect flight is driven by the advantages that it presents relative to the rotary and fixed airfoils. For example, flapping MAVs should have a greater maneuverability, develop more lift, produce less noise and get benefit from their small size and biomimetic behavior. The major disadvantages of flapping airfoils are, on one hand, the difficulty to conceive the mechanisms developed by insects during complex maneuvers [1], and on the other hand, the necessity of using low computational embedded systems, tiny sensors and actuators to ensure the free autonomous flight. The progress in microelectronic technologies, materials and communication tools, in addition to the researches held all over the world is helping the development of such airfoils. MAVs have several applications like surveillance, seismic and high voltage lines monitoring, intervening in dangerous environments, searching and rescuing, spying, investigating or just gaming.

The present work lies within the scope of the OVMI project, which aims to design and develop a flapping wings micro robot (see Fig. 1) capable of autonomous flight, by taking into account fluid mechanics and energetic aspects.

The goal of the present paper is to develop control laws able to ensure the flight in 3D space, i.e. the attitude stabilization and the linear displacements. Few of previous works have treated this problem. State feedback controllers are proposed, one acting directly on the position [2] and the other, bounded, acting on the vertical force and torques, is based on poles placement using the linearized dynamics of the system [3]. In [4], a Linear Quadratic Gaussian (LQG) optimal state feedback controller is determined. Couplings between the pitch and the forward movements on one hand, the roll and the lateral movements on the other hand are considered [2], [3].

In the present work, bounded state feedback nonlinear control laws of the flapping body's position and orientation are proposed. In fact, the linear control laws used in previous works are not sufficiently robust with respect to external disturbances like a drop of rain, wind, etc. Moreover, the proposed control is bounded in order to respect the maximum limit of the actuators driving the flapping wings. Besides, the attitude is determined by the quaternion [5], which presents more simplicity in computation and prevents numerical singularities induced by Euler angles.

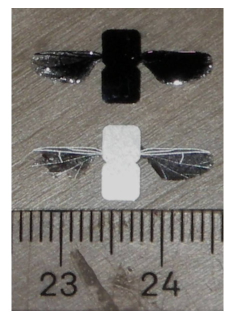

Fig. 1: Centimetric scale prototype of the "OVMI" project

The paper is organized as follows. Section II presents a simplified model of the flapping MAV with limited wings kinematics. In section III, bounded non linear control laws of the MAV's orientation and position are proposed, and the global stability is proven based on averaging and systems cascade theories. These control laws are validated by some simulations and robustness tests in section IV. Finally, section V presents conclusions and introduces future works.

II. INSECT FLIGHT MODELING

A simplified model of a flapping MAV with limited wings kinematics is proposed in this work in order to validate the control strategy. Translational and rotational motions of the body are computed based on the body's dynamic equations and the aerodynamic theory.

A. Wings movement parametrization

A wing is considered as a rigid body associated to a frame $\mathbb{R}^w(\overrightarrow{r}, \overrightarrow{t}, \overrightarrow{n}, \psi, \phi, \theta)$ (see Fig. 2). The axis $\overrightarrow{r}$ is oriented

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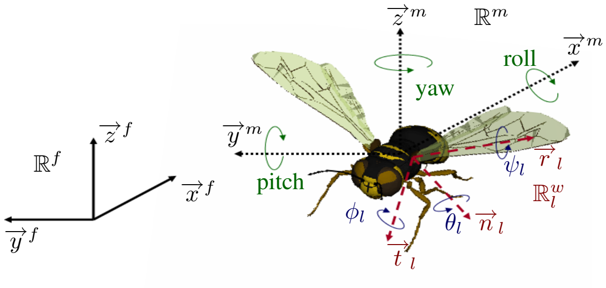


Fig. 2: Frames and wings angles

from the wing base to its tip, the axis $\overrightarrow{t}$ is parallel to the wing chord, oriented from trailing to leading edge and the axis $\overrightarrow{n}$ is perpendicular to the wing plane oriented so that the three-sided frame $(\overrightarrow{r}, \overrightarrow{t}, \overrightarrow{n})$ is direct. The angles (ψ, ϕ, θ) are used to specify the position of the wing through three rotations about the wings Euler axes $(\overrightarrow{r}, \overrightarrow{t}, \overrightarrow{n})$ respectively. The flapping angle, ϕ, defines an up and down movement of the wing. The rotation angle, ψ, defines a rotation of the wing about its main axis. The deviation angle, θ, defines a forward and backward movement of the wing parallel to the MAV's body. Angles ϕ and ψ are assumed to vary according to saw tooth and pulse functions respectively, so that the wing changes its orientation at the end of each half stroke (see Fig. 3). In order to use actuators for 2 DOF only, the wings are supposed to beat in the mean stroke plane, therefore angle θ is taken to zero. Note that the reduction of the vehicle's size induces a reduction of its electronic and mechanical components. For instance, the smallest MAV prototype is driven by one actuated degree of freedom per wing [6].
The temporal variation of the wings trajectory is given by

