

# Active Shaping of an Unknown Rheological Object Based on Deformation Decomposition into Elasticity and Plasticity

Mitsuru Higashimori, Kayo Yoshimoto, and Makoto Kaneko

**Abstract**—This paper discusses an active shaping method for an unknown rheological object by considering the characteristics of viscoelasticity. By utilizing a four-element model for approximating the dynamic characteristics of object's deformation, we drive the deformation decomposition into the elastic response and the plastic one. For shaping the object, we then propose a two-phase strategy for controlling the resultant deformation; in the first phase the viscoelastic parameters are estimated with avoiding the over deformation, based on the elastic response; in the second phase the desired resultant deformation is generated by actively managing the integral force, based on the plastic response. This strategy has an advance that the handling time of the robot is given by a finite time, while the desired resultant deformation is theoretically completed in the infinite time. We finally show experimental results for confirming the validity of the proposed strategy.

## I. INTRODUCTION

When human grasps and manipulates an object, she (or he) appropriately makes the finger motions and manages the contact force, with sensing the size, the shape, and the stiffness of the object. In order to realize such operations in engineering, there have been various works discussing manipulation by robot hand. In this field, not only for rigid objects but also for deformable objects, many works are studied [1]–[14]. Deformable objects can be classified into three groups; one is viscoelastic object, one is the plastic object, and the other one is rheological object [1]. In the above classification, the viscoelastic object deforms by the grasping force and the deformation is completely recovered after releasing. In the plastic object the whole deformation is maintained. In the rheological object, some part of deformation recovers and the other part remains as shown in Fig. 1(c).

The deformation characteristics of rheological objects, such as clay and food, can be expressed by the combination of viscous and elastic elements [15]. Depending upon how to give the contact force, the distribution between the elastic deformation (temporary deformation) and the plastic deformation (permanent deformation) is changed. The final shape of the object is directly connected to the plastic deformation. While the plastic deformation of food may become an important point not only for good appearance

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but also for feels in eating for consumers, the industrial technology for controlling the plastic deformation has not been put into practice in the food processing field.

This work discusses an active shaping method of a rheological object. Based on the deformation characteristics of the object, we propose a handling strategy for generating the desired resultant deformation. We first derive the approximation of the dynamic characteristics of the object's deformation based on a four-element viscoelastic model as shown in Fig. 1(b), and formulate the relationship between the applied contact force and the deformation response. We show the deformation decomposition of whole deformation into the elastic deformation response and the plastic deformation response. We then propose a two-phase strategy to generate the desired resultant deformation for shaping an unknown object as shown in Fig. 1(a). This strategy is composed of two phases; in the parameter estimation phase the viscoelastic parameters as shown in Fig. 1(b) are estimated with avoiding the over deformation based on the elastic response, and in the shaping phase the desired resultant deformation after releasing is generated as shown in Fig. 1(c), by actively managing the integral force based on the plastic response. In the proposed method, the handling time, in which the robot actually makes contact with the object, is given by a finite time, while the deformation of the object is theoretically completed in the infinite time. Therefore, compared with the frame based method by the position control, the proposed method has a great advantage from the viewpoint of reducing the operation time. Such a reduction of the operation time is expected to contribute to improvement of the processing efficiency in mass production. We finally confirm the validity of the proposed method by experiments.

This paper is organized as follows. In Section II, we briefly review related works. In Section III, we give the analytical model of a rheological object. In Section IV, we introduce a two-phase strategy composed of the parameter sensing phase and the shaping phase. In Section V, we show experimental results. In Section VI, we give the conclusion of this work.

## II. RELATED WORKS

For the viscoelastic object, Ono *et al.* have discussed the handling method for picking a cloth and expanding it by the robot hand [2]. Wakamatsu *et al.* have proposed the handling method for a linear object which can deform [3]. Masey and Caldwell have designed and developed the robot end-effector using the roller and blade gripper for handling the flexible sheet of pasta [4]. Taylor has discussed the issues

on handling of deformable materials, such as fabrics, paper, and food. To be used for them, mechanical grippers, intrusive grippers, and attraction grippers are explained [5], [6]. Chua *et al.* have reviewed the current state of development in the handling of non-rigid food products [7]. Shibata *et al.* have studied the control laws for simultaneously manipulating the position and the deformation of the viscoelastic object [8]. For the rheological object, Tokumoto and Hirai have discussed the motion planning for shaping the rheological object by pressing and expanding [9]. Tokumoto *et al.* have discussed how to arrange the viscous and the elastic elements so that deformation characteristics are nicely expressed [10]. Kimura *et al.* have discussed the modeling method for the general expression among the viscoelastic, the plastic, and the rheological objects [1]. Noborio *et al.* have treated the modeling for the rheological object and discussed the parameter estimating method for real objects [11]. Authors' group has discussed the design of food handling robot so that the remained deformation after releasing results in the minimum [12], [13]. Tsai and Kao have applied the latency model to viscoelastic objects for expressing the dynamic characteristics [14].

