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PID type robot joint position regulation with prescribed performance guaranties

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***Abstract*—This paper proposes a PID type regulator that achieves not only the global asymptotic convergence of the robot joint velocities and position errors to zero but it also guarantees a prescribed performance for the position error transient that is independent of system constants and control parameters. The proportional term of the control input uses a transformed error (TP) which incorporates the desired performance function; given sufficiently high proportional and damping gains, the proposed TPID controller ensures the position error's prescribed performance irrespective of constant disturbances and choice of control gains. Control parameter selection is merely confined in achieving admissible input torques. Simulation results for a three dof spatial robot confirm the theoretical analysis and illustrate the robustness of the prescribed performance regulator in case of time-variant bounded disturbances.**

I. Introduction

The stabilization property of PID controllers for robot manipulators has been established since the early '90 through a number of publications that deal solely with the stability problem rather than the system transient performance [1]–[7]. Some works discuss appropriate PID gain tuning procedures that are dependent on some robot prior knowledge but transient performance guaranties are not given. Although it has been early recognized that transient performance guaranties deserves further research, see for example [8], there are in general few theoretical works that study this problem for uncertain nonlinear systems. A sliding mode controller with transient performance guaranties which are dependent on control gains and system constants are proposed for systems in semi-strict feedback form in [9], [10] and applied in robot manipulators in [11]. A control methodology that achieves tracking with prescribed performance for a class of nonlinear systems with known relative degree is proposed in [12] and applied in robots for a combined joint position and velocity tracking error in [13]. Recently, prescribed performance guaranties are achieved in [14] for uncertain MIMO nonlinear systems by introducing an error transformation in a feedback linearizable controller and by exploiting the approximation capabilities of neural networks.

The idea of error transformation drawn from [14] has been exploited in the controllers proposed in [15]–[18]. In fact, prescribed performance adaptive controllers that are model based are proposed in [15], [16] for the robot position/force tracking and in [17] for the robot joint position tracking under bounded disturbances. Last, a PID type robot regulator is proposed in [18] that guarantees prescribed performance of the joint position error transient and a maximum steady state error that cannot however be zero. In this work, we have modified the error transformation and the control input of [18] in order to ensure error asymptotic convergence to zero as well as transient performance guaranties under constant disturbances irrespective of system constants and control parameters.

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II. Problem Description and Preliminaries

Consider a n degrees of freedom robot and let $q \in \Re^n$ be the vector of the generalized joint variables. The dynamic model of the robot is given by the following nonlinear differential equation:

$$H(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) + \delta = u \tag{1}$$

where $H(q) \in \Re^{n\times n}$ is the positive definite robot inertia matrix, $C(q,\dot{q})\dot{q} \in \Re^n$ is the vector of Coriolis and centripetal forces, $g(q)$ is the gravity vector, δ is any constant disturbance and u is the vector of applied torques. Notice that $\dot{H}(q) - 2C(q,\dot{q})$ is a skew-symmetric matrix and $g(q) = \frac{\partial U(q)}{\partial q}$ where $U(q)$ denotes the potential energy due to the gravity field. The aim of this work is to design an input control law that achieves the regulation of the joint position to the desired setpoint position $q_d \in \Re^n$ in terms of asymptotic stability as well as guaranteed prescribed performance for the joint position error $e(t) = q(t) - q_d$.

A. Properties of the Robot Dynamic Model

Some basic properties of the robot dynamic model are given below:

Property 1: The positive definite inertia matrix $H(q)$ satisfies the following inequalities:

$$\lambda_h I \leq H(q) \leq \lambda_H I \tag{2}$$

where $\lambda_H = \max_{q\in\Re^n}\left[\lambda_M\left(H(q)\right)\right]$ and $\lambda_h = \min_{q\in\Re^n}\left[\lambda_m\left(H(q)\right)\right]$ with $\lambda_M(\cdot)$ and $\lambda_m(\cdot)$ denoting the maximum and minimum eigenvalue of a square matrix respectively.

