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# On the solvability of the localization problem in robot networks

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*Abstract*— **This paper contributes to the problem of deterministic localization of robot networks using local and relative observations only. This is an important issue in collective and cooperative robotics where global positioning systems are not available, and the basic premise is the localization ability of the group.**

**We prove that, giving a set of relative observations made by the robots, the unique non ambiguous pose estimation of the robot network in a deterministic way, is a NP-hard problem. This means that no polynomial-time algorithm can deterministically solve the unique pose estimation problem based on relative observations. The consequence is that no guaranty can be provided, in a polynomial time, that the possibly estimated poses of the robots, will correspond to the effective (actual) ones. The proof is based on complexity theory. We build appropriate polynomial-time reductions acting on the localization problem and leading to well known NP-hard problems. The paper gives some tracks to overcome this issue.**

## I. Introduction

The group localization is a preliminary requisite to many collective robotic tasks (mapping, surveillance, manipulation, sample extraction, mining, etc). It is often resolved by extending single robot localization techniques to handle multi-robot pose (i.e. position and orientation) estimation. Collaborative localization approaches integrate relative observations aiming to fuse interdependent (homogeneous or heterogeneous) sensory information in a single estimate of the group pose. It has been demonstrated [1], [2] that localization uncertainty in groups of robots is lower compared to the situation where individual robots estimate separately their pose.

Various techniques have been proposed to integrate relative observations, like maximum likelihood estimation [3], extended Kalman filters [4], [5], particle filters and Monte-Carlo simulation [6], and cooperative strategies constraining the motion of the group [7], [8]. Although the designs of the previous schemes have led to practical implementations and have demonstrated their effectiveness in certain settings through extensive simulations or limited experiments, some fundamental questions have not been addressed.

Only few attempts [9], [10], [11] have proposed some theoretical foundations to the problem of localization in multi-robot formations. The work [10] is based on results in kinematics of planar mechanisms where the problem is expressed in a system of non linear closure equations and the robot formation is modelled as a closed kinematic chain. In [11], the authors aimed to derive necessary and sufficient conditions for *completely* localizing a formation of three or more robots equipped with omnidirectional cameras and wireless communication. The approaches in [9], [11] are based on graph rigidity and results on the localization in sensor networks.

An interesting approach to theoretic sensor network localization was investigated by Aspnes et al. [12], [13]. The authors construct grounded graphs to model sensors configuration and apply graph rigidity theory to test the conditions for unique localizability of the network. This was applied to construct uniquely localizable networks, in which some nodes know their locations and other nodes determine their locations by measuring the distances to their neighbors.

There is no trivial way to derive $SE(2)$ (position and orientation) localization from results on point formation $\mathbb{R}^2$, and many ambiguity cases annul and invalidate the proposed adaptations. In this paper, we address the fundamental question: *What is the computational complexity of robot network localization using relative observations*? Although various algorithms have been proposed, the computational complexity of determining the pose of the robots in a uniquely localizable network has not been investigated.

We hereafter consider the most challenging scenario where the robot network localization problem can only be inferred by relative observations. In such a framework, the absolute position of the robots are not directly given but should be measured or inferred from the cooperative localization process using robot interactions and observations. As a matter of fact, we provide a theoretical proof that shows the NP-hardness of the robot formation localization, *i.e.*, no deterministic algorithm can solve the robot network localization problem in a polynomial time.

The paper is organized as follows. Section II introduces the model of the robot network localization problem. In the same section, we also state the problem and the result addressed in this paper. The formal proof is given in Section III. Our result is situated relatively to few existing works in sensor networks in Section IV. We conclude this paper in Section V.

## II. The Robot Network Localization Problem

Self-positioning could be achieved by endowing each robot with a global positioning system (GPS), by triangulation with fixed known landmarks or by proprio-localization if the initial positions of robots are known. A GPS solution is costly and would not work in indoor environments and where

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the line-of-sights between the receiver and the satellites are broken because of obstructions (like dense vegetation, buildings). Furthermore, in most real environments, robots are deployed without their pose information known in advance, and there is no known external landmarks to support their location estimation.

