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UAV Attitude Estimation by Vanishing Points in Catadioptric Images

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Abstract— **Unmanned Aerial Vehicles (UAV) are the subject of an increasing interest in many applications and a key requirement is the stabilization of the vehicle. Some previous works have suggested using catadioptric vision, instead of traditional perspective cameras, in order to gather much more information from the environment and therefore improve the robustness of the UAV attitude estimation. This paper belongs to a series of recent publications of our research group concerning catadioptric vision for UAVs. Currently, we focus on the estimation of the complete attitude of a UAV flying in urban environment. In order to avoid the limitations of horizon-based approaches, the difficulties of traditional epipolar methods (such as rotation-translation ambiguity, lack of features, retrieving motion parameters from matrix decomposition, etc...) and improve UAV dynamic control, we suggest computing infinite homography. We show how catadioptric vision plays a key role to: first, extract a large number of lines, second robustly estimate the associated vanishing points and third, track them even during long video sequences. Therefore it is not only possible to estimate the relative rotation between consecutive frames but also compute the absolute rotation between two distant frames without error accumulation. Finally, we present some experimental results with ground truth data to demonstrate the accuracy and the robustness of our method.**

I. INTRODUCTION

Stabilizing a UAV is one of the most important steps towards their autonomy. In order to guarantee the orientation of the vehicle, a common solution is to increase the number of sensors and then apply data fusion methods. It is now well established that traditional navigation equipments like Global Positioning System or Inertial Navigation System suffer from many limitations. For example, GPS is sensitive to signal dropout and hostile jamming. The drawback of INS is that its position error compounds over time and may cause large localization errors. In order to overcome main disadvantages of both methods, many researchers suggested a vision-based approach of the navigation problem, which helps estimating localization and/or orientation of a UAV when GPS or inertial guidance is not available [1][2][3][4]. Most of the existing works use conventional cameras which have a relative small field of view, which drastically limits the information we could get from the environment (Fig 1). That is why some works have proposed using catadioptric sensors and applied them for various tasks [5][6][7].

Concerning UAV applications, Hrabar and Sukhatme were the first to apply catadioptric cameras for autonomous aerial vehicles. However they do not explicitly take characteristics of catadioptric images into account (e.g. distortions), require artificial visual targets and can control the vehicle only in 2D [8][9] . At the contrary, our research group aims to estimate the UAV 3D attitude, which has led to a series of recent publications [10][11][12][13] . The cited [10] and [11] methods compute the roll and pitch angles after extracting the horizon in catadioptric images by adapted Random Markov Field (MRF) or maximizing RGB-based Mahalanobis distance. However, they assume the horizon is clearly visible in the image. Moreover they cannot estimate the complete attitude of the UAV since horizon-based methods do not permit to calculate yaw angle. In [12], we have developed a hybrid approach that combines horizon-based technique with planar homography. Whereas we were able to estimate the 3 rotation angles, it requires a planar scene (high altitude or flat terrain) and strongly relies on the horizon extraction to reset the error accumulation. In order to get rid of the horizon dependency, we have developed a line-based method that can run in urban environment [13]. By considering the vertical direction as the normal vector of the ground plane, we were able to apply the same technique than in [10] and [11] to estimate the roll and pitch angles. However it still suffers from some important limitations. First, it does not permit to compute the yaw angle. Second, sky regions are needed to distinguish the vertical direction among the others and thus cannot be applied in dense urban areas or indoor environment. Finally, it requires separating sky and ground pixels, which is not an easy task. The present paper aims to overcome these difficulties while keeping the advantages of the previous method: applicable in urban environment and free from error accumulation. Our proposed approach consists in: first, extracting the vanishing points in catadioptric images and second, estimating the infinite homography between a pair of images. Independently of the vision system that is used, there are numerous advantages of such an approach. First, the infinite homography provides the full attitude of the UAV. Second, infinite homography provides a single rotation solution contrary to planar homography whose decomposition is not unique. Furthermore, in a control point of view, [14] has shown that points at infinity, and thus infinite homography, yields rotation decoupling properties that respect the UAV dynamics and results in a better behaved control solution. Whereas similar methods have already been applied to traditional perspective cameras [15], this paper is the first one to follow this approach for catadioptric vision. The inherent wide field of view of such sensors provides some important extra advantages. First, the image contains a much larger number of lines, thus we do not suffer from lack of information when extracting parallel lines. Second, the vanishing point, i.e. the intersection of parallel lines, lies in the image and therefore its estimation is more robust. Finally, a vanishing direction can be tracked during a very long sequence. As will be explained, this is a very important property because it permits to calculate the absolute rotation of the UAV without error accumulation. This paper is divided into five main parts. After introducing catadioptric projection, we derive the infinite homography equations for catadioptric vision. In the third part we explain how to extract lines and vanishing points in catadioptric images. Then we present how to compute relative and absolute orientations of the UAV. Finally we carry some experiments on real video sequences and compare the results with ground truth data.