Property 2: There exists a positive constant c_H so that the following inequality holds

$$\|C(q,\dot{q})\dot{q} - \dot{H}(q)\dot{q}\| \leq c_H \|\dot{q}\|^2 \tag{3}$$

where $\|\cdot\|$ denotes the Euclidean norm of a vector and the corresponding induced matrix norm.

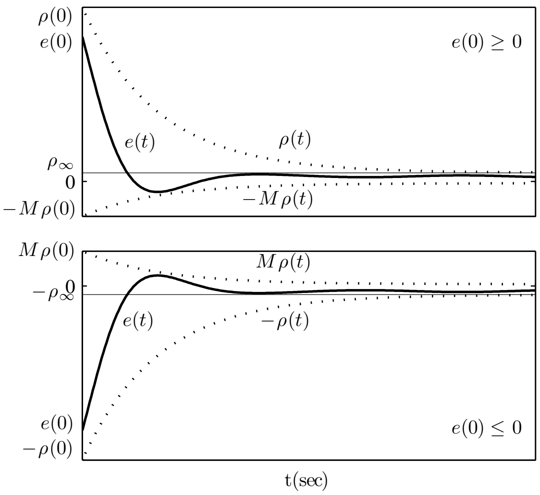

Fig. 1. Performance bounds

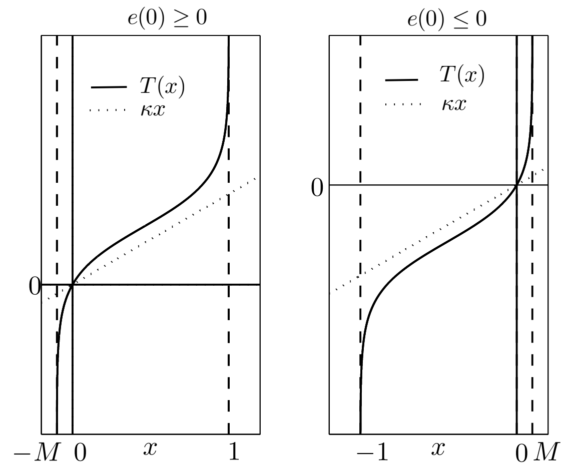

Fig. 2. Error transformation with natural logarithm

Property 3: There exists a positive constant c_g so that the following inequalities simultaneously hold:

$$U(q) - U(q_d) - e^T g(q_d) \geq -c_g \|e\|^2 \tag{4}$$

$$e^T\left(g(q) - g(q_d)\right) \geq -c_g \|e\|^2 \tag{5}$$

B. Prescribed Performance

By prescribed performance, we mean that the minimum speed of convergence, the maximum steady state error and the maximum allowed overshoot are set a priori. In fact, we define the overshoot index M with $0 < M \leq 1$ and a smooth, strictly positive and decreasing function of time $\rho(t)$, called the performance function. An ideal performance function is the exponential function given by:

$$\rho(t) = (\rho_0 - \rho_\infty)\exp(-lt) + \rho_\infty \tag{6}$$

where $\rho_0 = \rho(0)$, l defines the minimum speed of convergence and ρ_∞ is the maximum allowed steady state error that can not be zero. The mathematical expression of the prescribed performance for $e(t)$ is given by the following inequalities:

$$-M\rho(t) < e_i(t) < \rho(t) \quad \forall t \quad \text{in case of} \quad e_i(0) \geq 0 \tag{7}$$

$$-\rho(t) < e_i(t) < M\rho(t) \quad \forall t \quad \text{in case of} \quad e_i(0) \leq 0 \tag{8}$$

where $e_i(t) \in \Re$ denotes the i-th component of the joint position error $e(t)$ and is illustrated in Fig. 1 for a scalar $e(t)$. Notice that $\rho(t)$ and $M\rho(t)$ define the performance bounds within which the error should evolve. It is implied that ρ_0 is chosen so that $\rho_0 > |e_i(0)|$, where $|\cdot|$ denotes the absolute value of a scalar.

