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Updating Control Modes Based on Terrain Classification

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Abstract— **The need for terrain-dependent control systems on AGVs is evident when considering the variety of outdoor terrains many AGVs encounter. Although the idea of using terrain classification algorithms to identify the terrain and then update the control modes is well-established, the problem of how to intelligently update the control modes based on classifications has been left relatively unaddressed. This paper presents a simplistic rule, called the *update rule*, which decides when to change control modes based on past and present terrain classifications and is tuned using empirical data. Using experimental data from the eXperimental Unmanned Vehicle (XUV) mobile robot, this update rule is shown here to be both robust to misclassifications as well as sensitive to terrain transitions. This paper also develops and implements a sliding horizon approach to reaction-based terrain classification for improved sensitivity to terrain transitions. The update rule structure presented here is applicable to reaction- and vision-based terrain classification of individual terrains.**

I. INTRODUCTION

Today's autonomous ground vehicles (AGVs) are expected to operate in a variety of environments including deserts, beaches, forests, and swamps. The nature of these environments requires implementing unique control strategies in order to safely and efficiently traverse the environment. It is for this reason AGV control strategies should be terrain-dependent. In order to achieve terrain-dependent control it is necessary to develop appropriate control settings for each of the considered terrains as well as systems that correctly identify the terrain. This paper develops a rule for when to switch the control mode, termed the *update rule*, which is based on past and present terrain detections. An overview of relevant research in terrain classification and terrain-dependent control are given below.

Terrain classification for AGVs can be performed through what is seen visually, felt through the vehicle reactions during traversal or a combination of both vision and vehicle reactions. Several vision-based techniques have been used to describe the surrounding environment. These techniques use three dimensional maps to determine navigability [1] and identify vegetation, shrubs and trees [2], [3], stereo imagery to detect unexplored terrains [4], and image processing to detect surface characteristics such as roughness, slope and hardness [5]. However, the works of [1], [2], [3], [4], [5] are focused on characterizing the terrain instead of surface identification, which is more relevant to the research presented in this paper. Key vision-based techniques in detecting individual terrain surfaces include the work of [6], which shows the effectiveness of color-based features and the work of [7] which analyzes the effectiveness of several types of visual sensing in detecting mud. The work of [8] uses a laser line striper to classify sand, grass, gravel and asphalt terrains using image texture and spatial frequency domain features.

Reaction-based terrain classification is most often conducted using vehicle vibrations, which have been shown to have terrain signatures in the frequency domain [9], [10], [11]. As discussed in [12], [13], [14] the origin of vibration terrain signatures is the terrain signatures in the spatial frequency response of the terrain profiles. Several different types of classifiers have been used to perform vibration-based terrain classification [9], [10], [15], [16], [17], [18], leading to the comparison of techniques in [19], [20]. These comparisons indicate that support vector machines (SVMs) most often lead to high accuracy performance while other techniques can be shown to have advantages of SVMs such as classification time or training time. Additionally, the works of [12], [14], [21], [22] have sought to alleviate speed dependency, e.g. the need to train separate classifiers based on speed, using vehicle models or interpolation techniques.

Terrain classification that is robust to a wide variety of terrains, environmental conditions, and vehicle operating conditions will almost certainly require the symbiosis of vision-based and reaction-based methods. The primary works on fusing multiple terrain classification schemes include [23] which seeks to classify previously unseen terrains, and [24] which combines several vision-based schemes to classify sand, soil, grass, gravel, wood chips, asphalt and mixed terrains. However, current research fails to appropriately fuse vision- and reaction-based methods in order to accurately classify individual terrains in circumstances that would normally cause either vision- or reaction-based classification to fail.

