

Towards constrained motion planning of mobile manipulators

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Abstract—This paper addresses a constrained motion planning problem for mobile manipulators. The constraints are included into the system model by means of a sort of penalty function, and then processed in accordance with the endogenous configuration space approach. Main novelty of this paper lies in deriving a constrained Jacobian motion planning algorithm with the following features: inequality constraints are included into an extended kinematics model using a smooth approximation of the plus function, the model is then regularized against singularities, and the resulting imbalance in error equations is handled as a perturbation of an exponentially stable linear dynamic system. The operation of the constrained motion planning algorithm is illustrated by a motion planning problem of a mobile manipulator with bounds imposed on a platform variable. Performance of the algorithm is tested by computer simulations.

I. INTRODUCTION

A mobile manipulator is a robotic system that consists of a nonholonomic mobile platform carrying on board a holonomic manipulator. The kinematics of the mobile manipulator are represented by a control system with outputs. The output function describes position and orientation of the end effector with respect to an inertial frame. The end point map of this control system defines the kinematics map. For the reason that this map transforms an infinite dimensional space of platform controls and joint positions of the on board manipulator into a finite dimensional task space, the kinematic redundancy of a mobile manipulator goes to infinity. Given the kinematics map, the motion planning problem of a mobile manipulator amounts to determining a control of the platform and a joint position of the on board manipulator such that the end effector assumes a prescribed position and orientation. Such a problem, referred to as unconstrained, is equivalent to the inverse kinematic problem. The inverse kinematic problem for mobile manipulators is usually solved by means of Jacobian algorithms. A derivation procedure of Jacobian algorithm utilizes the continuation method [1]. A systematic application of this method to mobile manipulators has constituted the endogenous configuration space approach [2]. Within this approach, basically each concept and algorithm existing for holonomic manipulators can be adapted to mobile manipulators. The fundamental concept of the endogenous configuration space includes all admissible controls of the platform, and joint positions of the on board manipulator.

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If the mobile manipulator is supposed to accomplish tasks in a physical environment, its motion planning needs to take into account some motion constraints, reflecting the existence of obstacles in the environment, the presence of singularities, bounds on admissible values of system variables or bounds on controls. This converts the unconstrained motion planning problem into a problem with constraints. There are two main methods of incorporating constraints into a Jacobian inverse kinematics algorithm. A strategy proposed in [3] relies on multiplying the vector fields representing the system kinematics by a function vanishing in a "forbidden" region of system variables, so preventing the system trajectory from entering there. An alternative, presented in [4], recommends the use of exterior penalty functions. Examples of application of this method have been dealt with in [5]. Both these strategies guarantee that the constraints will be satisfied exactly. A somewhat less demanding strategy exploits the freedom that exists in the Jacobian kernel of a mobile manipulator. In this way, the inverse kinematics algorithm may generate motion in a desirable direction within the endogenous configuration space. An application of this method to obstacle avoidance in mobile manipulators has been shown in [6]. Its further development results in a motion planning algorithm able to accomplish several tasks with different priorities, applied in [7] to the motion planning of an ocean ship respecting a bound on the rudder angle. The construction of the motion planning algorithm with task priorities generalizes to mobile manipulators the ideas presented in [8]. An incorporation of constraints into a motion planning algorithm based on an extended Jacobian inverse kinematics algorithm is proposed in [9]. That algorithm extends to mobile manipulators the ideas developed in [10]. A straightforward modification of the Jacobian pseudo inverse algorithm resulting in constraining platform control functions has been done in [11]. Last but not least, a desirable behavior of a motion planning algorithm can be obtained by designing an algorithm that approximates a given pattern of behavior. Conceptual tools related to this objective have been provided in [12] and developed in [13].

This paper addresses the constrained motion planning problem for mobile manipulators, following the methodology proposed recently in [14]. The constraints are included into the system model by means of a sort of penalty function, and then processed in accordance with the endogenous configuration approach. Main novelty of this paper lies in devising a constrained Jacobian motion planning algorithm with the following features:

- the constraints are handled via a dynamic extension of the kinematics model,

- inclusion of constraints employs a smooth approximation of the plus function,
- the extended kinematics model is regularized against singularities,
- an imbalance in error equations due to the regularization is handled as a perturbation of an exponentially stable linear dynamic system.

