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Sensor-Based Tracking of Environmental Level Sets by a Unicycle-Like Mobile Robot

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Abstract— **We consider a single Dubins-like mobile robot traveling with a constant longitudinal speed in a planar region supporting an unknown field distribution. A single sensor provides the distribution value at the current vehicle location. We present a new sliding mode control method for tracking environmental level sets: the controller drives the vehicle to the set where the distribution assumes a pre-specified value and ensures that the vehicle circulates along this set afterwards. The proposed control algorithm does not employ gradient estimation and is non-demanding with respect to both computation and motion. Its mathematically rigorous analysis and justification are provided. The effectiveness of the proposed guidance law is confirmed by illustrative examples and computer simulations.**

I. Introduction

In the last decades, an extensive body of research was devoted to the problem of monitoring environmental boundaries by means of mobile robots and robotic sensor networks, see e.g., [3], [5]–[7], [11]–[14], [18], [20], [24] and the literature therein. This involves designs of algorithms that ensure detecting, tracking, and displaying the boundary of an environmental region by a single robot or a team of them. Examples of applications where such missions are of interest include monitoring forest fire [5], [20], contaminant clouds [24], sea temperature and salinity, detection of harmful algae blooms [13], [18] and oil spills [7], avoidance of dangerous operational zones, etc. Tracking of environmental level sets refers to the case where the boundary of interest is determined as the set where a certain field distribution assumes a specific value. This field may represent the concentration of a chemical or biological agent, a distribution of a physical quantity, like thermal, magnetic, electric, or optical field distributions, etc. Typically, this distribution is not known, and the motion control is based on its sensing at the vehicle current location.

Available strategies for environmental boundary detection and tracking can be classified according to different aspects. Recently much attention was given to cooperative tracking by means of mobile sensor networks, see e.g., [3], [5]–[7], [11], [13], [14], [20], [24] and the literature therein. A team of vehicles has extended capabilities to accomplish tasks as compared with a single vehicle, even more capable than any team member, via, e.g., accessing extended knowledge of the environment thanks to collaborative sensing and sensor data exchange. However, even in this case, limitations on communication may require the vehicle to operate autonomously for considerable time and distance. In the single vehicle scenario, information constraints are a real concern. In this respect, most of related publications can be divided into two categories [12] depending on whether the gradient of the field distribution is available to the on-board controller (see, e.g., [15], [19], [21], [26]) or not (see e.g., [1]–[3], [5]). In the latter case, access to data from numerous sensors providing the distribution values at various locations promotes gradient estimation. The situation where the multiple sensor information is unavailable is more challenging. A typical method to compensate for the lack of data is to get extra information via extra maneuvers by e.g., 'dithering' the sensor position [4], [8], [25]. However, systematic costly and superfluous maneuvers may be required to collect rich enough data, whereas the complementary multiple sensor scenario means more complicated and costly hardware. It should be also noted that for the single sensor scenario, there in fact is no fully completed and justified solution for the problem of navigation for tracking environmental level sets. Most works rely, more or less, on heuristics and offer no rigorous and completed justification of the proposed control laws. Furthermore, the implications of the non-holonomic nature of the vehicle were not much examined up to now.

In this paper, a single non-holonomic vehicle modeled as unicycle is considered. It is controlled by the time-varying angular velocity limited by a given constant and travels with a constant longitudinal speed in a planar region supporting an unknown field distribution. Modulo natural and partly unavoidable assumptions, the distribution is arbitrary. A single sensor provides the distribution value at the current vehicle location, and the vehicle is also capable to access the rate at which this measurement evolves as time progresses, but no further sensing capabilities are assumed. A navigation strategy is proposed for tracking environmental level sets: this strategy steers the vehicle to the environmental set where the distribution assumes a pre-specified value and afterwards ensures circulation of the vehicle along this set at the prescribed speed. We note that the proposed controller, is motivated by the equiangular navigation guidance (ENG) law which navigates a wheeled robot towards an unknown target using the range and range-rate measurements [22], [23]. The control law does not employ gradient estimates and related systematic exploration maneuvers, as well as extended data processing. Instead, we propose a discontinuous sliding mode controller, non-demanding with respect to both computation

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and motion. Unlike most papers in the area, mathematically rigorous analysis and justification of the proposed strategy are offered. The applicability of the proposed navigation law is confirmed by computer simulations.

