

Topology Control of Dynamic Networks in the Presence of Local and Global Constraints

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Abstract—Formation flying (FF) is a critical element in NASA’s future deep-space missions. Terrestrial Planet Finder (TPF), NASA’s first space-based mission to directly observe planets outside our own solar system, will rely on FF to achieve the functionality and benefits of a large instrument using multiple lower cost smaller spacecraft. Many key network design problems for such FF missions can be formulated as optimization problems with local and global constraints.

We develop a topology control algorithm that can be used for many network problems in the presence of local constraints, such as collision avoidance, and global constraints, such as network connectivity. The presence of contradictory objectives in topology control problems motivated a game-theoretic approach. We demonstrated that a game-theoretic technique could provide a framework for design and analysis of many topology control problems in dynamic networks. In particular, the problem of motion planning for formation reconfiguration in the presence of constraints on network connectivity and inter-spacecraft collisions is studied.

I. INTRODUCTION

Formation flying (FF) has been identified as a critical, enabling technology for future NASA space missions such as the Terrestrial Planet Finder (TPF) and Stellar Imager. Under the FF concept, spatially distributed spacecraft fly in formation with the capability of interacting and collaborating with one another, and work as a single unit, exhibiting a system-wide capability to accomplish shared objectives.

A representative formation flying mission is TPF, where it will use formation flying spacecraft to synthesize a large-baseline interferometer operating in the infrared wavelength region. JPL has studied and produced designs for TPF-Emma mission concept that uses a rectangular layout for the telescope spacecraft and an out-of-plane combiner spacecraft. Figure 1 shows the concept as studied in reference [7]. Our goal is to develop algorithms and simulation tools that can be directly applied to formation flying missions such as TPF.

The objective of *topology control* problems is to modify the underlying network topology to optimize local and global metrics. One of the most important global properties of any network is its connectivity. In wireless ad hoc networks, there exists a trade-off between the connectivity and the energy consumption in the network. Lowering the sensing/communication power will have an adverse effect on the connectivity of the network. In mobile sensor networks, there

exist similar trade-offs between connectivity and coverage, communication cost and agent mobility. Each spacecraft in the formation network must use the network properties to make local decisions that collectively guarantee the connectivity in the global sense. Such trade-offs motivate posing and addressing a *network design problem* to achieve a balance interplay between local and global objectives.

The presence of contradictory objectives and trade-offs in network design problems motivates a game-theoretic approach, where the aim is to design algorithms that optimize a network-wide cost among decision-making locally-informed agents. The agents are decision-makers with both local and global objective. For instance, the desired global objective could be constructing a connected network, with the local objective of having small number of neighbors, or consuming as little fuel as possible. Selfish agent behaviors could have disruptive effect on topology control protocols, unless adequate countermeasures are taken.

Our contribution is to demonstrate that a game-theoretic approach could provide a *framework* for design and analysis of many topology control problems in dynamic networks. In particular, we show that the motion planning for a formation reconfiguration can be done in a distributed fashion using a game approach, where we model each spacecraft as a player in the game with local and global objectives.

In the control community, the main concern in mobile sensor networks is the problem of motion planning while preserving connectivity of mobile networks [5], [9], [14], [15], [20], [2], [16], [19]. The connection between two mobile robots is lost either as a result of separation [2],

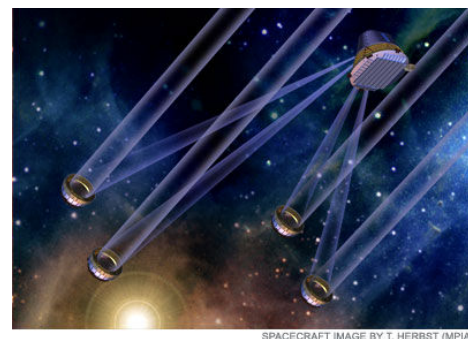


Fig. 1. TPF-Emma concept design.

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[20] or because of loss of line-of-sight [12], [16]. The problem of generating connected networks starting from disconnected initial conditions is addressed in [3], [12]. Spanos and Murray [14] introduced the concept of *robust connectivity* which locally characterized the connectedness of the network in terms of the relative positions of the neighboring agents. Potential field techniques were applied in [20], [2] for maintaining connectivity of networks of mobile robots.