In the related works treating the rheological object [10]–[14], they have focused on how to accurately reproduce the object's deformation in computer, by sensing the applied force and the deformation and by estimating the viscoelastic parameters. These problems are in direct problem, and the plastic deformation generated passively by given contact force. On the other hand, this work is regarded as the inverse problem, where the contact force is actively managed in order to control the plastic deformation.

### III. ANALYTICAL MODEL

#### A. Modeling

In general, in order to express the uniaxial dynamic characteristic of the rheological object with viscoelastic model, four elements (elasticity $\times 2$ , viscosity $\times 2$ ) are required [1]. In the former works by authors, the four-element Maxwell model as shown in Fig. 1(d) and the four-element Burger model as shown in Fig. 1(e) have been utilized [12], [13]. In this work, we utilize a four-element viscoelastic model as shown in Fig. 1(b) for approximating the deformation of the rheological object. The meanings of symbols in Fig. 1(b) are as follows:

- $x_h$  the displacement of the gripper;
- $x$  the whole deformation of the object;
- $f$  the contact force between the gripper and the object;
- $k_i$  the elastic coefficient of the  $i$ -th elastic element ( $i = 1, 2, 3$  and  $k_i > 0$ );
- $c_i$  the viscous coefficient of the  $i$ -th viscous element ( $i = 1, 2, 3$  and  $c_i > 0$ );
- $f_1$  the force applied to  $c_1$ ;
- $x_1$  the displacement of  $c_1$ ;
- $f_2$  the force applied to the  $c_2$  (or  $k_2$ );
- $x_2$  the total displacement of  $c_2$  and  $k_2$ ;
- $x_{k2}$  the displacement of  $k_2$ ;
- $x_{c2}$  the displacement of  $c_2$ ;

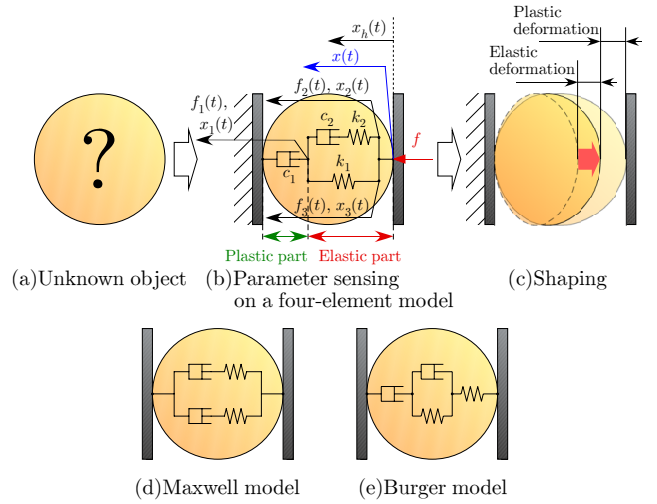


Fig. 1. The basic concept of the active shaping of an unknown rheological object: In the rheological object, depending upon how to give the contact force, the distribution between the elastic deformation (temporary deformation) and the plastic deformation (permanent deformation) is changed as shown in (c). After estimating the viscoelastic parameters in a four-element model as shown in (b), the plastic deformation which determines the final shape is actively generated as shown in (c). The model in (b) is equivalent to the Maxwell model in (d) and the Burger model in (e).

- $f_3$  the force applied to  $k_1$ ;
- $x_3$  the displacement of  $k_1$ .

Now, let us consider that the contact force  $f$  acts on the model as shown in Fig. 1(b) by the gripper's motion. We obtain the following equations:

$$f(t) = f_1(t) = f_2(t) + f_3(t) \quad (1)$$

$$x(t) = x_1(t) + x_2(t) = x_1(t) + x_3(t) \quad (2)$$

$$x_2(t) = x_{c2}(t) + x_{k2}(t) \quad (3)$$

$$f_1(t) = c_1 \dot{x}_1(t) \quad (4)$$

$$f_2(t) = c_2 \dot{x}_{c2}(t) = k_2 x_{k2}(t) \quad (5)$$

$$f_3(t) = k_1 x_3(t). \quad (6)$$

By removing  $f_i$ ,  $x_i$  ( $i = 1, 2, 3$ ),  $x_{k2}$ , and  $x_{c2}$  from (1)–(6), we can derive the following equation:

$$a_2 \ddot{x}(t) + a_1 \dot{x}(t) = b_2 \ddot{f}(t) + b_1 \dot{f}(t) + f(t) \quad (7)$$

where

$$a_2 \triangleq \frac{c_1 c_2 (k_1 + k_2)}{k_1 k_2} \quad (8)$$

$$a_1 \triangleq c_1 \quad (9)$$

$$b_2 \triangleq \frac{c_1 c_2}{k_1 k_2} \quad (10)$$

$$b_1 \triangleq \frac{c_1 k_2 + c_2 (k_1 + k_2)}{k_1 k_2}. \quad (11)$$

Equation (7) is the differential equation expressing the relationship between the contact force applied to the object from the gripper and the whole deformation of the object. By introducing the state variables of  $x = [x \ \dot{x}]^T$ , we transform (7) to the following state equation:

$$\dot{x} = Ax + bu \quad (12)$$

where

$$\mathbf{A} \triangleq \begin{bmatrix} 0 & 1 \\ 0 & -\frac{a_1}{a_2} \end{bmatrix} \quad (13)$$

$$\mathbf{b} \triangleq \begin{bmatrix} 0 \\ \frac{1}{a_2} \end{bmatrix} \quad (14)$$

$$u(t) \triangleq b_2 \ddot{f}(t) + b_1 \dot{f}(t) + f(t). \quad (15)$$

From (12), we can obtain

$$\mathbf{x}(t) = e^{\mathbf{A}t} \mathbf{x}(0) + \int_0^t e^{\mathbf{A}(t-\tau)} \mathbf{b} u(\tau) d\tau. \quad (16)$$

With  $\mathbf{c} \triangleq [1 \ 0]$ , we finally obtain

$$\begin{aligned} x(t) &= \mathbf{c} \mathbf{x}(t) \\ &= x(0) + \frac{a_2}{a_1} \left(1 - e^{-\frac{a_1}{a_2} t}\right) \dot{x}(0) \\ &\quad + \frac{1}{a_1} \int_0^t \left(1 - e^{-\frac{a_1}{a_2} (t-\tau)}\right) u(\tau) d\tau. \end{aligned} \quad (17)$$

Equation (18) shows the whole deformation response of the object  $x(t)$  under the contact force input  $u(t)$ .

### B. Deformation Decomposition into Elasticity and Plasticity

As shown in Fig. 2(a), let us consider the initial state where the contact force  $f$  is zero at  $t = 0$ . Now, we suppose that the contact force  $f$  applied to the object by the gripper is given as follows, as shown in Fig. 2(i)–(iv).

- (i) Loading ( $0 \leq t < t_1$ ): Increasing the contact force for  $f = f_c$ .
- (ii) Constant load ( $t_1 \leq t < t_2$ ): Maintaining the contact force with  $f = f_c$ .
- (iii) Unloading ( $t_2 \leq t \leq T_h$ ): Decreasing the contact force for  $f = 0$ .
- (iv) No load ( $T_h < t \leq \infty$ ): Maintaining the contact force with  $f = 0$ .

The constant force  $f_c$  is defined as the allowable load which never destroys the object. Also, the handling time  $T_h$  indicates the actual period of time in which the gripper makes contact with the object. Fig. 2 shows the relationship among the deformation of the object and the viscoelastic elements, where Fig. 2(a), (b), and (c) correspond with the state at the beginning of loading ( $t = 0$ ), the state at the end of the constant load ( $t = t_2$ ), and the state when the infinite time passes ( $t = \infty$ ), respectively. In addition to them, Fig. 2(d) shows the relationship between the input force  $f(t)$  and the deformation response  $x(t)$  given by (18). From Fig. 2(d), we can see that the four-element model can nicely express the characteristic of the rheological object, where some of the deformation recovers and some of that remains after unloading ( $t > T_h$ ). Now, focusing on roles of four elements in Fig. 2(a), let us separate the whole deformation into two components; one is the displacement of  $x_2 (= x_3)$  coming from the unit composed of  $k_1$ ,  $k_2$ , and  $c_2$ , and the other one is the displacement of  $x_1$  coming from the element of  $c_1$ . In the former component, the deformation  $x_2$  generated by  $f > 0$  is recovered after unloading by the work of  $k_1$  and  $k_2$ , and this deformation completely

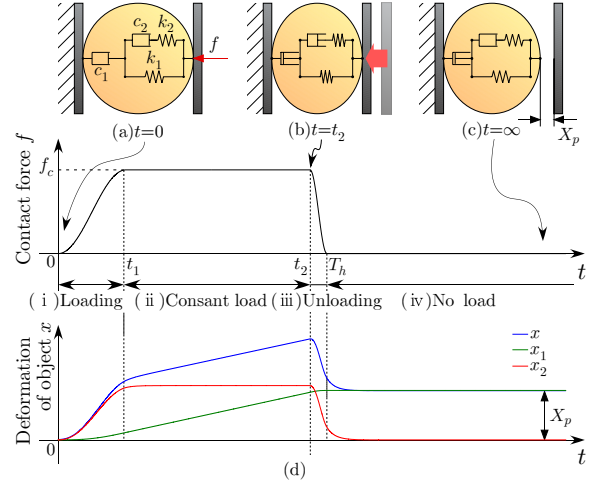


Fig. 2. Decomposition of object's deformation response: in the rheological object, whole deformation response  $x$  by the contact force  $f$  is decomposed into the elastic response  $x_1$  and the plastic one  $x_2$ . The elastic deformation completely recovers after unloading, while the plastic deformation keeps its amount at the end of unloading.