The body of the paper is organized as follows. Section II offers system description and problem setup, whereas Section III discusses assumptions. The main result is stated in Section IV and illustrated in Section V in the simplified yet instructive case of the central distribution. Illustrative examples and computer simulations are given in Section VI.

II. System description and problem setup

We consider a planar mobile robot modeled as unicycle. It travels with a constant longitudinal speed v and is controlled by the angular velocity u limited by a given constant $\overline{u}$. The robot travels in the area supporting an unknown field distribution $D(\mathbf{r})$. Here $\mathbf{r} = \mathbf{col}(x,y)$ is the vector of Cartesian coordinates x,y on the plane $\mathbb{R}^2$. The control objective is to steer the vehicle to the isoline where the distribution $D(x,y)$ takes a given value d_0, i.e., to the level curve $D(x,y) = d_0$, and make the vehicle circulating along this curve at the given speed v afterwards, thus displaying the isoline. The robot has access to the concentration $d(t) := D(x,y)$ at its current position $x = x(t), y = y(t)$ and is capable to access the rate $\dot{d}(t)$ at which this measurement evolves as time t progresses.

The kinematic model of the robot is as follows:

$$\begin{array}{lcl} \dot{x} = v\cos\theta & & x(0) = x_0 \\ \dot{y} = v\sin\theta & , & y(0) = y_0 \\ \dot{\theta} = u \in [-\overline{u}, \overline{u}]. & & \theta(0) = \theta_0 \end{array}, \qquad (1)$$

where θ gives the robot orientation. The equations (1) can describe the kinematics of tactical missiles, UAVs or wheeled mobile robots; see e.g. [9], [16], [17].

The problem is as follows. Find a controller that asymptotically drives the vehicle to the required level curve $D(x,y) = d_0$ and ensures that after a transient, the vehicle travels approximately along this curve at the given speed v. In brief, the objective is to ensure that $D[x(t),y(t)] \xrightarrow{t\to\infty} d_0$.

In this paper, we examine the following navigation law:

$$u(t) = \operatorname{sgn}\left\{\dot{d}(t) + \chi[d(t) - d_0]\right\}\overline{u},\ d(t) := D[x(t),y(t)], \quad (2)$$

where $\operatorname{sgn} a$ is the sign of a ($\operatorname{sgn} 0 := 0$) and $\chi(\cdot)$ is a linear function with saturation:

$$\chi(p) := \begin{cases} \gamma p & \text{if } |p| \le \delta \\ \operatorname{sgn}(p) v_* & \text{otherwise} \end{cases}, \quad v_* := \gamma\delta. \qquad (3)$$

The gain coefficient $\gamma > 0$ and the saturation threshold $\delta > 0$ are design parameters, which should be chosen to ensure the control objective.

III. Assumptions

We start with a natural technical requirement.

Assumption 3.1: The function $D(\cdot)$ is twice continuously differentiable.

To proceed, we introduce the matrix of the second derivatives D'' and that Φ_α of rotation at angle α:

$$D''(\mathbf{r}) := \begin{pmatrix} D''_{xx}(\mathbf{r}) & D''_{xy}(\mathbf{r}) \\ D''_{yx}(\mathbf{r}) & D''_{yy}(\mathbf{r}) \end{pmatrix},\ \Phi_\alpha := \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix},$$

as well as the standard inner product $\langle\cdot;\cdot\rangle$ in the plane

$$\langle \mathbf{r}; \rho\rangle := r_x\rho_x + r_y\rho_y \quad \text{for} \quad \begin{array}{l} \mathbf{r} = \mathbf{col}(r_x, r_y) \\ \rho = \mathbf{col}(\rho_x, \rho_y) \end{array}$$

and the related norm $\|\mathbf{r}\| := \sqrt{\langle \mathbf{r};\mathbf{r}\rangle}$.