Current research in terrain-dependent control settings is significantly less developed than that of terrain classification systems. The Land Rover LR3 and Freelander commercial vehicles make use of a terrain-dependent control system called "Terrain Response," which has five different terrain modes: 1) general driving for everyday driving, 2) grass/gravel/snow, 3) mud and ruts, 4) sand and 5) rock crawl [25]. These terrain modes change the settings on

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several vehicle systems including the anti-lock braking system (ABS), the traction- and stability-control systems, the locking action of the differentials, the shift schedule of the transmission, and the throttle response of the engine in order to improve, traction, steering and fuel efficiency. Ideally, an AGV control system should have a higher and lower level of control for handling changes in terrains. The higher level of control should work in coordination with robot planning in order to place limits on vehicle turning radius, speed, and acceleration/deceleration in order to reduce the likelihood of control loss through tipping, wheel slip, vehicle immobilization, etc. The lower level of control is more reactive in nature and includes systems like ABS, traction control and stability control. In commercial vehicles these systems are typically designed for worst-case scenarios such as wet or icy roads [26] or the most commonly encountered slippery terrain [27]. One work that implements both lower and higher levels of control is [28] which shows the effectiveness of using a dynamic model (which is terrain dependent) to improve path planning and traction control. It should also be noted that as the Land Rover LR3 and Freelander have shown, platforms other than AGVs may benefit from terrain-dependent control, such as landscaping vehicles and electric powered wheelchairs [29].

AGVs are without a human driver and must therefore use an automated process, or rule to switch between control modes. This paper seeks to clearly define this automation problem and present an appropriate solution. The most relevant research for the creation of such a rule is presented in [30], and applies an adaptive Bayesian filter using past and present terrain detections in order to filter out misclassifications. This work also mentions that the number of past detections considered in the adaptive Bayesian filter will directly affect how well and how quickly the system can detect terrain transitions, i.e., the system sensitivity. This method also has the ability to easily ignore falsely detected terrain transitions between terrains that are unlikely to occur in nature, e.g., a transition between sand and tile floors.

However, there are several key areas where the work of [30] can be improved. Although [30] mentions the need to consider sensitivity, the presented adaptive Bayesian filter does not easily allow the achievement of a specific level of sensitivity, in particular, detection of a new transition within a specific amount of time. Additionally, the work of [30] uses concatenated one second samples from individual terrains in order to asses the method's performance, whereas the research in this paper is created and validated using experimental data from terrain transitions.

This paper is organized as follows. The problem of control mode switching is clearly defined in Section II. Section III describes the robotic platform used to collect data for tuning and validating the control mode update process. Section IV details the tuning process, while Section V provides experimental results based on this tuning process. Final conclusions and future work are presented in Section VI.

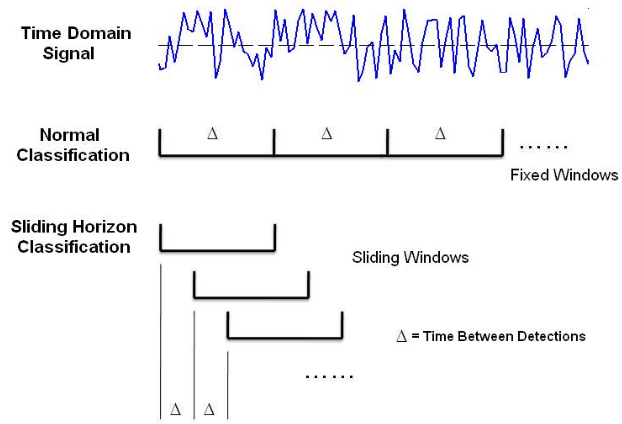


Fig. 1. Comparison between normal classification and sliding horizon classification.

II. Control Mode Updates

The two most important considerations in determining when to switch between control modes are sensitivity and robustness. When an AGV traverses from one terrain to a different terrain, a sensitive control update system is expected to quickly switch to the appropriate control mode. Conversely, as some misclassifications cannot be avoided, a robust control update system should be able to ignore a moderate number of misclassifications. The problem with creating a system that is both robust and sensitive is the trade-off between robustness and sensitivity. If the control system is too sensitive to the detected terrain it will switch control modes based on misclassifications, but a control system that is extremely robust may switch control modes too slowly. Therefore, the research presented in this paper will detail how to achieve a control switching rule, called the *update rule*, that balances the relationship between sensitivity and robustness.

