

Cooperative Control Synthesis for Moving-Target-Enclosing with Changing Topologies

Jing Guo Gangfeng Yan Zhiyun Lin

Abstract—A moving-target-enclosing problem is investigated in the paper, where the velocity of the target is unknown and the neighbor topologies may change over time. Each robot only uses the relative position information of the target and its neighbors that may dynamically change over time. An adaptive scheme is proposed to estimate the velocity of the target. Then a distributed control law for each robot is presented, which consists of two parts: One amounts to ensuring the convergence of the distance between the robots and the target to the desired one and the other is used to achieve the uniform distribution when enclosing the target in motion. Lyapunov-based techniques and graph theory are brought together for rigorous analysis of the convergence and stability properties. Our control strategy is practically implementable with only onboard sensors. Simulations are provided to illustrate our results.

I. INTRODUCTION

Multi-robot systems have received considerable attention in the fields of robotics and system control in recent years. A group of autonomous mobile robots are capable of accomplishing certain missions which are difficult or time consuming for a single robot. The problems such as formation control [1]–[3], rendezvous [4]–[6], consensus [7], [8], and target-enclosing [9]–[11] are widely studied. Cooperative target-enclosing as one of the common problems in multi-robot systems requires the group members to follow the target and surround it. Sometimes, the group members are also demanded to keep nominated distances or form a formation with other group-mates. Therefore, the solution to the problem of cooperative target enclosing can be used to entrap or attack a target [12] by reducing the escape windows or protect an object by blocking the intrusion ways of adversary agents [13], [14].

With the objective of moving unmanned aerial vehicles (UAVs) scattered in an environment in the presence of obstacles towards a target of interest while avoiding collision with other UAVs and obstacles and rotating around the target, the problem of close target reconnaissance is studied in [9] by forming an equilateral triangle. Under the assumption that a group of robots can be identified, algorithms based on cyclic pursuit strategies are developed to solve the target-enclosing problem in [10] and [15], respectively. In [11] Kobayashi et al. divide the task of capturing a target into an enclosing behavior and a grasping behavior. In addition, in [16] Lan et al. assume that the target is static and propose a novel approach based on hybrid control for cooperative target

tracking with multiple unicycle-type robots, and in [17] Gazi and Ordóñez develop an algorithm for capturing a moving target via sliding mode control.

In this paper, we consider a target moving in the plane with a piecewise constant velocity. The objective is to find a distributed control law for a group of robots so that they enclose the moving target and meanwhile attain an inter-robot formation. In our setup, no robot knows the moving velocity of the target, but it is assumed that every robot is able to measure the position of the target in its own local frame, as well as the positions of its neighbors, who are defined in the sense that there is no other robots between them by rotating a ray (originating from the target) from one to the other clockwise or counterclockwise. Thus, the neighbor relationships may change over time depending on the positions of the robots and the target. We propose a control law consisting of two parts: One deals with the task of following the target with a specific distance and the other addresses the task of rotating around the target and achieving equal distances from its neighbors. In addition, since the target's velocity information is unknown to any robots, an adaptive scheme is designed so that each robot can reconstruct the target's velocity with the error between the estimation and the velocity of the target converging to zero. The adaptive scheme is preferable to a simple differentiation filter to estimate the target's velocity as no absolute position of the target is available in an inertial coordinate frame. Lyapunov-based techniques and graph theory are brought together for rigorous analysis of the convergence and stability properties. Our results extend the work of [10] and [11] in the following aspects. First, the target is not stationary and its moving velocity is unavailable to any robots. Second, it is not required to label the robots and changing topologies are allowed. Third, the control law is practically implementable with only onboard sensors and the robots do not need a common sense of direction. Fourth, the adaptive strategy we propose assures that the robots are able to entrap the target adaptively even though the target has an abrupt change on its velocity.

Throughout the paper, we use the notation $\mathbf{1}$ to represent the vector with all 1 components and we use the notation \otimes to denote the Kronecker product.