Here is the outline of the paper. The problem statement is defined in Section II. Since the presented approach to topology control is based on game theory, its fundamentals are summarized in Section III. In Section IV the problem of formation reconfiguration is solved using the game-theoretic framework, followed by the concluding remarks in Section V.

II. NETWORK TOPOLOGY CONTROL PROBLEMS

Consider a network of n agents. The state (position) of agent i is represented as a point in the agent's configuration space S_i . The state space of all the agents, S , is defined as $S = S_1 \times S_2 \times \dots \times S_n$. The trajectory of agent i is represented as a mapping $s_i : [0, T] \rightarrow S_i$, which evolves according to the *state transition equation*,

$$\dot{s}_i(t) = f(s_i(t), a_i(t)) \quad (1)$$

where $a_i(t)$ is chosen from a set of control actions. The control action $a_i(t)$, transitions agent i from s_i to a subset of its configuration set, $R(s_i) \subset S_i$ (see Figure 2).

Let d_0 be the minimum safe distance between any two agents. We define the collision region between pair (i, j) as the following set of states:

$$S_{coll}^{ij} = \{s \in S_1 \times \dots \times S_n \mid \|s_i - s_j\| \leq d_0\}.$$

The *collision subset* is now defined as

$$S_{coll} = \bigcup_{i \neq j} S_{coll}^{ij}.$$

Given the valid configuration space

$$S_{valid} = S_1 \times \dots \times S_n - S_{coll},$$

the action set of agent i is restricted to:

$$S'_i = R(s_i) \cap S_{valid} \subset S_i. \quad (2)$$

Let $\mathcal{N}_i(s_i)$ be the neighborhood around s_i within which agent i can sense/communicate with other agents. $\mathcal{N}_i(s_i)$ depends on the limited sensor range and collision avoidance properties of agent i . We represent the set of agents and the sensing links between them as a graph $G(s)$, where $s = (s_1, \dots, s_n) \in S$. We define the adjacency matrix corresponding to $G(s)$ as a mapping $A : S \rightarrow \mathcal{M}$, from the configuration space S to \mathcal{M} , defined as the set of $n \times n$ symmetric matrices with each entry either 0 or 1 and the diagonal terms equal 0. The set of adjacency matrices corresponding to connected graphs is denoted by \mathcal{M}_c . Now, we can define the *connectivity constraint set* as

$$\Omega := \{s \in S : A(s) \in \mathcal{M}_c\}.$$

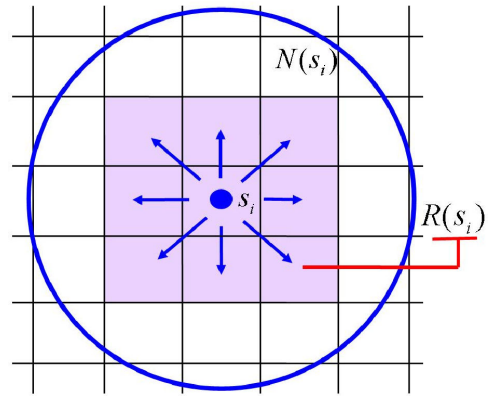


Fig. 2. The mobility model of Restricted SAP for an agent located at s_i . The search for the next best action is restricted to $R(s_i)$.

A. Problem Statement

During science experiments using a formation flying of spacecraft such as TPF-Interferometer, reconfigurations are needed to re-target the formation between the observations, while respecting a number of local and global constraints. Some *local constraints* are avoiding inter-spacecraft collisions, avoiding Sun-exposure of sensitive instruments, and maintaining inter-spacecraft sensing/communication links. *Global constraints* are typically maintaining connectivity of the formation, and high quality of observations. Optimizing the consumption of fuel and energy is also crucial for maximizing the life of the spacecraft formation. Thus, reconfiguration maneuvers for TPF-I typically involve designing path planners that optimize some performance index such as fuel or time while generating trajectories that satisfy the desired local and global constraints.

