# Planning and Control of an Internal Point of a Deformable Object

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Abstract— Manipulative operation of a target point inside a deformable object by a robotic system is necessary in many medical and industrial applications. However, this is a challenging problem because of the difficulty of imposing the motion of the target point by a finite number actuation points located at the boundary of the deformable object. In this paper, an approach towards positioning operation of an internal target point of a deformable object to the desired location by a system of three actuators is presented. First, we design an optimization technique that minimizes the total force applied to the object to determine the location of actuation points to effect the desired motion. Then a position-based PI controller is developed to control the motion of the actuators. A passivity observer and a passivity controller are developed to guarantee the stability of the whole system. The simulation results demonstrate the efficacy of the proposed method.

## I. INTRODUCTION

**D**EFORMABLE objects are used in many applications such as automobiles, aerospace, leather, packaging and surgery [1-4]. Most of the tasks involving the handling of deformable objects is done manually that make them labor intensive and time consuming. Thus such operations require skilled human operators for fast and accurate manipulation of material. In majority of the existing literature, however, the objects to be grasped and manipulated were considered to be rigid [5-6]. Less effort has been made in investigating the manipulation of deformable objects [7-11].

There are several important manipulative operations dealing with deformable objects such as whole body manipulation [10], shape changing [11], biomanipulation [12] and internal point manipulation [3, 7] that have practical applications. The main focus of this paper is the internal point manipulation of a deformable object. For instance, a positioning operation called linking in the manufacturing of seamless garments [7] requires manipulation of internal points in deformable objects. Mating of a flexible part in electric industry also results in the positioning of mated points on the object. In many cases these points cannot be manipulated directly since the points of interest in a mating part is inaccessible because of contact with a mated part. Additionally, in medical field, many diagnostic and therapeutic procedures require accurate needle targeting. In case of needle breast biopsy [3] and prostate cancer brachytherapy [4], needles are used to access a designated area to remove a small amount of tissue or to

implant radio-active seed at the targeted area. The deformation causes the target to move away from its original location. In these cases, we cannot manipulate the targeted areas directly because they are internal to the organs. They must be manipulated by controlling some other points where forces can be applied. Thus, there are many applications where the development of an automated mechanism to control an internal point of a deformable object could be beneficial, which is the focus of this paper.

Modeling of deformable objects has been studied in detail [13, 14]. However, works on controlling an internal point in a deformable object are rare. Mallapragada *et al.* [3] developed an external robotic system to position the tumor in image-guided breast biopsy procedures. In their work, three linear actuators manipulate the tissue phantom externally to position an embedded target inline with the needle during insertion. In [7], Wada *et al.* developed a robust control law for positioning the internal points of extensible cloths by manipulating the boundary points using robotic fingers. These works are important to the present application, but they did not address the optimal contact locations on the boundary of the object to effect the desired motion of the internal target point, which we present in this work.

Extensive literatures are available in determining the optimal contact locations by multi-fingered gripper for rigid objects [15, 16] with various stability criteria. A few literatures are also available in finding the optimal contact locations for handling deformable object [17, 18]. However, position control of an internal target point in a deformable object by multi-fingered gripper has not been attempted. In our intended application, we address the issue of determining the optimal contact locations for manipulating a deformable object such that the internal target point can be positioned to the desired location by three linear actuators using minimum applied forces.

In this paper, a position-based PI controller is developed to control the motion of the actuators such that the internal target point is positioned to the desired location. However, the controller for target position control is non-collocated since the internal target point is not directly actuated by the actuators. It is known in the literature that non-collocated control of a deformable object is not passive, which may lead to instability [19]. Thus, we present a new passivitybased non-collocated controller for the actuators to ensure safe and accurate position control of the internal target point. We develop a passivity observer (PO) and a passivity controller (PC) based on [20] for individual actuators. Our approach extends the concept of PO and PC in [20] to multipoint contacts with the deformable object.

The remainder of this paper is organized as follows:

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problem description is stated in Section II. Section III outlines the deformable object modeling. A framework for optimal contact locations is presented in Section IV. The control method is discussed in Section V. The effectiveness of the derived control law is demonstrated by simulation in Section VI. Finally, the contributions of this work and the future directions are discussed in Section VII.

