2010 IEEE International Conference on Robotics and Automation
Anchorage Convention District
May 3-8, 2010, Anchorage, Alaska, USA

Human-in-the-Loop: Terminal Constraint Receding Horizon Control with Human Inputs

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Abstract— **This paper presents a control theoretic formulation and optimal control solution for integrating human control inputs subject to linear state constraints. The formulation utilizes a receding horizon optimal controller to update the control effort given the most recent state and human control input information. The novel solution to the corresponding finite horizon optimal control problem with terminal constraint is derived using Hilbert space methods. The control laws are applied to two planar human-driven mass-cart pendula, where the task is to synchronize the pendula's oscillations.**

I. INTRODUCTION

Despite the advances in autonomous robotics and automation, some tasks still require human intervention or guidance to mediate uncertainties in the environment or to execute the complexities of a task that autonomous robots are not yet equipped to handle. Therefore, it is desirable to design robot controllers that utilize the strengths of both autonomous agents, adept at handling lower level control tasks, and humans, superior at handling higher-level cognitive tasks. Researchers in the Human-Robot Interaction (HRI) field refer to this as mixed initiative interaction or human-in-the-loop (e.g., [1],[2]). It can also be referred to as shared control, as in [3] and [4], since both controllers (human and computer) act on the same dynamic system.

Earlier work in mixed initiative interaction and human-in-the-loop control have focused on graphical user interfaces or haptic feedback to relay task-dependent data to the human and to relay human control information to an automatic controller or autonomous agent (e.g., [5], [6]). In this paper, we largely ignore this issue. Instead we focus on the design of the actual control laws.

In [7], a mixed initiative control utilizing navigation function based controllers is combined with human input to drive a differential drive robot around obstacles. The navigation functions are cost functions with a global minimum so the controllers drive to a specific goal state. When human input is incorporated into the controller, the human user can drive the robot away from the planned path and once the user stops issuing commands, the controller will drive the system towards the goal state again. Our paper's approach differs in that the goal state is not known a priori. On the contrary, the goal state is chosen by the controller to be a state that is closest to where the human input would drive that satisfies the state constraint.

Previous work on designing shared control laws include mobility aids, where the humans propel and steer a walker in the desired direction, while the walker is equipped to steer and apply brakes in order to prevent obstacle collisions and falls [4]. These algorithms are rule-based, and need to be experimentally tuned in order to smooth transitions between user control and automatic control. In [3], shared control for vehicle steering was examined using a motorized steering wheel and human driver. An automatic controller applied torque to the steering wheel to maintain a vehicle heading that follows the road while the driver must overcome this torque to make any corrections to the steering angle, relying on the physical human force to impart the intended behavior on the system. In this paper, we invert this relationship in that the controller is guiding the human to reach a target set. This allows the automatic controller to ensure that state constraints on the system are satisfied, while the human can direct the system to a solution, within the constraints, that is suitable from a higher-level point-of-view.

Specifically, a novel control theoretic formulation of the human-in-the-loop problem is presented by framing it as a receding finite horizon optimal control problem with a terminal state constraint and presenting a Hilbert Space-based solution to the corresponding optimal control problem. This resulting control law is then applied to the problem of two humans each driving a mass-cart pendula such that their pendula oscillations are synchronized.

The outline of the paper is as follows: Section 2 introduces the receding horizon problem formulation that incorporates human control input and a linear state constraint, while Section 3 details the solution to this optimal control problem using Hilbert's projection theorem. In Section 4, we show that the control indeed converges to a solution that satisfies the terminal constraint. This is followed by Section 5, in which the control law is applied to a simulation of two planar mass-cart-pendula, each partially controlled by a human operator seeking to synchronize the two pendula oscillations.