II. PROBLEM STATEMENT

Consider a group of n identical mobile robots which are modeled as point-masses freely moving in the plane, i.e.,

$$\dot{p}_i = u_i, \quad i = 1, \dots, n, \quad (1)$$

The authors are with the Department of Systems Science and Engineering, Zhejiang University, 38 Zheda Road, Hangzhou, 310027 P. R. China.

The work is supported by National Natural Science Foundation of China under Grant 60875074.

where $p_i \in \mathbb{R}^2$ is the position of robot i in the plane and $u_i \in \mathbb{R}^2$ is the velocity control input of robot i . Suppose now that a target moves in the plane with a piecewise constant velocity. That is,

$$\dot{p}_0 = v_0,$$

where p_0 is its position in the plane and v_0 is a piecewise constant.

Suppose that every robot carries an onboard sensor so that it is able to sense the relative positions of the target and its neighbors. However, the velocity of the target is not known to any robots. With these local measured information, it is expected that all robots can follow and enclose the target with evenly spaced circle formation. The problem is called the *moving-target-enclosing* problem in the paper and stated formally as follows:

Problem 2.1: Find a distributed control law for each robot using only local available information such that

- 1) each robot follows and maintains a desired distance d with the target,
- 2) all the robots are evenly spaced when enclosing the target.

A desired configuration is given in Fig. (1).

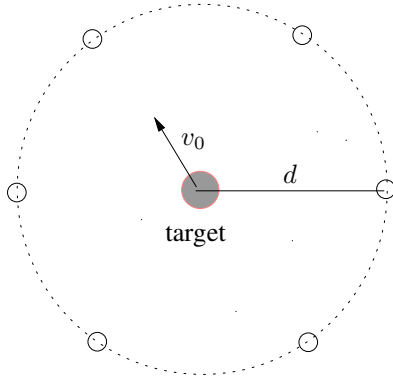


Fig. 1. Desired enclosing configuration.

III. CONTROL SYNTHESIS FOR MOVING-TARGET-ENCLOSING

In this section, we develop a control strategy for the moving-target-enclosing problem and then provide rigorous analysis for convergence.

As described in Problem 2.1, two specifications should be accomplished. We consider a control law having two terms, that is,

$$u_i = u'_i + u''_i.$$

Moreover, we consider the relative position of robot i in the local frame attached to the target, so we define $z_i = p_i - p_0$. Thus, we have

$$\dot{z}_i = u'_i + u''_i - v_0, \quad i = 1, \dots, n. \quad (2)$$

First, let us present a trivial observation in the following lemma, which will be used to design the first term u'_i in our controller.

Lemma 3.1: If $\dot{z}_i = -z_i(\|z_i\|^2 - d^2)$, then $\|z_i(t)\|$ tends to d as $t \rightarrow \infty$ for any $z_i(0) \neq 0$.

Proof: Let $\rho_i = \|z_i\|$. It can be easily obtained that

$$\dot{\rho}_i = -\rho_i(\rho_i^2 - d^2).$$

As this is a scalar differential equation, it is trivial to conclude that $\rho_i(t)$ tends to d for any initial condition $\rho_i(0) \neq 0$. ■

Lemma 3.1 suggests that when the target is stationary, we can select $u_i = -z_i(\|z_i\|^2 - d^2)$ such that every robot can eventually attain the desired distance d with the target. Hence, for our moving-target-enclosing problem, we choose the first term u'_i in our controller as follows:

$$u'_i = -z_i(\|z_i\|^2 - d^2), \quad i = 1, \dots, n, \quad (3)$$

Second, considering the form of (2), we propose that $u''_i - v_0$ should be orthogonal to u'_i in order not to affect the convergence of attaining the desired distance d when u'_i is applied (see Fig. 2). Now let us consider the polar

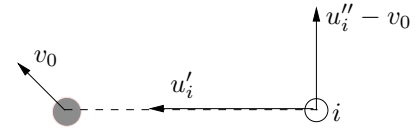


Fig. 2. Illustration for control decoupling.