Fig. 1. Compared to traditional perspective cameras (left), catadioptric systems (right) can gather much more information from the environment.

II. CATADIOPTRIC VISION AND IMAGE FORMATION

Intuitively, wider the field of view is, more information we can gather from the environment and more precise and robust the pose estimation will be. Obviously, an imaging system that is able to see "in all direction" could play a key role. Such kind of sensors is simply called omnidirectional systems and provides a wide field of view. Catadioptric cameras are a specific kind of omnidirectional systems. They are devices which use both mirrors (catoptric elements) and lenses (dioptric elements) to form images through a conventional camera [16]. Such systems usually have a field of view greater than 180 degrees and are getting both cheaper and more effective. While they have long been used in telescopes (to focus light from stars onto the eye of the observer), only recently they gained in popularity together with other omnidirectional vision systems based on fish-eye lenses or clusters of outwards looking cameras. Baker and Nayar classified catadioptric sensors into two categories depending on the number of viewpoints [17]. Sensors with a single viewpoint, named central catadioptric sensors, permit a geometrically corrected reconstruction of the perspective image from the original catadioptric image. Geyer and Daniilidis have demonstrated the equivalence for the single viewpoint category with a two-step projection via a unitary sphere centered on the focus of the mirror (the single viewpoint) [16]. This two-step projection consists first in projecting a real 3D point P_w to a point P_s from the center of the sphere O_c (Fig 2). The second step projects point P_s to point P_i in the image plane from a specific point O_p. In order to apply the equivalence, it is necessary to know the intrinsic parameters of the camera and two additional parameters namely ξ and φ which are respectively equal to distances $|O_cO_p|$ and $|O_cO_i|$. Parameters ξ and φ define the shape of the mirror and can be estimated by calibration [18]. Whereas it does not seem obvious at the first sight, the sphere equivalence is interesting for several reasons. First, it greatly simplifies the formalism to take catadioptric distortions into account. Indeed catadioptric images are highly distorted because of the mirror projection, which complicates the image analysis. Working in the sphere permits to manipulate those distortions much more easily. A second reason is that it allows working in a general framework, i.e. the sphere, independently on the fact that a hyperbolic or a parabolic mirror is used. Finally, the sphere space provides some very interesting projection properties. For example, it has been proved that a line is simply projected as a great circle in the sphere [16]. Therefore most of algorithms of line detection for catadioptric images are based on this important characteristic, especially our algorithm that we use in the present paper [19]. In our previous work [10], we have also shown that the sphere equivalence simplifies the horizon detection in aerial flights. Indeed, if we consider the horizon as the image of the occluding contour of the Earth sphere, the horizon is projected as a small circle on the equivalent sphere, which imposes interesting constraints.

III. INFINITE HOMOGRAPHY FOR CATADIOPTRIC VISION

Homography is a common and important tool to estimate the motion of the camera from coplanar real feature points detected in a pair of images [20]. On the contrary, infinite homography computes the motion based on the correspondence of points lying at the infinity, such as vanishing points. To the best of our knowledge, whereas there exist a few papers about homography for catadioptric vision [21], there is none dealing with infinite homography.