$$\begin{array}{rcl} \phi(t) & = & \begin{cases} \phi_0(1 - \frac{2t}{\kappa T}) & 0 \le t \le \kappa T \\ \phi_0(2\frac{t-\kappa T}{(1-\kappa)T} - 1) & \kappa T < t \le T \end{cases} \\ \psi(t) & = & \psi_0 \,\mathrm{sign}(\kappa T - t) \qquad 0 \le t \le T \\ \theta(t) & = & 0 \qquad 0 \le t \le T \end{array} \tag{1}$$

where sign designates the classical sign function, T is the wingbeat period, κ is the ratio of downstroke duration to the wingbeat period, ϕ_0 and ψ_0 are respectively the amplitudes of flapping and rotation angles. The last two parameters considered for both left and right wings, will be taken as control variables, as explained in the following.

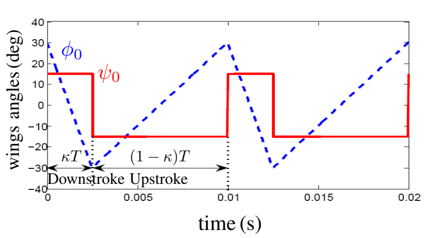


Fig. 3: Wings angles configuration over two wingbeat periods: flapping angle ϕ (dashed line) and rotation angle ψ (continuous line)

B. Body's dynamics

The equations of motion of a rigid body subject to external forces and torques are given by

$$\dot{P}^f = V^f \tag{2}$$

$$\dot{V}^f = \frac{1}{m} R^T(q) f^m - cV^f - g \tag{3}$$

$$\begin{pmatrix} \dot{q}_0 \\ \dot{\overrightarrow{q}} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} -\overrightarrow{q}^T \\ I_3 q_0 + \hat{q}^T \end{pmatrix} w^m \tag{4}$$

$$\dot{\omega}^m = J_m^{-1}(\tau^m - \omega^m \wedge J_m \omega^m) \tag{5}$$

$P^f \in \mathbb{R}^3$ and $V^f \in \mathbb{R}^3$ are respectively the linear position and velocity of the body's center of gravity relative to the fixed frame $\mathbb{R}^f$. ω^m is the angular velocity with respect to the mobile frame $\mathbb{R}^m$ attached to the insect's body on its center of gravity. c is the viscous damping coefficient and g the gravity vector. $f^m \in \mathbb{R}^3$ and $\tau^m \in \mathbb{R}^3$ are respectively the aerodynamic force and torque. $J_m \in \mathbb{R}^{3\times3}$ is the inertia matrix of the body relative to $\mathbb{R}^m$ and I_3 is the identity matrix. q is the quaternion defining the attitude of the body relative to $\mathbb{R}^f$ [5], $q = [\cos\frac{\nu}{2} \ (\overrightarrow{e}\sin\frac{\nu}{2})]^T = [q_0 \ \overrightarrow{q}^T]^T$ consisting of a rotation of angle ν about the Euler axis $\overrightarrow{e}$. $q_0 \in \mathbb{R}$ is the scalar part and $\overrightarrow{q} = [q_1 \ q_2 \ q_3]^T \in \mathbb{R}^3$ the vector part of the quaternion. $q \in \mathbb{H}$ where $\mathbb{H} = \{q \mid q_0^2 + \overrightarrow{q}^T\overrightarrow{q} = 1\}$ is the Hamilton space. $R(q) \in SO(3) = \{R(q) \in \mathbb{R}^{3\times3} : R^T(q)R(q) = I, \ \det R(q) = 1\}$ is the rotation matrix from the fixed frame $\mathbb{R}^f$ to the mobile frame $\mathbb{R}^m$, $R(q) = (q_0^2 - \overrightarrow{q}^T\overrightarrow{q})I_3 + 2(\overrightarrow{q}\,\overrightarrow{q}^T - q_0\hat{q}^T)$. $\hat{q}$ is the skew symmetric tensor associated to $\overrightarrow{q}$.

