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Intrinsic Repeatability: a new index for repeatability characterisation

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Abstract— **The paper deals with the question of robot precision and how to characterise repeatability. Hence ISO and ANSI repeatability indexes advantages and drawbacks are analysed. A new intrinsic repeatability index is proposed that can estimate the robot endpoint position variability satisfying the non-bias and convergence conditions. Computation of this index is performed using simulated straight and drifting trajectories. Influence of load on repeatability is studied using an experimental determination of an angular covariance matrix. Therefrom intrinsic repeatability can be computed in every workspace location using only this covariance matrix and the stochastic ellipsoid theory.**

Index Terms— **Stochastic Ellipsoids, Repeatability, Robot Accuracy, Industrial Robot**

INTRODUCTION

In the field of industrial robots, precision is an important issue. Precision is characterized by two different indicators: accuracy and repeatability. If the target is always the same, and the move is repeated several times, repeatability measures the dispersion between final points. Accuracy characterises the distance between the cloud of points and the commanded position as explained in ISO9283 [1] or ANSI R15.05-1[2].

In the first section, robot precision indices are presented. Within them, the ISO and ANSI repeatability indices are compared. In the second section, we study the pros and cons of the two different approaches. The concept of a repeatability sphere is discussed. The repeatability estimation is analysed in a mathematical point of view concerning bias and convergence. The sample size and the drift influence are specified.

In the third section, a new repeatability index is proposed to overcome some disadvantages of the usual procedures: it is called intrinsic repeatability index and can unify the ISO and ANSI approach. Simulations are made to illustrate the concept revealing intrinsic repeatability as a very good statistical estimator.

In the fourth section, we give the main lines to evaluate intrinsic repeatability using only the covariance matrix. We display an experimental procedure to estimate the angular covariance matrix and show that it is possible to evaluate load influence on repeatability index from this covariance matrix. Then, it is possible to evaluate workspace location influence on the repeatability index.

I. USUAL PRECISION INDICES

1) Repeatability and accuracy: The estimation of industrial robot precision is based on a test where the robot is set up to attain a commanded point and come back, this cycle being repeated several times in the same conditions. Measurements of the final robot positions show that they are near the commanded point and all the final positions constitute a cloud of points. Precision is then declined in accuracy and repeatability as displayed in fig.1. In the ISO procedure, the distance between the mean of the different final positions and the commanded position will caracterise accuracy. The ANSI definition is slightly different as it considers different locations on a standard path. For each location, the distance between the final position and the commanded position, called the deviation is measured. The accuracy index is then the mean of all the deviations.

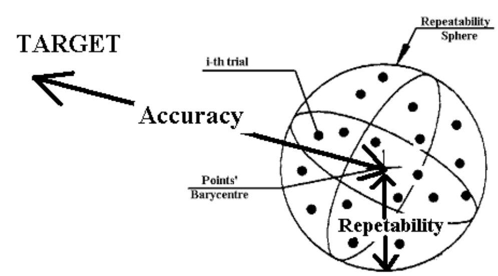

Fig. 1. ISO approach of accuracy and repeatability

2) ISO and ANSI repeatability indices: The ISO definition of repeatability index is the formula $REP_{ISO} = \overline{D} + 3S_D$ where $D = \sqrt{(x_i - \overline{x})^2 + (y_i - \overline{y})^2 + (z_i - \overline{z})^2}$ is the random variable (RV) "distance between the point (x_i, y_i, z_i) and the barycentre $(\overline{x}, \overline{y}, \overline{z})$". In this method, the repeatability is estimated at a given location. Therefore in order to evaluate repeatability variability in the workspace, it is necessary to estimate this repeatability index in different locations in the workspace.

The ANSI definition is slightly different because three different locations distributed at the extremities and the middle of the standard path have to be considered and the repeatability index is a mean of the distances between the target and the final position barycentre. So somewhere the influence of workspace location is summed up already in the ANSI repeatability index. To make it more clear, there should be several ISO repeatability indices for a robot but only one ANSI repeatability index.