II. PROBLEM STATEMENT

Given a discrete-time linear dynamic system,

$$x_{k+1} = Ax_k + Bv_k \tag{1}$$

with $x_k \in \mathbb{R}^n$, a human operator issues the commands $v_k \in \mathbb{R}^m$. However, if part of the task is to satisfy certain linear state constraints, commanding the system to do so may

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not be a trivial task. On the other hand, it is certainly possible to design an automatic controller that can handle the task of satisfying linear state constraints, as in [8]. However, the problem we wish to address in this paper is to implement a controller that drives the system in such a way that both the state constraint is satisfied and the human operators intentions for the system behavior are respected as much as possible.

In order to preserve the human operators' intentions for the system behavior, we wish to design a control law that minimizes deviations from the human input while also ensuring that the linear state constraint will be satisfied.

To accomplish this, we must predict where the human operator intends to drive the system which, in turn, requires a prediction on future human operator input. We linearly extrapolate the human input over a time horizon of N discrete-time steps, based on the current and previous input values. Thus, at every time instant, we have a predicted sequence of human input values, $\{v_k\} = \{v_k, \ldots, v_{k+N-1}\}$, for which we now want to find a control sequence, $\{u_k\} = \{u_k, \ldots, u_{k+N-1}\}$, that minimizes its deviations from $\{v_k\}$, while ensuring that the state resulting at the end of the sequence satisfies the linear constraint. Now, the linear state constraint can be defined as a terminal state constraint for the following receding horizon optimal control problem, $\mathcal{P}_N$:

$$\min_{\{u_k\}} V_N(\{v_k\}, \{u_k\}) \tag{2}$$

where

$$V_N(\{v_k\}, \{u_k\}) = \sum_{i=k}^{k+N-1} L(v_i, u_i), \tag{3}$$

$$L(v_i, u_i) = (u_i - v_i)^T (u_i - v_i), \tag{4}$$

with $\{v_k\} = \{v_k, \ldots, v_{k+N-1}\}$ and $\{u_k\} = \{u_k, \ldots, u_{k+N-1}\}$ denoting the control sequences, such that

$$x_{k+1} = Ax_k + Bu_k, \tag{5}$$

subject to

$$x_{k+N} \in \mathbb{X}_f = \{x \mid Mx = b\}, \tag{6}$$

with $x_k \in \mathbb{R}^n$, $M \in \mathbb{R}^{l \times n}$, $b \in \mathbb{R}^l$, and $u_k \in \mathbb{R}^m$. The control, u_k, applied to (5) is the first element in the sequence $\{u_k\}$

In this formulation, the cost penalizes deviations from the human command over the time horizon. The terminal constraint (6) guarantees that the state constraint, which was required for the task, is enforced at the end of the control horizon. Without the terminal constraint, (6), the control would simply equal the predicted human input. However, the terminal constraint may cause the control to deviate from the human input in order to ensure that the terminal constraint is satisfied. The receding horizon also allows for recalculating the control at each time instant when new human input and state information is available.

The choice of finite horizon, N, is crucial in that a large N requires that the linear approximation of the human input be accurate, otherwise the control input will not reflect the intent of the user. If N is too small, the control effort attempts to reach the constraint set within a small amount of time, so deviations from the human input can be large. As such, N must be chosen short enough such that the linear approximation of the user input is valid and long enough that user intention for the state is maintained.

III. CONTROL LAW DERIVATION

Methods for utilizing Hilbert Spaces as representations of control signals to find solutions to optimal control problems, as found in [9] and [10], are used in this derivation. The following Hilbert Space solution to the finite horizon optimal control problem within the receding horizon formulation is an augmentation of the work in [9].

The solution begins by defining the human input and control input sequences over the horizon, N, as being in the Hilbert space, $l_2^l[k, k+N-1]$, which we will denote as l_2 from now on.