The new concept of constrained motion planning algorithm is illustrated with a motion planning problem of a mobile manipulator with bounds imposed on a platform variable. A proof of convergence of the algorithm is sketched. Performance of the algorithm is tested by computer simulations involving a kinematic car-type platform endowed with an RTR on board manipulator.

This paper is organized as follows. Section 2 develops the basic theory, culminating in the motion planning algorithm. Section 3 shows an application of this algorithm to a mobile manipulator. The papers concludes with section 4.

II. BASIC CONCEPTS

We shall study the kinematics of a mobile manipulator, represented by a driftless control system with outputs [2]

$$\begin{cases} \dot{q} = G(q)u \\ y = k(q, x), \end{cases} \quad (1)$$

where $q \in R^n$, refers to platform coordinates, $x \in R^l$ denotes joint position of the on board manipulator, $y \in R^r$ describes the task coordinates, and $u \in R^m$ stands for the control of the platform. The control functions will be assumed Lebesgue square integrable on a time interval $[0, T]$, and denoted as $u(\cdot) \in L_m^2[0, T]$. Let $q(t) = \varphi_{q_0, t}(u(\cdot))$ be a platform trajectory started from q_0 . Then, the end point map

$$K_{q_0, T}(u(\cdot), x) = k(q(T), x)$$

of the system (1) is identified with the kinematics of the mobile manipulator. The pairs $(u(\cdot), x)$ constitute the endogenous configuration space $\mathcal{X} = L_m^2[0, T] \times R^l$.

The following motion planning problem with state constraints will be addressed in the system (1): given $y_d \in R^r$, find an endogenous configuration $(u_d(\cdot), x_d) \in \mathcal{X}$ such that $K_{q_0, T}(u_d(\cdot), x_d) = y_d$, while a certain platform coordinate remains bounded, $q_{lb} \leq q_k(t) \leq q_{ub}$. Our solution to this problem relies on a classic application of the endogenous configuration space approach [2], with some modifications accounting for the constraints. First of all, to incorporate the constraints into the system (1), we shall use the plus function $(x)_+ = \max\{x, 0\}$, so the inequality constraints will be satisfied, when the functions $(q_k(t) - q_{ub})_+$ and $(-q_k(t) + q_{lb})_+$ are zero for every $t \in [0, T]$. Because the plus function is nonnegative, this will be satisfied, whenever the sum of integrals over $[0, T]$ of these functions is zero. Next, it is well known [15] that the plus function can be efficiently approximated by a smooth function

$$(x)_+ \cong p(x, \alpha) = x + \frac{1}{\alpha} \ln(1 + \exp(-\alpha x)), \quad (2)$$

parameterized by α . The function (2) approaches $(x)_+$ when α increases to $+\infty$, furthermore, it turns out that the approximation is satisfactory even for moderate values of α . Utilizing this approximation, we shall augment the control system (1) with an extra state variable q_{n+1} by setting

$$\dot{q}_{n+1} = p(q_k - q_{ub}, \alpha) + p(-q_k + q_{lb}, \alpha), \quad q_{n+1}(0) = 0.$$

We let $q_e = (q, q_{n+1})$ and $y_e = (y, q_{n+1})$. Then, the extended system (1) will become an affine control system of the form

$$\begin{cases} \dot{q}_e = f_e(q_e) + G_e(q_e)u \\ y_e = k_e(q_e, x), \end{cases} \quad (3)$$

where

$$f_e(q_e) = \begin{pmatrix} 0 \\ p(q_k - q_{ub}, \alpha) + p(-q_k + q_{lb}, \alpha) \end{pmatrix},$$

$$G_e(q_e) = \begin{bmatrix} G(q) \\ 0 \end{bmatrix},$$

and the new output function $k_e(q_e, x) = (k(q, x), q_{n+1})$. Consequently, the extended kinematics

$$K_{q_{e0}, T}^e(u(\cdot), x) = k_e(q_e(T), x).$$

In the extended system the original motion planning problem can be given the following formulation: find an endogenous configuration $(u_d(\cdot), x_d)$ such that $K_{q_{e0}, T}^e(u_d(\cdot), x_d) = y_{ed}$, where $y_{ed} = (y_d, 0)$. This motion planning problem is unconstrained, so it can be solved by a Jacobian inverse kinematics algorithm. A derivation of such an algorithm begins with choosing an initial configuration $(u_0(\cdot), x_0) \in \mathcal{X}$. If $K_{q_{e0}, T}^e(u_0(\cdot), x_0) = y_{ed}$, the problem is solved. Otherwise, we define a smooth curve $(u_\theta(\cdot), x(\theta))$, $\theta \in R$, and compute the error