To satisfy both regulation and prescribed performance for the joint position error we incorporate an error transformation initially proposed in [14]. More specifically, we define:

$$\varepsilon(t) = T\left(\frac{e(t)}{\rho(t)}\right) \tag{9}$$

where $\varepsilon \in \Re^n$ is the transformed error and $T(\cdot)$ is applied element wise with respect to $e(t)$ and it is a smooth, strictly increasing function defining an onto mapping:

$$T : (-M, 1) \to (-\infty, \infty) \quad \text{in case of} \quad e_i(0) \geq 0$$
$$T : (-1, M) \to (-\infty, \infty) \quad \text{in case of} \quad e_i(0) \leq 0$$

Additionally, we here require $T(0) = 0$ in contrast to [18]; hence, $T(x)$ belongs to the sector $[\kappa, +\infty)$ where κ is a positive constant. The following functions

$$T[x(t)] = \ln\left(\frac{M + x(t)}{M[1 - x(t)]}\right) \text{ in case of } x(0) \geq 0 \tag{10}$$

$$T[x(t)] = \ln\left(\frac{M[1 + x(t)]}{M - x(t)}\right) \text{ in case of } x(0) \leq 0 \tag{11}$$

possess all required properties and thus may be used in (9). A graphical illustration of these function is provided in Fig. 2. Notice that the inverse transformation $T^{-1}(\cdot)$ exists and is bounded as follows: $|T^{-1}(\cdot)| < 1$. Also notice that M and ρ_0 can not be set equal to zero and $|e_i(0)|$ in order to avoid infinite initial values of the transformed error ε. Clearly, owing to the properties of the error transformation, we satisfy the prescribed performance (7), (8) for the position error transient, by keeping ε bounded. Notice that the magnitude of this bound is not related to the evolution of the tracking error $e(t)$ that is solely defined by (7) and (8). The controller should therefore be designed to guarantee the boundedness of ε in order to achieve the prescribed performance.

The derivative of the transformed error can be calculated as follows:

$$\dot{\varepsilon} = \partial T \left[\dot{q} + \alpha(t) e\right] \tag{12}$$

$$\text{where} \quad \partial T \triangleq \frac{\partial T}{\partial (e/\rho)} \frac{1}{\rho} > 0 \quad \text{a diagonal matrix} \tag{13}$$

$$\text{and} \quad \alpha(t) \triangleq -\frac{\dot{\rho}(t)}{\rho(t)} \quad \text{with} \quad \dot{\alpha}(t) = -\frac{\ddot{\rho}(t)}{\rho(t)} + \alpha^2(t) \tag{14}$$

The positive diagonal matrix ∂T can be regarded as a normalized Jacobian of the transformed error (9) over the performance function. Notice that ∂T^{-1} is bounded.

Using (6) and (14) we can easily prove that: i) $\alpha(t)$ is a bounded positive function with $\lim_{t\to+\infty} \alpha(t) = 0$, ii) $\dot{\alpha}(t)$ is a bounded negative function with $\lim_{t\to+\infty} \dot{\alpha}(t) = 0$, as well as:

$$|\dot{\rho}(t)| \leq l(\rho_0 - \rho_\infty) \tag{15}$$

$$\alpha(t) \leq \alpha(0) < l \tag{16}$$

$$|\dot{\alpha}(t)| < \frac{l^2}{4} \tag{17}$$

Using (10), (11) the following inequalities can be easily proved:

$$|\varepsilon_i| > \frac{4}{(M+1)\rho(t)} |e_i| \tag{18}$$

$$\partial T_i > \frac{4}{(M+1)\rho(t)} \tag{19}$$

where ε_i and ∂T_i denote the i-th element of ε and of the diagonal matrix ∂T respectively. Using (18) and/or (19) the following inequalities can be proved:

$$e^T \partial T \varepsilon \geq c\|e\|^2 \tag{20}$$

$$\|\varepsilon\|^2 \geq c\|e\|^2 \tag{21}$$

where $c = \left(\frac{4}{\rho_0(M+1)}\right)^2$.