Definition 3.1: The value d_* of D is said to be *trackable* if the related level set $L(d_*) := \{\mathbf{r} : D(\mathbf{r}) = d_*\}$ is a regular planar curve, i.e., $\nabla D(\mathbf{r}) := \mathbf{col}[D'_x(\mathbf{r}), D'_y(\mathbf{r})] \neq 0$ for all $\mathbf{r} \in L(d_*)$, and the robot is capable to track this curve, i.e., the curvature radius $R_{\text{curv}}(\mathbf{r})$ of the curve at any of its point $\mathbf{r} \in L(d_*)$ exceeds the minimal turning radius of the robot:

$$R_{\text{curv}}(\mathbf{r}) = \frac{\|\nabla D(\mathbf{r})\|^3}{\left|\left\langle D''(\mathbf{r})\Phi_{\frac{\pi}{2}}\nabla D(\mathbf{r}); \Phi_{\frac{\pi}{2}}\nabla D(\mathbf{r})\right\rangle\right|} > R := \frac{v}{\overline{u}}. \quad (4)$$

We assume that $a/0 := \infty$ for $a > 0$. So the inequality from (4) necessarily holds whenever the denominator equals zero.

The following assumption means that the control objective is realistic.

Assumption 3.2: The examined value d_0 is trackable.

Moreover, we need an extended version of this assumption. To introduce it, we start with the following.

Definition 3.2: The *initial circle* is any of two circles of the radius R that pass the initial position $\mathbf{col}(x_0, y_0)$ of the robot with the tangential angle θ_0 from (1). The *initial disc* is that encircled by an initial circle.

In other words, the initial circle is the trajectory of the vehicle driven by the constant control $u \equiv \overline{u}$ or $u \equiv -\overline{u}$.

Assumption 3.3: There exists an interval $[d_-, d_+]$ such that the following statements hold:

i) Any value $d \in [d_-, d_+]$ is trackable;
ii) This interval contains both the examined value d_0 and the value $d = D(\mathbf{r})$ for all $\mathbf{r}$ from any of the initial discs.

In other words, at any point $\mathbf{r}$ from the region

$$\mathcal{R} := \{\mathbf{r} : d_- \le D(\mathbf{r}) \le d_+\}, \qquad (5)$$

the gradient $\nabla D(\mathbf{r})$ is nonzero, and the curvature radius of the isoline passing through $\mathbf{r}$ exceeds the minimal turning radius of the robot, i.e., (4) holds for any $\mathbf{r} \in L(d_*)$ and $d_* \in [d_-, d_+]$. Our last assumption follows from the previous one in the case where the region (5) is bounded. So it is in fact required only if this region is unbounded.

Assumption 3.4: In the region (5), the gradient $\nabla D(\mathbf{r})$ is separated from zero:

$$\|\nabla D(\mathbf{r})\| \ge \kappa > 0 \qquad \forall \mathbf{r} \in \mathcal{R}. \qquad (6)$$

There exist $\Delta > 0$ and $\lambda \in (0,1)$ such that the following extension of (4) holds in the region (5), i.e., for all $\mathbf{r} \in \mathcal{R}$:

$$\frac{\|\nabla D(\mathbf{r})\|^3}{\left|\left\langle D''(\mathbf{r})\Phi_{\frac{\pi}{2}+\alpha}\nabla D(\mathbf{r}); \Phi_{\frac{\pi}{2}+\alpha}\nabla D(\mathbf{r})\right\rangle\right|} \ge \frac{R}{\lambda} \quad \forall |\alpha| \le \Delta. \quad (7)$$

Remark 3.1: By reducing Δ if necessary, we can assume without any loss of generality that

$$\Delta \leq \arccos \lambda. \tag{8}$$

IV. The Main Result

Now we are in a position to state the main result of the paper.

Theorem 4.1: Suppose that Assumptions 3.1—3.4 hold and the parameters $\gamma > 0$ and $\delta > 0$ of the controller (2) are chosen so that

$$v_* := \gamma\delta \leq v\kappa \sin\Delta, \qquad \gamma v_* < \kappa v \overline{u}\big[\cos\Delta - \lambda\big], \tag{9}$$

where Δ, κ, and λ are taken from Assumption 3.4, and v is the robot speed from (1). Then the controller (2) drives the vehicle to the required level curve $D = d_0$ and ensures its asymptotic tracking, i.e., $d(t) \to d_0$ as $t \to \infty$.

The proof of this theorem will be given in the full version of this paper.