$$e(\theta) = K_{q_{e0}, T}^e(u_\theta(\cdot), x(\theta)) - y_{ed}. \quad (4)$$

We want that the error decreases exponentially, with a prescribed decay rate $\gamma > 0$, so that

$$\frac{de(\theta)}{d\theta} = -\gamma e(\theta). \quad (5)$$

By differentiation of (4) we arrive at a Wazewski-Davidenko equation

$$J_{q_{e0}, T}^e(u_\theta(\cdot), x(\theta)) \begin{pmatrix} \frac{du_\theta(\cdot)}{d\theta} \\ \frac{dx(\theta)}{d\theta} \end{pmatrix} = -\gamma e(\theta), \quad (6)$$

where the Jacobian operator

$$J_{q_{e0}, T}^e(u(\cdot), x) \begin{pmatrix} v(\cdot) \\ w \end{pmatrix} = C(T) \int_0^T \Phi(T, t) B(t) v(t) dt + D(T) w. \quad (7)$$

The fundamental matrix $\Phi(t, s)$ satisfies the evolution equation

$$\frac{\partial \Phi(t, s)}{\partial t} = A(t) \Phi(t, s)$$

with initial condition $\Phi(s, s) = I_{n+1}$, and the matrices

$$A(t) = \frac{\partial}{\partial q_e} \left(f_e(q_e(t)) + G_e(q_e(t)) u(t) \right), \quad B(t) = G_e(q_e(t)),$$

$$C(t) = \frac{\partial k_e(q_e(t), x)}{\partial q_e}, \quad D(t) = \frac{\partial k_e(q_e(t), x)}{\partial x}$$

come from the linear approximation of (3) along the triple $(u(t), x, q_e(t))$. The Moore-Penrose generalized inverse of the Jacobian (7) is defined as

$$(J_{q_{e0}, T}^{\#e}(u(\cdot), x)\eta)(t) = \begin{bmatrix} B^T(t)\Phi^T(T, t)C^T(T) \\ D^T(T) \end{bmatrix} \mathcal{D}^{-1}\eta, \quad (8)$$

where

$$\mathcal{D} = C(T) \int_0^T \Phi(T, t)B(t)B^T(t)\Phi^T(T, t)dtC^T(T) + D(T)D^T(T)$$

is called the dexterity matrix of (3). Using this inverse, we transform the Wazewski-Davidenko equation into a dynamic system in the endogenous configuration space

$$\frac{d}{d\theta} \begin{pmatrix} u_\theta(\cdot) \\ x(\theta) \end{pmatrix} = -\gamma J_{q_{e0}, T}^{\#e}(u_\theta(\cdot), x(\theta))e(\theta) \quad (9)$$

whose limit trajectory

$$\lim_{\theta \rightarrow +\infty} \begin{pmatrix} u_\theta(\cdot) \\ x(\theta) \end{pmatrix} = \begin{pmatrix} u_d(\cdot) \\ x_d \end{pmatrix}$$

provides a solution to the constrained motion planning problem.

Obviously, the solution exists on condition that the dexterity matrix is invertible. This appears not to be the case in the system (3) as long as the constraints are satisfied. In order to overcome this difficulty we propose to add to the differential equation in (3) an extra regularizing term $r(q_{rk}) = r(q_k)$. In this way we obtain a regularized system

$$\begin{cases} \dot{q}_r = f_r(q_r) + G_r(q_r)u \\ y_r = k_r(q_r, x), \end{cases} \quad (10)$$

where $q_r = q_e$, $G_r(q_r) = G_e(q_r)$, $k_r(q_r, x) = k_e(q_r, x) = (k(q, x), q_{rn+1})$, and

$$f_r(q_r) = \begin{pmatrix} 0 \\ r(q_{rk}) + p(q_{rk} - q_{ub}, \alpha) + p(-q_{rk} + q_{lb}, \alpha) \end{pmatrix}.$$