## II. PROBLEM STATEMENT

Consider a case in which multiple actuators or multifingered grippers are manipulating a deformable object in a 2D plane. Before we discuss the design of control law, we present a result from [7] to determine the number of actuation points required to position the target at an arbitrary location in a 2D plane. The following definitions are given according to the convention in [7].

*Manipulation points*: are defined as the points that can be manipulated directly by robotic fingers. In our case, the manipulation points are the points where the external actuators apply forces on the deformable object.

*Positioned points*: are defined as the points that should be positioned indirectly by controlling manipulation points appropriately. In our case, the internal target point is the position point.

The control law to be designed is non-collocated since the internal target point is not directly actuated by actuators. The following result is useful in determining the number of manipulation points.

Result [7]: The number of manipulated points must be greater than or equal to that of the positioned points in order to realize any arbitrary displacement.

In our present case, we assume that the number of positioned points is one, since we are trying to control the position of the target. Hence, ideally the number of contact points would also be one. But practically, we assume that there are two constraints: (1) we do not want to apply shear force on the deformable object to avoid the damage to the surface, and (2) we can only apply control force directed into the deformable object. We cannot pull the surface since the actuators are not attached to the surface. Thus we need to control the position of the target by applying only unidirectional compressive force.

However, there exists a theorem on the force direction closure in mechanics that helps us to determine the equivalent number of compressive forces that can replace one unconstrained force in a 2D plane.

Theorem [21]: A set of wrenches W can generate force in any direction if and only if there exists a three-tuple of wrenches  $\{\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3\}$  whose respective force directions

$$f_1, f_2, f_3$$
 satisfy:

i) Two of the three directions  $f_1$ ,  $f_2$ ,  $f_3$  are independent. ii) A strictly positive combination of the three directions is zero.

$$\alpha f_1 + \beta f_2 + \gamma f_3 = 0 \tag{1}$$

where  $\alpha$ ,  $\beta$ , and  $\gamma$  are constants. The ramification of this theorem for our problem is that we need three control forces

distributed around the object such that the end points of their direction vectors draw a non-zero triangle that includes their common origin point. With such an arrangement we can realize any arbitrary displacement of the target point. Thus the problem can be stated as:

*Problem statement*: Given the number of actuators, the initial target and its desired locations, find appropriate contact locations and control action such that the target point is positioned to its desired location by controlling the boundary points of the object with minimum force.

## III. DEFORMABLE OBJECT MODELING

Consider a schematic in Fig. 1 where three actuators are positioning an internal target in a deformable object to the desired location. We assume that all the end-effectors of the actuators are in contact with the deformable object such that they can apply only push on the object as needed.



Fig. 1: Schematic of the actuators manipulating a deformable object.

We model the deformable object using discrete networks of mass-spring-damper system. The point masses are located at the nodal points and a voigt element is inserted between them. Fig. 2 shows a single layer of the deformable object. Each element is labeled as  $E_j$  for  $j = 1, 2, \dots, NE$ , where NE is total number of elements in a single layer.



Fig. 2: Model of a deformable object with interconnected mass-spring-damper. k is the spring stiffness, c is the damping coefficient.

## IV. FRAMEWORK FOR OPTIMAL CONTACT LOCATIONS

We develop an optimization technique that satisfies the force closure condition for three fingers planar grasp. The

resultant wrench for frictionless contacts of three actuators is given by

$$\mathbf{w} = \sum_{i=1}^{3} f_i \mathbf{n}_i(\mathbf{r}_i), \ (\forall \mathbf{w} \in \Re^2) (\exists f_i \ge 0, 1 \le i \le 3)$$
(2)

where,  $\mathbf{n}_i(\mathbf{r}_i)$  is the unit inner normal of i-th contact and  $f_i$  denotes the i-th actuator's force. We need to find three distinct points,  $\mathbf{r}_1(\theta_1)$ ,  $\mathbf{r}_2(\theta_2)$ , and  $\mathbf{r}_3(\theta_3)$ , on the boundary of the object such that Eq. (2) is satisfied. Here,  $\theta_1$ ,  $\theta_2$ , and  $\theta_3$  are the three contact point locations measured anticlockwise with respect to x axis as shown in Fig. 3.