disappears at the time of  $t \rightarrow \infty$ . On the other hand, in the latter component, the deformation  $x_1$  is generated during only the contact force  $f > 0$ , and the deformation at the end of unloading is permanently maintained. Therefore, the deformation  $x_1$  corresponds to the plastic deformation. Taking the above characteristics into account, the response of the whole deformation of the rheological object  $x(t)$  can be decomposed into the plastic deformation  $x_1(t)$  and the elastic deformation  $x_2(t)$ , as shown in Fig. 1(b). From (1) and (4), we can derive  $x_1(t)$  and  $x_2(t)$  as follows:

«Plastic deformation response»

$$x_1(t) = \frac{1}{c_1} \int_0^t f(\tau) d\tau \quad (19)$$

«Elastic deformation response»

$$\begin{aligned} x_2(t) &= x(t) - x_1(t) \\ &= x(t) - \frac{1}{c_1} \int_0^t f(\tau) d\tau. \end{aligned} \quad (20)$$

By using the above equations,  $x_1(t)$  and  $x_2(t)$  can be calculated as shown in Fig. 2(d).

In the four-element Maxwell model as shown in Fig. 1(d) and the four-element Burger model as shown in Fig. 1(e), we can also derive the differential equations equivalent to (7), while the distributions of viscoelastic parameters are different. We would like to note that the four-element model in Fig. 1(b) is more convenient expression than models as shown in Fig. 1(d) and (e), from the viewpoint of easy understanding in the physical meaning. We can intuitively see the relationship between the arrangement of elements and the decomposition of response.

## IV. ACTIVE SHAPING OF AN UNKNOWN RHEOLOGICAL OBJECT

For shaping a rheological object whose viscoelastic parameters are unknown, we discuss a strategy for controlling

the resultant deformation which remains after all gripper's actions. In order to manage both the elastic deformation and the plastic one, the viscoelastic parameters are required. Based on this consideration, we introduce a two-phase strategy for achieving the desired resultant deformation  $X_p^d$ ; in the first phase the viscoelastic parameters are estimated with the contact force and the deformation during loading and unloading, as shown in Fig. 3(a), and in the second phase the desired resultant deformation  $X_p^d$  is generated, as shown in Fig. 3(b).

#### A. Phase 1: Parameter Sensing Phase

Suppose the case that we give an arbitrary contact force  $f$  to the object, such as Section III-B, under the condition where the viscoelastic parameters of the object are unknown. In this case, the distribution of the deformations cannot be predicted. Therefore, the object may deform too much, and as a result the deformation at the stable state may become greater than the desired one. Since the gripper as shown in Fig. 2 cannot generate a pulling force, and restore the deformation, we have to avoid the over deformation under unknown parameters. On the other hand, in order to accurately estimate the dynamic characteristics of the object's deformation, it is desirable to observe a deformation as large as possible. Based on the above considerations, in this phase we generate the deformation of the object by the position-input where the maximum displacement is given by the desired resultant deformation  $X_p^d$ . After unloading, the deformation of the object is recovered by the elasticity based on (21). Thus, it is guaranteed that the object never deforms greater than the desired resultant deformation regardless of parameter values. We can avoid the risk of the over deformation even under the full plastic object with  $k_1 \simeq 0$  and  $k_2 \simeq 0$ .

In practice, the gripper is controlled by the position-input to grasp and to release the object during  $0 \leq t < T_{e1}$ , as shown in Fig. 3(a), under the maximum displacement with  $X_p^d$ . Then, with no load during  $T_{e1} \leq t < T_{e2}$ , some of deformation of the object is restored by the effect of the elastic deformation. We now would like to note that, by starting the release with  $f(T_{e1}) > 0$ ,  $x(T_{e2}) < X_p^d$  is always satisfied. By utilizing the contact force data

$$\mathbf{f} \triangleq [f(0), \dots, f(T_{e2})]^T \in \mathbb{R}^{N \times 1} \quad (22)$$

the object's deformation data

$$\mathbf{x} \triangleq [x(0), \dots, x(T_{e2})]^T \in \mathbb{R}^{N \times 1} \quad (23)$$

and the differential equation (7), we obtain the following equation

$$\mathbf{M}\mathbf{p} = \mathbf{q} \quad (24)$$

where

$$\mathbf{M} \triangleq \left[ \mathbf{x}, \int \mathbf{x} dt, -\mathbf{f}, -\int \mathbf{f} dt \right] \in \mathbb{R}^{N \times 4} \quad (25)$$

$$\mathbf{p} \triangleq [a_2, a_1, b_2, b_1]^T \in \mathbb{R}^{4 \times 1} \quad (26)$$

$$\mathbf{q} \triangleq \iint \mathbf{f} dt^2 \in \mathbb{R}^{N \times 1}. \quad (27)$$