A. Sliding Horizon

A sliding horizon approach for classification can be used to improve the sensitivity of the update rule without effecting the robustness of the terrain classification system. As reaction-based terrain classification requires multiple samples to classify the terrain, it typically requires 1-2 seconds to collect the necessary samples. This approach results in being able to detect a new terrain only every 1-2 seconds. In this paper the approach of waiting to collect entirely new samples before reclassifying the terrain will be referred to as *sequential horizon classification*. A sliding horizon essentially uses both new and previously collected samples to classify the terrain, instead of waiting to collect all new samples. This means that the time between detections, denoted here as Δ, is decreased. This approach is illustrated in Fig. 1.

In theory Δ can be as small as the sampling time t_s when using a sliding horizon. However, this is not always achievable since the time needed to perform the computations may be greater than t_s. In most cases vision-based classification is not expected to utilize a sliding horizon approach, as it does not utilize a time domain signal in order to classify the terrain. Although using a sliding horizon can increase the sensitivity when using reaction-based terrain classification, it is still subject to misclassifications. This means that an approach should be created that improves robustness while sacrificing little in terms of sensitivity. The following subsection details the development of an update rule that is both robust to misclassifications and sensitive to terrain transitions.

B. Problem Derivation

To formally describe the problem of determining a control update rule consider the case in which the terrain classification algorithm has been determined. Assume that a large set of terrain classification data (e.g., vibration signals) has been collected and that this data corresponds to all of the terrains that the AGV is expected to encounter, including terrain transition data (e.g., data that transitions between grass and asphalt). Let $\mathbf{R}$ denote the set of all control update rules. Note that $\mathbf{R}$ contains every rule that can be defined by one of the infinite number of rule structures (an example of which is given below) and hence contains an infinite number of (possibly infinite) rule subsets.

Now, let $\delta \geq 0$ denote the average time between control updates for all cases in which a terrain transition occurs and let $\rho \geq 0$ denote the average percentage of (undesirable) control updates that occur when traversing the terrains considered by the classification system. Reducing δ corresponds to increased sensitivity, while reducing ρ corresponds to increased robustness with $\rho = 0$ corresponding to perfect robustness for the data set under consideration. An ideal control update rule is one that simultaneously minimizes ρ and δ over $\mathbf{R}$, the set of all rules. However, there are two problems with this optimization problem. First, it is in general not possible to find a rule $R \in \mathbf{R}$ that simultaneously minimizes ρ and δ due to the inherent tradeoff between sensitivity and robustness. Second, it is virtually impossible to formulate practical optimization approaches for the infinite (and probably uncountable) set $\mathbf{R}$. To accommodate this latter problem, the set $\mathbf{R}$ may be replaced with a subset $\mathbf{R}_s \subset \mathbf{R}$ that leads to tractable optimization problems. The former problem is accommodated by searching for Pareto optimal solutions [31] over the reduced rule set $\mathbf{R}_s$.

To approach finding Pareto optimal solutions, define the two rule sets

$$\mathbf{R}_{\bar{\rho}} \triangleq \{R \in \mathbf{R}_s : \rho < \bar{\rho}\}, \quad \mathbf{R}_{\bar{\delta}} \triangleq \{R \in \mathbf{R}_s : \delta < \bar{\delta}\}, \tag{1}$$

The set of rules that simultaneously satisfy $\rho < \bar{\rho}$ and $\delta < \bar{\delta}$ is denoted by $\mathbf{R}_{\bar{\rho},\bar{\delta}}$ and is given by

$$\mathbf{R}_{\bar{\rho},\bar{\delta}} = \mathbf{R}_{\bar{\rho}} \cap \mathbf{R}_{\bar{\delta}}. \tag{2}$$

Now, define

$$\bar{\rho}^* \triangleq \inf\{\bar{\rho} : \ \mathbf{R}_{\bar{\rho},\bar{\delta}} \neq \emptyset\}, \quad \bar{\delta}^* \triangleq \inf\{\bar{\delta} : \ \mathbf{R}_{\bar{\rho},\bar{\delta}} \neq \emptyset\} \tag{3}$$