The delayed stall is considered as the unique aerodynamic force applied on the wing's aerodynamic center, which coincides with its center of mass in practical cases. The force is perpendicular to the wing and has the opposite direction of the wing's velocity. The module of the force is considered proportional to the square of the wing's velocity relative to $\mathbb{R}^m$. This assumption has to be checked on the prototype under construction. The module f^w of the force is given by

$$f^w = -\frac{1}{2}\rho C_w S_w v^w |v^w| \tag{6}$$

ρ is the air density, S_w is the wing's surface, v^w is the wing's velocity, C_w is a coefficient of the aerodynamic force applied on a wing. $C_w = C(1 + C_f)$ during downstroke and $C_w = C(1 - C_f)$ during upstroke, where $C \approx 3.5$ is the delayed stall force coefficient derived empirically in [7], [8] and C_f is a damping coefficient chosen so that the aerodynamic force is 20% greater during downstroke than during upstroke. This dissymmetry between the two half-strokes can be justified based on [1]. During downstroke, the dorsal side of the wing is opposite to the air flow. The supination opposes the ventral side of the wing to the flow. Consequently, the effective area of the wing is reduced and the orientation of air circulation about the wing reverses, leading to a wing camber alteration. Therefore, downstroke lift is likely to be higher than that of upstroke, so that the averaged force over a single wingbeat period should at least balance the body's weight.

The aerodynamic force relative to $\mathbb{R}^m$ is the sum of the forces developed by the left and right wings

$$f^m = f_l^m + f_r^m \tag{7}$$

The aerodynamic force has two components, the thrust that ensures a forward movement of the MAV, and the lift that ensures a vertical movement.
The aerodynamic torque relative to $\mathbb{R}^m$ is defined as the vectorial product of the force f^m and the wing's aerodynamic center's position p^m (p^m is the projection in $\mathbb{R}^m$ of the position $p^w = [L\ 0\ 0]$ relative to $\mathbb{R}^w$). Angular viscous torques are negligible with respect to aerodynamic torques.

$$\tau^m(t) = p_l^m(t) \wedge f_l^m(t) + p_r^m(t) \wedge f_r^m(t) \tag{8}$$

The wing's velocity is the center of mass position's derivative $v^m = \dot{p}^m$, v^w is the projection of v^m in $\mathbb{R}^w$ [9].
Note that this simplified model is developed only to test the control strategy.

C. Averaged model - Control constraints

The averaging theory has been largely studied in [10], [11], [12] for high frequency systems. Flapping bodies are an application of it [13]. The aerodynamic forces and torques, generated by the high frequency beating wings, affect the body's dynamics only by their averaged values over a wingbeat period, since the wingbeat frequency is much higher than the bandwidth of the body's dynamics. Moreover, a stabilizing control law calculated using the averaged model will stabilize the oscillating model too for sufficiently high flapping frequency. Therefore, the averaged dynamics of the time varying model (1-8) were calculated [9]

$$(\overline{f}_x, \overline{f}_z, \overline{\tau}_x, \overline{\tau}_z) = \Lambda(\phi_0^l, \phi_0^r, \psi_0^l, \psi_0^r) \tag{9}$$

The control of the flapping MAV's position and orientation is ensured by the aerodynamic force and torque applied to the body over a wingbeat period.
The attitude stabilization of the MAV is ensured by the roll, pitch and yaw control torques ($\overline{\tau}_x$, $\overline{\tau}_y$, $\overline{\tau}_z$). The MAV is supposed to move forward due to a thrust control force $\overline{f}_x$, vertically due to a lift control force $\overline{f}_z$ ($\overline{f}_x$ and $\overline{f}_z$ are expressed in $\mathbb{R}^m$), sideway due to a coupling between the roll and the vertical movements. The thrust and lift forces as well as the roll and yaw torques are generated by the flapping wings (9), whereas the pitch control torque $\overline{\tau}_y$ is supposed to be generated by a small mass moving inside the insect body, and changing its center of gravity. Considering

$$\begin{array}{rcccl} 0 & \leq & \phi_0 & \leq & \overline{\phi_0} \\ -\overline{\psi_0} & \leq & \psi_0 & \leq & \overline{\psi_0} \end{array} \tag{10}$$

