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Geometric Hand-Eye Calibration for an Endoscopic Neurosurgery System

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Abstract— **We present an algorithm in the Conformal Geometric Algebra framework which computes the transformation (rotation and translation) relating the coordinate system of an endoscopic camera, with the coordinate system of the markers placed on it. Such markers are placed in order to track the camera's position in real time by using an optical tracking system. The problem is an adaptation of the so-called hand-eye calibration, well known in robotics; however we name it the *endoscope-tracking system* calibration. By this way, we can relate the preoperative data (3D model and planing of the surgery), with the data acquired in real time during the surgery by means of the optical tracking system and the endoscope. Results obtained with our approach are compared with other approach which computes separately (in a two-step algorithm) the rotation and translation, and they are promising.**

I. INTRODUCTION

In general, the *registration* process consists on establishing a common geometric reference frame between two or more data sets. These data sets can be data taken using different modalities, or the same modality but in different times. In surgery, the registration is made with the purpose of having more preoperative and intraoperative information for diagnostic and navigation. That is, with registration one can relate the position of surgical instruments tracked in real time by an optical tracking system, with the virtual model created with preoperative images, display both of them in a common reference frame, compare information obtained before and during the surgery, etc.

In the operating room, there are multiple local coordinate systems that must be related in order to show to the surgeon a virtual model of what is happening in the real world. The figure 1 illustrates the scenario. When using information obtained from endoscopy or neuro-sonography, we must relate what is been observed by the endoscopic camera (or ultrasound system), with the virtual model. Therefore, calibration techniques are also used in order to register all the equipment needed in surgery.

In this paper we are proposing a formulation of the hand-eye calibration problem, which is well known in robotics community [1], [2], [3], [4], but we define it in terms of the Conformal Geometric Algebra in order to calculate the transformation (rotation and translation) between the coordinate system of the endoscopic camera, and the coordinate system of the spherical markers placed to the endoscope. We call this task the *endoscope-tracking system* calibration. The method proposed in this paper computes both, rotation and translation, at the same time, in contrast to [1]; in addition, our formulation avoids to take care of certain particularities which we need to take into account in other methods [2], [3], [4], making simpler to understand and to compute them, as will be explained in sections III and IV. However, before that, we present in section II a brief introduction to geometric algebra, and how to express rigid body transformations in this mathematical framework. Experimental results are devoted to section V, and they show that the presented method is promising.

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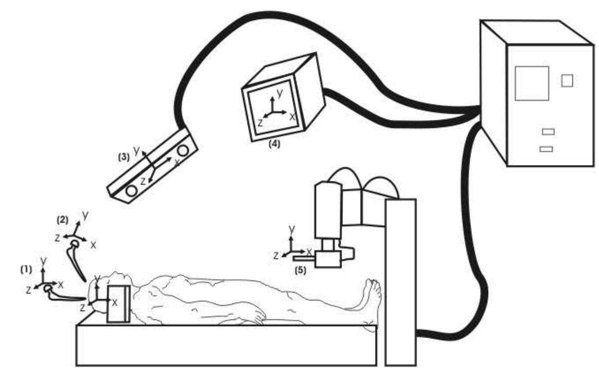

Fig. 1. Some of the different coordinates systems present in the operating room: surgical instruments (1,2), optical tracking system (3), virtual model(4), endoscopy (5).

II. GEOMETRIC ALGEBRA

The Geometric or Clifford Algebra was introduced by William K. Clifford (1845–1879) and there has been interesting proposals using it in areas as robotics, computer vision, etc. Roughly speaking, geometric algebra (GA) is a mathematical framework which allows us to treat geometric objects as algebraic entities that can be easily treated with algebraic tools.