coordinates of z_i . Denote $z_i = [z_i^x \ z_i^y]^T$. Then we define

$$\rho_i = \sqrt{(z_i^x)^2 + (z_i^y)^2} \text{ and } \theta_i = \arctan2(z_i^y, z_i^x),$$

where $\arctan2$ represents a two-argument arctangent function returning the angle of point z_i as a numeric value between 0 and 2π radians. Let

$$\mathbf{r}_i = \begin{bmatrix} \cos \theta_i \\ \sin \theta_i \end{bmatrix} \text{ and } \mathbf{s}_i = \begin{bmatrix} -\sin \theta_i \\ \cos \theta_i \end{bmatrix}.$$

It can be noted that \mathbf{s}_i is formed by counterclockwise rotating \mathbf{r}_i of $\pi/2$ radians and so it is orthogonal to \mathbf{r}_i . With the polar coordinates and these new notations of \mathbf{r}_i and \mathbf{s}_i , the formula of u'_i can be written as

$$u'_i = -\rho_i(\rho_i^2 - d^2)\mathbf{r}_i.$$

In order to make $u''_i - v_0$ orthogonal to u'_i and in order to achieve a uniform distribution when enclosing the target, we expect to have

$$u''_i - v_0 = \rho_i(\theta_{i_+} - \theta_i + \zeta_i)\mathbf{s}_i, \quad (4)$$

where

$$\zeta_i = \begin{cases} 0 & \text{if } \theta_{i_+} - \theta_i \geq 0, \\ 2\pi & \text{if } \theta_{i_+} - \theta_i < 0. \end{cases}$$

In the equation above, the subscript i_+ denotes the label of the *pre-neighbor* of robot i according to the following criteria:

- 1) If no other robot locates on the same ray originating at the target pointing towards robot i , then the pre-

neighbor of robot i is defined to be the one nearest to the target and first met by rotating the ray counter-clockwise;

- 2) If there are other robot locating on the same ray originating at the target pointing towards robot i , then the pre-neighbor of robot i is the one on the same ray closest to robot i but having longer distance to the target than robot i .

The pre-neighbor of robot i is denoted by i_+ . If robot j is a pre-neighbor of robot i (namely, $i_+ = j$), we also call robot i the *next-neighbor* of robot j and denote it by j_- . An illustration is given in Fig. 3. For example, the pre-neighbor

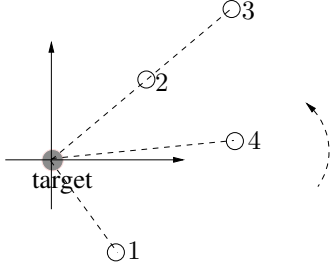


Fig. 3. Changing neighbor topology.

of robot 4 is robot 2, the pre-neighbor of robot 2 is robot 3, the pre-neighbor of robot 3 is robot 1, and the pre-neighbor of robot 1 comes back to be robot 4. By this definition, the neighboring topology is a unidirectional ring and we call it *interaction graph*. Note that this topology may dynamically change over time as robots move around. So we use L_t to denote the Laplacian of the interaction graph which is actually a piecewise constant matrix. It is worth to point out here that since at any time t , the interaction graph is always a unidirectional ring, the following holds:

- 1) $\mathbf{1}^T L_t = 0$;
- 2) $L_t^T + L_t$ has a simple eigenvalue at 0 with an associated eigenvector $\mathbf{1}$ and all other eigenvalues are positive.

After defining the neighboring topology and graph Laplacian, we come back to check the feasibility of implementing (4). Notice that to satisfy (4), the second term u_i'' should equal to $v_0 + \rho_i(\theta_{i_+} - \theta_i + \zeta_i)\mathbf{s}_i$, which means each robot should access the velocity of the target. However, the target's velocity v_0 is unknown to robots, so we introduce a new variable v_i for each robot i and construct an adaptive control so that v_i can adaptively converge to v_0 . Thus, we take u_i'' of the form

$$u_i'' = \rho_i(\theta_{i_+} - \theta_i + \zeta_i)\mathbf{s}_i + v_i, \quad i = 1, \dots, n, \quad (5)$$

where the dynamics of v_i will be designed later.