A. Notation and Homography

Before defining infinite homography for catadioptric vision, let us define important entities and notations. Let P_w be a world point whose coordinates are $(x_w, y_w, z_w)^T$ in the coordinate frame of the first image and is projected onto the associated sphere at $(x_s, y_s, z_s)^T = \lambda(x_w, y_w, z_w)^T$

where

$$\lambda = \frac{1}{\sqrt{x_w^2 + y_w^2 + z_w^2}} \quad (1)$$

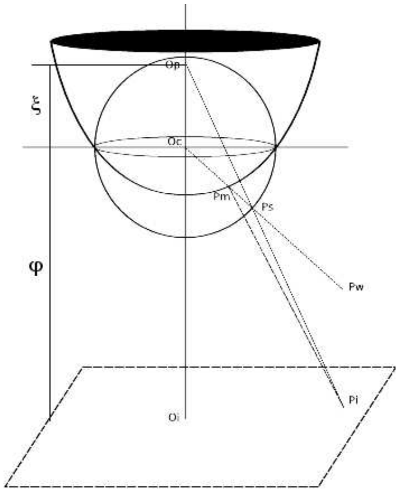

Fig. 2. Equivalence between the catadioptric projection and the two-step mapping via the sphere

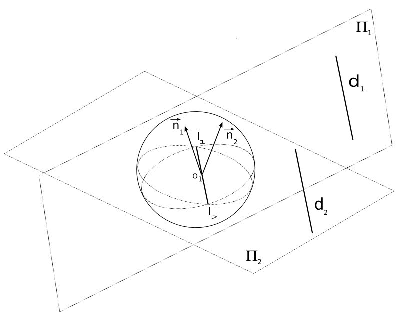

Fig. 3. The direction of two parallel lines corresponds to the direction of their associated vanishing points in the sphere.

Subsequently, P_w is represented by $(x'_w, y'_w, z'_w)^T$ in the coordinate frame of the second image and is projected onto the associated sphere at $(x'_s, y'_s, z'_s)^T = \lambda'(x'_w, y'_w, z'_w)^T$

where

$$\lambda' = \frac{1}{\sqrt{{x'_w}^2 + {y'_w}^2 + {z'_w}^2}} \tag{2}$$

In [21], homography for traditional perspective cameras has been extended towards catadioptric vision and defined as follows:

$$\begin{pmatrix} x'_s \\ y'_s \\ z'_s \end{pmatrix} = \frac{\lambda'}{\lambda} H \begin{pmatrix} x_s \\ y_s \\ z_s \end{pmatrix} \text{ where } H = R + T\tilde{n}^T \tag{3}$$

The entity $\tilde{n}$ is the normal to the plane such that $\tilde{n} = n/d$ and where d is the distance from the center of the sphere to the plane.

B. *Infinite Homography*

As defined in [20], the infinite homography H_∞ is the homography induced by the plane at infinity. Formally, for two corresponding points at infinity X and X' in a pair of perspective images, H_∞ verifies:

$$X' = H_\infty X \text{where } H_\infty = KRK^{-1} \tag{4}$$

When a pixel on the catadioptric image plane is projected onto the equivalent sphere, the calibration process (similarly to matrix K) is applied. Therefore, based on same geometric considerations than in [21], the infinite homography for two infinite point correspondences in the sphere can be easily defined as:

$$\begin{pmatrix} x'_s \\ y'_s \\ z'_s \end{pmatrix} = H_\infty \begin{pmatrix} x_s \\ y_s \\ z_s \end{pmatrix} \text{ where } H_\infty = R \tag{5}$$