The Geometric Algebra $G_{p,q,r}$ is constructed over the vector space $\mathcal{V}^{p,q,r}$, where p, q, r denote the signature of the algebra; if $p \neq 0$ and $q = r = 0$, the metric is Euclidean; if only $r = 0$, the metric is pseudo euclidean; if $p \neq 0, q \neq 0, r \neq 0$, the metric is degenerate. In this algebra, we have the *geometric product* which is defined as in (1) for two vectors a, b, and have two parts: the inner product $a \cdot b$ is the symmetric part, while the wedge product $a \wedge b$ is the antisymmetric part.

$$ab = a \cdot b + a \wedge b. \quad (1)$$

The dimension of $G_{n=p,q,r}$ is 2^n, and G_n is constructed by the application of the geometric product over the vector basis e_i. This results in a basis for G_n containing elements of different grade called *blades* (e.g. scalars, vectors, bivectors, trivectors, etc): $1, e_1...e_{12}...e_{123}...I$, which is called *basis blade*; where the element of maximum grade is the pseudoscalar $I = e_1 \wedge e_2... \wedge e_n$. Given a multivector M (linear combination of blades), if we are interested in extracting only the blades of a given grade, we write $\langle M \rangle_r$ where r is the grade of the blades we want to extract (obtaining an homogeneous multivector M' or a r-vector).

A. Conformal Geometric Algebra

To work in Conformal Geometric Algebra (CGA) $G_{4,1,0}$ means to embed the Euclidean space in a higher dimensional space with two extra basis vectors which have particular meaning; in this way, we represent particular objects of the Euclidean space with subspaces of the conformal space. The vectors we add are e_+ and e_-, which square to $1, -1$, respectively. With these two vectors, we define the null vectors

$$e_0 = \frac{1}{2}(e_- - e_+); \quad e_\infty = (e_- + e_+), \tag{2}$$

interpreted as the origin and the point at infinity, respectively. From now and in the rest of the paper, points in the 3D-Euclidean space are represented in lowercase letters, while conformal points in uppercase letters. To map a point $x \in \mathcal{V}^3$ to the conformal space in $G_{4,1}$, we use

$$X = x + \frac{1}{2}x^2 e_\infty + e_0. \tag{3}$$

In a similar fashion, other geometric entities can be defined in CGA; for example, a sphere is defined as

$$S = c + \frac{1}{2}(c^2 - \rho^2)e_\infty + e_0 \tag{4}$$

In fact, some authors refer to points in CGA as spheres of radius zero. Circles are defined as the intersection of two spheres: $Z = S_1 \wedge S_2$. Lines are defined as

$$\begin{aligned} L &= rI_E + e_\infty m I_E & (5)\\ & r = a - b; m = a \wedge b & (6) \end{aligned}$$

All these entities and their transformations can be managed easily using the rigid motion operators described in section II-B.

B. Rigid body motion in CGA

In CGA, rotations are represented by the *rotors*, which are defined as

$$R = e^{\frac{1}{2}\mathbf{b}\theta} = \cos\frac{\theta}{2} + \mathbf{b}\sin\frac{\theta}{2} \tag{7}$$

where $\mathbf{b}$ is the bivector dual to the rotation axis, and θ is the rotation angle. Rotation of an entity is carried out by multiplying it by the left with the rotor R, and by the right for the reversion of the rotor $\tilde{R}$: $X' = RX\tilde{R}$. Translation is carried out by the so called *translator*

$$T = e^{\frac{e_\infty t}{2}} = 1 + \frac{e_\infty t}{2} \tag{8}$$

where $t \in \langle G_3 \rangle_1$ is the translation vector. Note that this operator can be interpreted as a special rotor, expressed in a null space because $e^2_{infty} = 0$. Translations are applied in a similar way to rotations: $X' = TX\tilde{T}$.