Let $\mathcal{H} = l_2$ with the norm squared defined as $\langle y, y \rangle = \sum_{i=k}^{k+N-1} y_i^T y_i$ and the inner product given as $\langle y, w \rangle = \sum_{i=k}^{k+N-1} y_i^T w_i$ for all $y, w \in \mathcal{H}$. The following steps are taken to find the projection of the human input signal, point $v = \{v_k\}$, on the affine variety representing the space of control inputs for which the terminal state constraint is satisfied, V_α. We will, first, find a subspace of $\mathcal{H}$, V_0, that is parallel to V_α and then find a subspace that is orthogonal to V_0, which is also orhogonal to V_α. Then, we can translate that subspace so that it passes through point v. The point that lies in both V_α and the translated orthogonal space is the unique minimizer. This is shown graphically in Figure (1) as in [9].

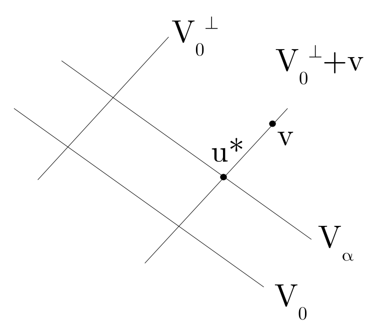


Fig. 1. Hilbert Space Diagram showing the unique minimizer, u^*

Now that we have point v, the next step is to define the constraint space as the affine variety. The state at the end of the control horizon N is given by

$$x_{k+N} = A^N x_k + \sum_{i=k}^{k+N-1} A^{k+N-1-i} B u_i \tag{7}$$

and it is required that this state satisfy the linear terminal constraint (6). The linear operator $L : l_2 \rightarrow \mathbb{R}^n$ is defined such that

$$\begin{aligned} Lu &= \sum_{i=k}^{k+N-1} A^{k+N-1-i} B u_i, \\ x_{k+N} &= A^N x_k + Lu \end{aligned}$$

with $u = \{u_k\}$. Hence, we can rewrite (6) as

$$MLu = b - MA^N x_k. \tag{8}$$

Let $L^* : \mathbb{R}^n \rightarrow l_2$ denote the adjoint operator

$$L_i^* = B^T (A^{k+N-1-i})^T$$

for $i = k, \ldots, k+N-1$. Furthermore, the grammian LL^* is given by

$$LL^* = \sum_{i=k}^{k+N-1} A^{k+N-1-i} BB^T (A^{k+N-1-i})^T.$$

Next, (8) will be used to construct a subspace and the affine variety. Let V_0 be defined as a subspace of $\mathcal{H}$ and V_α be the affine variety such that

$$\begin{aligned} V_0 &= \{u \mid MLu = 0\}, \\ V_\alpha &= \{u \mid MLu = b - MA^N x_k\}. \end{aligned}$$

We now have the point v, the affine variety V_α, and the subspace V_0, so next, we will find a subspace that is orthogonal to V_0, which is also orthogonal to V_α. The orthogonal complement $V_0^\perp$ to V_0 is

$$V_0^\perp = \{s \mid \langle u, s \rangle = 0, \ \forall u \in V_0\}.$$

$V_0^\perp$ is obtained by letting d be any point in $\mathbb{R}^l$,

$$\begin{aligned} 0 = \langle MLu, d \rangle_{\mathbb{R}^l} &= \langle u, L^* M^T d \rangle_{l_2} \text{ i.e.} \\ V_0^\perp &= \{s \mid s = L^* M^T d, \ d \in \mathbb{R}^l\}. \end{aligned}$$

The orthogonal complement can be translated by $v = \{v_k\}$, giving

$$\begin{aligned} V_0^\perp + v &= \{w \mid w = s + v, \ s \in V_0^\perp\} \text{ such that} \\ V_0^\perp + v &= \{w \mid w = L^* M^T d + v, \ d \in \mathbb{R}^l\}. \end{aligned}$$

Now, to find the unique minimizer, the intersection of $V_0^\perp + v$ and V_α gives

$$\begin{aligned} MLw &= b - MA^N x_k \qquad (9) \\ ML(L^* M^T d + v) &= b - MA^N x_k \\ MLL^* M^T d &= b - MA^N x_k - MLv. \end{aligned}$$