We consider the question of the self-localization of a network of robots, using relative observations only. This means that robots positions and orientations will only be inferred from their interactions and relative observation exchanges. To have a global localization however, it is necessary that at least one robot (as a reference robot) has absolute positioning capabilities. This condition is also necessary to have the error in global localization bounded.

### A. Model

We consider a group of $n$ robots denoted $\mathscr{R} = \{r_1, r_2, \cdots, r_n\}$ equipped with heterogeneous sensors and moving in a planar world, i.e. the two-dimensional special Euclidean group $SE(2)$. We assume a distributed system of robots, i.e. each robot performs sensing and communication with all of its spatial neighbors (present in its field of view). Robots can be in movement, but the estimation of the robot formation is static and supposes the immediate acquisition of the relative observations. A body-reference frame $F_{r_j}$[1] is attached to each robot $r_j$, where relative observations of neighboring robots are measured. A global reference frame $F_{r_0}$ is defined by considering a virtual robot or by fixing the position of an added robot (a reference robot) $r_0$.

We extend the *point-formation* concept reviewed in [13] to handle the robot distance and bearing observations. We define a ***robot-**formation* $\mathscr{F}_r$ as the set of $n$ robot poses $\mathscr{Q} = \{q_1^t, q_2^t, ..., q_n^t\}$, $q_i = (x_i, y_i, \theta_i)$ in $\mathbb{R}^2 \times SO(2)$ together with a set $\mathscr{L}$ of $k$ links, labelled $(i, j)$, defining the existence and the nature of the observations between robots $r_i$ and $r_j$, where $i$ and $j$ are distinct integers in $\{1, 2, ..., n\}$. The links $\rho_{ij}$ or $\varphi_{ij}$ (or, both) label those specific robot pairs whose inter-robot observations (distance or bearing) are given (see figure 1 for an example).

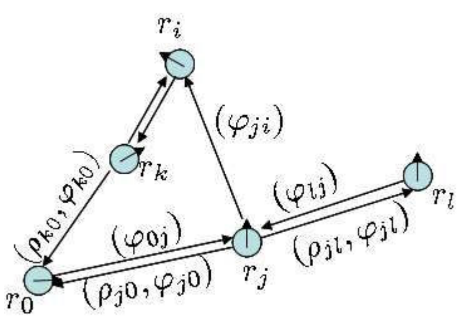


Fig. 1. An example of a robot network with sensory information modelling

The robot network localization problem is to uniquely estimate the robot-formation $\mathscr{F}_r$ in any reference frame attached to one robot. Obviously, if the robot-formation is known in a reference frame $F_{r_i}$, it could be estimated in any other reference frame $F_{r_j}$ by a rigid transformation where $r_j$ is a robot contained in the formation.

[1]We will also note the robot frame $r_j$ when there is no possible confusion.

### B. Relative Observations

Relative observations provide different kinds of positional constraints on the robot network formation, such as the distance between two robots ($\rho_{ij}$) or their relative bearing ($\varphi_{ij}$). This can spell in the form of non linear equations in the parameters of the relative observations and robot poses ($p_{ji} = (x_{ji}, y_{ji})^t$ for relative $r_i$ position in the robot $r_j$ frame, and $\theta_{ji}$ for relative $r_i$ orientation relatively to the $r_j$ frame):

$$\rho_{ik} = \sqrt{(p_{ji} - p_{jk})^T \cdot (p_{ji} - p_{jk})} \quad (1)$$

$$\varphi_{ji} = \tan^{-1}(y_{ji}/x_{ji}) \quad (2)$$

$$\varphi_{ik} - \varphi_{ki} + \pi = \theta_{ik} = \theta_{jk} - \theta_{ji} \quad (3)$$

$$\varphi_{ij} - \varphi_{ik} = \cos^{-1} \frac{(p_{ji} - p_{jj})^t (p_{ji} - p_{jk})}{\|p_{ji} - p_{jj}\| \cdot \|p_{ji} - p_{jk}\|} \quad (4)$$

Let $Z$ denoting the measurements set. We have $Z = \{(\rho_{1,2}; \varphi_{1,2}), \cdots, (\rho_{1,n}; \varphi_{1,n}), (\rho_{2,1}; \varphi_{2,1}), \cdots, (\rho_{n,n-1}; \varphi_{n,n-1})\}$. Obviously, if $r_j$ do not sense $r_i$ then $(\rho_{j,i}; \varphi_{j,i}) \notin Z$. Further if $r_j$ sense only either $\rho_{ji}$ or $\varphi_{ji}$ with $r_j$ then either $(\rho_{j,i}; null) \in Z$ or $(null; \varphi_{j,i}) \in Z$.