The problem we consider here concerns maintaining the connectivity of a mobile network during reconfiguration, while generating optimal, collision-free, trajectories.

P: *Given an initial and final connected configurations for a network of n spacecraft, how should they move such that the intermediate configurations remain connected?*

There has been some prior work on optimal-fuel collision-free motion planning [17], [13]. But the issue of maintaining connectivity in a spacecraft formation has not been studied thoroughly. The authors in [11] proposed to reorient the formation as a virtual rigid body so that each spacecraft retains its current neighbors during reconfiguration. However, this solution may not be energy and fuel optimal. Spanos and Murray [15] used the notion of connectivity robustness to study the *feasibility* of connectivity-preserving motions, and their approach did not address the motion-planning aspect of the problem. Stump et al. [16] studied the problem of maintaining connectivity during a scout mission using a multiple mobile robots. The problem of preserving connectivity in a multi-agent system during flocking and coordinated motion was studied in [2], [20].

Now, here is a formal definition of the problem:

P': *Given $s^{initial} \in \Omega$ and $s^{final} \in \Omega$ we wish to find the shortest path $s(t)$ so that $s(0) = s^{initial}$ and $s(T) = s^{final}$*

and $\mathbf{s}(t) \in \Omega$ for some $T > 0$ and for all $0 \leq t \leq T$.

The above problem can be written as the following optimization problem:

$$\begin{aligned} \min_{\mathbf{s}} \quad & \|\mathbf{s}(t) - \mathbf{s}(T)\| \\ \text{subject to} \quad & \mathbf{s}(t) \in \Omega, \quad \forall t \in (0, T) \end{aligned} \quad (3)$$

The above problems can be formulated as multi-objective optimization problems. Recently, game-theoretic techniques have proved to be useful tools for solving multi-objective optimization problems such as network design problems [4], [10]. Komali [4] developed a game-theoretic power-based protocol for energy minimization, while Resta et al. [10] developed a game-theoretic MST-based protocol. However, applications of game theory to mobile networks are still very much under-developed. Early results are presented in [1], [6]. Before presenting our game-theoretic solution, we review fundamental concepts of Game Theory.

III. GAME THEORETIC APPROACH

A. Game Theory Fundamentals

The framework of studying formation control is laid out by defining the following three components, $\langle N, S, U \rangle$:

- 1) The agent set $N = \{1, \dots, n\}$ where n is the number of agents in the game.
- 2) The action (or strategy) set $S = S_1 \times \dots \times S_n$ where S_i is the set of actions of agent i . The strategy vector $\mathbf{s} = (s_1, \dots, s_n) \in S$ is also denoted by $\mathbf{s} = (s_i, s_{-i})$ where s_i is the action of agent i and s_{-i} denotes the actions of all other agents.
- 3) The utility set $U = \{u_1, \dots, u_n\}$ where $u_i : S \rightarrow \mathbb{R}$ is the utility function representing the desirable properties of the network and the cost associated with them resulting from the actions of agent i .

In a spacecraft formation, the set of spacecraft constitute the agent set, and the individual action sets S_i are considered the set of possible translational (or rotational) motions of each spacecraft. The two main aspects of a cooperative system design using a game-theoretic approach are (a) designing the agents' utilities that are compatible with some global cost function (Section III-B), and (b) developing a multi-agent learning algorithm¹ and addressing its informational and computation requirements (Section III-C). Convergence, computational efficiency and equilibrium selection are properties that need to be addressed (Section III-D).

Each agent, having a distinct utility function and information set, selects a feasible action that maximizes its utility u_i given the state of other agents in the formation. Such a game is played in an iterative fashion. Assume that the game is repeated at discrete times $k \in \{0, 1, 2, \dots\}$, and we are interested in the asymptotic behavior of the system. For arbitrary games, an equilibrium may not exist or multiple equilibria may exist. However, a priori knowledge about the equilibrium can help us design games that are more likely to

¹In the game theory, learning is the process of *how* agents reach the equilibrium.

converge. The equilibrium corresponding to a configuration where each agent has no incentive to unilaterally change its action is called Nash Equilibrium (NE).