III. Controller Design

We propose the following independent joint controller:

$$u = -K_v \dot{q} - K_\varepsilon \partial T \varepsilon(t) - K_I \int_0^t y(\tau) d\tau \tag{22}$$

where K_v, K_ε, K_I are positive definite diagonal gain matrices, ε and ∂T are defined by (9) and (13) respectively, and y is given by:

$$y = \dot{q} + (\alpha(t) + \beta) e \tag{23}$$

where β is a positive control constant. Notice that comparing with the conventional PID global asymptotically stable controller [3], the proportional term in (22) involves the transformed error ε instead of the actual error and a time varying gain $K_\varepsilon \partial T$ while the output vector y is enriched with the term $\alpha(t)e$ that vanishes at infinity.

Substituting the input control law (22) into the robot dynamic equation (1) and adding and subtracting the gravity vector at the desired target i.e. $g(q_d)$ we can write the closed loop system as follows:

$$H(q)\ddot{q} + C(q,\dot{q})\dot{q} + K_v \dot{q} + K_\varepsilon \partial T \varepsilon + [g(q) - g(q_d)] + K_I z(t) = 0 \tag{24}$$

where

$$z(t) = \int_0^t y(\tau) d\tau + K_I^{-1} \left(g(q_d) + \delta\right) \text{ with } \dot{z}(t) = y(t) \tag{25}$$

Taking the inner product of the closed loop system (24) with y given by (23) we get:

$$\frac{dV}{dt} + W = 0 \tag{26}$$

where

$$\begin{aligned} V =& \frac{1}{2}\dot{q}^T H(q)\dot{q} + [\alpha(t) + \beta]\, e^T H(q)\dot{q} \\ &+ \frac{1}{2}[\alpha(t) + \beta] e^T K_v e + \frac{1}{2} z^T K_I z \\ &+ \frac{1}{2}\varepsilon^T K_\varepsilon \varepsilon + U(q) - U(q_d) - e^T g(q_d) \end{aligned} \tag{27}$$

and

$$\begin{aligned} W =& \dot{q}^T \left[K_v - (\alpha(t) + \beta) H(q)\right] \dot{q} \\ &+ (\alpha(t) + \beta) e^T [C(q,\dot{q})\dot{q} - \dot{H}(q)\dot{q}] \\ &+ (\alpha(t) + \beta)\, e^T \left[g(q) - g(q_d)\right] + \beta e^T K_\varepsilon \partial T \varepsilon \\ &- \dot{\alpha}(t) e^T H(q)\dot{q} - \dot{\alpha}(t)\frac{1}{2} e^T K_v e \end{aligned} \tag{28}$$

Notice that the first two terms of V can be written as follows:

$$\begin{aligned} &\frac{1}{2}\dot{q}^T H(q)\dot{q} + [\alpha(t) + \beta]\, e^T H(q)\dot{q} = \\ &\quad \frac{1}{4}\left[\dot{q} + 2(\alpha(t) + \beta)e\right]^T H(q)[\dot{q} + 2(\alpha(t) + \beta)e] \\ &\quad - (\alpha(t) + \beta)^2 e^T H(q) e + \frac{1}{4}\dot{q}^T H(q)\dot{q} \end{aligned} \tag{29}$$

Splitting the term $\frac{1}{2}\varepsilon^T K_\varepsilon \varepsilon$ of (27) into two equal parts and using (4), (21) and (29), we can lower bound V as follows:

$$\begin{aligned} V \geq& \frac{1}{4}\dot{q}^T H(q)\dot{q} + \frac{1}{4}\varepsilon^T K_\varepsilon \varepsilon + \frac{1}{2} z^T K_I z \\ &+ \frac{1}{2}(\alpha(t) + \beta)\, e^T \left[K_v - 2(\alpha(t) + \beta)\lambda_H I_n\right] e \\ &+ e^T \left(\frac{c}{4} K_\varepsilon - c_g I_n\right) e \end{aligned} \tag{30}$$