In order to state the motion planning problem in the regularized system (10) we need to introduce the desirable output in the form

$$y_{rd} = \left(y_d, \int_0^T r(q_{rk}(t))dt \right).$$

Let $K_{q_r, T}^r(u(\cdot), x)$ denote the kinematics of (10). Given a curve $(u_\theta(\cdot), x(\theta)) \in \mathcal{X}$, we obtain the error

$$e_r(\theta) = K_{q_r, T}^r(u_\theta(\cdot), x(\theta)) - y_{rd}(\theta), \quad (11)$$

and compute its derivative

$$\frac{de_r(\theta)}{d\theta} = J_{q_{r0}, T}^r(u_\theta(\cdot), x(\theta)) \begin{pmatrix} \frac{du_\theta(\cdot)}{d\theta} \\ \frac{dx(\theta)}{d\theta} \end{pmatrix} - \frac{dy_{rd}(\theta)}{d\theta} = -\gamma e_r(\theta), \quad (12)$$

where $J_{q_{r0}, T}^r(u(\cdot), x)$ denotes the Jacobian of the regularized system. The regularizing term should be chosen in such a way that the regularized Jacobian is invertible. To proceed, we shall skip from the middle part of (12) the term $\frac{dy_{rd}(\theta)}{d\theta}$, obtaining the Wazewski-Davidenko equation

$$J_{q_{r0}, T}^r(u_\theta(\cdot), x(\theta)) \begin{pmatrix} \frac{du_\theta(\cdot)}{d\theta} \\ \frac{dx(\theta)}{d\theta} \end{pmatrix} = -\gamma e_r(\theta), \quad (13)$$

that, thanks to invertibility of the regularized Jacobian, defines the dynamic system

$$\begin{pmatrix} \frac{du_\theta(\cdot)}{d\theta} \\ \frac{dx(\theta)}{d\theta} \end{pmatrix} = -\gamma J_{q_{r0}, T}^{\#r}(u_\theta(\cdot), x(\theta))e_r(\theta). \quad (14)$$

Plugged back to the error equation (12), the solution of (14) yields

$$\begin{cases} \frac{de_{ri}(\theta)}{d\theta} = -\gamma e_{ri}(\theta), & \text{for } i = 1, \dots, r, & \text{and} \\ \frac{de_{rr+1}(\theta)}{d\theta} = -\gamma e_{rr+1}(\theta) + \pi(\theta), \end{cases} \quad (15)$$

where

$$\pi(\theta) = -\frac{d}{d\theta} \int_0^T r(q_{rk}(t))dt$$

$(q_{rk}(t))$ coming from the system (10) driven by $u_\theta(\cdot)$ denotes an imbalance term in the error equations.

Suppose that the trajectory $(u_\theta(\cdot), x(\theta))$ exists for every θ . Then, the following consequence of a theorem by Desoer and Vidyasagar [16] defines the behavior of the errors (15).

Theorem 1: Let $\pi(\cdot) \in L_\infty$, and $\lim_{\theta \rightarrow +\infty} \pi(\theta) = 0$. Then $e_r(\cdot) \in L_\infty$, $\frac{de_r(\cdot)}{d\theta} \in L_\infty$, and $\lim_{\theta \rightarrow +\infty} e_r(\theta) = 0$.

In the next section the ideas developed above will be applied to a constrained motion planning problem for a mobile manipulator composed of a kinematic car-type mobile platform carrying on board an RTR manipulator.

III. MOBILE MANIPULATOR

Let us consider a mobile manipulator composed of a kinematic car-type nonholonomic mobile platform and an RTR-type holonomic on board manipulator, shown in figure 1. Denote the platform coordinates by $q = (x, y, \varphi, \psi) \in R^4$,

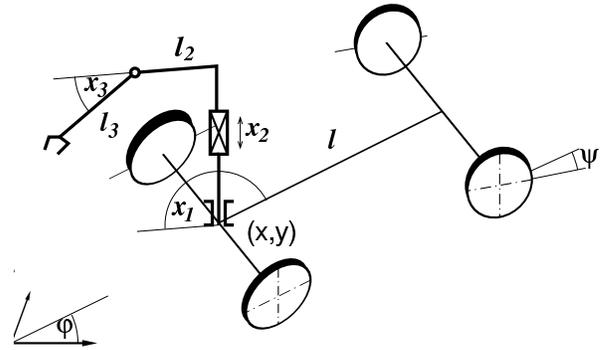


Fig. 1. The mobile manipulator

the input vector by $u = (u_1, u_2) \in R^2$, the vector of joint positions of the on board manipulator by $x = (x_1, x_2, x_3) \in R^3$, and the vector of the end effector Cartesian coordinates by $y = (y_1, y_2, y_3) \in R^3$. The meaning of the notations has been explained in the figure. The length of the car $l = 1$. With these notations, the kinematics of the mobile manipulator are represented as the following driftless control system with