Considering u_i' in (3) and u_i'' in (5), we can write the dynamics of ρ_i and θ_i as

$$\begin{cases} \dot{\rho}_i &= -\rho_i(\rho_i^2 - d^2) + \mathbf{r}_i^T(v_i - v_0), \\ \dot{\theta}_i &= (\theta_{i_+} - \theta_i + \zeta_i) + \frac{1}{\rho_i}\mathbf{s}_i^T(v_i - v_0). \end{cases} \quad (6)$$

Let

$$\begin{aligned} \rho &= [\rho_1, \dots, \rho_n]^T \in \mathbb{R}^n, \\ \theta &= [\theta_1, \dots, \theta_n]^T \in \mathbb{R}^n, \\ v &= [v_1^T, \dots, v_n^T]^T \in \mathbb{R}^{2n}. \end{aligned} \quad (7)$$

Moreover, we define

$$D = \begin{bmatrix} \rho_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \rho_n \end{bmatrix}, \quad h(\rho) = \begin{bmatrix} \rho_1^2 - d^2 \\ \vdots \\ \rho_n^2 - d^2 \end{bmatrix},$$

$$R = \begin{bmatrix} \mathbf{r}_1^T & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \mathbf{r}_n^T \end{bmatrix} \in \mathbb{R}^{n \times 2n},$$

and

$$S = \begin{bmatrix} \mathbf{s}_1^T & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \mathbf{s}_n^T \end{bmatrix} \in \mathbb{R}^{n \times 2n}.$$

Finally, we let $e = v - \mathbf{1} \otimes v_0$ and let $\zeta = [\zeta_1, \dots, \zeta_n]^T$. According to our setup and the definition of ζ_i , we know that there is only one ζ_i equal to 2π and all others are 0, which means $\mathbf{1}^T \zeta = 2\pi$. Then we write system (6) in a vector form, which is

$$\begin{cases} \dot{\rho} = -Dh(\rho) + Re, \\ \dot{\theta} = -L_t\theta + \zeta + D^{-1}Se. \end{cases} \quad (8)$$

Next, we adopt a controlled Lyapunov function approach to find the dynamics for v_i . Let us consider the following Lyapunov function candidate

$$V = \frac{1}{4}h(\rho)^T h(\rho) + \frac{1}{2}(-L_t\theta + \zeta)^T (-L_t\theta + \zeta) + \frac{1}{2}e^T e.$$

From the definition, we know that the function V is continuous, but not continuously differentiable everywhere. However, it is piecewise continuously differentiable. Hence, in what follows, we use the right upper Dini derivative D^+V for our analysis. More details can be found in [18]. Let t^+ represent the time instant just after the switching at t . Then, along the solution of system (8), one obtains

$$\begin{aligned} D^+V &= \frac{1}{2}h(\rho)^T \frac{\partial h}{\partial \rho} \dot{\rho} - (-L_{t^+}\theta + \zeta)^T L_{t^+} \dot{\theta} + \dot{e}^T e \\ &= h(\rho)^T D[-Dh(\rho) + Re] + \dot{e}^T e \\ &\quad - (-L_{t^+}\theta + \zeta)^T [L_{t^+}(-L_{t^+}\theta + \zeta + D^{-1}Se)] \\ &= -h(\rho)^T D^2 h(\rho) \\ &\quad - \frac{1}{2}(-L_{t^+}\theta + \zeta)^T (L_{t^+} + L_{t^+}^T)(-L_{t^+}\theta + \zeta) \\ &\quad + [h(\rho)^T DR - (-L_{t^+}\theta + \zeta)^T L_{t^+} D^{-1}S + \dot{e}^T] e. \end{aligned}$$

Note that $\dot{v} = \dot{e}$. So we set

$$\dot{v} = -R^T Dh(\rho) + S^T D^{-1} L_{t^+}^T (-L_{t^+}\theta + \zeta)$$

so that $h(\rho)^T DR - (-L_{t^+}\theta + \zeta)^T L_{t^+} D^{-1}S + \dot{e}^T = 0$. In

other words,

$$\dot{v}_i = -\rho_i(\rho_i^2 - d^2)\mathbf{r}_i - \frac{1}{\rho_i}\varphi_i\mathbf{s}_i,$$

where $\varphi_i = (\theta_i - \theta_{i-} + \zeta_{i-}) - (\theta_{i+} - \theta_i + \zeta_i)$.