This equation can be obtained in another way. In perspective case, let a scene line have vanishing point v_i in the first view and v'_i in the second view [20]. The vanishing point v_i has direction d_i measured in the first cameras coordinate frame, and the corresponding vanishing point v'_i has direction d'_i measured in the second cameras coordinate frame. These directions can be computed from the vanishing points, for example, $d_i = K^{-1}v_i/\|K^{-1}v_i\|$ where the normalizing factor is included to ensure that it is a unit vector. The directions d_i and d'_i are related by the camera rotation as $d'_i = Rd_i$. Actually, similar properties can be derived for catadioptric images (Fig 3). Indeed, in [13] we have proved the following property: if d_1 and d_2 are two parallel lines with unit vector u then their equivalent great circles C_1 and C_2 on the unitary sphere intersect into two antipodal points I_1 and I_2. These points depend only on direction u and verify $I_1I_2 = u$ where u is the associated vanishing direction.

This property means that the direction of a vanishing point in the sphere space is the same than the direction of its associated lines in the world. As the same conditions than in the perspective case are obtained, the similar equation can be applied: $d'_i = Rd_i$ where d_i and d'_i are the vanishing directions in sphere space.

C. *Infinite Homography Computation*

Infinite homography has 3 degrees of freedom, corresponding to the number of DOF of the rotation matrix. Each vanishing point correspondence in the sphere space (noted X_i and X'_i)provides two equations, which means that two infinite point correspondences are sufficient. Using pitch (γ), roll (β) and yaw (α)-based rotation angles, we obtain the following system:

$$X'_i = H_\infty X_i, i = 1, 2, \ldots \tag{6}$$

where

$$H_\infty = R_{\alpha,\beta,\gamma} = \begin{bmatrix} \cos\alpha\cos\beta & -\sin\alpha\cos\gamma + \cos\alpha\sin\beta\sin\gamma & \sin\alpha\sin\gamma + \cos\alpha\sin\beta\cos\gamma \\ \sin\alpha\cos\beta & \cos\alpha\cos\gamma + \sin\alpha\sin\beta\sin\gamma & -\cos\alpha\sin\gamma + \sin\alpha\sin\beta\cos\gamma \\ -\sin\beta & \cos\beta\sin\gamma & \cos\beta\cos\gamma \end{bmatrix} \tag{7}$$

Traditionally, this system is non-linear with respect to the three rotation parameters (α,β,γ). However if quaternions are used to represent rotations, as suggested by Horn [22], we can easily obtain a closed-form solution.

IV. EXTRACTION OF LINES AND VANISHING POINTS

In order to estimate the infinite homography, some points lying at infinity are required. The most common infinite points are called vanishing points and correspond to the intersection of parallel lines. Contrary to traditional perspective vision, catadioptric sensors provide two great advantages for extracting vanishing points: first, the image contains much more parallel lines and second, the vanishing points lie in the image plane, which makes their estimation more robust. In [19], we have introduced a complete method that automatically extracts lines, parallel lines and vanishing points. In this section, we remind the main steps of the algorithm for convenience of the readers and clarity of the paper.

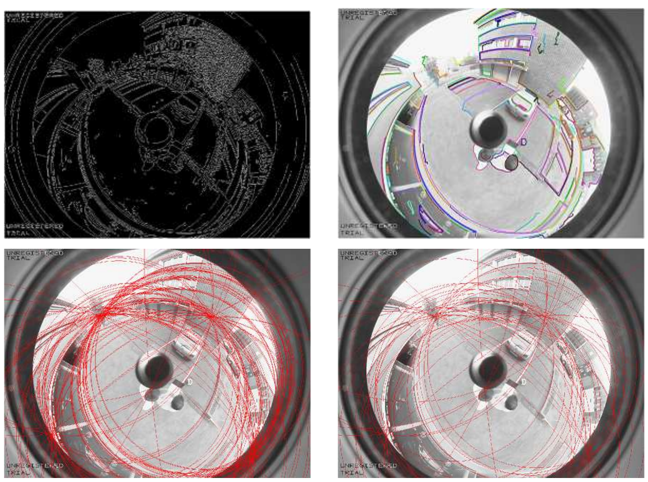

Fig. 4. The 4 main steps of our line detection algorithm: edge detection by Canny (top-left), edge chaining (top-right), before (bottom-left) and after (bottom-right) the merging step.