To express rigid body transformations, rotors and translators are applied consecutively. The result is called *motor*:

$$M = TR \tag{9}$$

Such operator is applied to any entity of any dimension by multiplying the entity by the operator from the left, and by the reverse of the operator from the right: $X' = MX\tilde{M}$. The motor M is a special multivector of even grade. To see its components, let us carry out the multiplication of R and T

$$\begin{aligned} M &= TR \\ &= (1 + \frac{1}{2}e_\infty t)(\cos(\frac{\theta}{2}) + \mathbf{b}\sin(\frac{\theta}{2})) \\ &= \cos(\frac{\theta}{2}) + \mathbf{b}\sin(\frac{\theta}{2}) + \frac{1}{2}e_\infty(t(\cos(\frac{\theta}{2}) + \mathbf{b}\sin(\frac{\theta}{2}))) \\ &= R + R' & (10) \end{aligned}$$

Since the multiplication of a vector $t \in \langle G_3 \rangle_1$ by a bivector $\mathbf{b} \in \langle G_3 \rangle_2$ results in a multivector of the form $\lambda_1 e_1 + \lambda_2 e_2 + \lambda_3 e_3 + \lambda_4 e_{123}$, and since $t\cos(\frac{\theta}{2}) \in \langle G_3 \rangle_1$, we can rewrite (10) as

$$\begin{aligned} M &= \cos(\frac{\theta}{2}) + \mathbf{b}\sin(\frac{\theta}{2}) + \frac{1}{2}e_\infty(t\cos(\frac{\theta}{2}) + t\mathbf{b}\sin(\frac{\theta}{2})) \\ &= \cos(\frac{\theta}{2}) + \mathbf{b}\sin(\frac{\theta}{2}) + \frac{1}{2}e_\infty(t\cos(\frac{\theta}{2}) + & (11)\\ & \lambda_1 e_1 + \lambda_2 e_2 + \lambda_3 e_3 + \lambda_4 e_{123} \\ &= \cos(\frac{\theta}{2}) + \mathbf{b}\sin(\frac{\theta}{2}) + e_\infty(t' + \lambda e_{123}) \\ &= \cos(\frac{\theta}{2}) + \mathbf{b}\sin(\frac{\theta}{2}) + e_\infty t' + \lambda e_{\infty 123} & (12) \end{aligned}$$

where $t' \in \langle G_3 \rangle_1$ and $\lambda = \frac{1}{2}\lambda_4$. Note that $e_\infty t'$ is a bivector with components $e_{\infty 1}, e_{\infty 2}, e_{\infty 3}$. If we take only the bivectorial parts of the motor M, we obtain

$$\begin{aligned} \langle M \rangle_2 &= \langle R \rangle_2 + \langle R' \rangle_2 \\ &= \mathbf{m} + \mathbf{m'} \\ &= \sin(\frac{\theta}{2})\mathbf{b} + e_\infty t' & (13) \end{aligned}$$

Therefore, if we express the vector t' in terms of their dual bivector $t' = t'' I_E$, we can rewrite (13) as

$$\langle M \rangle_2 = b' I_E + e_\infty t'' I_E \tag{14}$$

If we see the representation of the lines in 6, we observe that the bivectorial part of the motor M is in fact a line and it corresponds to the screw axis in which is carried out the rotation and translation of the object.

III. Hand-Eye Calibration in CGA

The hand-eye calibration is the calculation of the relative pose (position and orientation) between a robotic hand and a rigid camera mounted on it. Using this camera, we can determine the position in its coordinate system of an objective

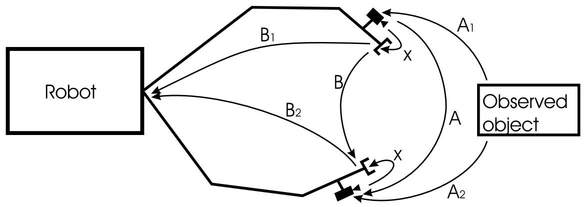

Fig. 2. The hand-eye calibration problem.

to catch or to reach; however, the commands of movements are in the coordinate system of the robotic hand; therefore, to know the hand-eye transformation can be of great utility in this kind of tasks.

The usual way to describe the hand-eye calibration is by means of homogeneous transformation matrices. Let be X the transformation from the camera to the robotic hand, A_i the transformation from the camera to the world coordinate system, and B_i the transformation from the robot base to the robotic hand in the i-th movement. Figure 2 illustrates the problem.