Thus, the complete adaptive control law for each robot i is given as

$$\begin{cases} u_i = -\rho_i(\rho_i^2 - d^2)\mathbf{r}_i + \rho_i(\theta_{i+} - \theta_i + \zeta_i)\mathbf{s}_i + v_i, \\ \dot{v}_i = -\rho_i(\rho_i^2 - d^2)\mathbf{r}_i - \frac{1}{\rho_i}\varphi_i\mathbf{s}_i. \end{cases} \quad (9)$$

Remark 3.1: Note that in (9), ρ_i is the distance from robot i to the target, $(\theta_{i+} - \theta_i + \zeta_i)$ is the angle between robot i and its pre-neighbor, $(\theta_i - \theta_{i-} + \zeta_{i-})$ is the angle between robot i and its next-neighbor, \mathbf{r}_i is the direction towards the target and \mathbf{s}_i is the perpendicular direction (see Fig. 4 for an illustration). All these information can be measured by an onboard sensor (e.g., camera) attached to robot i . Therefore,

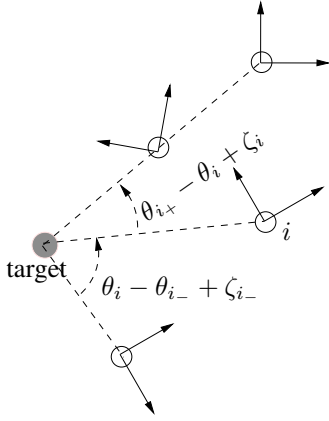


Fig. 4. Locally implementable control.

the control law (9) is locally implementable. Moreover, the group of robots do not require to have a common sense of direction. However, as also can be seen in (9), the knowledge of relative position is necessary for the proposed control law and it can not be loosen to just distances or just bearing angles.

Finally, we present our main result to show that the control law (9) solves the moving-target-enclosing problem.

Theorem 3.1: For a group of n robots with the control law (9), if none of $p_0(0), p_1(0), \dots, p_n(0)$ are co-located, then as $t \rightarrow \infty$,

- 1) $\rho_i(t) \rightarrow d$,
- 2) $(\theta_{i+}(t) - \theta_i(t) + \zeta_i) \rightarrow 2\pi/n$.

Proof: For the control law (9), it is obtained that

$$\begin{aligned} D^+V &= -h(\rho)^T D^2 h(\rho) \\ &\quad - \frac{1}{2}(-L_{t+}\theta + \zeta)^T (L_{t+} + L_{t+}^T)(-L_{t+}\theta + \zeta). \end{aligned}$$

Notice that $(L_{t+} + L_{t+}^T)$ is positive semi-definite. Hence, it follows that

$$D^+V \leq 0.$$

Moreover, recall that the kernel of $(L_{t+} + L_{t+}^T)$ is $\text{span}\{\mathbf{1}\}$.

So

$$\begin{aligned} E &= \{(\rho, \theta, v) | D^+V = 0\} \\ &= \{(\rho, \theta, v) | \rho_i = 0 \text{ or } d, -L_{t+}\theta + \zeta \in \text{span}\{\mathbf{1}\}\}. \end{aligned}$$

It can be checked further that the largest invariant set in E is

$$M = \{(\rho, \theta, v) | \rho_i = 0 \text{ or } d, -L_{t+}\theta + \zeta \in \text{span}\{\mathbf{1}\}, v = \mathbf{1} \otimes v_0\}.$$

Then from the non-smooth version of LaSalle's invariant principle [18], every trajectory approaches M as $t \rightarrow \infty$. For states in M , we have $-L_{t+}\theta + \zeta = a\mathbf{1}$ for some constant a . Recall that

$$\mathbf{1}^T(-L_{t+}\theta + \zeta) = 2\pi.$$

As a result, $\mathbf{1}^T a\mathbf{1} = 2\pi$, which implies $a = 2\pi/n$.