A. Line Detection

Our line detection method can be seen as an extension of the polygonal approximation approach and is based on the following geometrical property: a line in space is projected as a great circle on the equivalent sphere. Our algorithm is composed of three intuitive steps. First we extract edges and build edge chains. Second we project the chains on the sphere by using calibration parameters and then we check for each chain if it verifies the great circle constraint. More precisely, we consider the two spherical endpoints of a chain, X_s^1 and X_s^N, that define a unique great circle whose plane normal is $\vec{n} = (a, b, c) = \overrightarrow{OX_s^1} \times \overrightarrow{OX_s^N}$, where O is the sphere center. Then we consider that a chain point $X_s = (x_s, y_s, z_s)$ verifies the great circle constraint if $|ax_s + by_s + cz_s| \leq DistThresh$. In the case at least one chain point does not belong to the great circle, the third step consists in splitting the chain at the furthest chain point from the great circle, i.e.

$$\arg\max_i |ax_s^i + by_s^i + cz_s^i| \quad (8)$$

Finally, a line might not be continuously detected as an edge and thus might be divided into some parts during the splitting step. That is why the final step consists in merging detected lines having similar normals. Figure 4 depicts the different steps of the procedure.

B. Vanishing Points Extraction

A set of parallel lines intersect in two antipodal points in the sphere [16]. These two points corresponds to the vanishing directions and can be characterized by a unit vector u. Therefore the idea for detecting parallel lines consists in computing this vector u. Let n_1 and n_2 be the normal vectors of two great circles. Their intersection is calculated by $u = n_1 \times n_2$ and corresponds to the direction of the antipodal points. Then we consider that a great circle normal n_i is in the same direction than u if:

$$1 - |n_i \cdot u| \leq SimilarityThreshold \quad (9)$$

Doing the same way for each combination of two normals, we can compute the vector that corresponds to the highest number of parallel lines, that is to say the vanishing point of those lines, and the associated parallel lines. Following the same procedure, it is possible to detect the second and third dominant directions after removing from the combination list the normals that already belong to a detected dominant direction (Fig 5).

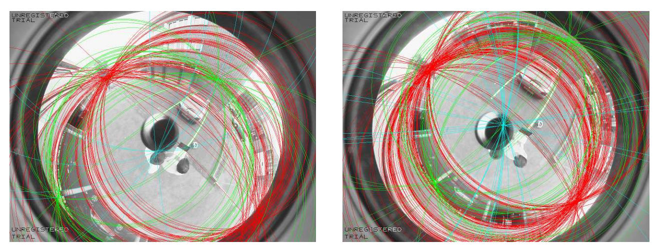

Fig. 5. Detection of the three most dominant directions and their associated parallel lines in an image pair. Each conic represents a detected line and its color corresponds to the set of parallel lines it belongs to. The vanishing points lie at the intersection of the associated parallel lines.

V. ATTITUDE ESTIMATION

In this section, we present two methods to compute the UAV attitude: common composition of relative rotations between two consecutive frames and direct attitude estimation by matching vanishing points of distant frames.

A. Common Approach

After extracting vanishing points, we need to build some correspondences in order to compute the infinite homography between two images. In our applications, the video is captured continuously so we can match vanishing points in a pair of consecutive images by a simple similarity measure. The i^{th} vanishing point v_i^t of frame t is matched to the vanishing point v_k^{t+1} of frame $t+1$ which is solution of:

$$\arg\max_k |v_i^t \cdot v_k^{t+1}| \quad (10)$$

Once at least two vanishing points have been matched, we can calculate the infinite homography, as explained in III-C, and we obtain the relative rotation between the two frames. If one wants to compute the absolute rotation, the composition of relative rotations must be computed. The attitude matrix R_i at the i^{th} frame is calculated by $R_i = R_0 \circ R'_1 \circ R'_2 \ldots \circ R'_i$ where R_0 corresponds to the initial absolute orientation matrix and R'_k the relative rotation obtained at frame k.