So, we can solve for d with

$$d = (MLL^* M^T)^{-1} (b - MA^N x_k - MLv). \tag{10}$$

Therefore, plugging (10) back into (9), the optimal control sequence is given by

$$u_i^* = L_i^* M^T (MLL^* M^T)^{-1} (b - MA^N x_k - MLv) + v_i$$

for $i = k, \ldots, k+N-1$. For the receding horizon formulation, only the first element of u_i^* is applied to (5). Thus, the optimal control law at time k is

$$u_k^* = L_k^* M^T (MLL^* M^T)^{-1} (b - MA^N x_k - MLv) + v_k. \tag{11}$$

This control law minimizes the predicted cost over the horizon and so, we state this as a theorem:

Theorem 3.1: Given the terminal constraint receding finite horizon optimal control problem, $\mathcal{P}_N$, and the predicted human input sequence, $\{v_k\}$, the optimal control law is given by (11).

However, since only the first element of the optimal control sequence is used, we need to guarantee that the state will actually converge to the state constraint.

IV. Proof of Convergence

This paper refers to previous works in model predictive/receding horizon control to formulate conditions and methods used to prove state convergence for the proposed optimal controller (e.g., [11], [12], [13], [14]). This paper differs from these earlier works in that the proposed control law does not result in asymptotic stability but in the convegence of the state to a terminal constraint set not containing an equilibrium point of the dynamic system.

Here, we adapt the stability proofs detailed in [11] and [12] to show that the optimal control will indeed drive the state to the terminal constraint set given by (6). Adapted from [11], the following conditions are required:

Conditions:

- C1 $L(v_k, u_k) \geq \gamma(\|(u_k - v_k)\|)$, where γ is a K-function and $L(0, 0) = 0$.
- C2 $L(v_k, u_k)$=0 for all $x_k \in \mathbb{X}_f$.
- C3 The set $\mathbb{X}_f$ is positively invariant under control v_i such that $Ax_i + Bv_i \in \mathbb{X}_f \ \forall x_i \in \mathbb{X}_f, v_i \in \mathbb{V}(x_i)$, where $\mathbb{V}(x_i)$ is an input constraint on the human input.
- C4 A solution to $\mathcal{P}_N$ exists for a set of initial states denoted by $\mathbb{F}$.

The input constraint set, $\mathbb{V}(x_i)$, is a subset of $\mathbb{R}^m$ and is a function of the state, x_i, in that the human input is only restricted to $\mathbb{V}(x_i)$ when $x_i \in \mathbb{X}_f$. Conditions C1 and C2 are clearly satisfied by our choice of stage cost (4). Conditions C2 and C3 ensure that once the system reaches the terminal set, the stage cost is zeroed and the system will not be driven out of the constraint set, i.e. $u_i = v_i$ and $x_{i+1} = Ax_i + Bu_i \ \in \mathbb{X}_f \ \forall \ x_i \in \mathbb{X}_f$. These two conditions imply a strong assumption in that we assume that the bounds on the human operator control and the ability of the operator is sufficient for keeping the state within the constraint set once this set has been reached. In other words, the human operator is trusted with the control to make $\mathbb{X}_f$ invariant. This is a reasonable assumption because once the system has converged to the state constraint set, it should be obvious to the human operator that large incorrect command inputs will force the system out of the constraint set.

The solution to $\mathcal{P}_N$ exists for $x_k \in \mathbb{F}$, where $\mathbb{F}$ is the set of initial states for which a feasible solution can be computed. From [12], a solution is feasible if the solution results in the satisfaction of the state and input constraints on the optimization problem. We should note that this does not imply that the solution is optimal. The optimal control law that solves $\mathcal{P}_N$, given $\mathcal{V}_k = \{v_0, v_1, \ldots, v_{N-1}\}$, results in the following control and state sequences,

$$\begin{aligned} \mathcal{U}_k^{opt} &= \{u_0^{opt}, u_1^{opt}, \ldots, u_{N-1}^{opt}\} \qquad (12)\\ \mathcal{X}_k^{opt} &= \{x_0^{opt}, x_1^{opt}, \ldots, x_{N-1}^{opt}, x_N^{opt}\}, \qquad (13)\end{aligned}$$

with $x_N^{opt} \in \mathbb{X}_f$.