Next we show that $\rho_i(t)$ does not converge to 0 by contradiction. Suppose there is a solution $\rho_i(t) \rightarrow 0$ for $\rho_i(0) \neq 0$. From the above proof, we know that $v_i(t)$ tends to v_0 . Also, from Lemma 3.1, it is known that every solution $\xi(t)$ of the system $\dot{\xi} = -\xi(\xi^2 - d^2)$ approaches d for any initial condition $\xi(0) \neq 0$. Combining these three observations, it follows that for any arbitrary small $\varepsilon > 0$ and $\delta > 0$, there exists a T such that for all $t > T$,

$$d - \varepsilon < \xi(t) < d + \varepsilon, \quad (10)$$

$$\rho_i(t) < \varepsilon, \quad (11)$$

$$|\mathbf{r}_i^T(v_i - v_0)| < \delta. \quad (12)$$

From (10) and (11), one obtains

$$|\rho_i(t) - \xi(t)| \geq d - 2\varepsilon \text{ for all } t \geq T. \quad (13)$$

On the other hand, we treat the dynamics of ρ_i as perturbed system of $\dot{\xi} = -\xi(\xi^2 - d^2)$ with initial time at T . From the perturbation theory (Theorem 9.1 in [19]), it follows that for all $t > T$,

$$|\rho_i(t) - \xi(t)| < ke^{-\gamma(t-T)}|\rho_i(T) - \xi(T)| + \beta\delta,$$

where k , γ and β are positive constants independent the choices of ε and δ . Since $ke^{-\gamma(t-T)}|\rho_i(T) - \xi(T)|$ converges to 0 exponentially, for any sufficiently small $\varepsilon' > 0$ there exists a $T' > T$ such that for all $t > T'$,

$$|\rho_i(t) - \xi(t)| < \varepsilon' + \beta\delta. \quad (14)$$

Note that both ε' and δ can be selected arbitrarily small and we choose them so that $\varepsilon' + \beta\delta < d - 2\varepsilon$. Thus, a contradiction is reached between (13) and (14). ■

Remark 3.2: From the proof of Theorem 3.1, the adaptive control not only ensures convergence to the desired configuration, but also guarantees that the system is stable as $D^+V \leq 0$.

Remark 3.3: Note that in our control strategy, the inter-agent collision avoidance is not considered. However, the inter-agent separation can be achieved by using the artificial potential function method. Since $u_i'' - v_0$ is orthogonal to u_i' , then an artificial function can be added in (5) to separate two agents that are too close and it does not affect the convergence properties.

IV. SIMULATION

In this section, we present a simulation to illustrate our results. We simulate eight robots tracking and enclosing a moving target where the velocity v_0 of the target is a piecewise constant signal changing its value at $t = 30s$. The desired distance between every robot and the target is $d = 50$. In the simulation, the initial positions of the eight robots are randomly generated in the plane. The simulated trajectories of the eight robots under our control law are given in Fig. 5. It can be seen that each of them converges

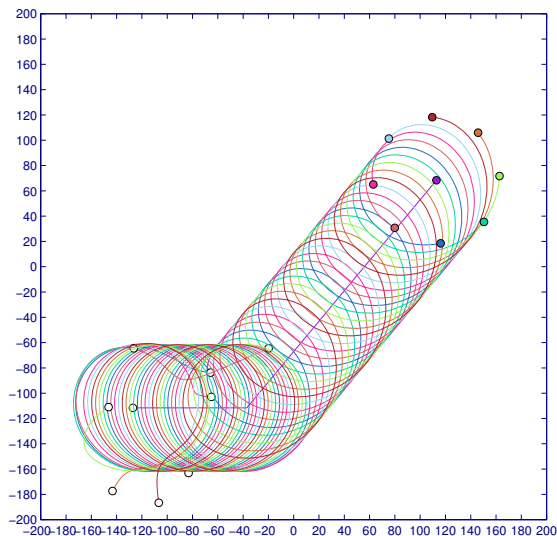


Fig. 5. The simulated trajectories of the eight robots enclosing a moving target.