B. Robust Approach

Obviously, small error on each rotation matrix, due to inaccuracy in vanishing points extraction, will accumulate over time. Actually, catadioptric vision provides a very interesting practical solution to this problem: thanks to the very wide field of view, a vanishing direction can be tracked during a long sequence, as will be depicted in the results section. Therefore, the vanishing points of the current image also exist in the initial image of the sequence. Instead of directly matching the vanishing points of two distant images, which is not an easy task, we first match the vanishing points in all consecutive images. Then, we simply backtrack the matching results from frame to frame, so finally we get the correspondence between the initial and the current image even if they are very distant. This is a great advantage because it avoids the composition of relative rotations and thus error does not accumulate. The validity of this approach is analyzed in the results section.

VI. EXPERIMENTAL RESULTS

In this section, we present some results issued from two different experiments. The first sequence contains 100 frames in which three vanishing points can be continuously detected (Fig 5). Figure 6 depicts the smoothness of the extraction of the 2 horizontal vanishing points. This video was recorded some months ago and at that time, we did not have the inertial sensor. In order to compare our results with ground truth data and demonstrate the robustness of our method, we have recorded a second sequence which contains IMU data and is much longer (520 frames at 4fps). Fig 7 depicts the fact that the vanishing points can be tracked along the whole video even with large camera motion. We have computed the infinite homography from 2 vanishing directions between the initial and the current frame to avoid the error accumulation problem which is traditionally inherent to long sequences. In order to qualitatively compare IMU and camera data, these two sensors must be calibrated. We have manually set a calibration corresponding to a rotation of 180 degrees along the vertical axis, consistently with the layout of our system. In a first test, we compare the normal vector of the ground plane obtained by the cross-product of the two horizontal vanishing directions and the vertical vector issued from the IMU. Results are depicted by Fig 8 and show that the vertical is accurately extracted. In the second test, we compare the evolution of the three rotation angles obtained by infinite homography and the IMU (Fig 9). We have obtained a mean/std error of 3.8/2.6 for roll angle, 2.3/1.8 for pitch angle and 4.6/3.5 for yaw angle, which demonstrates the robustness and accuracy of our system.

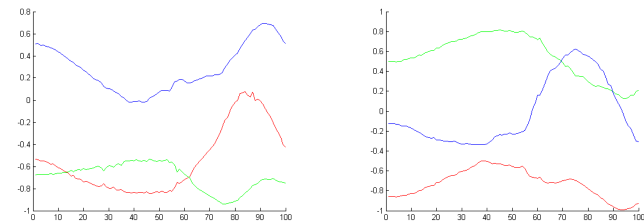

Fig. 6. Evolution of the 3D components v_x, v_y and v_z (red, green and blue) of the two horizontal vanishing directions (left and right graphs) corresponding to the sequence of figure 5. Note the smoothness of the evolution along the 100 frames.

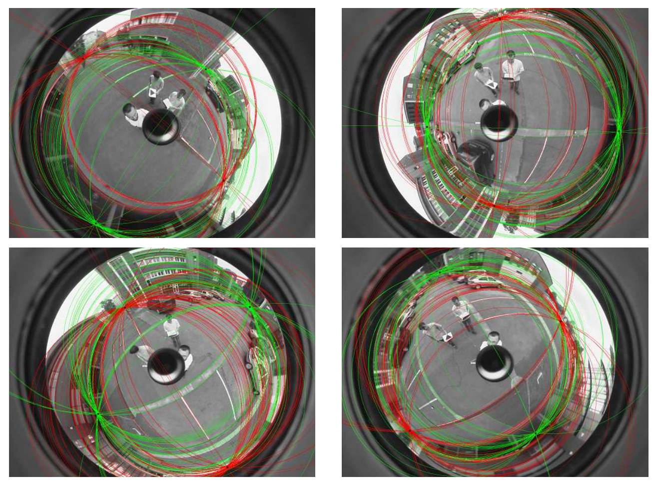

Fig. 7. The vanishing points can be robustly extracted and continuously tracked along the whole sequence despite large camera motion.