for left and right wings, system (9) defines a convex set Ω in the mean control variables ($\overline{f}_x$, $\overline{f}_z$, $\overline{\tau}_x$, $\overline{\tau}_z$) (see Fig. 4a and 4b, Ω_{τ_x,τ_z} and Ω_{f_x,f_z} are the projection of Ω on the planes ($\overline{\tau}_x, \overline{\tau}_z$) and ($\overline{f}_x, \overline{f}_z$) respectively). Therefore, anywhere in the set Ω, there exists a wing configuration (ϕ_0^l, ϕ_0^r, ψ_0^l, ψ_0^r) producing the mean desired forces and torques ($\overline{f}_x$, $\overline{f}_z$, $\overline{\tau}_x$, $\overline{\tau}_z$). Considering the mean behavior over a wingbeat period of system (2-5), the MAV is approximated by a rigid body subject to external forces and torques. Therefore, the averaged state of the time varying model $\overline{x}$ is equivalent to a rigid body state x^{rb}.
In this work, state feedback control laws, stabilizing the rigid body's position and orientation, will be proposed.

$$(\overline{f}_x, \overline{f}_z, \overline{\tau}_x, \overline{\tau}_z) = \mathcal{U}(x^{rb}) = \mathcal{U}(\overline{x}) \tag{11}$$

By inverting (9), (11) can be written

$$(\phi_0^l, \phi_0^r, \psi_0^l, \psi_0^r) = \Lambda^{-1}(\mathcal{U}(\overline{x})) \tag{12}$$

This control law should respect the saturation set Ω, ensuring that the amplitudes of the wings' angles $(\phi_0^l, \phi_0^r, \psi_0^l, \psi_0^r)$ remain within the saturation bounds (10).

III. Insect flight control

Different control laws existing in the literature can be used for flapping bodies. However, they present some disadvantages. For example, linear control laws are not sufficiently robust with respect to external disturbances. Thereby, nonlinear control should be used. Two techniques are widely used: the input output linearization and the backstepping control. While the first one brings the problem back to the linear case, the latter requires a knowledge of the system's inertial parameters which are difficult to identify.
In this section, a nonlinear bounded control strategy, that stabilizes system (2-5) and guarantees the wings angles limits, is proposed. It can be applied on a finer model of the flapping body; only the relation between the mean aerodynamic forces and torques over a wingbeat period and the wings angles amplitudes (9) is needed. The control is independent of the inertia matrix, robust with respect to system's parameters uncertainties, aerodynamic coefficient, etc.
The attitude stabilization is ensured by applying control torques for subsystem (4-5), taking the roll, pitch and yaw angles called (η_1, η_2, η_3) to 0. The control of the position in $\mathbb{R}^f$ (2-3) is ensured by the control thrust and lift. The lateral movement will be created by tilting the aircraft along the roll axis. Therefore, (2-5) can be considered as cascade of systems, it is of the form [14]

$$\left\{ \begin{array}{rcl} \dot{x} & = & f(x,y) \\ \dot{y} & = & g(y,u) \end{array} \right. \tag{13}$$

which means that the translational dynamics depend on the rotational ones, but the rotational dynamics are independent of the translational ones.

A. Attitude control

The control law applied in this paragraph is supposed to drive the body to a desired orientation q_d, while the angular velocity should vanishes: $q \to q_d$, $\omega^m \to 0$ as $t \to \infty$. The error between the current and desired orientations of the body is quantified by the quaternion error: $q_e = q \otimes q_d^{-1}$, where q^{-1} is the quaternion conjugate given by $q^{-1} = [q_0\ -\overrightarrow{q}^T]^T$, $\otimes$ is the quaternion product defined by $q \otimes Q = [(q_0 Q_0 - \overrightarrow{q}\overrightarrow{Q})\ (q_0\overrightarrow{Q} + Q_0\overrightarrow{q} + \overrightarrow{q} \wedge \overrightarrow{Q})^T]^T$, and $\wedge$ denotes the vectorial product.

The proposed attitude stabilizing control torque is a bounded state feedback based in its formulation on the model of a rigid body [15] (equivalent to the averaged model of the flapping body) and applied to the time variant model (flapping MAV). This control law is extremely simple and therefore suitable for an embedded implementation. Moreover, the control law is robust with respect to aerodynamic coefficient and does not require the knowledge of the body's inertia. Let $\overline{\tau} = [\overline{\tau}_x\ \overline{\tau}_y\ \overline{\tau}_z]^T$ be the roll, pitch and yaw control torques.