outputs

$$\begin{cases} \dot{q}_1 = u_1 \cos q_3 \cos q_4, \\ \dot{q}_2 = u_1 \sin q_3 \cos q_4, \\ \dot{q}_3 = u_1 \sin q_4, \\ \dot{q}_4 = u_2, \\ y = k(q, x) = \begin{pmatrix} q_1 + (l_2 + l_3 \cos x_3) \cos(q_3 + x_1) \\ q_2 + (l_2 + l_3 \cos x_3) \sin(q_3 + x_1) \\ x_2 + l_3 \sin x_3 \end{pmatrix}. \end{cases} \quad (16)$$

Given a control time horizon $T > 0$, the platform controls entering the system (16) will be chosen in the form of a truncated Fourier series

$$u_i(t) = \sum_{j=0}^{s_i} \lambda_{ij} \varphi_j(t), \quad i = 1, 2, \quad (17)$$

where $\varphi_j(t)$ are basic trigonometric functions defined on $[0, T]$, and $s_i + 1$ is length of the i th control series. The expression (17) implies that the controls can be written as $u(t) = P(t)\lambda$, for $\lambda \in R^{s_1+s_2+2}$, and a suitably defined block matrix $P(t)$.

A. Unconstrained Motion Planning

To illustrate the performance of the unconstrained motion planning algorithm, we examine the problem of reaching by the system (16) the desirable taskspace point $y_d = (0, 0, 2)$ in time $T = 1$ without any constraints on platform coordinates. The initial state of the mobile platform $q_0 = (20, 0, \frac{\pi}{2}, 0)$. The platform control functions are chosen in the form (17), where $\varphi_0(t) = 1$, $\varphi_1(t) = \sin 2\pi t$, $\varphi_2(t) = \cos 2\pi t$, $\varphi_3(t) = \sin 4\pi t$, $\varphi_4(t) = \cos 4\pi t$, with $s_1 = 2$ and $s_2 = 4$. It follows that $(\lambda, x) \in R^{11}$. The initial endogenous configuration $\lambda_0 = (-1, 0.5, -0.5, -0.2, 0.1, 0.1, 0.01, 0.01)$ and $x_0 = (0, 1, \frac{\pi}{2})$. The decay rate $\gamma = 0.1$. Plots representing the solution of the motion planning problem are shown in figure 2. It follows that the turn angle of the front wheels takes unrealistic values greater than $\frac{\pi}{2}$ that causes a cusp turn on the platform path.

B. Constrained Motion Planning

In order to restrict the turn angle of the front wheels, we shall reformulate the previous motion planning problem by adding a constraint $|q_4| < \frac{\pi}{3}$, so $q_{ub} = \frac{\pi}{3}$ and $q_{lb} = -\frac{\pi}{3}$. As the regularizing function we use $r(q_4) = q_4^2$. In consequence, the augmented and regularized system (16) will take the form (10) by setting

$$\dot{q}_5 = q_4^2 + p(q_4 - q_{ub}, \alpha) + p(-q_4 + q_{lb}, \alpha), \quad (18)$$

along with $q_5(0) = 0$, as well as by adding an output variable $y_4 = q_5$. Thus, in the regularized system $q_r = (q_1, \dots, q_5)$ and $y_r = (y_1, \dots, y_4)$. The motion planning problem will now be defined by the desirable point

$$y_{rd} = \left(y_d, \int_0^T q_4^2(t) dt \right).$$

Assuming that $u_\theta(t) = P(t)\lambda(\theta)$, we obtain the kinematics of the regularized system $K_{q_r, T}^r(\lambda(\theta), x(\theta))$, and define the error

$$e_r(\theta) = K_{q_r, T}^r(\lambda(\theta), x(\theta)) - y_{rd}(\theta). \quad (19)$$