A robot will be able to infer its position and orientation if some relative bearing and distance measurements are available of the *observed* (neighbor) robots. The previous relative observations (equations 1→4) when simultaneously combined together are sufficient to estimate, without ambiguity, the *observed* robot configuration in the *observer* robot frame. They do not constitute necessary conditions because the pose estimation could be filled using relative observations among more than two robots. Furthermore, in general framework, all the observations may not be available. We say that the team is *localizable* iff the measurement set $Z$ is enough to determine, in a unique manner, all the coordinates $\mathscr{Q}$ of all the robots in a system of any robot frame $r_j$. It is easy to see that if the cohort is localizable in $r_j$ system then it is localizable in all the local systems.

General techniques to multi-robot localization are built upon these basic equations to estimate the robot poses. Collecting available relative observations results in a redundant non linear system of constraints on the network geometric formation and contributes to its localization. An attempt to solve the localization problem could be by linearizing the system of constraints around an initial configuration, but would not guarantee the uniqueness of the definite solution. This approach was investigated by Zhang et al. [10] where the functional independence of the observation constraints was postulated as a necessary and sufficient condition for the localizability of the network. Even for rigid geometric formations, this condition is not sufficient to guarantee the uniqueness of the solution, and can even not be necessary to determine the configuration of the network.

*C. Problem and Result Statements*

The problem of robot network localization using relative observations can be stated as follows: *Can we deterministically provide a localization algorithm to uniquely estimate the robot group pose using relative observations only*?

*Theorem 2.1:* The Robot Network Localization Problem (*RNLP*) is NP-hard.

The theorem will be proved in the following sections. The consequence is that, using relative observations only, the uniqueness of a defined solution to the problem of localization could not be guaranteed and so for the robot formation in a non ambiguous way.

## III. Proof

*A. Preliminaries on Complexity Theory*

All definitions and problems described in this subsection are well-known in Complexity Theory (refer to [14]).

*A decision problem* is a problem whose the answer is either *yes* or *no*. *P* represents the set of decision problems that can be solved in polynomial time. Intuitively, *P* is the set of problems that can be solved quickly. *NP* is the set of decision problems with the following property: If the answer is *yes*, then there exists a proof of this fact that can be checked in polynomial time. Intuitively, *NP* denotes the set of problems where we can verify a *YES* answer quickly if we have the solution in front of us.

Obviously, if a problem is in *P*, then it is also in *NP*- to verify that the answer is *yes* in polynomial time, we can just throw away the proof and recompute the answer from scratch. One of the fundamental questions in theoretical computer science is whether or not $P = NP$? Nobody knows. Intuitively, it seems to be obvious that $P \neq NP$: A lot of problems can be extremely hard to solve, even when the solutions are obvious once you see them. However, no proof exists for this question.

*Definition 3.1:* A problem $\pi$ is said *NP*-Hard if a polynomial-time algorithm for $\pi$ would imply a polynomial-time algorithm for every problem in *NP* (i.e. $P = NP$).
*NP*-Hard problems are at least as hard as any problem in *NP*. Although this has never been proved, it is widely suspected that there exists no polynomial-time algorithms for *NP*-Hard problems.

The Partition Problem is an example of *NP*-Hard problems stated as follows:

*Definition 3.2: Partition Problem*, called shortly *PP*: Given a set $S$ of $n$ positive integers, does there exist any partition $\{A,B\}$ (called special partition) such that the statement in Equation 5 is true?

$$\sum_{a\in A} a = \sum_{b\in B} b \qquad (5)$$

*B. The proof formulation*

In order to prove that *RNLP* is NP-hard, we introduce the following problem.