Function V is positive definite with respect to $\dot{q}$, ε and z if K_v and K_ε gains are chosen sufficiently high to guarantee the following inequality:

$$K_v \geq 2(l + \beta)\lambda_H I_n \tag{31}$$

$$K_\varepsilon \geq \frac{4}{c} c_g I_n \tag{32}$$

Examining the sign of W, notice that when the error stays within the performance bounds, the second line of (28) can be written as follows:

$$[-\dot{\rho}(t) + \beta\rho(t)] \left(T^{-1}(\varepsilon)\right)^T [C(q,\dot{q})\dot{q} - \dot{H}(q)\dot{q}]. \tag{33}$$

Using the bound of the inverse transformation i.e. $\|T^{-1}(\varepsilon)\| \leq \sqrt{n}$, the Properties 1-3 as well the inequalities (20) and $-\dot{\alpha}(t)e^T H(q)\dot{q} \geq -|\dot{\alpha}(t)|\lambda_H \left(\frac{\|e\|^2}{2\xi} + \frac{\xi}{2}\|\dot{q}\|^2\right)$ for some positive ξ into (28), we can lower bound W as follows:

$$W \geq \dot{q}^T \left(K_v - d(t)I_n\right)\dot{q} + e^T\left[\beta c K_\varepsilon - (\alpha(t)+\beta)c_g I_n\right]e + \frac{1}{2}|\dot{\alpha}(t)|e^T\left(K_v - \lambda_H \xi^{-1} I_n\right)e \quad (34)$$

where

$$d(t) = (\alpha(t)+\beta)\lambda_H + (|\dot{\rho}(t)| + \beta\rho(t))c_H\sqrt{n} + \lambda_H|\dot{\alpha}(t)|\tfrac{\xi}{2} \quad (35)$$

is a positive function that can be upper bounded by using $\rho(t) < \rho_0$, (15)-(17) and setting $\xi = \frac{8(l+\beta)}{l^2}$ in order to simplify the results, as follows:

$$d(t) < 2(l+\beta)\lambda_H + l(\rho_0 - \rho_\infty)c_H\sqrt{n} + \beta\rho_0 c_H\sqrt{n}$$

Hence W is positive definite with respect to $\dot{q}$ and e provided that the following conditions hold:

$$K_v > \left\{2(l+\beta)\lambda_H + \left[l(\rho_0-\rho_\infty)+\beta\rho_0\right]c_H\sqrt{n}\right\}I_n \quad (36)$$

$$K_v > \frac{\lambda_H l^2}{8(\lambda+\beta)}I_n \quad (37)$$

$$K_\varepsilon > \frac{l+\beta}{c\beta}c_g I_n \quad (38)$$

Condition (36) is dominant with respect to (31) and (37); in particular, condition (31) is more restrictive than (37) since $l^2 < 16(l+\beta)^2$ and condition (36) is clearly more restrictive than (31). Furthermore, for compact presentation we can combine (32) and (38) into the following condition:

$$K_\varepsilon > \max\left[4, \beta^{-1}(l+\beta)\right]c_g c^{-1} I_n \quad (39)$$

If (36) and (39) hold, then function $V(\dot{q}, z, \varepsilon, e)$ is globally positive definite while its derivative $\frac{dV}{dt}$ is non-positive when $|e_i(t)| \leq \rho(t)$; hence, $V \leq V(0)$ which implies $V(t) \in \mathcal{L}_\infty$.

It is possible to prove by contradiction that $|e_i(t)| \leq \rho(t)\ \forall\ t \in \Re^+$, given $|e_i(0)| < \rho_0$. Let us assume that there exists a time instant t_b at which the solution $e_i(t)$ reaches the performance bounds and $\varepsilon(t_b)$ goes to infinity as implied by (10), (11). On the other hand, the continuity of the solution implies that during this time interval $\dot{V}(t) \leq 0$ and subsequently $V(t_b) \leq V(0)$ that in turn implies $\varepsilon(t_b)$ is bounded which contradicts $\varepsilon(t_b)$ is infinite. Hence, there exists no t_b for which $e_i(t)$ will cross the performance bound and thus V can be used as a Lyapunov-like function in order to prove the following global result:

Theorem 1: The control law (22) applied to system (1) achieves the global asymptotic convergence of joint velocity and position error to zero with guaranteed position error prescribed performance given by (7), (8) provided that the controller gains K_v and K_ε satisfy (36) and (39) for a choice of β.