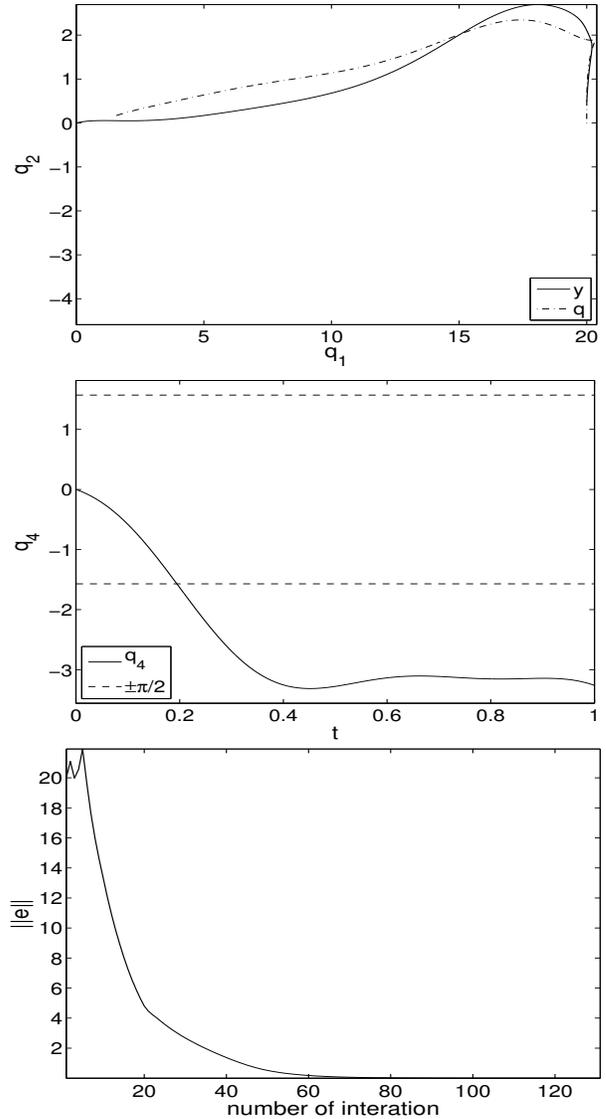


Fig. 2. Taskspace path, q_4 trajectory and error convergence for unconstrained planning

The Wazewski-Davidenko equation (13) takes the form

$$J_{q_r, T}^r(\lambda(\theta), x(\theta)) \begin{pmatrix} \frac{d\lambda(\theta)}{d\theta} \\ \frac{dx(\theta)}{d\theta} \end{pmatrix} = -\gamma e_r(\theta).$$

The Jacobian is represented by a matrix

$$J_{q_r, T}^r(\lambda, x) = [C(T, x) \int_0^T \Phi(T, t) B(t) P(t) dt, \quad D(T, x)],$$

where the $\Phi(t, s)$ satisfies the evolution equation $\frac{\partial \Phi(t, s)}{\partial t} = A(t)\Phi(t, s)$, and the matrices defining the linear approximation of the regularized system along $(u_\theta(t), x(\theta), q_{r\theta}(t))$ are the following

$$A(t) = \begin{bmatrix} 0 & 0 & -u_1 \sin q_3 \cos q_4 & -u_1 \cos q_3 \sin q_4 & 0 \\ 0 & 0 & u_1 \cos q_3 \cos q_4 & -u_1 \sin q_3 \sin q_4 & 0 \\ 0 & 0 & 0 & u_1 \cos q_4 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2q_4 + a(q_4) & 0 \end{bmatrix},$$

where

$$a(q_4) = \frac{\exp(-\alpha(-q_4+q_{lb})) - \exp(-\alpha(q_4-q_{ub}))}{(1+\exp(-\alpha(-q_4+q_{lb}))) (1+\exp(-\alpha(q_4-q_{ub})))},$$

$$B(t) = \begin{bmatrix} \cos q_3 \cos q_4 & 0 \\ \sin q_3 \cos q_4 & 0 \\ \sin q_4 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix},$$

$$C(t, x) = \begin{bmatrix} 1 & 0 & -(l_2 + l_3 \cos x_3) \sin(q_3 + x_1) & 0 & 0 \\ 0 & 1 & (l_2 + l_3 \cos x_3) \cos(q_3 + x_1) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$D(t, x) =$$

$$\begin{bmatrix} -(l_2 + l_3 \cos x_3) \sin(q_3 + x_1) & 0 & -l_3 \sin x_3 \cos(q_3 + x_1) \\ (l_2 + l_3 \cos x_3) \cos(q_3 + x_1) & 0 & -l_3 \sin x_3 \sin(q_3 + x_1) \\ 0 & 1 & l_3 \cos x_3 \\ 0 & 0 & 0 \end{bmatrix}.$$