Definition 3.1: A strategy vector \mathbf{s}^* is a Nash Equilibrium if $u_i(\mathbf{s}^*) \geq u_i(s_i, \mathbf{s}_{-i}^*)$ for all $i \in N$ and all $s_i \in S_i$.

A network corresponding to a Nash equilibrium is called *stable*. If the equilibrium also optimizes a (global) cost function, then the network is called *optimal*. A Nash equilibrium may or may not correspond to the optimal network. There is a trade-off between networks that are stable and those that are optimal. The ratio of the solution quality of the best Nash equilibrium relative to the optimal network is known as the *price of stability* (POS). Similarly, the ratio of the worst Nash equilibrium relative to the optimal network is called the *price of anarchy*. We are interested in bounding the price of anarchy and price of stability, and would like network formation games in which these measures are small. In general, the optimizer of the global cost function may not be the best Nash equilibrium, thus the bound on POS is not always tight.

The existence and convergence to a NE can be guaranteed for a special class of games called *potential games*, where each agent takes actions that constantly improve its utility, and as a result a dynamic process emerges. For such games, one defines a *potential function* that reflect how much each agents benefits from unilateral change in its strategy.

Definition 3.2: A strategic game $\Gamma = \langle N, S, U \rangle$ is a Potential Game if there exist a potential function $\Phi : S \rightarrow \mathbb{R}$ such that for all $i \in N$

$$\Phi(s'_i, s_{-i}) - \Phi(s_i, s_{-i}) = u_i(s'_i, s_{-i}) - u_i(s_i, s_{-i}) \quad (4)$$

for any alternative strategy $s'_i \neq s_i$.

In a more general case, we have an *ordinal potential game* where the equality in (4) is replaced with inequalities:

Definition 3.3: A strategic game $\Gamma = \langle N, S, U \rangle$ is an Ordinal Potential Game if there exist a potential function $\Phi : S \rightarrow \mathbb{R}$ such that for all $i \in N$

$$\Phi(s'_i, s_{-i}) - \Phi(s_i, s_{-i}) > 0 \iff u_i(s'_i, s_{-i}) - u_i(s_i, s_{-i}) > 0 \quad (5)$$

for any alternative strategy $s'_i \neq s_i$.

In an ordinal potential game, an improvement in the utility of each agent, when the other agents take no action, results in an improvement of the potential function. The structure of potential games guarantees the existence of NE [8].

Theorem 3.4: Every potential game has at least one Nash Equilibrium, namely the strategy \mathbf{s} that maximizes $\Phi(\mathbf{s})$.

B. Utility Design

The individual utility functions can be designed in a number of ways as described in [1]. If the utility of each agent is set to the global utility we have the *Identical Interest Utility* (IIU):

$$u_i(s_i, s_{-i}) = \Phi(s_i, s_{-i}) \quad (6)$$

In this case, continuous dissemination of global information is required among the agents. For IIU the optimal states

yield highest utilities, however, suboptimal Nash equilibria may still exist.

By setting the utility of each agent to the marginal contribution made by the agent to the global utility, one obtains the *Wonderful Life Utility* (WLU):

$$u_i(s_i, s_{-i}) = \Phi(s_i, s_{-i}) - \Phi(s_{-i}), \quad (7)$$

where $\Phi(s_{-i})$ is the value of the potential function in the absence of agent i .

Both IU and WLU lead to potential games with the global utility being the potential function. For more examples of utility design see [1]. The agent utilities cannot be designed independently of the negotiation mechanism employed by the agents. Thus, next we present the negotiation algorithm we will use for our solution.

C. Learning Algorithms

Any individual agent is cooperative with other agents only to the extent that cooperation helps the agent to maximize its own utility. Each agent will negotiate with other agents without any knowledge about the utilities of the other agents. This is because the agents may not have the same information regarding their environment. The advantage of this assumption is that it makes the agents truly autonomous in the sense that each agent is individually capable of making robust strategic decisions in uncertain environments.

Now we describe a learning algorithm that in the game-theory literature is known as *Spatially Adaptive Play* (SAP). At each time $k > 0$, an agent, say i , is randomly selected (with equal probability) to take its action and update its state $s_i[k]$. All other agents do not take any action such that $s_{-i}[k+1] = s_{-i}[k]$.