Remark 4.1: i) The controller (2) may exhibit a sliding motion. Theorem 4.1 addresses the equivalent dynamics [10].

ii) The only limitation on the choice of $d_\pm$ is that d_- should not exceed the examined value d_0 and the infimum of $D(\mathbf{r})$ on any of the initial discs, whereas d_+ should not be less than the corresponding supremum and d_0.

iii) In many practical situations, $D[\mathbf{r}] > 0$ and $D[\mathbf{r}] \to 0$ as $\|\mathbf{r}\| \to \infty$. Then for $d_- > 0$, the region (5) is necessarily compact and so Assumption 3.4 follows from Assumption 3.3. At the same time, the quantities κ, Δ and λ from Assumption 3.4 remain used in the conditions (9) under which the controller parameters v_* and γ inevitably ensure the control objective. These conditions can be rewritten without an aid of the mediators Δ and λ, as is shown by the following lemma.

Lemma 4.1: Suppose that Assumptions 3.1—3.3 hold and the region (5) is bounded. Then Assumption 3.4 is true. Furthermore, (9) is valid whenever

$$b := \frac{\gamma v_*}{\kappa v \overline{u}} < c - R \cdot \omega(c) \quad \text{for} \quad c := \sqrt{1 - \frac{v_*^2}{\kappa^2 v^2}}, \tag{10}$$

where

$$0 < \kappa \leq \min_{\mathbf{r} \in \mathcal{R}} \|\nabla D(\mathbf{r})\| \quad \text{and} \quad \omega(c) :=$$

$$\max_{|\alpha| \leq \arccos c} \max_{\mathbf{r} \in \mathcal{R}} \frac{\left|\left\langle D''(\mathbf{r})\Phi_{\frac{\pi}{2}+\alpha}\nabla D(\mathbf{r}); \Phi_{\frac{\pi}{2}+\alpha}\nabla D(\mathbf{r})\right\rangle\right|}{\|\nabla D(\mathbf{r})\|^3}. \tag{11}$$

The proof of this lemma will be given in the full version of this paper.

Remark 4.2: i) In fact, the conditions (9) and (10) are equivalent if a common κ is employed in them.

ii) The variables b and c from (10) can be viewed as new independent controller parameters with the ranges $b > 0, 0 \leq c < 1$. The parameters v_*, γ, and δ can be restored from b, c by the formulas:

$$v_* = \kappa v \sqrt{1 - c^2}, \qquad \gamma = \frac{b\overline{u}}{\sqrt{1 - c^2}}, \qquad \delta = \frac{v_*}{\gamma}.$$

iii) The function $\omega(\cdot)$ is continuous and decreases. So the right hand side of the inequality from (10) increases as $c \uparrow$; its limit as $c \to 1 - 0$ is positive due to Assumption 3.3. Hence b and c satisfying (10) do exist.

iv) In (10), the maximum $\omega(c)$ from (11) can be replaced by its upper estimate $\overline{\omega}(c)$.

V. Central Distribution

Now we illustrate Theorem 4.1 and the controller design in the simplified yet instructive case of the central distribution:

$$D(\mathbf{r}) = \varphi(\|\mathbf{r}\|), \qquad \text{where} \tag{12}$$
$$\varphi(p) \geq 0, \quad \varphi(p) \to 0 \text{ as } p \to \infty,$$
$$\varphi'(p) < 0 \; \forall p > 0, \quad \varphi'(0) = 0.$$

The function $\varphi(\cdot)$ is twice continuously differentiable. For brevity of notations, the origin of the Cartesian coordinate system is co-located with the center of the distribution.

The level sets are circles centered at the origin. Those $\{\mathbf{r} : D(\mathbf{r}) = d_0\}$ with $d_0 < \varphi(R)$ have the radius $< R$ and so cannot be tracked. All other level sets can be tracked by means of the controller (2) provided that its parameters are chosen properly and the vehicle starts moving far enough from the center of the distribution, as is shown by the following.