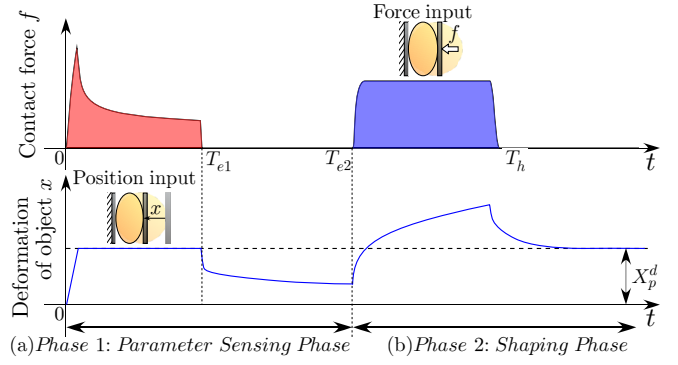


Fig. 3. Two-phase strategy for generating the desired resultant deformation  $X_p^d$  of an unknown object: In the first phase, the viscoelastic parameters are estimated with avoiding the over deformation. The gripper is controlled by the position-input with the maximum displacement of  $X_p^d$ . In the second phase, the desired resultant deformation is generated within a finite handling time, with actively managing the plastic deformation response. The gripper is controlled by the force-input.

$N$  is the number of sampling data during  $0 \leq t \leq T_{e2}$ . By utilizing the integral equation given by (24), we can reduce the noise caused by the differential operation of data. From (24), we can estimate the coefficient parameters of (7)  $\hat{\mathbf{p}} \triangleq [\hat{a}_2, \hat{a}_1, \hat{b}_2, \hat{b}_1]^T \in \mathbb{R}^{4 \times 1}$  by the least squares method as follows:

$$\hat{\mathbf{p}} = (\mathbf{M}^T \mathbf{M})^{-1} \mathbf{M}^T \mathbf{q}. \quad (28)$$

In the case where the time interval  $T_{e2} - T_{e1}$  is given long enough for the elastic deformation to be recovered completely as a stable state, we can independently estimate  $\hat{a}_1 (= \hat{c}_1)$ , which leads the plastic deformation, by

$$\hat{a}_1 = \frac{1}{x(T_{e2})} \int_0^{T_{e1}} f(\tau) d\tau \quad (29)$$

so that we can improve the accuracy of parameter estimation. With the coefficient parameters  $\hat{\mathbf{p}}$  obtained by the above procedure and (8)–(11), the viscoelastic parameters in Fig. 1(b) can be computed as follows:

$$\hat{k}_1 = \frac{\hat{a}_1^2}{\hat{a}_2 - \hat{a}_1 \hat{b}_1} \quad (30)$$

$$\hat{k}_2 = \frac{\hat{a}_2^2 - \hat{a}_1 \hat{a}_2 \hat{b}_1 + \hat{a}_1^2 \hat{b}_2}{(\hat{a}_2 - \hat{a}_1 \hat{b}_1) \hat{b}_2} \quad (31)$$

$$\hat{c}_1 = \hat{a}_1 \quad (32)$$

$$\hat{c}_2 = \frac{\hat{a}_1 (\hat{a}_2^2 - \hat{a}_1 \hat{a}_2 \hat{b}_1 + \hat{a}_1^2 \hat{b}_2)}{(\hat{a}_2 - \hat{a}_1 \hat{b}_1)^2}. \quad (33)$$

If the deformation of  $x = X_p^d$  is maintained by the large  $T_{e1}$  enough for the contact force to become  $f = 0$ , the potential energy stored in the elastic elements is completely dissipated by the deformation of the plastic deformation component, and the desired resultant deformation  $X_p^d$  can be achieved at this phase. In this case, however, time of  $T_{e1} \rightarrow \infty$  is theoretically required.



## B. Phase 2: Shaping Phase

Let us consider the resultant deformation which remains after completing the whole handling operation. We define the resultant deformation  $X_p$  as the deformation when the infinite time passes and the deformation response results in the stable state under no load, as shown in Fig. 2(c) and (iv). In this case, we can express

$$X_p = x(\infty) \quad (34)$$

$$= x_1(\infty) + x_2(\infty). \quad (35)$$

In (35),  $x_2$  is the elastic deformation response and it results in  $x_2(\infty) = 0$  after unloading. On the other hand,  $x_1$  is the plastic deformation response. As shown in (19), since  $x_1$  changes during  $f(t) > 0$ , namely  $0 \leq t \leq T_h$ , we obtain  $x_1(\infty) = x_1(T_h)$ . Therefore, we derive the resultant deformation  $X_p$ , which finally remains on the object, as the following equation:

$$X_p = x_1(T_h) \quad (36)$$

$$= \frac{1}{c_1} \int_0^{T_h} f(t) dt. \quad (37)$$

Equation (37) means that the resultant deformation  $X_p$  is determined by the plastic response, and it can be calculated by the viscous coefficient  $c_1$  and the integral force. Focusing on this point, from (37) and  $\hat{c}_1$ , the integral force  $F^d$  leading to the desired resultant deformation  $X_p^d$  is expressed by the following equation:

$$F^d(X_p^d) = \hat{c}_1 X_p^d - \int_0^{T_{e1}} f(t) dt. \quad (38)$$

The second term of the right side in (38) is the integral force that is already applied to the object in *Phase 1*.