The state transition for agent i is $s_i[k+1] = f(s_i[k], a_i[k])$. If agent i chooses the strategy of selecting the best action s_i^+ such that $u_i(s_i^+, s_{-i}) \geq u_i(s_i, s_{-i})$ for all $s_i \in S_i[k]$, then the dynamics of the potential game evolves according to the *best-response dynamics*. In a best-response dynamics, given the configuration of the formation, each agent moves to a new state $s_i \in S_i[k]$ that maximizes its utility, i.e.

$$s_i^+[k+1] = \arg \max_{s_i \in S_i[k]} u_i(s_i, s_{-i}[k]). \quad (8)$$

Such best-response dynamics implies a greedy algorithm that always converges to a NE [4], identified by the potential maximizers. Algorithm 1 formalizes such a greedy best-response algorithm.

Algorithm 1 Best-Response Algorithm(s) \rightarrow s*

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while  $\hat{s}$  is not a NE do
  for  $i \in N$  do
    find  $S'_i \subset S_i$ 
     $s_i^+ = \arg \max_{s_i \in S'_i} u_i(s_i, s_{-i})$ 
  end for
end while

```

Note that Algorithm 1 is a *distributed* algorithm. The action of each agent is also restricted to a subset of its configuration space as given by (2).

D. Existence, Convergence and Computation Complexity of Equilibria

In some games, best response dynamics always converge quickly, but in many games it does not. For *potential games* we have the following result:

Theorem 3.5: [18] In any potential game, best response dynamics always converge to a Nash equilibrium.

In some games the potential function can be optimized in polynomial time, but in others the optimization problem is NP-hard. The problem of finding equilibria in potential games is closely related to the problem of finding local optima in optimization problems. Tardos and Wexler [18] showed that finding a Nash equilibrium in potential games is a Polynomial Local Search (PLS) problem (i.e PLS-complete), assuming that the best response of each agent can be found in polynomial time.

Theorem 3.6: [18] Finding a pure Nash equilibrium in potential games, where best response can be computed in polynomial times, is PLS-complete.

In our scenario, the computational complexity depends on the scale of the grid of the action space, the number of agents, and the number of collision checkings.

Because only one agent updates its proposal at a given negotiation step, the convergence of negotiations may be slow when there are large number of agents. Multiple agents can be allowed to update their strategies at a given step as long as they do not have a common link. Allowing such multiple updates may potentially speed up the negotiations substantially. In summary, a restricted SAP can be a very effective negotiation mechanism in our topology game because it would have low computational burden on each agent and it would lead to (locally) optimal solutions in potential games.

IV. A DISTRIBUTED SOLUTION