Fig. 3: Three fingers grasp of a planar object.

A physically realizable grasping configuration can be achieved if the surface normals at three contact points positively span the plane so that they do not all lie in the same half-plane [22]. Therefore, a realizable grasp can be achieved if the pair-wise angles satisfy the following constraints

$$\theta_{\min} \leq |\theta_i - \theta_j| \leq \theta_{\max} , \theta_{\text{low}} \leq \theta_i \leq \theta_{\text{high}}, \ i, j = 1, 2, 3, \ i \neq j$$
(3)

A unique solution to realizable grasping may not always exist. Therefore, we develop an optimization technique that minimizes the total force applied on to the object to obtain a particular solution. The optimal locations of the contact points would be the solution of the following optimization problem.

 $\mathbf{f}^T \mathbf{f}$ 

subject to 
$$\mathbf{w} = \sum_{i=1}^{3} f_i \mathbf{n}_i(\mathbf{r}_i)$$
  
 $\theta_{\min} \leq |\theta_i - \theta_j| \leq \theta_{\max}, \ i, j = 1, 2, 3, \ i \neq j$  (4)  
 $f_i \geq 0, \ i = 1, 2, 3$   
 $0 \leq \theta_i \leq 360^{\circ}, \ i = 1, 2, 3$ 

Once we get the optimal contact locations, all three actuators will take their respective positions to effect the desired motion at those contact points.

## V. DESIGN OF THE CONTROLLER

After getting the optimal contact locations, the planner generates the desired reference locations for these contact points by projecting the error vector between the desired and the actual target locations in the direction of the applied forces. All actuators are controlled by their individual controllers using the following control law

$$f_i = K_{Pi} \mathbf{n}_i^T \mathbf{e} + K_{Ii} \int \mathbf{n}_i^T \mathbf{e} dt , \ i = 1, 2, 3$$
(5)

where,  $K_{Pi}$ , and  $K_{Ii}$  are the proportional and integral gains, **e** is the position error between the desired and the actual locations of the target. Forces applied by the actuators on the surface of the deformable object are calculated by projecting the error vector in the direction of the applied forces.

The Eq. (5) does not guarantee that the system will be stable. Thus a passivity-based control approach based on energy monitoring is developed to guarantee the stability of the system. The basic idea is to use a PO to monitor the energy generated by the controller and to dissipate the excess energy using a PC when the controller becomes active [23]. We develop a 2-port network with PO and PC as shown in Fig. 4. Here, we consider that the plant is passive. In Fig. 4,  $v_i$  is the output velocity of the i-th actuator and  $f_i$  is the controller output of the i-th actuator. The desired velocity of the i-th actuator is given by  $v_{di}$ . Now we design a PO for sufficiently small time-step  $\Delta T$  as:



Fig. 4: Series configuration for 2-port networks.  $\alpha_{1i}$  and  $\alpha_{2i}$  are the adjustable damping elements at each port.

$$E_{i}(t_{k}) = \Delta T \sum_{j=0}^{k} (f_{i}(t_{j}) v_{di}(t_{j}) - f_{i}(t_{j}) v_{i}(t_{j}))$$
(6)

where,  $\Delta T$  is the sampling period and  $t_j = j \times \Delta T$ . In normal passive operation,  $E_i(t_j)$  should always be positive. In case when  $E_i(t_j) < 0$ , the PO indicates that the i-th controller is generating energy and going to be active. The sufficient condition to make the whole system passive can be written as

$$\Delta T \sum_{j=0}^{k} f_{i}(t_{j}) v_{di}(t_{j}) \geq \Delta T \sum_{j=0}^{k} f_{i}(t_{j}) v_{i}(t_{j}), \ \forall t_{k} \geq 0, i = 1, 2, 3$$
(7)

where k means the k-th step sampling time. The input and output energy can be computed as [24]