In practice, by measuring the contact force  $f$  in real-time by a sensor system and monitoring the integral force based on (38), in this phase, we actively manage the plastic deformation response included in the whole deformation. The gripper is controlled by the force-input to grasp and to releases the object during  $T_{e2} \leq t < T_h$ , as shown in Fig. 3(b), under the maximum force of  $f_c$ . At the time of  $t = T_h (= t|_{F=F^d(X_p^d)})$ , the gripper releases the object completely. During  $t > T_h$ , the deformation of the object is automatically recovered to the desired resultant deformation  $X_p^d$  by the elastic response.

We would like to note that, while the infinite time  $t \rightarrow \infty$  is theoretically required for the stable state with  $x = X_p^d$ , the gripper operation can be completed in the finite handling time of  $T_h$ . Compared with the frame based method by the position-input, which is mentioned in the last paragraph of Section III-B, the proposed method has a great advantage from the viewpoint of reducing the total operation time.

## V. EXPERIMENTS

### A. Experimental Setup

Fig. 4 shows an overview of the experimental system. Fig. 5 shows a photo of the gripper and the object. The gripper is composed of two parallel plates, where one is fixed at

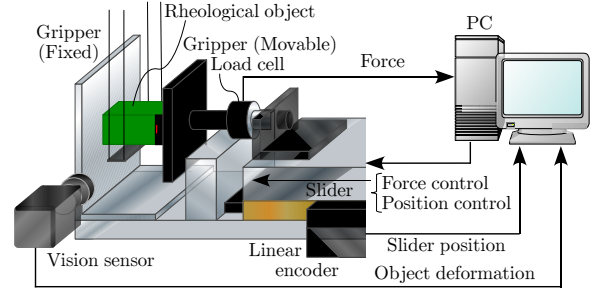


Fig. 4. An overview of experimental system.

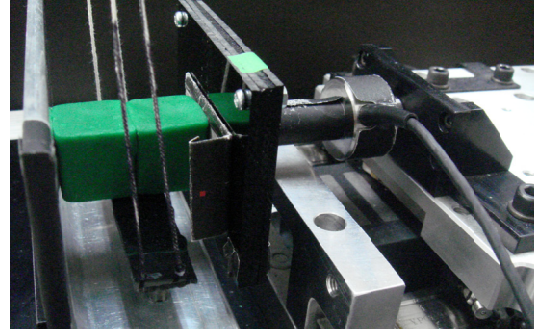


Fig. 5. The wheat clay sandwiched in the gripper.

the base and the other is attached at the linear slider which is controlled by PC. By using the position  $x_h(t)$  measured by the linear encoder, the slider can be controlled by the position feedback. By using the contact force  $f(t)$  measured by the load cell connected to the slider axis, the slider is also controlled by the force feedback and the integral force  $F$  can be calculated. In addition, the whole deformation of the object  $x(t)$  is measured by the vision sensor which can detect the marker attached to the edge of the object by the vision sensor. The paper is pasted at the side of the object, so that the adhesive effect between the object and the gripper is disappeared and the marker for the vision tracking is attached. The object is the rectangular-shaped wheat clay with the width of 44 mm, the depth of 22 mm, and the height of 22 mm, respectively. According to the proposed strategy, the slider is controlled by the position feedback in *Phase 1* and by the force feedback in *Phase 2*, respectively. We give the reference trajectory of the position input  $x_h^r(t)$  and that of the force input  $f^r(t)$  as follows:

$$x_h^r(t) = \begin{cases} \frac{1}{2} X_p^d \left( 1 - \cos\left(\frac{2\pi}{T_e} t\right) \right) & (0 \leq t < \frac{T_e}{2}) \quad (39) \\ X_p^d & (\frac{T_e}{2} \leq t \leq T_{e1}) \quad (40) \\ 0 & (T_{e1} < t < T_{e2}) \quad (41) \end{cases}$$

$$f^r(t) = \begin{cases} \frac{1}{2} f_c \left( 1 - \cos\left(\frac{2\pi}{T_s} t\right) \right) & (T_{e2} \leq t < \frac{T_s}{2}) \quad (42) \\ f_c & (\frac{T_s}{2} \leq t \leq T_h) \quad (43) \\ 0 & (T_h < t) \quad (44) \end{cases}$$

$$T_h = t|_{F=F^d(X_p^d)}. \quad (45)$$

TABLE I  
EXPERIMENTAL RESULTS UNDER  $X_p^d = 5.0$  MM

	$\hat{c}_1$ Ns/mm	$\hat{c}_2$ Ns/mm	$\hat{k}_1$ N/mm	$\hat{k}_2$ N/mm	$X_p$ mm
I	5.13	1.49	1.04	1.76	5.1
II	3.16	0.928	0.963	1.19	5.0
III	3.94	0.981	0.864	1.25	5.0
IV	3.55	0.910	0.911	1.54	4.8
V	4.77	1.17	1.02	3.98	5.5
Avg	4.11	1.10	0.960	1.944	5.1