Then, $R \in \mathbf{R}_{\bar{\rho}^*,\bar{\delta}}$ is a Pareto optimal solution that solves

$$\min_{R \in \mathbf{R}_s} \rho \ \text{ subject to } \ \delta \leq \bar{\delta}, \tag{4}$$

while $R \in \mathbf{R}_{\bar{\rho},\bar{\delta}^*}$ is a Pareto optimal solution that solves

$$\min_{R \in \mathbf{R}_s} \delta \ \text{ subject to } \ \rho \leq \bar{\rho}. \tag{5}$$

To present a possible rule structure as the basis of $\mathbf{R_s}$ let the *history window* $\mathbf{W}$ be the set consisting of the present terrain classification w_0 and the $n-1$ previous terrain classifications, i.e., w_{-i} for $i = 1,2,\ldots,n-1$. Hence, $\mathbf{W}$ consists of n terrain classifications and is given by,

$$\mathbf{W} = \{w_{-n+1}, w_{-n+2}, \ldots, w_{-1}, w_0\}. \tag{6}$$

Let n_a denote the number of classifications of Terrain a in $\mathbf{W}$. Then the controller will switch to the control mode for Terrain a if for some $\eta \in (0.5, 1]$

$$\frac{n_a}{n} \geq \eta. \tag{7}$$

This rule structure has two parameters that must be determined: n and η. Section IV describes how to determine these two parameters by essentially solving the Pareto optimization problem of (4) or (5) using empirical data. Note that reducing η or increasing n, increases the system sensitivity and reduces robustness. It should also be noted that the choice of n and η is expected to depend on the vehicle speed. This is because the faster the speed, the further away the terrain corresponding to the classification w_{-i} is from the current position.

Although (6) and (7) present a single rule structure, future research will also consider rule structures based on Hidden Markov Modeling and a Bayesian temporal filter similar to the work of [30]. Ultimately, the performance of these other rule possibilities and the method presented here will be compared in the hopes of choosing the most appropriate update rule structure.

III. Robotic Platform and Classification Approach

The tuning process and results of Section IV and V are obtained using data from the eXperimental Unmanned Vehicle (XUV). The XUV, pictured in Fig. 2, is a four-wheel, Ackerman steered, all wheel drive, autonomous vehicle, weighing approximately 3000 lbs with a wheelbase of 1.88 m and track width of 1.91 meters. The XUV is able to record vibration measurements of vertical acceleration $\ddot{z}$, roll rate ω_{roll}, and pitch rate ω_{pitch} at a sampling rate of 50 Hz using the XUV's Inertial Reference Unit (IRU). However, the XUV is equipped with a suspension system, which can sometimes dampen the vehicle vibrations and therefore make vibration-based terrain classification more difficult.

Terrain data for the XUV tests consists of $\ddot{z}(t)$, $\omega_{roll}(t)$, and $\omega_{pitch}(t)$ recorded at speeds of 5.26 mph and 8.41 mph over grass, gravel and paved terrains as well as a terrain transition between gravel and pavement. This transition can be seen in Fig. 2. The paved terrains consist of approximately

Fig. 2. The eXperimental Unmanned Vehicle (XUV) on a gravel and asphalt (pavement) transition

95% asphalt and 5% concrete. The inclusion of concrete in this terrain is due to the limitations of the testing location.

For classification, the recorded terrain data was processed and classified as described in [13] which utilizes a FFT, PCA and Parzen window estimation with a radial basis function window. This process first entails applying a FFT to 2 second segments of the $\ddot{z}(t)$, $\omega_{roll}(t)$, and $\omega_{pitch}(t)$ measurements, resulting in the frequency responses $\ddot{z}(j\omega)$, $\omega_{roll}(j\omega)$, and $\omega_{pitch}(j\omega)$. These frequency responses are then rearranged into training patterns $\mathbf{x}$, defined by

$$\mathbf{x} = [\ \ddot{z}(j\omega) \quad \omega_{roll}(j\omega) \quad \omega_{pitch}(j\omega)\ ], \tag{8}$$