*Definition 3.3: Unique Partition Problem*, called shortly $UPP$: Given a set $S$ of $n$ positive integers having at least one special partition $\{A,B\}$, has it a unique special partition?

The proof of the localization solvability is based on two reductions:

1) Firstly, we prove that $UPP$ is NP-hard. The proof is based on a polynomial transformation acting on the well known NP-hard problem, the Partition Problem (PP).
2) Secondly, we construct a reduction of *RNLP* to the introduced $UPP$ and deduce the NP-hardness of *RNLP*.

In the sequel, we demonstrate the theorem (3.4), followed consequently by the proof of theorem 2.1.

*Theorem 3.4:* $UPP$ is NP-Hard.

*C. Proof of Theorem 3.4*

From Definition 3.1, we can easily deduce that if a polynomial-time algorithm $\alpha$ imply a polynomial-time algorithm for a known *NP*-hard problem, then $\alpha$ is *NP*-hard too. In this way, the proof consists by showing the following statement:

$$UPP \in P \Longrightarrow PP \in P$$

Given an instance $S_{pp}$ of *PP*, we can transform it into an instance $S_{upp}$ of $UPP$ having at least one special partition by applying the following method according to the three cases:

1) $\sum_{s\in S_{pp}} s$ is odd then consider $S_{upp} = S_{pp} \cup \{\sum_{s\in S_{pp}} s\}$
2) $\frac{1}{2}\sum_{s\in S_{pp}} s \in S_{pp}$ then consider $S_{upp} = S_{pp}$
3) otherwise consider
$S_{upp} = S_{pp} \cup \{\frac{1}{2}\sum_{s\in S_{pp}} s, 2\sum_{s\in S_{pp}} s, \frac{5}{2}\sum_{s\in S_{pp}} s\}$

In the three cases, note that $S_{upp}$ is an instance of $UPP$ because $S_{upp}$ is a set of positive integers ($\frac{1}{2}\sum_{s\in S_{pp}} s, 2\sum_{s\in S_{pp}} s, \frac{5}{2}\sum_{s\in S_{pp}} s$ are integers because $\sum_{s\in S_{pp}} s$ is even) and have at least one special partition $\{A,B\}$ (case1: $A = S_{pp}, B = \{\sum_{s\in S_{pp}} s\}$, case2 $A = S_{pp} - \{\frac{1}{2}\sum_{s\in S_{pp}} s\}, B = \{\frac{1}{2}\sum_{s\in S_{pp}} s\}$, case3 $A = S_{pp} \cup \{2\sum_{s\in S_{pp}} s\}, B = \{\frac{1}{2}\sum_{s\in S_{pp}} s, \frac{5}{2}\sum_{s\in S_{pp}} s\}$).

*Lemma 3.5:* In case1, $S_{upp}$ has a unique special partition iff $S_{pp}$ has not a special partition.

*Proof:* Clearly, $S_{upp}$ has a unique special partition $\{A,B\}$ with $A = S_{pp}$ and $B = \{\sum_{s\in S_{pp}} s\}$. Further $S_{pp}$ has not a special partition because $\sum_{s\in S_{pp}} s$ is odd. ■

*Lemma 3.6:* In case2, $S_{upp}$ has a unique special partition iff $S_{pp}$ has a special partition.

*Proof:* $S_{upp}$ and $S_{pp}$ have the same special partition $\{A,B\}$ with $A = S_{pp} - \{\frac{1}{2}\sum_{s\in S_{pp}} s\}, B = \{\frac{1}{2}\sum_{s\in S_{pp}} s\}$. Further this partition is unique. Indeed assume by contradiction there exists another special partition $\{C,D\}$. Without loss of generality, suppose $\frac{1}{2}\sum_{s\in S_{pp}} s \in C$. Since $C \neq B$, there are at least two elements in $C$. Hence $\sum_{d\in D} d < \frac{1}{2}\sum_{s\in S_{pp}}$.