$$E_{i}^{T}(k) = \begin{cases} E_{1i}^{T}(k-1) + f_{i}(k)v_{di}(k) & \text{if } f_{i}(k)v_{di}(k) > 0 \\ \vdots & \vdots \end{cases}$$
(8)

$$E_{1i}(k) = \begin{cases} E_{1i}^{T}(k-1) & \text{if } f_{i}(k)v_{di}(k) \le 0 \\ (\nabla^{T}(k-1)) & (D_{i}(k)) = 0 \end{cases}$$

$$E_{2i}^{T}(k) = \begin{cases} E_{2i}^{T}(k-1) - f_{i}(k)v_{di}(k) & \text{if } f_{i}(k)v_{di}(k) < 0\\ E_{2i}^{T}(k-1) & \text{if } f_{i}(k)v_{di}(k) \ge 0 \end{cases}$$
(9)

$$E_{1i}^{P}(k) = \begin{cases} E_{1i}^{P}(k-1) - f_{i}(k)v_{i}(k) & \text{if } f_{i}(k)v_{i}(k) < 0\\ E_{1i}^{P}(k-1) & \text{if } f_{i}(k)v_{i}(k) \ge 0 \end{cases}$$
(10)

$$E_{2i}^{P}(k) = \begin{cases} E_{2i}^{P}(k-1) + f_{i}(k)v_{i}(k) & \text{if } f_{i}(k)v_{i}(k) > 0\\ E_{2i}^{P}(k-1) & \text{if } f_{i}(k)v_{i}(k) \le 0 \end{cases}$$
(11)

where,  $E_{1i}^{T}(k)$  and  $E_{2i}^{T}(k)$  are the energy flowing in and out at the trajectory side of the controller port, respectively, whereas  $E_{1i}^{P}(k)$  and  $E_{2i}^{P}(k)$  are the energy flowing in and out at the plant side of the controller port, respectively. So the time domain passivity condition is given by

$$E_{1i}^{T}(k) + E_{1i}^{P}(k) \ge E_{2i}^{T}(k) + E_{2i}^{P}(k), \ \forall k \ge 0$$
(12)

In order to dissipate the excess energy of the control system, a damping force should be applied that obeys the following constitutive equation.

$$f = \alpha v \tag{13}$$

where, f is the damping force, v is the velocity of the actuator, and  $\alpha$  is the damping coefficient. The algorithm used for a 2-port network with impedance causality (i.e., velocity input, force output) at each port is given by the following steps:

$$E_{i}(k) = E_{1i}^{T}(k) - E_{2i}^{P}(k) + E_{1i}^{P}(k) - E_{2i}^{T}(k) + \alpha_{1i}(k-1)v_{di}(k-1)^{2} + \alpha_{2i}(k-1)v_{i}(k-1)^{2}$$
(14)

where the last two terms are the energy dissipated at the previous time step.

2) Two series PCs are designed for several cases as given below:

Case 1: If  $E_i(k) \ge 0$ , i.e., if the output energy is less than the input energy, there is no need to activate any PCs.

Case 2: If  $E_i(k) < 0$ , i.e., if the output energy is more than the input energy, i.e.,  $E_{2i}^P(k) > E_{1i}^T(k)$ , then we need to activate only the plant side PC.  $\alpha_i(k) = 0$ 

$$\alpha_{1i}(k) = 0$$
(15)
$$\alpha_{2i}(k) = -E_i(k) / v_i(k)^2$$

Case 3: Similarly, if  $E_i(k) < 0$ ,  $E_{2i}^T(k) > E_{1i}^P(k)$ , then we need to activate only the trajectory side PC.

$$\alpha_{1i}(k) = -E_i(k) / v_{di}(k)^2$$
(16)

$$\alpha_{2i}(k) = 0$$

3) The contributions of PCs are converted into power variables as

$$f_i'(k) = \alpha_{1i}(k)v_{di}(k) \tag{17}$$

 $f_i^{p}(k) = \alpha_{2i}(k)v_i(k)$ 4) Modified outputs are  $f_i^{T}(k) = f_i(k) + f_i^{t}(k)$   $f_i^{p}(k) = f_i(k) + f_i^{p}(k)$ (18)

where,  $f_i^t(k)$  and  $f_i^p(k)$  are the PCs' outputs at trajectory and plant sides of the controller ports, respectively.  $f_i^T(k)$ and  $f_i^p(k)$  are the modified outputs at trajectory and plant sides of the controller ports, respectively.