which are said to contain the terrain signatures. Next, the training patterns are separated into different sets based on speed. These sets correspond to each of the aforementioned training speeds, with an acceptable speed tolerance of ± 0.75 mph. Separate tuning processes are then conducted using each of the resulting sets to determine the 2 tuning parameters: the PCA energy percentage and σ the width of the radial basis function window. By tuning the PCA energy percentage, the dimension of the feature space is reduced while maintaining as much of the feature variability as possible. The tuning of σ is analogous to determining the influence of each of the training patterns on the estimation of the probability density functions. In order to reduce the likelihood of over training, this tuning process is performed using 10-fold cross validation. Testing of new terrain data is then conducted using the PCA-Parzen window classifier combination whose speed is closest to the speed of the test sample. Using this classifier, the class ω_j which corresponds to the highest conditional probability $p(\omega_j|\mathbf{x})$ is selected as the class of the test pattern. It should also be noted that for testing purposes a sliding horizon with $\Delta = 0.04$ secs is used, which is much larger than the time required to classify a test pattern (2.3 msecs).

Although the results based on this data obviously lead to a classification method that is categorized as reaction-based, it is important to emphasize that the tuning approach described

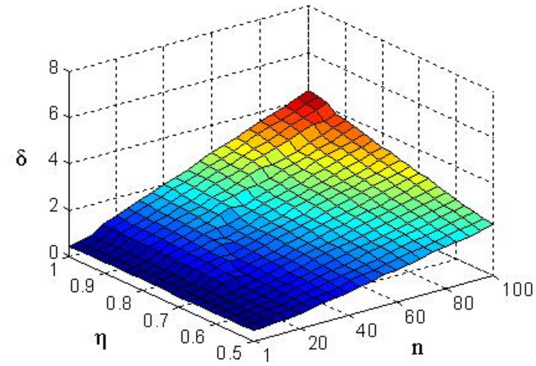


Fig. 3. The time delay, δ that results from variation of the parameters n and η

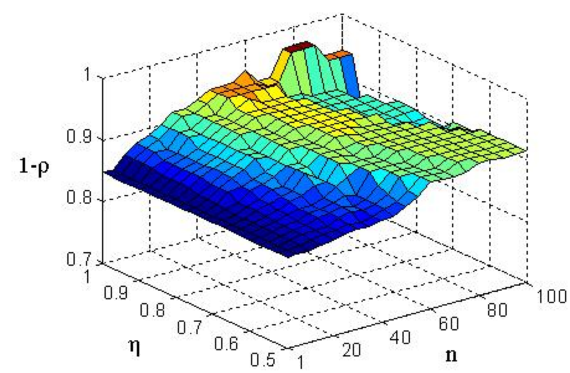


Fig. 4. The frequency of correct control mode usage $(1-\rho)$ that results from variation of the parameters n and η

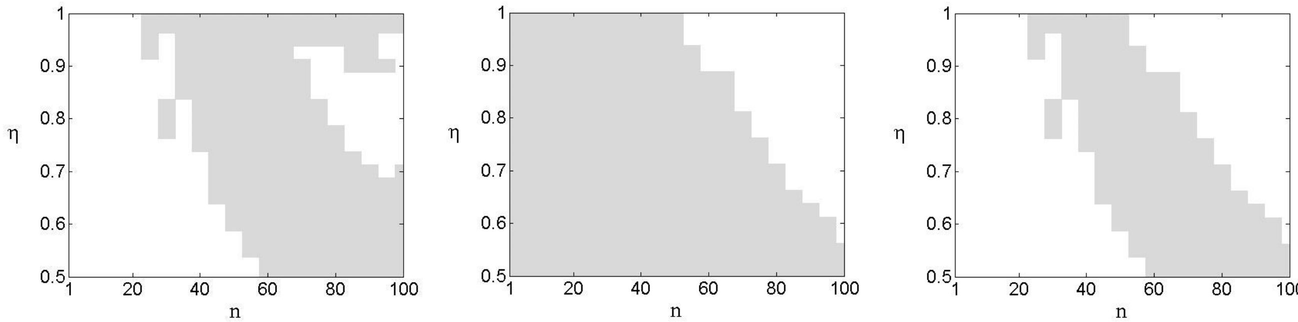
in Section IV can be applied to both reaction- and vision-based techniques such as [8], [24], which were developed for classification of terrain surfaces.