Consequently, $\sum_{c\in C} c \neq \sum_{d\in D} d$ and $\{C,D\}$ is not a special partition. ■

*Lemma 3.7:* In case3, $S_{upp}$ has a unique special partition iff $S_{pp}$ has not a special partition.

*Proof:*

- $S_{upp}$ has a unique special partition $\Longrightarrow$ $S_{pp}$ has not a special partition.
  By contradiction, assume $S_{pp}$ has a special partition $\{A,B\}$. By definition $S_{upp}$ has a special partition $\{C,D\}$ with $C = S_{pp} \cup \{2\sum_{s\in S_{pp}} s\}, D = \{\frac{1}{2}\sum_{s\in S_{pp}} s, \frac{5}{2}\sum_{s\in S_{pp}} s\}$. Further $\{E,F\}$ with $E = \{\frac{5}{2}\sum_{s\in S_{pp}} s\} \cup A$ and $F = \{\frac{1}{2}\sum_{s\in S_{pp}} s, 2\sum_{s\in S_{pp}} s\} \cup B$ is a special partition different to $\{A,B\}$. Hence $S_{upp}$ has not an unique special partition.
- $S_{pp}$ has not a special partition $\Longrightarrow$ $S_{upp}$ has a unique special partition.
  $S_{upp}$ has a least one special partition $\{A,B\}$ with $A = S_{pp} \cup \{2\sum_{s\in S_{pp}} s\}, B = \{\frac{1}{2}\sum_{s\in S_{pp}} s, \frac{5}{2}\sum_{s\in S_{pp}} s\}$. Assume by contradiction there exists another special partition $\{C,D\}$ different to $\{A,B\}$. Without loss of generality, suppose that $\frac{5}{2}\sum_{s\in S_{pp}} s \in C$.
  - $2\sum_{s\in S_{pp}} s \notin C$. Indeed, if $2\sum_{s\in S_{pp}} s \in C$, then $\sum_{c\in C} c > \sum_{d\in D} d$.
  - $\frac{1}{2}\sum_{s\in S_{pp}} s \notin C$. Indeed, if $\frac{1}{2}\sum_{s\in S_{pp}} s \in C$, then:
    1) either, $C = \{\frac{1}{2}\sum_{s\in S_{pp}} s, \frac{5}{2}\sum_{s\in S_{pp}} s\} \Longrightarrow C = B$, a contradiction;
    2) or, $C = \{\frac{1}{2}\sum_{s\in S_{pp}} s, \frac{5}{2}\sum_{s\in S_{pp}} s\} \cup X$ with $X \neq \emptyset$. In this case, $\sum_{c\in C} c > \sum_{d\in D} d$.
  - $C \neq \{\frac{5}{2}\sum_{s\in S_{pp}} s\}$, otherwise $\sum_{c\in C} c < \sum_{d\in D} d$.
  - $C \neq \{\frac{5}{2}\sum_{s\in S_{pp}} s\} \cup S_{pp}$, otherwise $\sum_{c\in C} c > \sum_{d\in D} d$.
  - $C \neq \{\frac{5}{2}\sum_{s\in S_{pp}} s\} \cup X$, with $\emptyset \neq X \subset S_{pp}$. Indeed assume by contradiction that $C = \{\frac{5}{2}\sum_{s\in S_{pp}} s\} \cup X$ with $\emptyset \neq X \subset S_{pp}$. Let $Y = Spp - X$. Then, we have: $D = \{\frac{1}{2}\sum_{s\in S_{pp}} s, 2\sum_{s\in S_{pp}} s\} \cup Y$. So, $\sum_{x\in X} x = \sum_{y\in Y} y$ with $X \cap Y = \emptyset$ and $X \cup Y = S_{pp}$. Therefore, $\{X,Y\}$ is a special partition of $S_{pp}$. A contradiction.

Finally we cannot construct any other special partition $\{C,D\}$. ■

Clearly in the three cases, the transformations are in polynomial time. If there exists a polynomial time algorithm for *UPP*, called *AlgoUPP*, we have immediately a polynomial time algorithm from Lemmas 3.5, 3.6 and 3.7 for *PP* (refer to Algorithm 1).