II. Discussion on usual repeatability indices

Repeatability sphere ? Concerning repeatability, both procedures are considering distances between final positions and the barycentre. ISO considers the mean plus three times the standard deviation. We could think that this definition is inspired by the Gaussian distribution. The 3σ interval could implicitly mean that 99.7% of the final positions are bounded in a sphere, whose radius would be the repeatability index. In the ANSI procedure, repeatability index is simply the mean of the different distances between final position and the barycentre. In this last case, it is explicitly said that a sphere with a radius equal to $\overline{r}_{REP}+3S_{REP}$ will enclose 99.7% of the results, where $\overline{r}_{REP}$ is the total mean repeatability and S_{REP} is the standard deviation. But in fact, both procedures do not give enough information about the spatial distribution. They have a pragmatic approach and they sum up the cloud in one indicator. Consequently the spatial distribution can not be apprehended in three dimensions. Having just one figure and this cloud representation, we may think of a repeatability sphere. But this image is wrong and ellipsoids should be used [3]. So the 99.7% of the results bounded in the repeatability sphere may be wrong in most of the cases.

Bias and convergence ? If we consider the estimation of repeatability as a classical estimation problem, we would search for an estimation indicator that would have two important properties: no bias and convergence. Let us see how the ISO and ANSI repeatability indices accomodate with these two criteria. The question of having a non-biased index is essentially linked with the warm-up delay. ISO suggests to begin measures after several moves to warm up the robot and to wait for a stabilization of the final endpoint positions. When the robot begins the cycles, because of thermic dilatation, we observed a drift phenomena. We remarked that the necessary delay to obtain stabilisation can be long (one or several hours), and moreover it is difficult to caracterise the time when stabilization is obtained. This is a real disadvantage in an industrial application because of wasted time in the process setup and it is a real disadvantage also for robot specification because we do not know when to begin the experiments to have a representative result. The danger is that both repeatability and drift may be caracterised together. ISO and ANSI have a special procedure to caracterise warm-up delay and then it would be interesting if the repeatability index could be estimated without being mixed with the warm-up effects.

The sample size. The question of convergence is also a crucial issue. The confidence interval for the repeatability depends on the size of the data sample. In the ISO procedure, the size of the sample is 30. In [4], the case of an isotropic Maxwell distribution is studied, and we computed the 0.95 confidence interval for the repeatability estimation and it was $\pm 16\%$. If the size increased to 100 samples, the confidence interval reduced to $\pm 8\%$. So in theory the larger the size of the sample, the thinner the uncertainty about repeatability estimation. But in practise, if the sample was larger, the drift would bring a bias on the result.

Both problems can be illustrated by the example of fig.2 where a linear drift affects the trajectory. If the repeatability index is calculated using sets of size 30, the effect of linear drift will always bias the final estimation. We may think that a better result could be obtained computing mean of the consecutive repeatability indexes (by set of size 30), but even in this case the influence of the drift will always subsist. This is quite annoying as the quantity of information about the process being larger, the estimation precision should be better. ISO does not consider the question of computing the mean of consecutive repeatability indexes though it could be interesting to improve the estimation precision despite the bias is still there. As this repeatability index variability is quite important, the danger is that robot manufacturer would choose the best figure in the whole experiments and the displayed repeatability would rather be a minimum of the repeatability indexes than a mean !

In the ANSI procedure, the sample size is 500 after stabilisation. With this large sample size, the uncertainty width which is proportional to $\frac{1}{\sqrt{N}}$ is thin. One drawback is that experiments would require time and another difficult is being sure stabilization is obtained.

The drift. In fact both norms want to estimate repeatability when experimental conditions are such that every try can be considered as a realisation of a unique random variable, which means in practise that the probability density function is the same for the whole trajectory. We have studied this property and it is not clear if such an hypothesis is valid. In fact when studying the process in a statistical point of view, it reveals that the process is stationary for the second order, ie the standard deviation can be considered constant. But the first order stationarity was not always established in our experimental work as explained in [5].

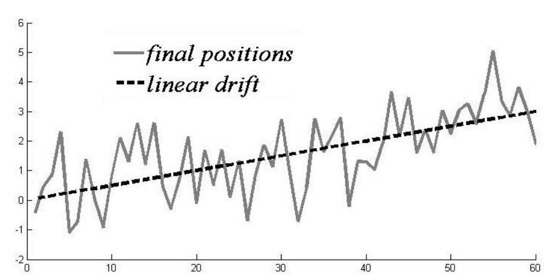

Fig. 2. Simulation of a linear drift leading to a repeatability index bias

To sum up, a correct repeatability estimation should have no bias and should converge. The experimental procedures proposed in the ISO and ANSI norm suffer from this point of view. The need for a mathematical estimator respecting both non-bias and convergence conditions is crucial.