$$\overline{\tau}_i = -\alpha_i \sigma_{M_{2,i}}(\lambda_i[\gamma_i \overline{\omega}_i + \text{sign}(q_{e0})\sigma_{M_{1,i}}(\overline{q}_{e_i})]) \quad (14)$$

where $i \in \{1, 2, 3\}$, $\text{sign}(q_0)$ takes into account the possibility of 2 rotations to drive the body to its equilibrium orientation; the one of smaller angle is chosen. $\overline{\omega}_i$ and $\overline{q}_i$ are the averaged angular velocities and quaternion over a single wingbeat period (averaged state of the rotational subsystem $\overline{x}_r = \{\overline{\omega}_i, \overline{q}_i\},\ i = 1, 2, 3$) representing the time varying angular velocities and quaternion of a rigid body. $\alpha_i,\ \lambda_i,\ \gamma_i$ are positive parameters. Differently from [15], γ_i has been added in order to slow down the convergence of the torque relative to the angular velocity. Moreover, the general case represented by the quaternion error (instead of the current quaternion) is considered. $\sigma_{M_{1,i}}$ and $\sigma_{M_{2,i}}$ are saturation functions with $M_{1,i}$ and $M_{2,i}$ the saturation bounds: $M_{1,i} \geq 1$, $M_{2,i} \geq \lambda(2M_{1,i} + \epsilon_i)$ and $\epsilon_i > 1$. The $M_{2,i}$'s are chosen in order to respect input saturations: wings Euler angles and body's length. Based on (9,10), the maximum flapping and rotation amplitudes, $\overline{\phi}_0$ and $\overline{\psi}_0$, define a set Ω_{τ_x,τ_z} of admissible torques (see Fig. 4a). The saturation bounds $M_{2,1}$ and $M_{2,3}$ are adjusted in (14) so that $\overline{\tau}_x$ and $\overline{\tau}_z$ remain in the limits of Ω_{τ_x,τ_z}, which guarantees not to exceed the maximum angles. $M_{2,2}$ should respect the saturation induced by the length of the body, since the pitch torque is generated by a small mass moving inside it.

The asymptotic stability of the closed loop system has been shown in [15] for rigid bodies using the following Lyapunov function (the added parameter γ_i and the use of the quaternion error do not change the proof)

$$Vr = \frac{1}{2}{\omega^{rb}}^T J_m \omega^{rb} + \kappa((1 - q_{e_0}^{rb})^2 + {\overrightarrow{q}_e^{rb}}^T \overrightarrow{q}_e^{rb}) \quad (15)$$

Therefore, $\overline{\omega} \to 0$ and $\overline{q} \to \overline{q}_d$ (based on the rigid body case). By means of the averaging theory, $\|\omega - \overline{\omega}\| < k_1 T$ and $\|q - \overline{q}\| < k_2 T$ for $k_{1,2} > 0$ and T the wingbeat period.

B. Position control

The subsystem (2-3) can be considered as a chain of integrators by neglecting the drag force represented by the term cV^f, therefore considering the system at low speeds. It is considered as a disturbance term in simulations. Supposing that, after a sufficiently long time, the subsystem (4-5) is stabilized over the pitch and yaw axis ($\eta_2 = 0,\ \eta_3 = 0$), then the normalized translational subsystem described by equations (2-3) can be written ($P^f = [P_x\ P_y\ P_z]^T$)

$$\begin{cases} \dot{p}_1 = p_2 \\ \dot{p}_2 = v_x \end{cases} \quad (16)$$

$$\begin{cases} \dot{p}_3 = p_4 \\ \dot{p}_4 = -v_h \sin(\eta_1) \\ \dot{p}_5 = p_6 \\ \dot{p}_6 = v_h \cos(\eta_1) - 1 \end{cases} \quad (17)$$

$p = (p_1, p_2, p_3, p_4, p_5, p_6) = (\overline{P}_x, \overline{V}_x, \overline{P}_y, \overline{V}_y, \overline{P}_z, \overline{V}_z)$ is the averaged state of the translational subsystem, $\overline{x}_t = p$, $v_x = \frac{\overline{f}_x}{mg}$, $v_h = \frac{\overline{f}_z}{mg}$, η_1 the roll angle and 1 is the normalized gravity.
As in paragraph III-A, a bounded state feedback control law, calculated using the averaged model over a wingbeat period (equivalent to a rigid body model), is tested on the time variant model. The proposed controller is extremely low cost for an embedded implementation. Moreover, it is robust to measurement delays and to system model uncertainty [16].