The transformation A_i is obtained by extrinsic calibration techniques [5]. The transformation B_i is given by the robot's direct kinematics. The formulation of the hand-eye problem is well-known

$$AX = XB \tag{15}$$

where $A = A_2 A_1^{-1}$ and $B = B_2^{-1} B_1$. In order to solve this problem, at least two movements are required with non parallel rotation axes, and several methods have been proposed to find the solution: some people estimate the rotation at first and then the translation [1], while other make it simultaneously [2]. Daniilidis [3] presents a solution based on dual quaternions, while [4] proposes the use of the motor algebra $G_{3,0,1}$.

We formulate the problem in terms of motors of the conformal geometric algebra framework. Following the formulation of [4] for the hand-eye calibration problem in (15), it will be expressed as

$$M_A M_X = M_X M_B \tag{16}$$

where $M_A = A + A'$, $M_B = B + B'$ and $M_X = R + R'$ (Sect. II-B). In [4], it is shown that the angle and pitch of the gripper (endoscope in our case) are equal to the angle and pitch of the camera (they remain invariant under coordinate transformations, which is known as the screw congruence theorem); therefore, the problem is solved using only the lines defined by the motors

$$\begin{aligned} L_A &= \mathbf{a} + \mathbf{a}' \\ &= M_X L_B \tilde{M}_X \\ &= (R + R')(\mathbf{b} + \mathbf{b}')(\widetilde{R + R'}) \\ &= R\mathbf{b}\tilde{R} + e_\infty (R\mathbf{b}\tilde{R}' + R\mathbf{b}'\tilde{R} + R'\mathbf{b}\tilde{R}) \end{aligned} \tag{17}$$

where $\mathbf{a}$, $\mathbf{a}'$, $\mathbf{b}$, $\mathbf{b}'$ are bivectors (like in (13)). By separating the real part and the part multiplied by e_∞, we have

$$\mathbf{a} = R\mathbf{b}\tilde{R} \tag{18}$$

$$\mathbf{a}' = R\mathbf{b}\tilde{R}' + R\mathbf{b}'\tilde{R} + R'\mathbf{b}\tilde{R} \tag{19}$$

Multiplying from the right by R and using the relationship $\tilde{R}R' + \tilde{R}'R = 0$, the following relationships are obtained

$$\mathbf{a}R - R\mathbf{b} = 0 \tag{20}$$

$$(\mathbf{a}'R - R\mathbf{b}') + (\mathbf{a}R' - R'\mathbf{b}) = 0 \tag{21}$$

which can be expressed in matrix form as

$$\begin{bmatrix} \mathbf{a} - \mathbf{b} & [\mathbf{a} + \mathbf{b}]_\times & \mathbf{0_{3\times1}} & \mathbf{0_{3\times3}} \\ \mathbf{a}' - \mathbf{b}' & [\mathbf{a}' + \mathbf{b}']_\times & \mathbf{a} - \mathbf{b} & [\mathbf{a} + \mathbf{b}]_\times \end{bmatrix} \begin{bmatrix} R \\ R' \end{bmatrix} = 0 \tag{22}$$

We call D to this 6×8 matrix; the unknown vector $[R, R']^T$ is 8-dimensional. The notation $[u]_\times$ represents the skew-symmetric matrix formed with the vector u, which is given by

$$\hat{u} = \begin{bmatrix} 0 & -u_3 & u_2 \\ u_3 & 0 & -u_1 \\ -u_2 & u_1 & 0 \end{bmatrix} \tag{23}$$

The matrix D is composed only by bivectors (blades of any other grade are not included), therefore we can use the SVD method to find $[R \;\; R']^T$ as the kernel of D.