Consider the optimization problem (3), where given connected initial and final topologies, we would like to have a motion planning algorithm that respects local and global constraints such as collision-avoidance and connectivity. Our goal is to find the solution to (3) in a distributed fashion. In this work, it is assumed that agents have access to the global knowledge regarding the connectivity of the network, but act locally, and their cooperation is to the extend that it helps them maximize their utilities.

Let $f(\mathbf{s}) : S \rightarrow \mathbb{R}$ be the connectivity indicator function:

$$f(\mathbf{s}) = \begin{cases} 1 & \text{if } \mathbf{s} \in \Omega \\ 0 & \text{otherwise.} \end{cases} \quad (9)$$

Function $f(\mathbf{s})$ can be computed using the k -connectivity matrix defined as:

$$C_k(\mathbf{s}) = I + A(\mathbf{s}) + A(\mathbf{s})^2 + \dots + A(\mathbf{s})^k, \quad (10)$$

The (i, j) entry of $C_k(\mathbf{s})$ can be interpreted as the number of communication paths of k -hop or less that connect agent

i to agent j . Therefore $(n - 1)$ -hop connectivity matrix, $C_{n-1}(\mathbf{s})$, can be used for evaluation of connectivity, because the maximum possible length of a path between pair (i, j) is $n - 1$.

A nonzero entry (i, j) of $C_{n-1}(\mathbf{s})$ represents that there is a path from agent i to j . Thus, agent i can check the connectivity of $G(\mathbf{s})$ by checking the number of nonzero entries of the i th row of $C_{n-1}(\mathbf{s})$, which is denoted by its l_0 -norm:

$$f(\mathbf{s}) = \begin{cases} 1 & \text{if } \|[C_{n-1}(\mathbf{s})]_i\|_0 \\ 0 & \text{otherwise.} \end{cases} \quad (11)$$

Let $d_i(t) = \|s_i(t) - s_i(T)\|$ represent the distance between the position of agent i at time t to its desired final position. Optimization (3) is equivalent to:

$$\begin{aligned} \min_{\mathbf{s}} \quad & \sum_{i=1}^n d_i(t) \\ \text{subject to} \quad & f(\mathbf{s}) = 1 \end{aligned} \quad (12)$$

Now one can solve optimization (12) in a *distributed* fashion as a potential game where the utility function of each agent is

$$u_i(\mathbf{s}) = \kappa_i \cdot (f(\mathbf{s}) - 1) - d_i(t). \quad (13)$$

By setting $\kappa_i = \|s_i(0) - s_i(T)\| = d_i(0)$ one can guarantee that the utility of each agent monotonically increases by moving towards its desired final position, as long as $G(\mathbf{s})$ remains connected.

Then, the game $\Gamma = \langle N, S, U \rangle$ is a potential game with the potential function:

$$\Phi(\mathbf{s}) = \sum_{i=1}^n u_i(\mathbf{s}). \quad (14)$$

It is easy to see that the utility function is a Wonderful Life Utility (WLU) defined by (7). The greedy best-response algorithm 1, presented in Section III-C, can now drives the agents to a NE characterized by the state that maximizes (14).

Figure 3 shows the simulation results from a reconfiguration maneuver between two configurations for TPF-I mission. If each spacecraft takes the shortest path (straight line) between its initial position, Fig. 3(a), and its desired final position, Fig. 3(b), the network becomes disconnected. However, the network topology game, defined in this section, generates a set of collision-free trajectories that preserve the connectivity of the network, as the algebraic connectivity² of the network remains positive during the entire maneuver (see Fig. 3(d)). The NE corresponds to the state that maximizes the potential function (14), i.e. the desired final configuration. Note that the performance and convergence of the learning algorithms depend on the choice of the utility and potential functions.