A. Feasibility

We will show that for all $x_k \in \mathbb{F}$, the successive state resulting from the first control in the optimal control sequence at time k, x_{k+1}, also has a feasible solution (i.e. $x_{k+1} \in \mathbb{F}$). Written another way, at time k, u_k^{opt} is applied to the system with state x_k, to produce x_{k+1}, for which a feasible solution can be found given a human input sequence. A feasible control and state sequence at time $k+1$, given the human input sequence $\mathcal{V}_{k+1} = \{v_1, v_2, \ldots, v_{N-1}, v_N(x_N^{opt})\}$, is

$$\begin{aligned} \mathcal{U}_{k+1}^{fea} &= \{u_1^{opt}, u_2^{opt}, \ldots, u_{N-1}^{opt}, v_N(x_N^{opt})\} \qquad (14)\\ \mathcal{X}_{k+1}^{fea} &= \{x_1^{opt}, x_2^{opt}, \ldots \\ &\ldots \quad , x_{N-1}^{opt}, x_N^{opt}, Ax_N^{opt} + Bv_N(x_N^{opt})\}, \qquad (15)\end{aligned}$$

assuming v_1, from $\mathcal{V}_k$, is the human input at $k+1$. Recall that, by Condition C3, the human input control, $v_N(x_N^{opt})$, when x_N is in the state constraint set, provides $\mathbb{X}_f$ with invariance for all $x_N \in \mathbb{X}_f$, $v_N(x_N^{opt}) \in \mathbb{V}(x_N)$. Hence, all states starting in the set of feasible initial states, will always stay in the set of feasible initial states.

B. Convergence

Lyapunov analysis is employed to show that the state will converge to the terminal constraint set. The theroem and proof utilized by [12] proves asymptotic stability and we will use a similar approach to show convergence. The cost $V_N(\{v_k\}, \{u_k\})$ is used as a Lyapunov function and we will show that the following properties hold, which is sufficient to ensure convergence:

P1 $V_N(\{v_k\}, \{u_k\}) \geq \gamma(||u_k - v_k||)$ for some K-function $\gamma(.)$.

P2 $V_N(\{0\}, \{0\}) = 0$.

P3 $V_N^{opt}(\{v_{k+1}\}, \{u_{k+1}^{opt}\}) - V_N^{opt}(\{v_k\}, \{u_k^{opt}\}) \leq -\gamma(||u_k - v_k||)$ for all $x_k \notin \mathbb{X}_f$.

Note that in Property P3, the cost, $V_N^{opt}(\{v_{k+1}\}, \{u_{k+1}^{opt}\})$, is a result of applying the optimal control, u_k^{opt}, to the system at time k to get x_{k+1}.

Theorem 4.1: Given Conditions C1-C3, all states, for which a feasible solution exists, will converge to the constraint set, $\mathbb{X}_f$, as $k \to \infty$.

Proof: Properties P1 and P2 are satisfied by the choice of cost function (3) and Condition C1. It remains to show that Property P3 is satisfied. Using the sequences (12)-(15), we can show that the value function decreases by at least the initial stage cost,

$$V_N^{opt}(\mathcal{V}_{k+1}, \mathcal{U}_{k+1}^{opt}) - V_N^{opt}(\mathcal{V}_k, \mathcal{U}_k^{opt}) \leq -L(v_k, u_k),$$

by the following.