Having computed the Jacobian pseudo inverse $J_{q_{r0}, T}^{\#r}(\lambda, x)$, we obtain the dynamic system (14) determining the motion planning algorithm

$$\begin{pmatrix} \frac{d\lambda(\theta)}{d\theta} \\ \frac{dx(\theta)}{d\theta} \end{pmatrix} = -\gamma J_{q_{r0}, T}^{\#r}(\lambda(\theta), x(\theta)) e_r(\theta). \quad (20)$$

Outside singular configurations, the solution of the motion planning problem is defined as the limit $(\lambda_d, x_d) = \lim_{\theta \rightarrow +\infty} (\lambda(\theta), x(\theta))$.

Using the form of the controls, it can be shown that the imbalance term appearing in the last error equation in (15) assumes the form

$$\pi(\theta) = v^T \frac{d\lambda(\theta)}{d\theta} + 2\lambda^T(\theta) Q \frac{d\lambda(\theta)}{d\theta},$$

where v is a constant vector depending on $q_4(0)$, and Q denotes a constant symmetric matrix. Now, assuming that a solution of (20) exists for every θ , we get the boundedness of $(\lambda(\theta), x(\theta))$ as well as of the derivative $(\frac{d\lambda(\theta)}{d\theta}, \frac{dx(\theta)}{d\theta})$, and deduce immediately that $\pi(\cdot) \in L_\infty$. Furthermore, it turns out that outside singular configurations the second order derivative $\frac{d^2\lambda(\theta)}{d\theta^2}$ is also bounded, because of the boundedness of

$$\begin{pmatrix} \frac{d^2\lambda(\theta)}{d\theta^2} \\ \frac{d^2x(\theta)}{d\theta^2} \end{pmatrix} = -\gamma \left(D \left(J_{q_{r0}, T}^{\#r}(\lambda(\theta), x(\theta)) e_r(\theta) \right) + I_{11} \right) \times \begin{pmatrix} \frac{d\lambda(\theta)}{d\theta} \\ \frac{dx(\theta)}{d\theta} \end{pmatrix} - \gamma J_{q_{r0}, T}^{\#r}(\lambda(\theta), x(\theta)) \begin{pmatrix} 0 \\ \pi(\theta) \end{pmatrix},$$

D denoting the derivative with respect to (λ, x) . We conclude that $\lim_{\theta \rightarrow +\infty} \frac{d\lambda(\theta)}{d\theta} = 0$, using the Barbalat's lemma [17]. By virtue of Theorem 1, the error (19) vanishes to zero, so the motion planning algorithm has solved the constrained motion planning problem.

Performance of this algorithm has been illustrated by computer simulations. The same motion planning problem as in subsection III-A has been solved for identical initial conditions, except that the following bounds have been imposed on q_4 variable: $q_4 \in [-\frac{\pi}{3}, \frac{\pi}{3}]$, and $q_4 \in [-\frac{\pi}{6}, \frac{\pi}{6}]$. In simulations the value of $\alpha = 50$. The results are demonstrated, respectively, in the figures 3 and 4.