²Algebraic connectivity is the second smallest eigenvalue of the Laplacian matrix corresponding to graph $G(\mathbf{s})$.

V. SUMMARY AND FUTURE WORK

In this paper, the problem of formation reconfiguration is formulated as a constraint optimization problem with both local constraints - such as collision avoidance - and global constraints - such as network connectivity. We demonstrated that game-theoretic techniques could provide a framework for design and analysis of such topology control problems in a dynamic network. The game-theoretic framework allows us to obtain distributed solutions, where the agents use global information and make decisions locally.

The game-theoretic approach can be classified as either a potential function method or as a randomized algorithm. In the deterministic case, when the decision of each agent is a pure strategy, it involves the minimization of a local utility function - as such, it can be considered as a local potential function method. In the case, when the strategy of the agent is “mixed”, that is, based on randomization and maximizing the utility in expected value, it can be considered as a randomized algorithm. Development of a randomized algorithm is the subject of future work. There are advantages to using a game-theoretic approach such as the existence of pure equilibria, and the fact that best-response dynamics are guaranteed to converge. Also, the “price of stability” can be bounded using the potential function method. Other coordination and motion planning task such as coverage and rendezvous can be framed as multi-player games [6].

The computation time is a critical parameter for the success of a reconfiguration mission when fast formation changes are needed. Possible directions for future work would be studying the learning algorithm’s complexity and reducing the computation time. The extension of the results to more complicated dynamics in the presence of other constraints, such as limited field-of-view, is essential to the transfer of the technology to actual deep-space missions, and is an ongoing work.

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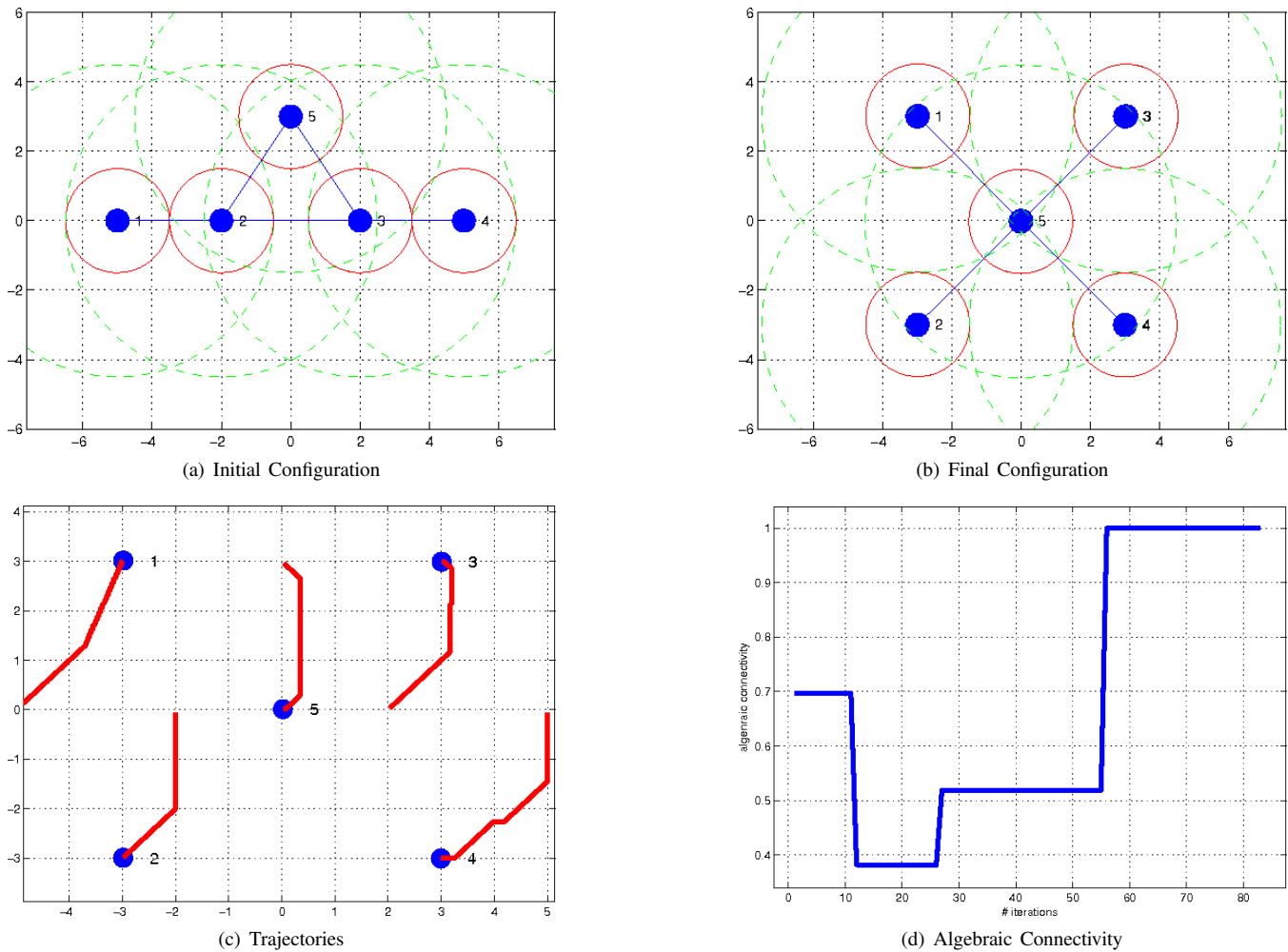


Fig. 3. Preserving connectivity during formation reconfiguration. The sensing radius is $r = 4.5m$ (dotted circles) and the collision avoidance radius is $r_{min} = 1.5m$ (solid circle) (a) Initial configuration, (b) Final configuration, (c) The trajectories of all agents during reconfiguration, where the circles are at the final positions, (d) The value of the algebraic connectivity ($\lambda_2(L(G))$) remains positive, indicating that G remains connected during reconfiguration.

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