Proof: Since $\frac{dV}{dt} = -W \leq 0$, $V(\dot{q}, z, \varepsilon, e) \leq V(0)$ and consequently $\dot{q},\ z,\ e,\ \varepsilon \in \mathcal{L}_\infty$. The boundedness of ε implies that e is bounded by the performance function according to (7), (8) as well as that ∂T is bounded. Since $\dot{q},\ z,\ e,\ \varepsilon \in \mathcal{L}_\infty$, from (24) we get $\ddot{q} \in \mathcal{L}_\infty$. The boundedness of $\ddot{q}$, and $\dot{q}$ implies that $\dot{q}$ and e are uniformly continuous. Furthermore, from (36), given (39) and (34), there exist positive constants γ_1 and γ_2 such that $W \geq \gamma_1\|\dot{q}\|^2 + \gamma_2\|e\|^2$ and hence by integrating $\frac{dV}{dt} \leq -\gamma_1\|\dot{q}\|^2 - \gamma_2\|e\|^2$ along the time interval $[0, +\infty)$ we get:

$$V(0) - V(+\infty) \geq \gamma_1\int_0^{+\infty}\|\dot{q}\|^2 d\tau + \gamma_2\int_0^{+\infty}\|e\|^2 d\tau$$

that clearly implies: $\dot{q}$, $e \in \mathcal{L}_2$. Since $\dot{q}$, e are uniformly continuous and belong to the $\mathcal{L}_2$ space, it follows from Desoer and Vidyasagar (1975) that $\dot{q} \to 0$ and $e \to 0$. ■

Alternatively, it is possible to use the integral of the joint error e instead of y in the control law (22) and to prove the following theorem:

Theorem 2: Consider the control input:

$$u = -K_v\dot{q} - K_\varepsilon \partial T \varepsilon(t) - K_I\int_0^t e(\tau)d\tau \quad (40)$$

and the closed loop system (24) where $z(t)$ is given by:

$$z(t) = \int_0^t e(\tau)d\tau + K_I^{-1}\left(g(q_d)+\delta\right) \text{ with } \dot{z}(t) = e(t) \quad (41)$$

and K_v, K_ε and K_I are positive definite diagonal matrices. If the control gains are chosen in order to satisfy condition (36) and:

$$K_\varepsilon > \max\left[4, \tfrac{16}{15}\beta^{-1}(l+\beta)\right]c_g c^{-1} I_n \quad (42)$$

$$K_I < \frac{\beta c}{16}K_\varepsilon \quad (43)$$

with β being a free design positive constant, then global asymptotical convergence of the joint velocity and position error to zero with guaranteed position error prescribed performance given by (7), (8) is achieved.

In fact, the inner product of y (23) with (24) yields $\frac{dV_e}{dt} + W_e = 0$ where:

$$V_e = V + e^T K_I z \quad (44)$$

$$W_e = W - e^T K_I e^T - \frac{1}{2}\dot{\alpha}(t) z^T K_I z \quad (45)$$

with W given by (28) and V having the form of (27) with the quadratic term of z (41) being replaced by $\frac{\alpha(t)+\beta}{2}z^T K_I z$. Following the same procedure as before, V_e and W_e can be lower bounded as follows:

$$V_e \geq \frac{1}{4}\dot{q}^T H(q)\dot{q} + e^T\left(\frac{c}{4}K_\varepsilon - c_g I_n\right)e + \frac{1}{2}(\alpha(t)+\beta)\,e^T\left[K_v - 2(\alpha(t)+\beta)\lambda_H I_n\right]e + \frac{1}{8}\varepsilon^T K_\varepsilon \varepsilon + \frac{1}{8}\begin{bmatrix} e \\ z \end{bmatrix}^T \begin{bmatrix} cK_\varepsilon & 8K_I \\ 8K_I & 4K_I\beta \end{bmatrix}\begin{bmatrix} e \\ z \end{bmatrix} \quad (46)$$