IV. Rule Tuning

In Subsection II-B it was determined that by controlling n and η, which control the sensitivity and robustness, it may be possible to find a solution to (4) or (5) using a previously trained classification scheme and data extracted from real terrain transitions. This section illustrates the process of tuning n and η using data from the XUV mobile robot on a terrain transition between gravel and asphalt as described in Section III.

In practice, given a (preferably large) set of experimentally based terrain classification data that contains several terrain transitions (e.g., from grass to asphalt), the values of ρ and δ can be computed for various combinations of n and η in some reasonable range. Here, this process is illustrated using vibration data from the XUV mobile robot traveling at 8.41 mph as it transitions from gravel to asphalt and asphalt to gravel. Varying n and η and using the update rule structure of Subsection II-B yields Fig. 3 and Fig. 4, which show the mean values of δ and $1-\rho$.

These figures reinforce the previously stated relationships



(a) Rho Constraint Met

(b) Delta Constraint Met

(c) Both Constraints Met

Fig. 5. Allowable values of n and η (shaded regions) for constraints $\delta \leq 2.5$ sec and $\rho \leq 0.10$)

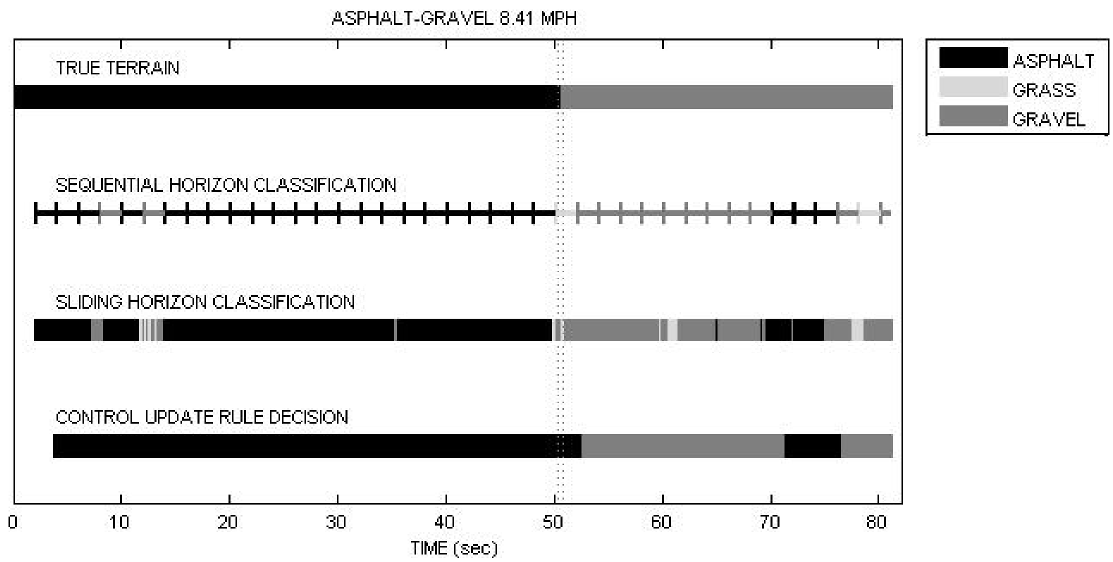


Fig. 6. Results for Terrain Classification, Sliding Horizon Classification and the Observed Update Rule on Asphalt to Gravel Transition

between n, η, sensitivity and robustness as stated in Subsection II-B. To show this, first consider Fig. 3. This figure shows that reducing η or n results in a faster transition time δ, which is analogous to increased sensitivity as previously discussed. Fig. 4 demonstrates that increasing n and η generally improves system robustness, which is shown through a larger value of $1-\rho$. Interestingly, Fig. 3 indicates a nearly linear relationship between n, η and sensitivity. However, this relationship is not necessarily expected to be linear when considering terrain transitions that are less abrupt than the transition between asphalt and gravel.