### VI. SIMULATION AND DISCUSSION

We perform extensive simulations of positioning an internal target point to a desired location in a deformable object by external actuators to demonstrate the feasibility of the concept. We consider a circular section of the deformable object of diameter 0.08 m. We assume that the initial

position of the target is at the center of the circular section i.e., (0, 0) mm. The goal is to position the target at the desired location (5, 5) mm with a smooth straight line trajectory. We discretize the circular section with elements of mass-spring-damper. A few elements are shown in Fig. 5. We choose *m*=0.006 kg for each point mass, *k*=10 N/m for spring constant and *c*=5 Ns/m for damping coefficient. In this simulation, we use  $K_{Pi}$  =1000 and  $K_{li}$  =1000, *i*=1,2,3. With this set up, optimal contact locations are determined (Eq. 4) as 38.5°, 204.9°, and 244.9°. The actuators will take their appropriate positions to perform the target positioning at the desired location. Two tasks are presented below.



Fig. 5: Optimal contact locations (points 1, 2 and 3) around the deformable object.

Task 1:

In this task, we consider a case when the PCs are not turned on and the target is following the desired straight line trajectory. A simple position based PI controller generates the control command based on the error between the desired and the actual location of the target. Fig. 6 shows that the target tracked the desired position trajectory. Actuators forces generated by the PI controller are presented in Fig. 7. However, the POs for actuators 2 and 3 are become negative as shown in Fig. 8. Negative values of the POs signify that the output energy of the 2-port network is greater than the input energy. Since the plant is considered to be passive, the only source of generating extra energy is the controller that makes the whole system unstable. So we must engage passivity controller to modify the controller output by dissipating the extra amount of energy.

## Task 2:

In task 2, the PCs are turned on and the actuators are commanded to effect the desired motion of the target. The PCs are activated when the POs cross zero from a positive value. The required damping forces are generated to dissipate only the excess amount of energy generated by the controller. In this case, the target tracks the desired straight line trajectory well with the POs remaining positive. Fig. 9 represents the actual and the desired trajectories of the target when PCs are turned on. It shows that to maintain stability, the performance of the controller is slightly degraded due to the addition of the damping forces with the forces generated by PI controller. For this case, the PCs on the plant side are only activated whereas the PCs on the trajectory side remain idle. Fig. 10 shows the PCs forces generated at the plant side when the POs cross zero. The total force required to move the target point can be obtained from Eq. 18. The POs become positive during interaction between the actuators and the object as shown in Fig. 11. Hence, the stability of the overall system is guaranteed.

#### VII. CONCLUSIONS AND FUTURE WORK

We have developed a control framework to achieve positioning operation of an internal target point in a deformable object by three external actuators. Here the actuators could be stand-alone actuators or a gripper with multiple fingers. To manipulate the target inside the deformable object, the boundary of the object is regulated to effect the desired motion of the target. An optimizationbased planning is introduced to determine the contact locations around the periphery of the object. A time-domain passivity control scheme with adjustable dissipative elements has been developed to guarantee the stability of the whole system. Extensive simulation results validate the optimal contact formulation and stable interaction between the actuators and the object.

Future work includes testing the controller with more complex shapes of the deformable object, 3-D objects, and verifying the methodology by experiments.

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Fig.6: The desired (red dashed line) and the actual (blue solid line) straight lines when PCs are not turned on.



Fig. 7: Controller forces when PCs are not turned on.



Fig. 8: POs for three actuators when PCs are not turned on.



Fig.9: The desired (red dashed line) and the actual (blue solid line) straight lines when PCs are turned on.



Fig. 10: Required forces supplied by PCs at the plant side when PCs are turned on



Fig. 11: POs for three actuators when PCs are turned on.