The optimal cost at $k+1$ is bounded above by the feasible cost as given by optimality (i.e $V_N^{opt}(\mathcal{V}_{k+1}, \mathcal{U}_{k+1}^{opt}) \leq V_N^{fea}(\mathcal{V}_{k+1}, \mathcal{U}_{k+1}^{fea})$), so the change in cost may be rewritten as

$$\begin{aligned} V_N^{opt}&(\mathcal{V}_{k+1}, \mathcal{U}_{k+1}^{opt}) - V_N^{opt}(\mathcal{V}_k, \mathcal{U}_k^{opt}) \leq \\ &V_N^{fea}(\mathcal{V}_{k+1}, \mathcal{U}_{k+1}^{fea}) - V_N^{opt}(\mathcal{V}_k, \mathcal{U}_k^{opt}).\end{aligned}$$

We can now continue by plugging in the sum of the stage costs,

$$\begin{aligned} & V_N^{opt}(\mathcal{V}_{k+1}, \mathcal{U}_{k+1}^{opt}) - V_N^{opt}(\mathcal{V}_k, \mathcal{U}_k^{opt}) \\ \leq\ & \sum_{i=k+1}^{k+N} L(v_i, u_i^{feas}) - \sum_{i=k}^{k+N-1} L(v_i, u_i^{opt}) \\ =\ & L(v_1, u_1^{opt}) + \cdots + L(v_{N-1}, u_{N-1}^{opt}) + L(v_N, v_N) - \\ \ldots\ -\ & L(v_0, u_0^{opt}) - L(v_1, u_1^{opt}) - \cdots - L(v_{N-1}, u_{N-1}^{opt}) \\ =\ & L(v_N, v_N) - L(v_0, u_0^{opt}).\end{aligned}$$

Given Condition 1, $L(v_N, v_N) = 0$, $v_k = v_0$, and $u_k = u_0$, the change in cost from time k to time $k+1$ is given by

$$\begin{aligned} V_N^{opt}(\mathcal{V}_{k+1}, \mathcal{U}_{k+1}^{opt}) - V_N^{opt}(\mathcal{V}_k, \mathcal{U}_k^{opt}) &\leq -L(v_k, u_k) \\ &\leq -\gamma(||u_k - v_k||).\end{aligned}$$

Hence, for all $x_k \in \mathbb{F}$, the state will converge to the constraint set, $\mathbb{X}_f$, as $k \to \infty$. ■

V. Simulation Results

A. Human Operation of Mass-Cart-Pendula

In order to demonstrate the viability of the presented problem formulation and optimal control law, we apply the control law to a MATLAB simulation of the two mass-cart-pendula synchronization problem, as discussed in [8]. However, in this problem, a human operator is issuing force commands to one mass-cart-pendulum, while another human issues commands to the other. The human commands have saturation limits while the automatic control effort does not. The human operators try to drive the pendula in a certain direction while the controller ensures the pendula oscillations synchronize with the carts maintaining a set distance apart. The pendula synchronization and cart formation is the linear state constraint for this problem. The operators visually monitor the progress of the system through a graphic display as seen in Figure 2. In the following subsections, the dynamics and control are detailed and then the simulation results are given for some specific simulated and actual human commands.

B. Dynamics and Control of Mass-Cart Pendula

As seen in Figure 3, the force, F applied in the P_x direction, is the control input, u, to the system. No damping force is considered in this model as pendula can be approximated as zero damping systems. The linearized continuous

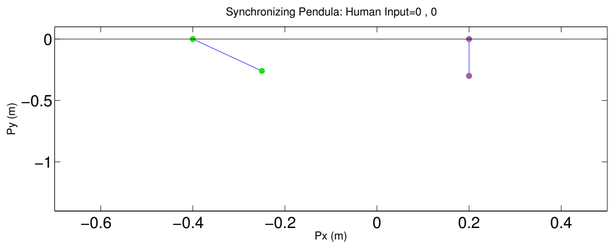


Fig. 2. Pendulum Graphic Display

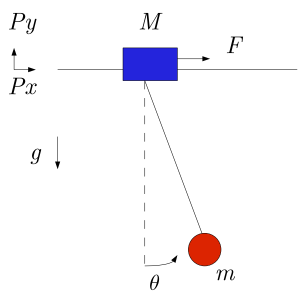


Fig. 3. Pendulum Diagram

dynamics about the hanging equilibrium point, gives the single pendulum system as $\dot{x}_i(t) = \tilde{A}_i x_i(t) + \tilde{B}_i u_i(t)$, where:

$$x_i = \begin{bmatrix} P_{x,i} \\ \dot{P}_{x,i} \\ \theta_i \\ \dot{\theta}_i \end{bmatrix}, \tilde{A}_i = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{-(M+m)g}{Ml} & 0 \end{bmatrix}, \tilde{B}_i = \begin{bmatrix} 0 \\ \frac{1}{M} \\ 0 \\ \frac{-1}{Ml} \end{bmatrix}.$$

Note that this pair, $(\tilde{A}_i, \tilde{B}_i)$, is controllable.

For a 2 planar pendula system, the system can be written as $\dot{x}(t) = \tilde{A}x(t) + \tilde{B}u(t)$, where

$$\begin{aligned} x' &= \begin{bmatrix} x_1' & x_2' \end{bmatrix}, u = \begin{bmatrix} u_1 & u_2 \end{bmatrix}', \\ \tilde{A} &= \begin{bmatrix} \tilde{A}_1 & 0 \\ 0 & \tilde{A}_2 \end{bmatrix}, \tilde{B} = \begin{bmatrix} \tilde{B}_1 & 0 \\ 0 & \tilde{B}_2 \end{bmatrix}. \end{aligned}$$

Note that in this paper, $\tilde{A}_1 = \tilde{A}_2$ and $\tilde{B}_1 = \tilde{B}_2$, since the pendula are assumed to be homogeneous. The system can then be rewritten as a discrete system with a time step, $dt =$ 0.01s. Hence, $A = e^{\tilde{A}dt}$ and $B = \int_0^{dt} \tilde{B}e^{\tilde{A}dt}$ giving the discrete system dynamics,

$$x_{k+1} = Ax_k + Bu_k \tag{16}$$

for $x_k \in \mathbb{R}^8$ and $u_k \in \mathbb{R}^2$.

The constraint for this problem is $P_{x,1,N} - P_{x,2,N} = d$, $\dot{P}_{x,1,N} - \dot{P}_{x,2,N} = 0$, $\theta_{1,N} - \theta_{2,N} = 0$, $\dot{\theta}_{1,N} - \dot{\theta}_{2,N} = 0$, i.e. $Mx_N = b$, where

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \end{bmatrix}, b = \begin{bmatrix} d \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

The users input force commands by using keyboard arrow keys to increment or decrement the force in 0.1 N increments with a maximum of 10 N and a minimum of −10 N. The up arrow key will zero the force input. The control law, (11), is applied to (16) given v being the sequence $\{v_k, (v_k - v_{k-1}) + v_k, 2(v_k - v_{k-1}) + v_k, \ldots, (N-1)(v_k - v_{k-1}) + v_k\}$. In other words, we approximate future human commands by linearly extrapolating to time $k + N - 1$.

C. Results

The following simulations were run with parameters: $d = -1\ m$, $l = 0.3\ m$, $m = 2\ kg$, $M = 3\ kg$, $g = 9.8\ m/s^2$, $N = 1.0\ s$, with a sample time of $0.1\ s$.

Figures 4 and 5 contain a set of plots resulting from two human subjects attempting to drive the mass-cart-pendula so that the right most pendulum position was approximately at the $10\ m$ mark.