Lemma 5.1: Suppose that the field distribution satisfies (12), initially, the distance from the vehicle to the distribution center exceeds trice the minimal turning radius $R_0 := \|\mathbf{r}(0)\| > 3R$, and the examined value d_0 meets the vehicle tracking capability: $d_0 < \varphi(R)$, where $R := v/\overline{u}$ is the minimal turning radius. Suppose also that the parameters of the controller are chosen so that $c < 1$ and

$$b < c - \frac{R}{R_-}, \; b < c - R \max_{R_- \leq \rho \leq R_+} \left|\frac{c^2}{\rho} + \frac{\varphi''(\rho)}{\varphi'(\rho)}(1 - c^2)\right|, \tag{13}$$

where

$$R_- \leq \min\{R_0 - 2R; R_*\}, \qquad R_+ \geq \max\{R_0 - 2R; R_*\} \tag{14}$$

and R_* is the root of the equation $\varphi(R_*) = d_0$, whereas b and c are defined in (10). Then the controller (2) asymptotically drives the vehicle to the required level curve $D = d_0$ and ensures its asymptotic tracking, i.e., $d(t) \to d_0$ as $t \to \infty$.

The proofs of the lemmas from this section will be given in the full version of this paper.

A. Central Gaussian Distribution

If the field distribution is caused by immersion of a certain substance from a pointwise source and its subsequent diffusion in an isotropic unbounded environment, the distribution profile at a given time is often Gaussian:

$$D(x,y) = \Phi e^{-\frac{(x-x_c)^2+(y-y_c)^2}{2\sigma^2}}.$$

In practice, the center (x_c, y_c), intensity Φ, and dispersion σ may be unknown. At the same time, they can be often estimated a priory

$$\sigma \in \big[\sigma_-, \sigma_+\big], \; \Phi \in \big[\Phi_-, \Phi_+\big], \; \sqrt{|x_c - x_*|^2 + |y_c - y_*|^2} \leq \rho_*,$$

where $\sigma_+ \geq \sigma_- > 0, \Phi_+ \geq \Phi_- > 0, x_*, y_*, \rho_* \geq 0$ are known. The next lemma offers a criterion for convergence of the vehicle to the required level set, which is stated in terms of only these known quantities.

Lemma 5.2: Suppose that the initial distance $R_{0,*}$ from the robot to the point (x_*, y_*) exceeds $3R + \rho_*$, and the examined value $d_0 < \Phi_- e^{-\frac{R^2}{2\sigma_-^2}}$, where $R := v/\overline{u}$ is the minimal turning radius. Suppose also that the parameters b and c of the controller (defined in (10)) are chosen so that

$$1 > c > \max_{\sigma \in [\sigma_-, \sigma_+]} \zeta(\sigma). \quad 0 < b < c - \frac{R}{R_-}, \tag{15}$$

where

$$\zeta(\sigma) := \left\{ \begin{array}{ll} \frac{R}{R_-} & \text{if } \sigma \geq \frac{\sqrt{R_- R_+ \left(1 - \frac{R^2}{R_-^2}\right)}}{\sqrt{1 + \frac{R_-}{R_+}}} \\ \sqrt{1 - \frac{\sigma^2}{R_- R_+}\left(1 + \frac{R_-}{R_+}\right)} & \text{otherwise} \end{array} \right\},$$

$$R_- := \min\left\{ R_{0,*} - \rho_* - 2R; \sigma_- \sqrt{2 \ln \frac{\Phi_-}{d_0}} \right\} \tag{16}$$

$$R_+ := \max\left\{ R_{0,*} + \rho_* - 2R; \sigma_+ \sqrt{2 \ln \frac{\Phi_+}{d_0}} \right\}$$

Then the controller (2) asymptotically drives the vehicle to the required level curve $D = d_0$ and ensures its asymptotic tracking, i.e., $d(t) \to d_0$ as $t \to \infty$.