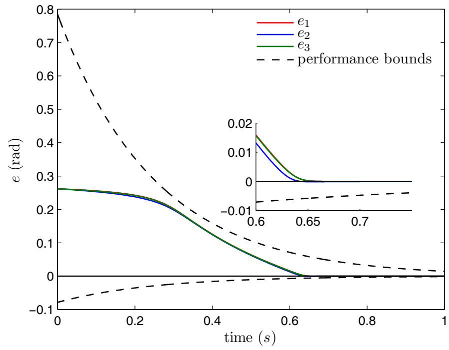

Fig. 3. Position error responses

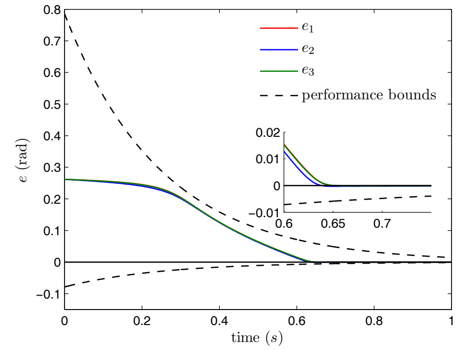

Fig. 6. Position error responses in case of disturbance

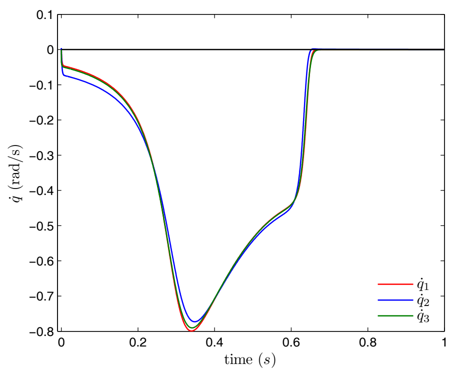

Fig. 4. Joint velocity responses

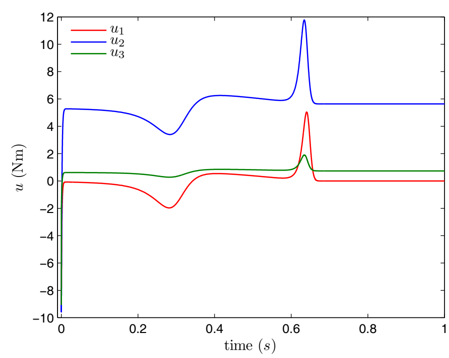

Fig. 5. Input torque responses in case

$$W_e \geq \dot{q}^T\big(K_v - d(t)I_n\big)\dot{q} + e^T\big[\tfrac{15}{16}\beta c K_\varepsilon - (\alpha(t)+\beta)c_g I_n\big]e + \frac{1}{2}|\dot{\alpha}(t)|e^T\left(K_v - \lambda_H \xi^{-1} I_n\right)e + e^T\big(\tfrac{1}{16}c\beta K_\varepsilon - K_I\big)e \tag{47}$$

Function V_e is positive definite and W_e is positive definite with respect to $\dot{q}$ and e provided that conditions (36), (42) hold and Theorem 2 can be proved following the proof of Theorem 1.

Setting a priori the prescribed performance indices, gain tuning in both controllers (22), (40) is merely confined to ensure that (36) and (39) or (42), (43) hold and that input torques are admissible. The choice of gains in order to satisfy (36) and (39) or (42), (43) is dependent on the approximate knowledge of the characteristic constants of the robot dynamic model λ_H, c_H and c_g as well as the known values of prescribed performance parameters ρ_0, l and M.