1) Stabilization of the forward movement: System (16) define a double integrator, and can be stabilized using the control developed in [16]. v_x can then be chosen as

$$v_x = \frac{\overline{v}_x}{\varepsilon_x + \varepsilon_x^2}(-\varepsilon_x \sigma(p_2) - \varepsilon_x^2 \sigma(\varepsilon_x p_1 + p_2)) \quad (18)$$

where ε_x is a positive parameter lower than 1 and $\sigma(.)$ is a twice differentiable function bounded between ± 1 parameterized by $0 < \mu < 1$ [17]

$$\sigma(s) = \begin{cases} -1 & s < -1 - \mu \\ e_1 s^2 + e_2 s + e_3 & s \in [-1 - \mu, -1 + \mu[\\ s & s \in [-1 + \mu, 1 + \mu] \\ -e_1 s^2 + e_2 s - e_3 & s \in]1 - \mu, 1 + \mu] \\ 1 & s > 1 + \mu \end{cases} \quad (19)$$

with $e_1 = \frac{1}{4\mu},\ e_2 = \frac{1}{2} + \frac{1}{2\mu},\ e_3 = \frac{\mu^2 - 2\mu + 1}{4\mu}$.

$\overline{v}_x$ should respect the saturation bound represented by the set Ω_{f_x,f_z} (see Fig. 4b) in order to guarantee admissible flapping and rotation angles. The asymptotic stability of (p_1, p_2) is then ensured (cf. proof [16]).

2) Stabilization of the lateral and vertical movements: System (17) associates the lateral movement of the MAV to a roll movement, inspired from the works on PVTOLs (Planar Taking Off and Landing) aircrafts [17]. η_1 is considered as an intermediate input for system (17) and should respect a desired angle η_{1_d}:

$$\eta_{1_d} = \arctan(\frac{-v_y}{v_z + 1}) \quad (20)$$

v_y and v_z will be determined later on.
The vertical normalized lift v_h is given by

$$v_h = \sqrt{v_y^2 + (v_z + 1)^2} \quad (21)$$

An appropriate control torque as defined in (14) will drive the roll angle η_1 to the desired value η_{1_d} defined by the corresponding quaternion q_d, hence system (17) will be transformed into the form of two independent second order integrators [17].

$$\begin{cases} \dot{p}_3 &= p_4 \\ \dot{p}_4 &= v_y \end{cases} \qquad \begin{cases} \dot{p}_5 &= p_6 \\ \dot{p}_6 &= v_z \end{cases} \tag{22}$$

Therefore, the stability of the lateral and vertical movements can be ensured following [16]:

$$v_y = \frac{\overline{v}_y}{\varepsilon_y + \varepsilon_y^2}(-\varepsilon_y \sigma(p_4) - \varepsilon_y^2 \sigma(\varepsilon_y p_3 + p_4)) \tag{23}$$

$$v_z = \frac{\overline{v}_z}{\varepsilon_z + \varepsilon_z^2}(-\varepsilon_z \sigma(p_6) - \varepsilon_z^2 \sigma(\varepsilon_z p_5 + p_6)) \tag{24}$$

$0 < \varepsilon_y, \varepsilon_z < 1$, and $\sigma(.)$ is defined as in (19). $\overline{v}_y$ and $\overline{v}_z$ are chosen such that:

$$\overline{v}_h = \sqrt{\overline{v}_y^2 + (\overline{v}_z + 1)^2} \tag{25}$$

and $\overline{v}_h$ should respect the saturation bounds represented by the set Ω_{f_x,f_z} (see Fig. 4b). The asymptotic stability of (p_3, p_4, p_5, p_6) is then ensured [16], [17].

3) Stability of the translational movement of the time varying system: Applying the proposed control law, $\overline{P}^f \to 0$ and $\overline{V}^f \to 0$ (based on the demonstration for rigid bodies). By means of the averaging theory, $\|P^f - \overline{P}^f\| < k_3 T$ and $\|V^f - \overline{V}^f\| < k_4 T$ for $k_{3,4} > 0$ and T the wingbeat period.

IV. SIMULATIONS

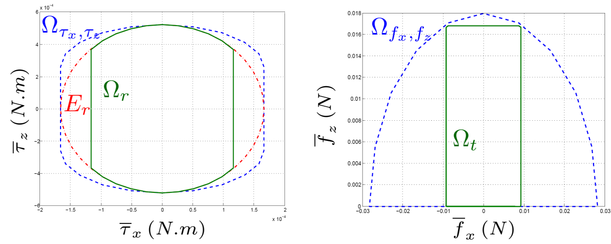
The numerical values, used in the simulations of the model proposed in this work, correspond to the Hymenoptera insect [18]. The wingbeat frequency is $100\,Hz$ and the body mass $500\,mg$. The wing surface ($S_w \approx 1.5\,cm^2$) is computed so that a vertical ascendant movement can be achieved with a flapping angle remaining lower than $\overline{\phi}_0 = 50\,°$ (maximum flapping amplitude for Hymenoptera). The rotation angle amplitude is taken to its maximum value $\overline{\psi}_0 = 90\,°$.