and maintains the desired distance to the moving target. Moreover, they are uniformly distributed when enclosing the target in motion. After the abrupt change of the target's velocity at $t = 30s$, the eight robots with our control law are capable of recovering to the desired circle formation. The evolutions of the distances between all the robots and the target (namely $\rho_i = \|p_i - p_0\|$, $i = 1, \dots, 8$) are plotted in Fig. 6 and Fig. 7, where Fig. 6 shows the first part for the time interval $[0, 30)$ and Fig. 7 shows the rest starting at $t = 30s$. From Fig. 7, we observe that the distances between the robots and the target are perturbed away from the desired distance because of an abrupt change from the target, but our control law enables the robots to resume the enclosing motion and to adapt to the new target's velocity after a short transient period. In addition, the evolution of the separation angles between any two neighboring robots is depicted in Fig. 8. From the figure, we can see that the separation angles between any two neighboring robots converge to a common value $2\pi/8$, which validates our theoretical result.

V. CONCLUSION

In this paper, we have discussed a moving-target-enclosing problem. Without knowing the target's velocity, each robot

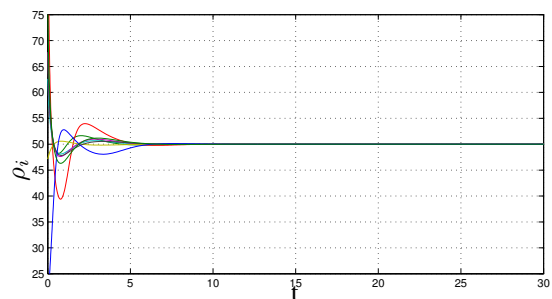


Fig. 6. The distances between the eight robots and the target for the time interval $[0, 30]$.

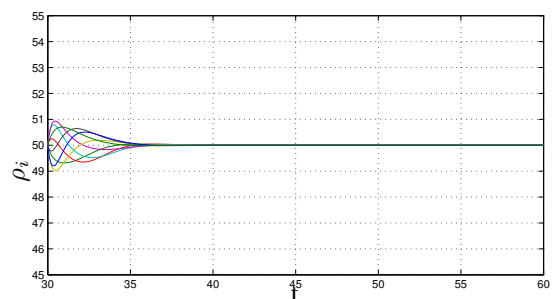


Fig. 7. The distances between the eight robots and the target for the time interval $[30, 60]$.

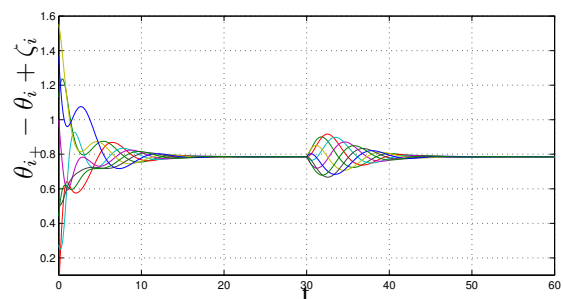


Fig. 8. The separation angles between any two neighboring robots.

only uses the local relative position information from the target and its neighbors. With the setup proposed, target tracking and inter-robot coordination are essentially decoupled. One part of the control law amounts to ensuring the convergence of the distance between the robots and the target to the desired one. The other part of the control law is used to achieve uniform distribution when enclosing the target in motion. Besides, an adaptive scheme is proposed to estimate the velocity of the target. Lyapunov-based techniques and graph theory are brought together for control synthesis, which allows for changing topologies. The convergence and stability properties are investigated in detail using a non-smooth version of LaSalle's invariance principle. Our control strategy is practically implementable by only onboard sensors. Our results extend the work of [10] in two aspects:

allowing for a moving target with an unknown velocity and tolerating with neighbor changing which removes the requirement of labels for the robots. However, in the paper, we have not discussed the situation in the presence of noisy measurements. It will be studied in future work.