This shows the correctness of Theorem 3.4—*i.e.*, *UPP* is NP-Hard.

### *D. Proof of Theorem 2.1*

Suppose that we have a polynomial-time algorithm that takes in input the measurement set $Z$ and return *yes/no* localizable. We will show that such an algorithm can be used to solve *UPP* in polynomial time.

To simplify the discussion, assume first that $W = SE(1)$. In other words, the robots are located on the same line. Without loss of generality, assume that the reference robot is $r_1$ and the $y-$coordinates $y_{1i} = 0$ for each robot $r_i$. So the robot formation is characterized by the variables $(x_{1i}, \theta_{1i})$ for each $r_i$. Obviously $(x_{11}, \theta_{11}) = (0, 2\pi)$. Given an instance $S = \{s_1, s_2, s_3, \cdots, s_n\}$ for *UPP*, consider *RNLP* in $SE(1)$ with $n$ robots where $Z = \{(\rho_{1,2}, \varphi_{1,2}); \ldots; (\rho_{i,i+1}, \varphi_{i,i+1}); \ldots; (\rho_{n-1,n}, \varphi_{n-1,n})\} \cup \{(\rho_{n,1}, \varphi_{n,1})\}$ and $(\rho_{i,i+1}, \varphi_{i,i+1}) = (s_i, 2\pi)$ with $1 \leq i \leq n-1$, $(\rho_{n,1}, \varphi_{n,1}) = (s_n, 2\pi)$.

So the distance between the first and the second robot is equal to the first integer in $S$, the distance between the second and the third is equal to the second integer in $S$, and so on and so forth. Finally, the distance between the last robot and the first robot is equal to the last integer in $S$. Locally, the bearing of $r_{i+1}$ in relation to $r_i$ is $2\pi$. Remark that if we can place the robots by folding the linear chain in a way that the distances between all adjacent robots are satisfied, the robots orientations can be deduced immediately without ambiguities.

*Lemma 3.8:* $Z$ is well an instance of *RNLP*

*Proof:* $S$ has at least one special partition $(A,B)$. Thus we have $\sum_{a\in A} a = \sum_{b\in B} b$. Hence $\sum_{a\in A} a - \sum_{b\in B} b = 0$. Consequently we can construct a linear chain in a way the distance between all adjacent robots are satisfied by applying this method: The distances corresponding to $A$ point to the right and the distances corresponding to $B$ point to the left. Once the robots placed, we can deduce the orientations. Hence there exists coordinate satisfying the constraint $Z$. $Z$ is an instance of *RNLP*. ■

From the Proof III-D we can easily deduce the following Lemma:

*Lemma 3.9:* $S$ has a unique special partition iff the coordinates satisfying $Z$ are unique in $SE(1)$.

Seeing transforming $S$ to $Z$ is in polynomial time, if there exists a polynomial-time algorithm for *RNLP* then, from Lemma 3.9, we have a polynomial algorithm for *UPP*. This concludes *RNLP* is *NP*-Hard in $SE(1)$. In $SE(2)$, *RNLP* is NP-hard too. We come down to the previous case $SE(1)$ by adding a virtual robot $r_0$ in the formation. Then we express the fact that the robots are aligned by adding in the measurement set, $Z$, the observations $(null, 2\pi) = (\rho_{0i}, \varphi_{0i})$ with $1 \leq i \leq n$. Finally, to ensure that the added robot $r_0$ wont influence the *yes/no* decision of localization of the $n-$robot formation, we make the $r_0$ robot localizable with regard to the reference robot $r_1$ by adding the following observations: $(\sum_{s\in S} s, 2\pi) = (\rho_{01}, \varphi_{01})$ and $(\sum_{s\in S} s, \pi) = (\rho_{10}, \varphi_{10})$. Note that the distance between the virtual robot and the reference one is chosen relatively large to not affect the permutations (if there exists) of the robots placement in the chain formation.