Considering that we have $n \geq 2$ movements, the following matrix is built

$$C = \begin{bmatrix} D_1^T & D_2^T & D_3^T & D_4^T & ... \end{bmatrix}^T \tag{24}$$

in order to apply the SVD method and to find the solution for $[R, R']^T$. Since the range of the matrix C is at most 6, the last right two singular vectors, v_7 and v_8 correspond to the two singular values whose value is zero or near to zero, and such vectors expand the null space of C. Therefore, as $[R, R']^T$ is a null vector of C, we can express it as a linear combination of v_7 and v_8. If we express these vectors in terms of two vectors of 4D $v_7 = (u_1, v_1)^T$ and $v_8 = (u_2, v_2)^T$, this linear combination can be expressed as

$$\begin{bmatrix} R \\ R' \end{bmatrix} = \alpha \begin{bmatrix} u_1 \\ v_1' \end{bmatrix} + \beta \begin{bmatrix} u_2 \\ v_2' \end{bmatrix} \tag{25}$$

Taking into account the constraints

$$R\tilde{R} = 1 \quad \text{and} \quad \tilde{R}R' + \tilde{R}'R = 0 \tag{26}$$

we obtain the following quadratic equations in terms of α and β

$$\alpha^2 u_1^T u_1 + 2\alpha\beta u_1^T u_2 + \beta^2 u_2^T u_2 = 1 \tag{27}$$

$$\alpha^2 u_1^T v_1 + \alpha\beta(u_1^T v_2 + u_2^T v_1) + \beta^2 u_2^T v_2 = 0 \tag{28}$$

In order to solve these equations, we make a change of variable, substituting in (28) $\mu = \alpha/\beta$ and we obtain two solutions for μ. Going back to (27) and replacing the relationship $\alpha = \mu\beta$, we obtain

$$\beta^2(\mu^2 u_1^T u_1 + \mu(2u_1^T u_2) + u_2^T u_2 = 1 \tag{29}$$

which takes two solutions for β.

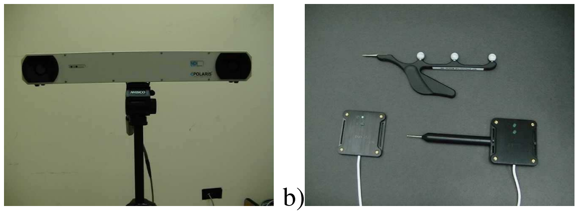

Fig. 3. Optical tracking system, *Polaris*.

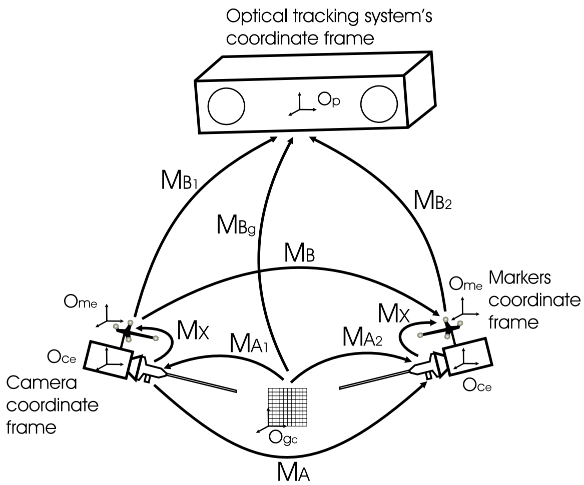

Fig. 4. The problem of the calibration between the endoscopic camera and the optical tracking system (Polaris).

IV. Endoscope-tracking system calibration

The optical tracking system used is the *Enhanced Hybrid Polaris System* (Fig. 3), which is labeled as (3) in the scenario shown in figure 1. This system emits infrared light that is reflected by markers placed to the object we are interested to track (Fig. 3.c); the reflected light is detected by the Polaris system, and then it estimates the 3D position of the different markers. We attach the local coordinate system of the markers to one of them. By this way, we know the transformation between the markers reference frame and the Polaris reference frame.

When using information obtained from endoscopy or neuro-sonography, we must relate what is been observed by the endoscopic camera (or ultrasound system), with the virtual model.

To solve this problem, we propose the formulation of the hand-eye calibration problem in terms of the Conformal Geometric Algebra, explained in section III, to calculate the transformation between the coordinate system of the endoscopic camera, and the coordinate system of the spherical markers placed to the endoscope. We call this task the *endoscope-tracking system* calibration. The scenario changes from the one shown in figure 2 to the one shown in figure 4, where the reader can see that there is a (rigid) transformation between the calibration grid, and the Polaris System, $\mathbf{M_{B_g}}$. Such transformation will be used to validate the results of the endoscope-tracking system calibration method. The transformations involved in the problem are expressed as motors of the CGA: $M = TR$.