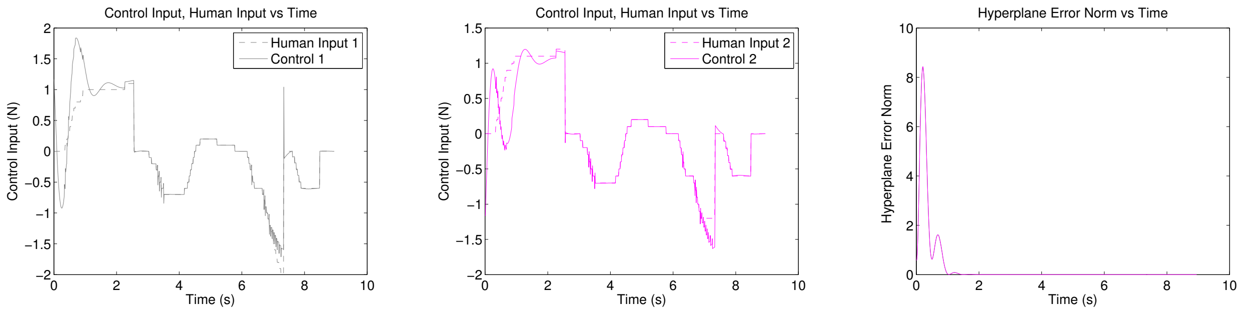
Note in Figures 4(c) and 5, the state converges to the constraint set within $2\ s$. In Figures 4(a) and 4(b), we can see how the control effort deviates from the human control input. In Figure 4(a), the control responds to the input given by Human Operator 2 at the $1\ s$ mark and swings away from the Human Operator 1 input. This of course will be disconcerting for the Human Operator during operation.

In Figure 5(a), the Human Operators were able to drive the right mass-cart-pendulum to the $10\ m$ position. The plots in Figures 5(a)-5(d) show that the system converges to the linear constraint where both pendula oscillations are syncronized and the carts distances are a fixed distance apart. In addition, the human operators can drive the synchronized pendula left or right, albeit not with the immediacy direct control would allow for.

It should be noted that with a longer time horizon, the applied control will more closely match the human input, however, the system will take longer to converge to the constraint set and large changes in human input delay convergence further. In this case, the predicted human input used in the controller is no longer an accurate approximation of the human input over that time.

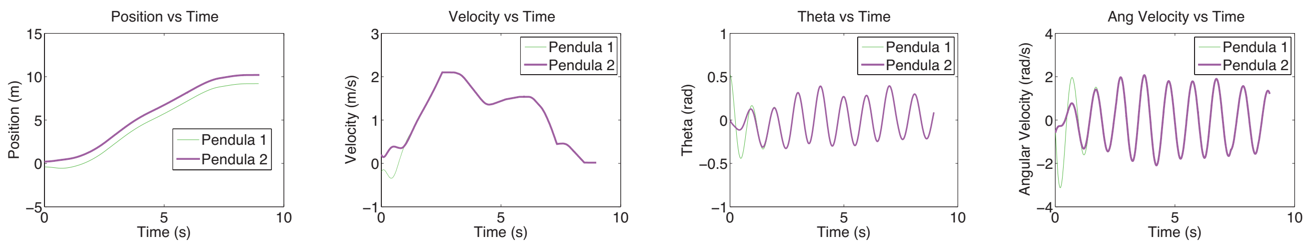
VI. Conclusions

This paper presented a receding horizon optimal controller that integrates human input signals into an automatic control signal such that a discrete linear system was driven to satisfy a state constraint set. The controller still allowed the human operators to control which solution within the constraint set the system drives to. We demonstrated the viability of this control law through a MATLAB simulation where human input was incorporated into a control signal that both synchronized pendula oscillations and allowed the human to drive the coordinated pendula to any cart position desired. Hence, a control theoretic solution to a specific class of mixed initiative human-robot interaction, where one part of the task can be modeled as a linear state constraint for a completely controllable linear system, was presented.



(a) Human 1 input vs Control 1 input. (b) Human 2 input vs Control 2 input. (c) Squared Norm of Error from Constraint Set

Fig. 4. Human Operator Input Pendula Result.



(a) Position. (b) Velocity. (c) Angle. (d) Angular Velocity.

Fig. 5. Human Operator Pendula Result.

Acknowledgments

This work was supported by the US National Science Foundation through Grant # 0820004. The authors wish to thank Amir Rahmani for helpful discussions.

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