VI. Simulation Results

In this section, we present some examples of computer simulations which demonstrate the performance of the navigation law (2). We simulate a unicycle robot governed by the kinematic equations (1) moving in a planar region which supports an unknown field distribution. The robot moves with constant linear velocity $v = 0.5\ m/s$ and its maximum angular velocity is limited to $\overline{u} = 1\ rad/s$. It is supposed to track the environmental level sets defined by an unknown distribution function. In the first simulation, we consider a Gaussian field caused by the distribution function $D(x,y) = 10e^{-((x-8)^2+(y-5)^2)/600}$. The controller parameters $\gamma = 1$, $\delta = 0.1$ and $v_* = \gamma\delta = 0.1$ are chosen to meet the conditions (10), (15) and (16). The filed distribution is shown in Fig. 1. Having applied the navigation law (2), the robot approaches and moves along the desired level set, shown in Fig. 2(a). Fig. 2(b) depicts the distribution value at the robot position which converges to the desired value $d_0 = 6m$. Another example is shown in figures 3 and 4 wherein the robot tracks several level sets caused by distribution function $D(x,y) = 10 - 0.5\sqrt{(x-8)^2 + (y-5)^2}/3$ and predefined with $d_0 = 3,4,5,6,7$ and $9m$.

Remark 6.1: On the sliding surface, the proposed controller generates high-frequency switching control signal. In practice, this may lead to undesirable chattering due to constraints on available switching frequency and delay. A way to overcome this problem is to replace the discontinuous

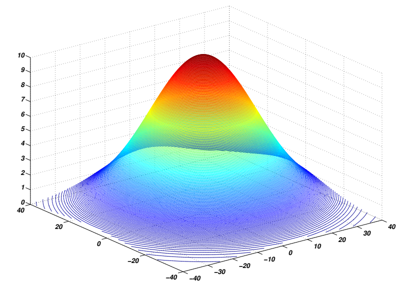
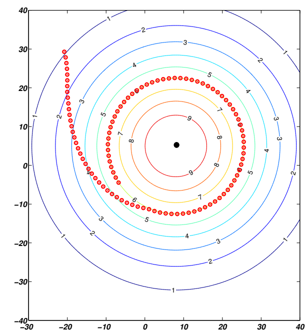
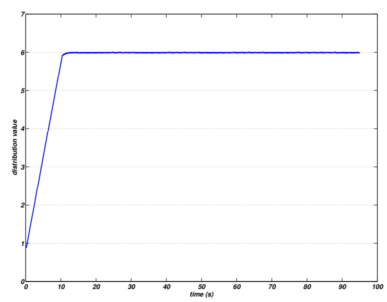


Fig. 1: The Gaussian field distribution generated by $D(x,y) = 10e^{-((x-8)^2+(y-5)^2)/600}$



(a)



(b)

Fig. 2: (a) Tracking a desired level set by a unicycle robot and (b) distribution value at the robot position.

sign function in the control law (2) by its continuous approximation. Our computer simulation employs this method with approximation by a linear function with saturation.

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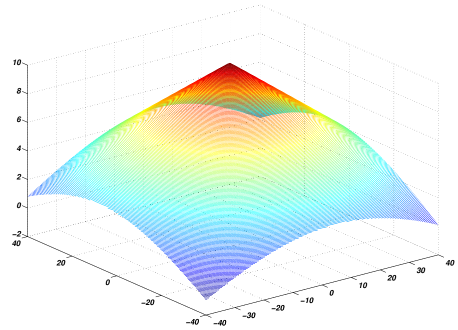
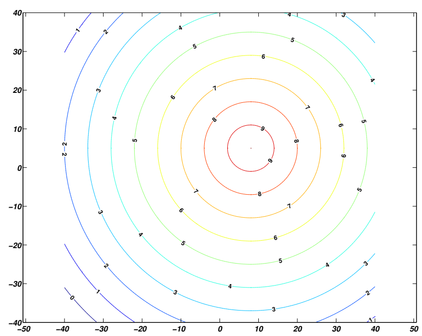


(a)



(b)

Fig. 3: The field distribution generated by $D(x,y) = 10 - 0.5\sqrt{(x-8)^2+(y-5)^2}/3$ and its level sets

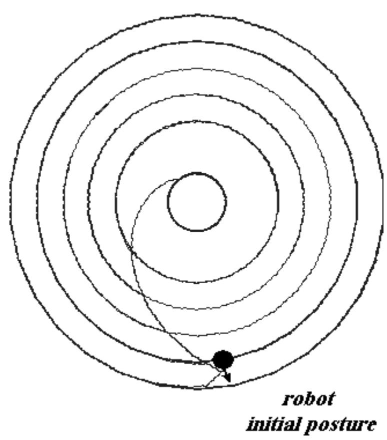


Fig. 4: Tracking the desired level sets by a unicycle robot

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