**Algorithm 1** Function $AlgoPP(S_{pp})$ solving the Partition Problem

**function** $AlgoPP(S_{pp})$: `boolean`
    **if** $\sum_{s \in S_{pp}} s$ is odd
    **then return** NOT($AlgoUPP(S_{pp} \cup \{\sum_{s \in S_{pp}} s\})$);
    **else**  **if** $\frac{1}{2} \sum_{s \in S_{pp}} s \in S_{pp}$
        **then return** AlgoUPP($S_{pp}$);
        **else return** NOT(AlgoUPP($S_{pp} \cup \{\frac{1}{2} \sum_{s \in S_{pp}} s, 2 \sum_{s \in S_{pp}} s, \frac{5}{2} \sum_{s \in S_{pp}} s\}$));

## IV. RELATED WORKS

The localization problem in sensor networks is often considered using graph rigidity [15], [16] and graph embeddability [17], [18]. The problem of sensor network localization with distance information is to determine the locations of all the nodes, given the connected graph of the network and the known positions of some nodes (the beacons).

An interesting approach in [16] formulates the problem of solvability in terms of injective mappings between the configuration space and the distance information space. The authors [16] addressed the problem of uniquely localizable networks using graph theory rigidity. Even though this theory provides many advances in sensor network localization, unfortunately the results apply in networks connected using homogeneous information only, namely the distance between network nodes in an undirected connected graph.

The computational complexity of graph embeddability has been investigated in case of general distance information graphs by Saxe [17] and unit disk graphs (with limited distance observability) by Breu and Kirkpatrick [18]. Saxe [17] has shown that testing the realizability of weighted graphs is NP-hard. Aspnes et al. [13] argued that realizing a graph is still hard, even if it is known that the graph is globally rigid and that it has a realization. They [12], [13] construct grounded graphs to model sensor network and apply graph rigidity theory to test the conditions for unique localizability and to construct uniquely localizable networks in which the positions of some nodes are known.

There is no comparable results for networks with multi-sensory information and no work at our knowledge dealing with distance and bearing ($SE(2)$) information in graphs, though there are partial results with distance information in 3-space. There is no trivial way to derive $SE(2)$ network localization from results in point formation $\mathbb{R}^n, n = 2; 3$, and many ambiguity cases invalidated our attempts of adaptations.

The NP-hardness proved for distance network localization does not trivially imply that the problem of robot localization is NP-hard, because bearing observations could add new constraints on the system that make it more tractable. An example is determining the position of a robot giving a distance information will result in an infinity of positions on a circle of radius the measured distance. However, adding a bearing measurement, will reduce the set of solutions to a point, the actual position of the observed robot.

## V. CONCLUSION

The general localization schemes of robot networks are mainly heuristic-based and a full theoretical foundation of network localization is lacking.

We have addressed the question of the non ambiguous group localization, using *relative observations* acquired by the robots, and proved that deterministically resolving the Robot Network Localization Problem (*RNLP*) is NP-hard. Thus, one could not provide a localization algorithm that uniquely estimate, in a polynomial time, the robot group pose using relative observations (distance and bearing information) only. To prove that *RNLP* is NP-hard, we construct polynomial time reductions based on a well known NP-hard problem, namely the set partition problem.

Note that the proposed results never consider mobility as an ability to determine the group poses. Some results in the current literature seem to be ignored ,e.g., [19]. However, the result presented in this paper is applicable to static sensor as well as mobile robot networks. Indeed, consider first that the algorithm is based on the decision to move made by the robot with respect to the set of measurements. In that case, a robot could decide to make a move to remove the localization ambiguity, but this means that the robot is able to determine whether the configuration is localisable, which is impossible. In another approach, the robots could always make moves to evaluate all the poses. But, since we consider *deterministic* systems only, starting from an ambiguous configuration, the collective movements could lead the robots in another ambiguous configuration.

This absence of a sufficient uniqueness condition permits the computation of erroneous positions that may in turn lead applications to produce flawed results. The work presented here is a first step towards some theoretical foundations to the robot network localization. In future works, we would like to characterize classes of measurement sets for which *RLNP* becomes polynomial. Such sets exist, *e.g.*, each robots sees the pose of all the others robots. Does there exist such classes with relaxed measurement requirements?


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