Based on these numerical values, the saturation sets Ω_{τ_x,τ_z} and Ω_{f_x,f_z} can be determined explicitly (10).
Ω_{τ_x,τ_z} has been approximated to the largest ellipse E_r that fits inside Ω_{τ_x,τ_z} (see Fig. 4a) for calculus simplification reasons. Therefore, the control torques $\overline{\tau}_x$ and $\overline{\tau}_z$ should respect an ellipsoidal saturation defined by

$$y^T P y = 1 \tag{26}$$

where $y = (\overline{\tau}_x\ \overline{\tau}_z)^T$ and P is a diagonal definite positive matrix representing the ellipse's semi-axes. Practically, if $\overline{\tau}_x \geq \alpha_1 M_{2,1}$, $\overline{\tau}_x$ could be saturated to $\alpha_1 M_{2,1}$ and $\overline{\tau}_z = 0$. To avoid a null yaw control torque in this case, 70% of $\alpha_1 M_{2,1}$ will be attributed to $\overline{\tau}_x$, $\overline{\tau}_z$ will be calculated by equation (26) defining a set Ω_r (see Fig. 4a). This choice is justified by the necessity to bring the MAV to its flat position (horizontal plane) first.

The admissible set of thrust and lift forces Ω_{f_x,f_z} is drawn on Fig. 4b. Since $\overline{v}_h$ will be decomposed in $mg\overline{v}_y$ and $mg\overline{v}_z$, rectangular saturation is chosen inside Ω_{f_x,f_z} in order to simplify the calculation of the bounds at each iteration. Saturation bounds are calculated so that more power is given to the lateral movement since it is associated to the roll



(a) Rotational saturation set (b) Translational saturation set

Fig. 4: Yaw torque versus roll torque (left), defining the saturation set Ω_{τ_x,τ_z} approximated to an ellipse E_r then to a set Ω_r. Lift versus thrust (right), defining the saturation set Ω_{f_x,f_z} approximated to the set Ω_t.

movement: the MAV can be brought to the horizontal plane rapidly. The final set is then given by Ω_t (see Fig. 4b).

The convergence of the control laws is tested in simulation. The MAV is taken to the equilibrium from an initial position $(2, -1, -3)(\,m)$ and orientation $(-25, 45, 30)\,°$. The robustness of the control law is tested with respect to external disturbances (forces of $(10^{-2}, 0, 10^{-2})$ and torques of $(10^{-4}, 10^{-4}, 10^{-4})$ in $\mathbb{R}^m$) applied at $t = 25\,s$. Control torques and forces act synergetically to overcome the disturbances and ensure the MAV's stability. The roll, pitch and yaw angles, angular velocities, control torques and position, linear velocities, thrust and lift control forces are plotted on Fig. 5, 6, 7 and 8. The robustness of the control has been successfully tested with respect to a bad estimation of $35\,\%$ of the aerodynamic coefficient C_w which is difficult to identify.

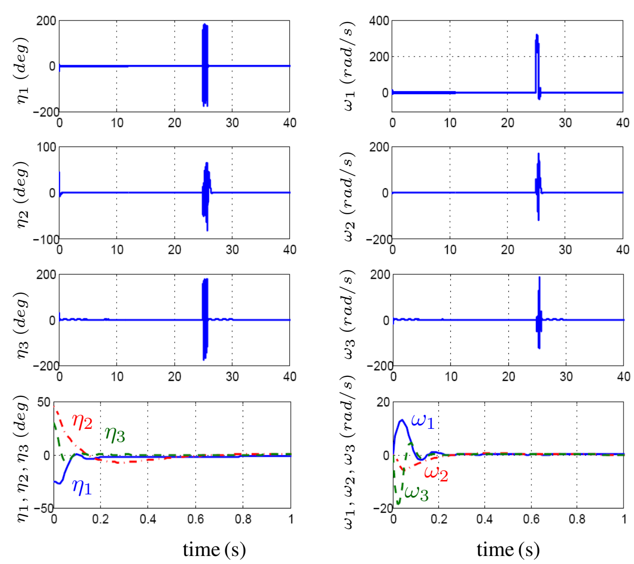


Fig. 5: The convergence of the roll, pitch and yaw angles from initial orientation $(-25, 45, 30)\,°$ (left) and the corresponding angular velocities (right) in presence of external disturbances applied at $t = 25\,s$ during 5 wingbeats. Angles and angular velocities zoomed to the first s (bottom).