TABLE II  
EXPERIMENTAL RESULTS UNDER  $X_p^d = 10.0$  MM

	$\hat{c}_1$ Ns/mm	$\hat{c}_2$ Ns/mm	$\hat{k}_1$ N/mm	$\hat{k}_2$ N/mm	$X_p$ mm
VI	2.56	1.11	0.983	1.16	9.8
VII	2.66	1.30	1.03	1.46	10.2
VIII	2.85	1.30	0.982	1.62	10.1
IX	2.74	0.94	1.15	2.20	9.8
X	2.36	1.05	0.966	1.18	10.2
Avg	2.63	1.14	1.022	1.52	10.0

$T_e$  and  $T_s$  are the cycle times of sine wave to determine the loading rates in *Phase 1* and *Phase 2*, respectively. By (43) and (45), the fast release motion is commanded at the time of  $F = F^d(X_p^d)$ .

### B. Experimental Results

Table I and Table II show the experimental results under the desired resultant deformation of  $X_p^d = 5.0$  mm and  $X_p^d = 10.0$  mm, respectively. In Table I and Table II, the estimated viscoelastic parameters and the resultant deformations in five trials for each  $X_p^d$  are shown. From Table I, we can see that the average value of  $\bar{X}_p = 4.7$  mm is generated for the desired value of  $X_p^d = 5.0$  mm, while the variations of the parameters are caused by the difference in the initial size error and the dryness of the object. Fig. 6 shows a series of photos during the trial I, where the solid line indicates the desired resultant deformation.<sup>1</sup> Fig. 6(a)–(c) show *Phase 1: Parameter Sensing Phase*. Fig. 6(d)–(f) show *Phase 2: Shaping Phase*. Fig. 7 shows the contact force  $f$  and the deformation of object  $x$  with respect to time in the trial I for  $X_p^d = 5.0$  mm under  $f_c = 5$  N, where the time labels of (a)–(f) in Fig. 7 are associated with Fig. 6(a)–(f), respectively. The dashed line in Fig. 7 shows the reconstructed deformation response  $x$  computed by using (12) with the parameters  $\hat{p}$  estimated in *Phase 1* and the contact force data  $f$ . While the viscoelastic parameters are estimated in *Phase 1*, the reconstructed deformation response nicely corresponds with the experimental one throughout all parts. In *Phase 2*, we can see that the object once has the over deformation, and then the deformation gradually recovers and approaches to the desired resultant deformation. While the time for achieving the desired deformation is  $t \simeq 40$  s as shown in Fig. 7(f), the actual time for the gripper to handle the object is just  $t = T_h = 16.9$  s as shown in Fig.

<sup>1</sup>The video attachment media file for this paper shows experiments.

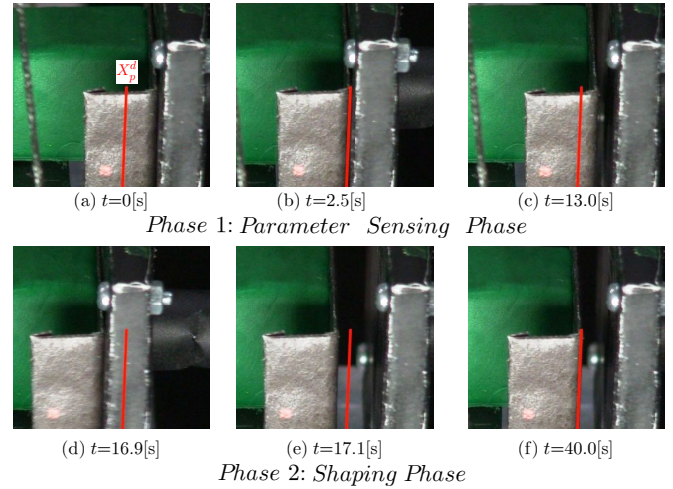


Fig. 6. A series of photos during an experiment under  $X_p^d = 5.0$  mm: In *Phase 1*, the object is deformed to  $X_p^d$  as shown in (b). Then some of the deformation recovers as shown in (c). In *Phase 2*, the object once deforms greater than  $X_p^d$  as shown in (d). However, the deformation gradually comes back to  $X_p^d$  after releasing as shown in (e) and (f).