Based on Fig. 3 and Fig. 4 reasonable constraints on ρ and δ can be determined, which will be used as a starting point for solving (4) or (5). In this example constraints of $\delta \leq 2.5$ sec and $\rho \leq 0.10$ are chosen. These choices are based on the obtainable ρ and the impact of δ on the vehicle stopping distance. Though for some vehicle control systems $\delta > 2.5$ may be acceptable, the vehicle stopping distance benefits of a terrain-dependent control system diminish as δ increases. Values of n and η that satisfy these constraints are then recorded and are displayed by the shaded regions of Figure 5. The intersection of Fig. 5(a) and Fig. 5(b) gives several possible choices of n and η which meet the described limits. These choices are shown in Fig. 5(c). If desired, the sensitivity or robustness can then be improved by incrementally reducing one of the constraints until any further reduction results in an empty set of n and η choices. This process essentially solves the optimization problem (4) or (5).

V. Results

By implementing the update rule derived from the described tuning process, the vehicle control system is expected to rarely switch control modes based on misclassifications. Using the tuning process started in Section IV and solving (4), that is finding the most robust update rule that satisfies $\delta \leq 2.5$, results in an update rule defined by the parameters $n = 45$ and $\eta = 0.95$. Fig. 6 and Fig. 7, which correspond to transitions of gravel and asphalt in different directions, show that the derived update rule rarely switches the control mode based on misclassified terrains. These figures also show the result of terrain classification using both sequential and sliding horizons for comparison purposes. Additionally, the dotted vertical lines in Fig. 6 and Fig. 7 represent the time

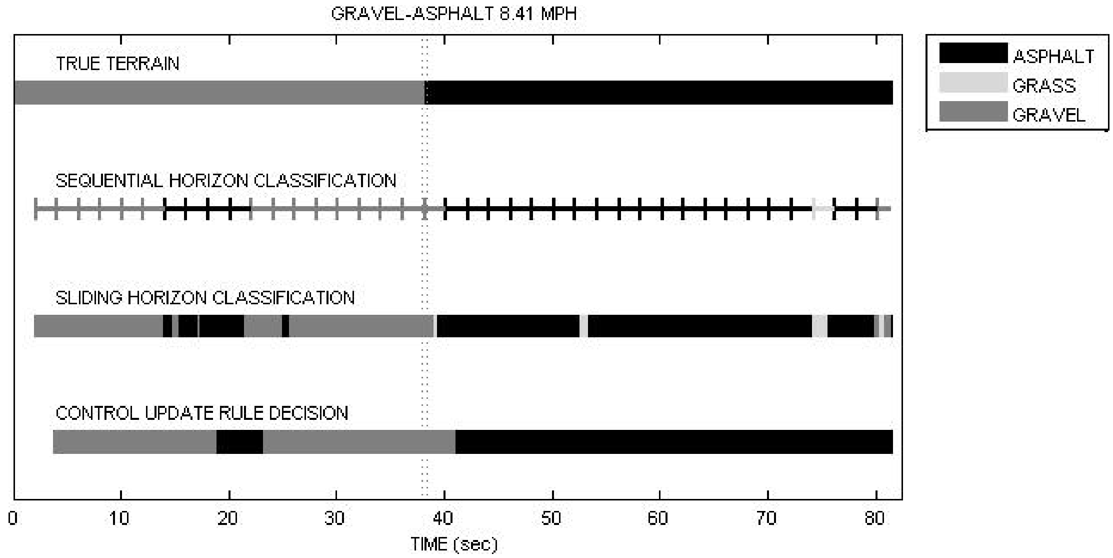


Fig. 7. Results for Terrain Classification, Sliding Horizon Classification and the Observed Update Rule on Gravel to Asphalt Transition

and distance required for the vehicle to transition between terrains, i.e. the *transition region*. While the asphalt and gravel transition is fairly abrupt, other terrain regions may be several feet long and therefore take several seconds to pass, especially at slow speeds. A classification of either terrain during the transition region is considered acceptable.