The procedure is summarized as follows

1) Given n movements of the endoscopic camera (we move it freely by hand to arbitrary positions), M_{B_i}, and their corresponding movements M_{A_i}, verify if their scalar parts are equal (screw congruence theorem with motors).
2) For the movements that fulfill the previous requirement, extract the directions and moments of the lines L_{A_i} and L_{B_i} defined by the motors. Build the matrix C as in (24).
3) Apply SVD to matrix C. Take the right singular vectors v_7 and v_8 corresponding to the two singular values nearest to zero (a threshold is applied by the noise).
4) Compute the coefficients for (28) and find the two solutions of μ.
5) For both values of μ, compute the value of $\mu^2 u_1^T u_1 + 2\mu u_1^T u_2 + u_2^T u_2$ and choose the one that gives the biggest value. Then, compute α and β.
6) The final solution is $\alpha v_7 + \beta v_8$

V. Experimental results

In order to validate the accuracy of the estimated transformation $\mathbf{M_X}$, we use the calibration grid used to calibrate the endoscopic camera by Zhang's method [5], and shown in Fig. 5.a. Let be X_g the set of points corresponding to the corners of the calibration grid, referred to the coordinate system $\mathbf{O_{g_c}}$. These coordinates are expressed in millimeters, according to the size of each square in the calibration grid, which has 1.25 mm by side in our case.

1) Taking the points X_g in the grid reference frame, apply the transformation $\mathbf{M_{A_i}}$ to express them in the camera's reference frame. Let be X_{A_i} the resulting points.
2) Project the points X_{A_i} to the image plane using
$$x_{A_i} = K \;\; [R_{M_A} \;\; t_{M_A}] \;\; X_{A_i} \tag{30}$$
These points should be projected on the corners of the squares in the calibration grid on the image (Fig. 5).
3) Taking the points X_g, apply the transformations $\mathbf{M_{B_g}}$, $\mathbf{M_{B_i}}$ and $\mathbf{M_X}$. Let be $X_{M_{B_i}X}$ the resulting points.
4) Project the points $X_{M_{B_i}X}$ onto the image plane using
$$x_{B_i} = K \;\; [R_{M_{B_i}X} \;\; t_{M_{B_i}X}] \;\; X_{M_{B_i}X} \tag{31}$$
In the ideal case (without noise), the projected points x_{A_i} should match with the projected points x_{B_i}. However, as a result of noise in the Polaris readings or noise in the estimation of transformations, a small linear displacement between x_{A_i} and x_{B_i} is possible (see Fig. 6.a). We can measure the error ϵ between the two projections as
$$\epsilon = \frac{\sum_{i=1}^{n} (x_{A_i} - x_{B_i})}{n} \tag{32}$$
5) In order to correct the displacement, the centroid of each point set is calculated: c_{A_i} and c_{B_i}. Then,

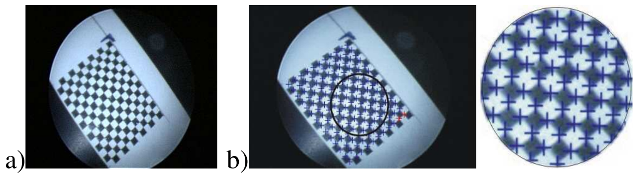

Fig. 5. a) Original image; b) Result of the projection using (30); it is included a zoom of the marked region for better visualization.