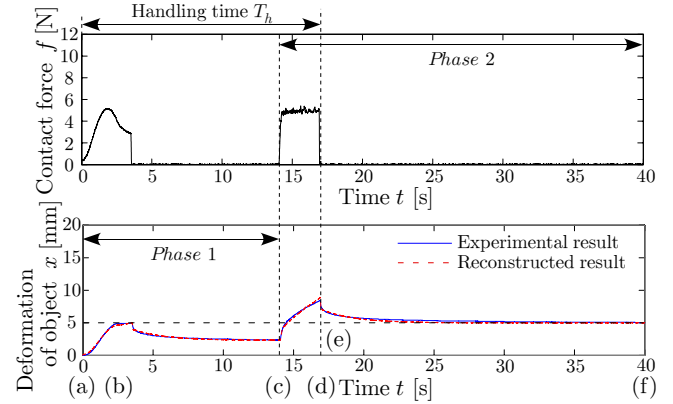


Fig. 7. Contact force and deformation of object with respect to time under  $X_p^d = 5.0$  mm.

7(d). In the same way, Fig. 8 shows the contact force  $f$  and the deformation of object  $x$  with respect to time in the trial VI for  $X_p^d = 10$  mm under  $f_c = 10$  N. We can see that the actual handling time is just  $T_h = 15.6$  s. Thus, the proposed method has an advantage that the handling time  $T_h$  can be reduced by actively managing the plastic deformation response included in the whole deformation response of the object.

### C. Discussion

We now consider the adaptability of the estimated parameters for different target deformation. From Table I and Table II, we can see that the estimated parameters depend upon the amount of deformation. This difference in parameters is caused by the non-linearity in large deformations. Fig. 9 (a-i) and (b-i) show the experimental results for  $X_p^d = 5$  mm and  $X_p^d = 10$  mm, respectively, where only *Phase 2: Shaping Phase* is executed with the constant viscoelastic parameters. These constant parameters were estimated in the

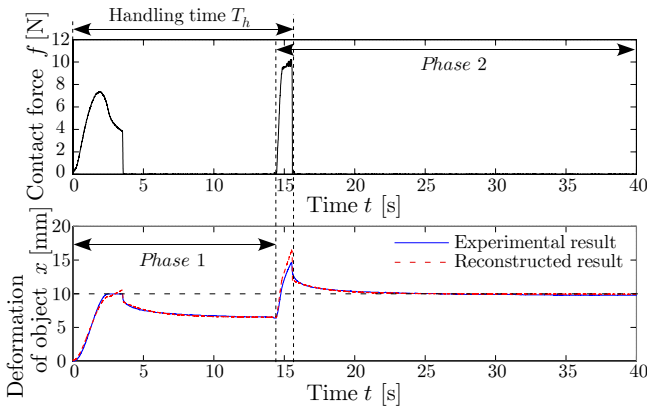


Fig. 8. Contact force and deformation of object with respect to time under  $X_p^d = 10.0$  mm.

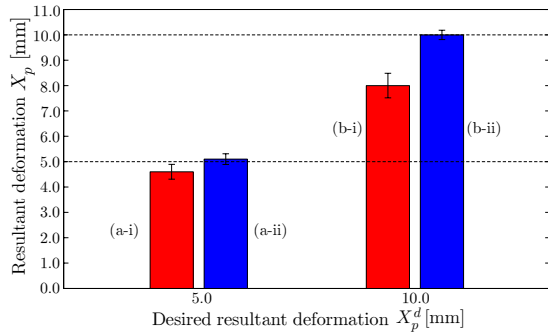


Fig. 9. Experimental results by two different methods; (i) is based on the constant parameters obtained by the deformation of 5 mm, and (ii) is based on the parameters depending upon the amount of the desired resultant deformation. The method (ii) introduced in this work is more useful for shaping the object precisely, by utilizing the dynamic parameter distribution.

preliminary experiments with the deformation of 5 mm. Fig. 9 (a-ii) and (b-ii) show the experimental results for  $X_p^d = 5$  mm and  $X_p^d = 10$  mm, respectively, where the proposed strategy with *Phase 1* and *Phase 2* is utilized. From Fig. 9(a-i) and (a-ii), we can see that both methods generate the resultant deformation with the same level. On the other hand, from Fig. 9(b-i) and (b-ii), we can see that the method (ii) obviously works better than the method (i). From the above observation, the method (ii) utilizing the dynamic parameter distribution is more useful for shaping the object precisely, while the parameters expressing the dynamic characteristic depend upon the amount of deformation.

## VI. CONCLUSION

We discussed an active shaping strategy for an unknown rheological object. The main results in this work are summarized as follows:

- We expressed the dynamic characteristics of rheological object's deformation by utilizing a four-element viscoelastic model, and formulated the deformation decomposition into the elastic response and the plastic one.

- We proposed a two-phase strategy for shaping the object: In the first phase, the viscoelastic parameters are estimated with avoiding the over deformation based on the elastic deformation response. In the second phase, the desired resultant deformation is generated with actively managing the plastic deformation response.
- We showed that, in the proposed strategy, the handling time in which the gripper actually works for the object can be formulated by a finite time, while the object's deformation is theoretically completed in the infinite time.
- We showed experimental results for confirming the validity of the proposed strategy.

In this work, we considered object's deformation in one-dimensional model. Actually, three-dimensional deformation is happened in real objects. In the future, we would like to extend the proposed method to three-dimensional model, with considering the suitability of real food objects.

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