Notice that the transformed error used in the proposed controllers (22), (40) is derived by a modified transformation function that maps zero position errors to zero transformed errors as compared to that used in [18] and thus ensures that the joint position error asymptotically vanishes. However, this modified transformation restricts the prescribed overshoot

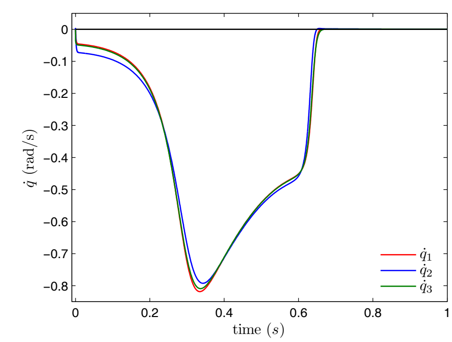

Fig. 7. Joint velocity responses in case of disturbance

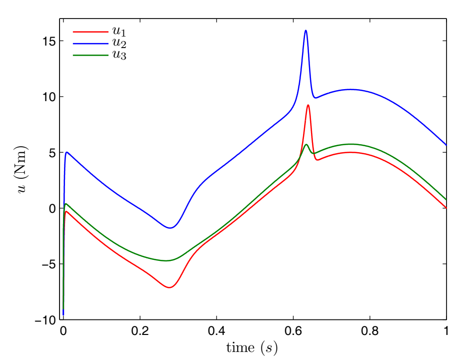

Fig. 8. Input torque responses in case of disturbance

index to arbitrarily small but non zero values. In [18] zero overshoot was possible but a positive although arbitrarily small maximum steady state error could only be guaranteed.

IV. Simulation Results

We consider the spatial 3 dof robotic arm with masses $m_1 = m_2 = 1$ kg$, m_3 = 0.5$ kg, lengths $l_2 = 1$ m$, l_3 = 0.5$ m, inertias $I_{z1} = 4.15\ 10^{-4}$ kgm^2, $I_{y2} = I_{z2} = 0.021$ kgm^2, $I_{y3} = I_{z3} = 0.0039$ kgm^2. The initial position of the joints is 15 deg and the desired setpoint is 0 deg. Hence the initial position error for each joint is $e_i(0) = 0.262$ (rad). The performance function is chosen as follows: $\rho(t) = (0.785 - 10^{-4})e^{-4t} + 10^{-4}$. An overshoot index $M = 0.1$ is chosen. Such a performance function implies a 1 s settling time and a maximum steady state error of 10^{-4} rad. We simulate the closed loop system using the following control gains' values: $K_v = 200I_3$, $K_\varepsilon = I_3$, $K_I = 0.5I_3$ and $\beta = 0.5$. The simulation results are shown in Figs. 3-5. Notice that the joint position errors stay within the prescribed position bounds and asymptotically vanish in less than 1 sec (Fig. 3) while velocities converge to zero in the same time period (Fig. 4). The desired goals have been achieved by admissible control inputs shown in Fig. 5. We further examine the robustness of the controller in time-dependent bounded sinusoidal disturbances by using $\delta(t) = 5\sin(2\pi t)[1\ 1\ 1]^T$; the results are given in Figs. 6-8. From Figs. 6 and 7, it is clear that both position errors and velocities are slightly affected by the introduction of the disturbance input. Notice how input torques (Fig. 8) adopt to disturbance in order to satisfy the prescribed performance of the position error and convergence of the velocities to zero. The simulation results of the closed loop system with $z(t) = e(t)$ are slightly affected as compared to the previously presented and hence their presentation is omitted.

V. Conclusions

This work proposes PID type robot joint controllers that use an appropriately transformed joint position error. The asymptotic convergence to zero and guaranties on prescribed performance transients for position errors has been proved under constant disturbances given that the gains exceed a minimum value that is system and performance related. In particular, the joint position error is guaranteed to evolve within the prescribed performance bounds irrespective of the values of robot parameters and gains. Performance bounds are constructed by a priory setting the values for the desired maximum overshoot and minimum speed of convergence. Simulation results with the proposed controllers confirm the theoretical findings and demonstrate their robustness in case of time-dependent input disturbances.

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