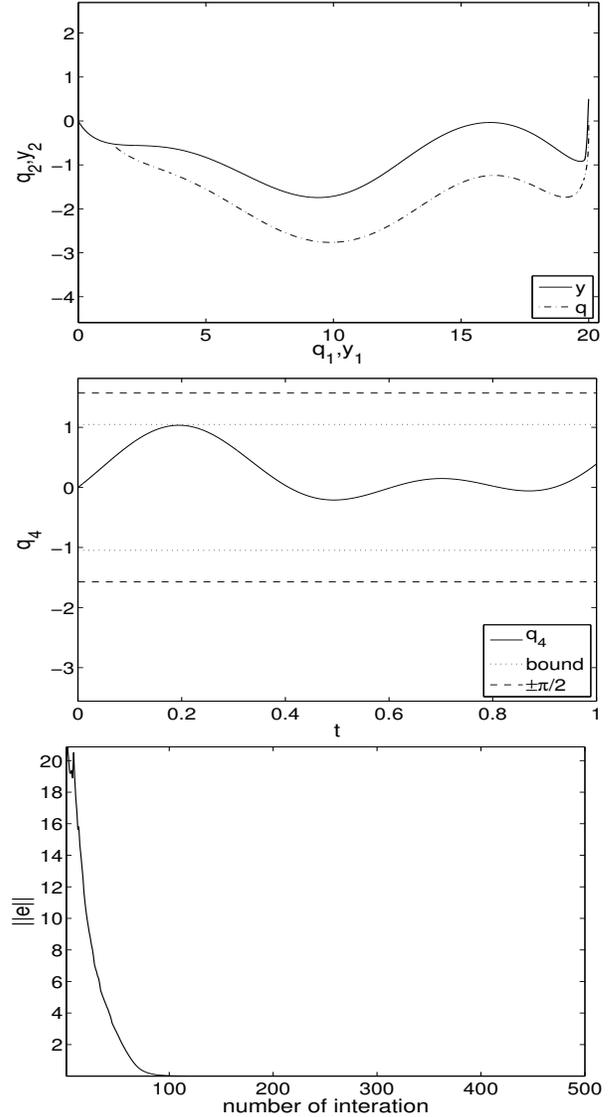


Fig. 3. Taskspace path, q_4 trajectory and error convergence for the constraint $q_4 \in [-\frac{\pi}{3}, \frac{\pi}{3}]$

IV. CONCLUSIONS AND FUTURE WORKS

The paper has introduced a new constrained motion planning algorithm for mobile manipulators. Computer simulations have demonstrated that the quality of motion produced by the algorithm as well as the resulting error convergence are satisfactory. Future research will be directed toward handling more involved constraints and developing rigorous proofs of error convergence. Another focus of future work could be a merge of motion planning and predictive control.

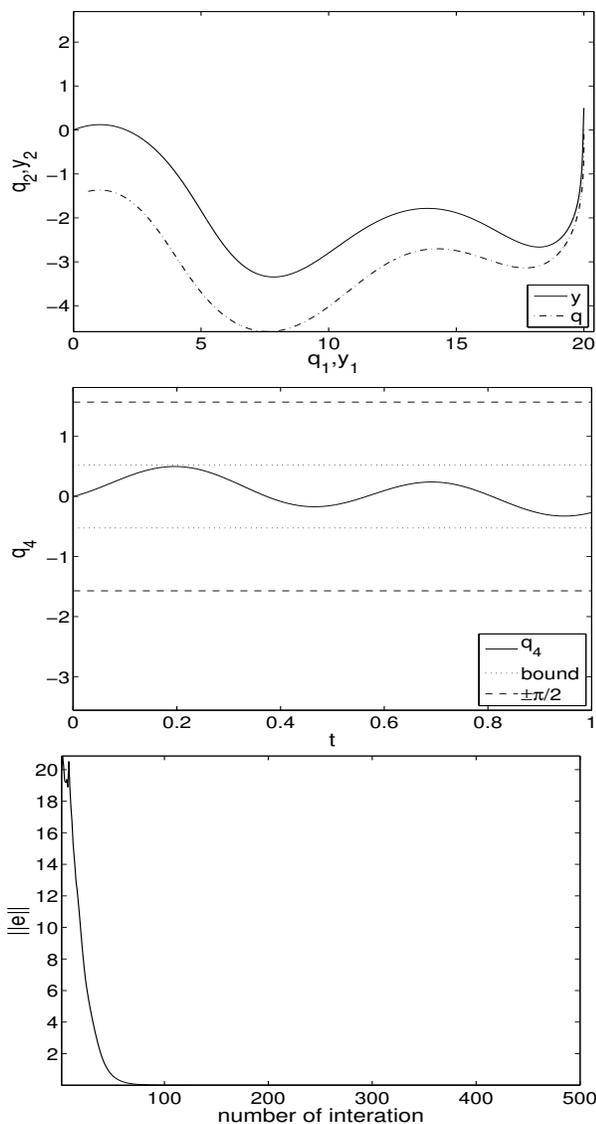


Fig. 4. Taskspace path, q_4 trajectory and error convergence for the constraint $q_4 \in [-\frac{\pi}{6}, \frac{\pi}{6}]$

In the domain of Jacobian motion planning algorithms ideas of predictive control have already been used in order to make the algorithm more "closed loop" [18] or more efficient at long distances between the starting and terminal endogenous configurations [19]. However, in both these cases the problem of constraints has not been of primary significance. A challenging alternative to the approach presented in this paper is to formulate the motion planning problem as an optimal control problem with constraints that can be solved by modern nonlinear model predictive control strategies [20], [21], [22].

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