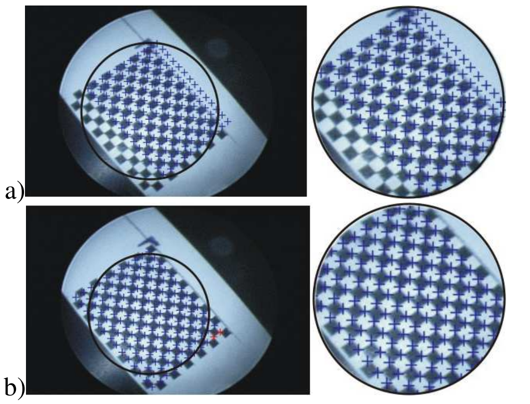

Fig. 6. a) Result of the projection using (31); b) Result applying (33)

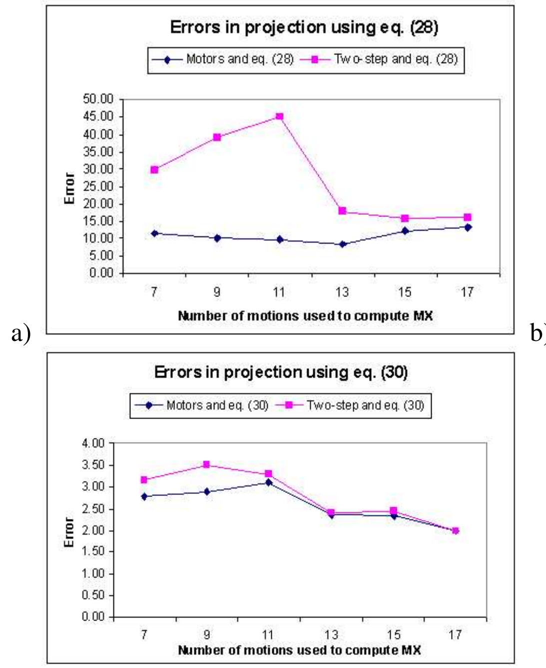

Fig. 7. Average errors obtained with our algorithm (labeled as Motors) and the method computing rotation-translation in a two-step algorithm, when varying the number of motions to compute the transformation. Error computed as the euclidean distance, measured in pixels as in (35).

the points x_{B_i} are displaced in such a way that the centroids match.

$$x'_{B_i} = x_{B_i} + (c_{A_i} - c_{B_i}) \tag{33}$$

After the displacement, the average error is calculated as

$$\epsilon' = \frac{\sum_{i=1}^{n} (x_{A_i} - x'_{B_i})}{n} \tag{34}$$

The figure 6.b shows the result after doing such correction to the points showed in figure 6.a.

The experiments were carried out using and endoscope from Karl Storz Endoscopy, which consists in a TeleCam SL II camera and an endoscope with a view angle of zero degrees. The movements were done by hand, to arbitrary positions in space. We test the algorithm computing the transformation M_X using a different number of motions each time. To compare our results with other standard method, we present the computation of the rotation and translation of the transformation M_X by the method proposed in [1], which are computed by a two-step algorithm, in contrast with our method which computes both at the same time. To measure the error between the projection using (30) and the one using (31) or (33), we take the Euclidean distance between each point x_{A_i} and its corresponding point x_{B_i}, or the distance between x_{A_i} and x'_{B_i}, when using (33), that is

$$Error = |x_{A_i} - x_{B_i}| \tag{35}$$

This error is a distance measured in pixels. In each motion, the image of the grid contains 154 points approximately.

The graph shown in figure 7.a shows the average errors obtained between x_{A_i} and x_{B_i} estimating the transformation with different number of motions for the two methods: the one using motors and the one called two-step in the graph; the graph shown in figure 7.b shows the average errors obtained between x_{A_i} and the displaced position x'_{B_i} (eq. (33)). Note in 7.a that our method makes since the beginning a more accurate computation, and although in the corrected version (fig. 7.b) the difference is smaller, it still gives a better approximation to x_{A_i}, taken as the expected value.

Figures 8.a shows the average error between x_{A_i} and x_{B_i} obtained for each one of five motions used to compute the transformation by the two methods; while figure 8.b shows the average error between x_{A_i} and x'_{B_i} for each one of the five motions. Figure 9 shows the results but for each one of nine motions. Figure 10 shows the average errors obtained using thirteen and seventeen motions, using (31).