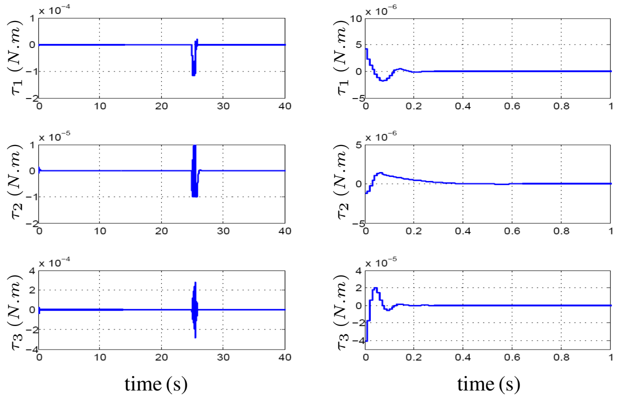


Fig. 6: The roll, pitch and yaw control torques in presence of external disturbances applied at $t = 25\,s$ during 5 wingbeats (left) zoomed to the first s (right).

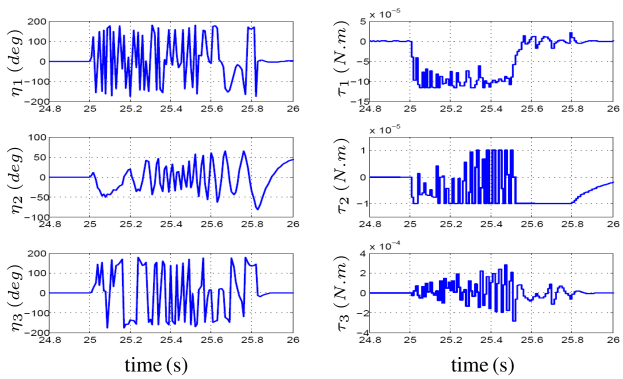


Fig. 7: The roll, pitch and yaw angles (left) and control torques (right) zoomed in presence of the disturbance.

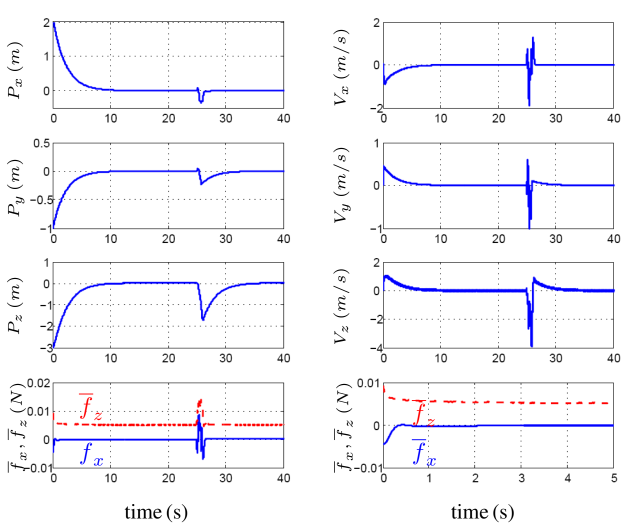


Fig. 8: The displacement of the MAV along the 3 axes in $\mathbb{R}^f$ from initial position $(2, -1, -3)$ (left) and the corresponding linear velocities (right) in presence of external disturbances applied at $t = 25\,s$ during 5 wingbeats. Thrust and lift control (in $\mathbb{R}^m$) (bottom left) zoomed to the first $5\,s$ (bottom right).

V. Conclusions and Future Works

The present paper has presented a control law to stabilize the flapping flight in a hovering position. The proposed control is based on the theory of cascade, aiming to stabilize the attitude of the MAV then to drive the body to a desired position based on the rotational stabilization. Bounded state feedback control torques and forces were applied. The proposed control laws are low cost in terms of computation, therefore suitable for an embedded implementation. The control law was tested with respect to aerodynamic errors and to external disturbances. The control law will be tested on a finer model, specifically the simulator "OSCAB" developed by the ONERA, France. Then, it will be tested on the prototype under construction.

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