REFERENCES

- [1] P. Ogren, M. Egerstedt, and X. Hu, "A control Lyapunov function approach to multiagent coordination," *IEEE Transactions on Robotics and Automation*, vol. 18, no. 5, pp. 847–851, 2002.
- [2] Z. Lin, M. Broucke, and B. Francis, "Local control strategies for groups of mobile autonomous agents," *IEEE Transactions on Automatic Control*, vol. 49, no. 4, pp. 622–629, 2004.
- [3] B. D. O. Anderson, C. Yu, S. Dasgupta, and A. S. Morse, "Control of a three-coleader formation in the plane," *Systems & Control Letters*, vol. 56, no. 9-10, pp. 573–578, 2007.
- [4] J. Lin, A. S. Morse, and B. D. O. Anderson, "The multi-agent rendezvous problem. part 1: The synchronous case," *SIAM Journal on Control and Optimization*, vol. 46, no. 6, pp. 2096–2119, 2007.
- [5] —, "The multi-agent rendezvous problem. part 2: The asynchronous case," *SIAM Journal on Control and Optimization*, vol. 46, no. 6, pp. 2120–2147, 2007.
- [6] D. V. Dimarogonas and K. J. Kyriakopoulos, "On the rendezvous problem for multiple nonholonomic agents," *IEEE Transactions on automatic control*, vol. 52, no. 5, pp. 916–922, 2007.
- [7] W. Ren, R. W. Beard, and E. M. Atkins, "A survey of consensus problems in multi-agent coordination," in *Proceedings of the 2005 American Control Conference*, Portland, OR, USA, 2005, pp. 1859–1864.
- [8] R. Olfati-Saber, J. A. Fax, and R. M. Murray, "Consensus and cooperation in networked multi-agent systems," *Proceedings of the IEEE*, vol. 95, no. 1, pp. 215–233, 2007.
- [9] I. Shames, B. Fidan, and B. D. O. Anderson, "Close target reconnaissance using autonomous UAV formations," in *Proceeding of 47th IEEE Conference on Decision and Control*, Cancun, Mexico, 2008, pp. 1729–1734.
- [10] T. H. Kim and T. Sugie, "Cooperative control for target-capturing task based on a cyclic pursuit strategy," *Automatica*, vol. 43, no. 8, pp. 1426–1431, 2007.
- [11] K. Kobayashi, K. Otsubo, and S. Hosoe, "Design of decentralized capturing behavior by multiple robots," in *IEEE Workshop on Distributed Intelligent Systems: Collective Intelligence and its applications*, 2006, pp. 463–468.
- [12] C. Schumacher, "Ground moving target engagement by cooperative UAVs," in *Proceedings of the 2005 American Control Conference*, Portland, OR, USA, 2005, pp. 4502–4505.
- [13] R. Aguilar-Ponce, A. Kumar, J. L. Tecpanecatli-Xihuitl, and M. Bayoumi, "A network of sensor-based framework for automated visual surveillance," *Journal of Network and Computer Applications*, vol. 30, no. 3, pp. 1244–1271, 2007.
- [14] A. Sarwal, D. Agrawal, and S. Chaudhary, "Surveillance in an open environment by cooperative tracking amongst sensor enabled robots," in *Proceeding of 2007 IEEE International Conference on Information Acquisition*, Jeju City, Korea, pp. 345–349.
- [15] S. Hara, T. H. Kim, Y. Hori, and K. Tae, "Distributed formation control for target-enclosing operations based on a cyclic pursuit strategy," in *Proceeding of the 17th IFAC World Congress*, Seoul, Korea, 2008, pp. 6602–6607.
- [16] Y. Lan, G. Yan, and Z. Lin, "A hybrid control approach to cooperative target tracking with multiple mobile robots," in *Proceeding of the 2009 American Control Conference*, St. Louis, MO, USA, 2009, pp. 2624–2629.
- [17] V. Gazi and R. Ordóñez, "Target tracking using artificial potentials and sliding mode control," in *Proceedings of the 2004 American Control Conference*, Boston, Massachusetts, USA, 2004, pp. 5588–5593.
- [18] N. Rouche, P. Habets, and M. Laloy, *Stability Theory by Liapunov's Direct Method*. Springer-Verlag, 1975.
- [19] H. K. Khalil, *Nonlinear Systems*, 3rd ed. Prentice Hall, 2002.