VI. Conclusions

In this work we have presented a method to estimate the transformation relating an endoscopic camera used in surgical procedures, with the set of spherical markers placed on it, which are tracked by an optical tracking system. By this way, we have all the transformations needed to appropriately compute a composed transformation relating the preoperative data (virtual model) with the intraoperative data obtained from the endoscopic camera.

The presented approach take advantage of the representation of rigid transformations (rotation and translation) in the conformal geometric algebra, which are expressed as versors called *motors*. The composition of such motors was analyzed, showing that they contain the line representing the screw axis in which is carried out the rotation and translation

a) b)

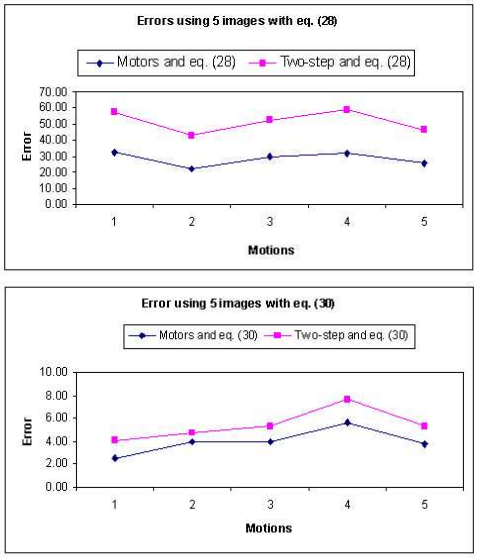

Fig. 8. Average errors obtained with (35) for each one of five images (five motions) comparing our algorithm (labeled as Motors) and the method computing rotation-translation in a two-step algorithm.

a) b)

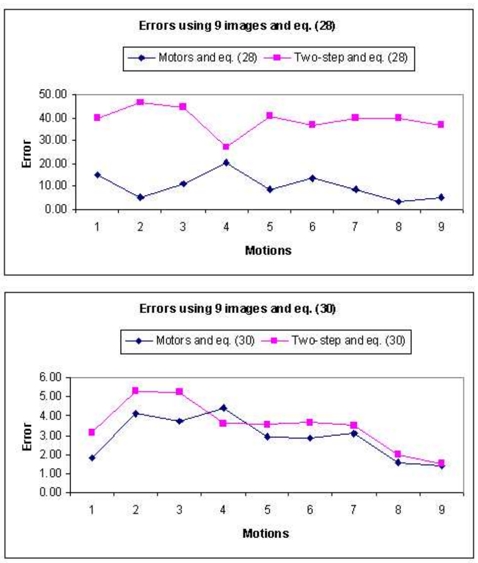

Fig. 9. Average errors obtained using (35) for each one of nine images (nine motions) comparing our algorithm (labeled as Motors) and the method computing rotation-translation in a two-step algorithm

of the object. Then the transformation is estimated based on the lines defined by the motors in different movements of the endoscopic camera. Numerical results show that the method is accurate enough and it is suitable to be used in real surgeries.

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a) b)

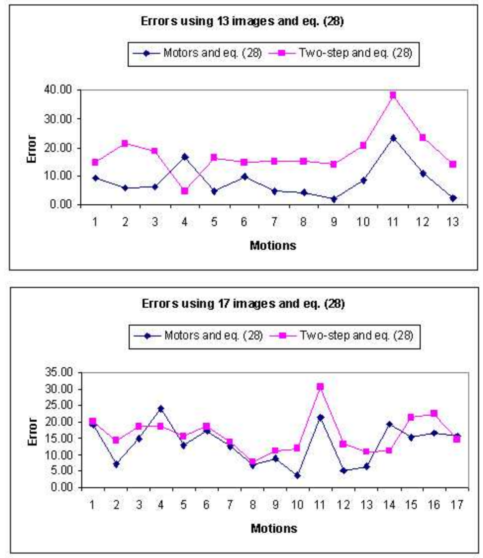

Fig. 10. Average errors obtained using (35) with 13 and 17 motions, comparing our algorithm and the method computing rotation-translation in a two-step algorithm

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