VII. CONCLUSION

A. Conclusions

The presented work aims to compute the complete attitude of a UAV flying in urban environment and is the latest paper of a series of publication of our research group involved in catadioptric vision for UAV applications. Whereas we had obtained interesting results, our previous methods suffered from some limitations: horizon needs to be largely visible, yaw angle cannot be estimated or sky region is required. In order to overcome these difficulties, we have derived infinite homography for catadioptric images based on vanishing points. Independently of the vision system, there are numerous advantages of such an approach: the complete UAV attitude can be estimated, the rotation solution is unique (matrix decomposition is not needed) and it leads to a better behaved control solution. We are aware that similar methods have already been applied to traditional perspective cameras but this paper is the first one to follow this approach for catadioptric vision and we have listed some important extra advantages that are provided by the inherent wide field of view of such sensors. First, the image contains a much

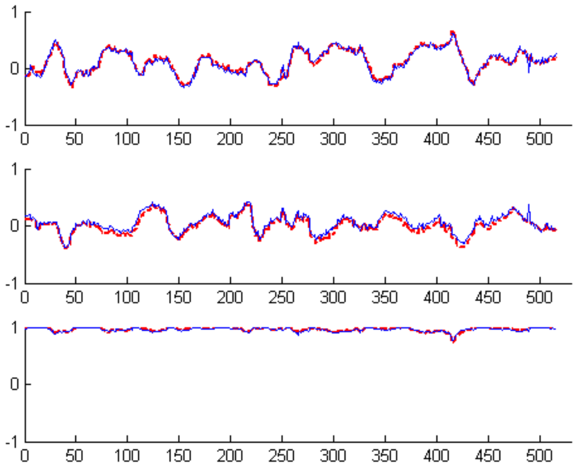

Fig. 8. Comparison of the normal vector of the ground plane (components xyz in top, middle, bottom figures) obtained by IMU (red dashed line) and by cross-product of the two horizontal vanishing directions (blue solid line) for the sequence of figure 7.

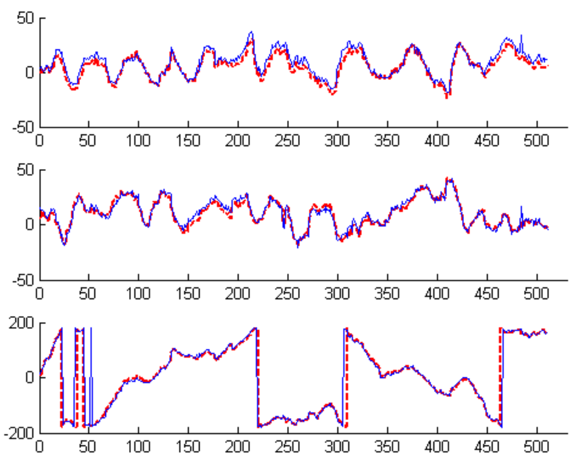

Fig. 9. Comparison of roll (top), pitch (middle) and yaw (bottom) angles obtained by IMU (red dashed line) and the infinite homography (blue solid line) for the sequence of figure 7. The "jumps" of the yaw angle are simply due to the discontinuities (-180°;180°).

larger number of lines, thus we do not suffer from lack of information when extracting parallel lines. Second, the vanishing points, i.e. the intersection of parallel lines, lie in the image and therefore their estimation is more robust. Finally, a vanishing direction can be continuously tracked during very long sequences. This is a very important property because it permits to directly calculate the infinite homography between the initial and the current frame, and thus the UAV attitude without error accumulation. Experiments on long video sequences and comparison with ground truth data have demonstrated the robustness and the validity of our approach. In future work, we plan to apply some filtering techniques on the UAV motion and develop some embedded algorithms for realtime onboard processing.

VIII. ACKNOWLEDGMENTS

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