Fig. 6 and Fig. 7 show that using sequential horizon classification, the detected terrain could only be updated every 2 seconds, whereas sliding horizon classification can detect the terrain in near real-time. However, using a sliding horizon does not address the issue of updating the control mode based on misclassifications. This means that using a sliding horizon without an update rule could result in several erroneous switches of the control system. In fact, when using sliding horizon classification the terrain is often misclassified based on Fig. 6 and Fig. 7, with these stretches lasting from approximately 0.1 seconds to 4 seconds. However, most stretches of misclassifications seem to last less than 2 seconds. It is these short segments of misclassifications which the update rule is designed to ignore. From Fig. 6 and Fig. 7 it can be seen that the update rule is able to achieve this goal with minimal change in the average delay in switching the control mode δ over sliding horizon classification. Although the update rule also causes a small start-up delay (approx. 2 secs), in order to populate $\mathbf{W}$. These delays are considered allowable based on the previously enforced constraint $\delta \leq 2.5$ secs. And as there is only one undesired control mode switch in each of these trials, the update rule is shown here to be both robust and sensitive. Interestingly, the stretch of terrain where the wrong control mode is chosen corresponds to rut-like features in the gravel that resulted from the AGV traversing this path multiple times. This means that this stretch may benefit from control settings closer to the asphalt control mode (the mode chosen to implement) due to the absence of loose rocks in the gravel surface. Fig. 6 and Fig. 7 also show that the update rule is effective regardless of the travel direction. That is, the update rule works well for transitions from asphalt to gravel as well as gravel to asphalt.

In order to obtain an update rule for when the vehicle is traveling at a speed of approximately 5.26 mph, the tuning process of (4) with $\delta \leq 2.5$ is applied to the gravel and asphalt transition data at 5.26 mph. The update rule parameters n and η as well as the performance characteristics δ and $1-\rho$, for the 5.26 mph and 8.41 mph update rules are presented in Table I.

TABLE I

XUV UPDATE RULES GRAVEL AND ASPHALT TRANSITIONS

Speed (mph)	n	η	δ (sec)	$1-\rho$	Change in ρ
5.26	55	1.00	2.24	92.2%	9.0%
8.41	45	0.95	2.11	93.6%	9.8%

Table I shows the potential of using an update rule to conduct online switching of vehicle control modes. The update rules at 5.26 mph and 8.41 mph were shown to use the best control mode 92.2% and 93.6% of the time respectively. These accuracies respectively correspond to improvements of 9.0% and 9.8% over switching the control mode based on sliding horizon terrain classification. This change in ρ clearly shows the potential robustness improvement that is obtainable using a well designed update rule. Additionally, the delay in switching the control system δ is found to be around 2.2 seconds at each speed. When considering that terrain classification without the use of a sliding horizon approach is only able to update the terrain every 2 secs on the XUV, a 2.2 second delay is not considered significantly different than what would be achieved using a typical classification approach. However, the update rule is obviously substantially more robust.

It was previously theorized that the update rule would be dependent on the vehicle speed. Table I seems to enforce this theory, as n and η are somewhat different based on speed. However, these results consist of the compiled results for only two trials on the same terrain transition. As n and η are not very different, additional trials and a variety of terrains should be considered before concluding that separate update rules will be needed based on vehicle speed. Data for additional trials and other terrain transitions is currently unavailable, but will be considered in future research.

VI. CONCLUSIONS AND FUTURE WORKS

One area of terrain-dependent control that has seen little discussion and research is in how to use a terrain classification system to update the control modes on an AGV. This paper presents research in this area and demonstrates how to achieve an update rule that balances robustness with respect to misclassifications and sensitivity to changes in the traversed terrain. Using experimental data from a large mobile robot and a reaction-based classification scheme, this method of determining an update rule is shown to have the ability to achieve the desired balance.

As previously mentioned, future work on development of an update rule should consider the use of data from multiple terrain transitions, both abrupt and gradual in nature. This should help to prove or disprove the theory that separate update rules should be used based on vehicle speed, a theory which the results of this paper seem to validate. As some terrain transitions are unlikely to exist in nature, future research should consider how to incorporate this information within the update rule framework.

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