III. Intrinsic repeatability, a new repeatability index

We are looking for an estimator that has no bias, converges and can be used as soon as the robot begins working without waiting for the warm-up delay.

In this purpose, we try to eliminate the drift influence. This is done first calculating the moving average $Mean(10)$ on the last ten attempts and studying the difference between this moving average and the measured position. The statistical distribution of this new RV is studied. It is obvious from fig. 3 that there is an improvement in the fact that the trajectory drift is already smaller. Then we tried to restrict the moving average $Mean(3)$ to the three last attempts, the trajectory for the new RV was even straighter. At last, the information concerning moving average was taken on the previous attempt $Mean(1)$ and the jump process was studied. A complete statistical analysis proved that the jump process was stationary and so it was the right RV to work with in order to estimate repeatability.

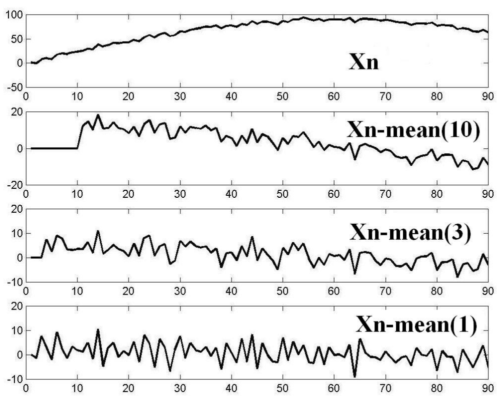

Fig. 3. Moving average to get rid of the drift effect

We find then a relation between the jump and position process. A statistical study of the position process show that the Gaussian distribution is a good modeling. So two parameters are necessary to caracterise this distribution: mean and variance. But the mean of the position distribution is affected by the drift and so was difficult to estimate. In fact, the mean does not really interest us in the case of repeatability because we have just said that we wanted to build a new index, independent of the drift.

Let $E(X_n)$ and σ_n (resp. $E(X_{n+1})$ and σ_{n+1}) be the mathematical expectation and standard deviation of the X_n Gaussian position distribution (resp. X_{n+1}). The mean $E(X_n)$ of the position process is not stationary but is affected by a drift illustrated in fig.4.

Experimentally the variations of $E(X_n)$ are small compared to the process standard deviation σ but are significant on a long time schedule: $E(X_{n+1}) \simeq E(X_n)$ and sometimes $|E(X_{n+30}) - E(X_{n+1})| \gg \sigma$. The standard deviation σ_n of the position process X_n is considered constant: $\sigma_n = \sigma$.

With these assumptions, the jump process $J_n = X_{n+1} - X_n$ is a centered Gaussian distribution with a standard

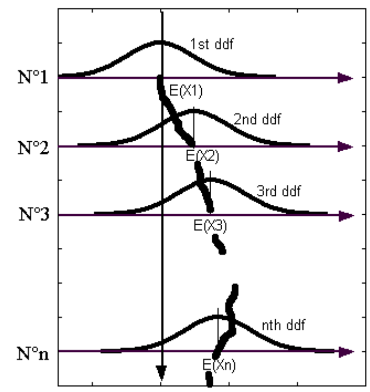

Fig. 4. Probability density function of the position process

deviation of $\sigma\sqrt{2}$.

This stochastic property is interesting as it is then possible to estimate the repeatability from the jump process taking into account wider parts of the trajectory. Estimation is no longer affected by the drift of the position process. Numerous shots increase the variance estimation precision as the uncertainty decreases with $\frac{1}{\sqrt{N}}$. Moreover experiments can begin early without waiting for stabilization and the temperature variations in the room do not affect the results. In 3-D, the jump process covariance matrix is equal to the position process covariance matrix multiplied by a factor 2. So if all the drift affecting the position process disappears, it is as if the jump and position spatial distributions are $\sqrt{2}$ ratio homothetic. The computation of the repeatability index from the jump process can be achieved in a similar way as in the ISO and ANSI norms but in the end, the result has to be divided by $\sqrt{2}$ to find the usual repeatability indices and be able to compare the results. Let J be the Euclidean norm of the 3-D jump process then the definitions of the jump process are:

$$REP_{intr} = \frac{1}{\sqrt{2}}\left[\overline{J} + 3 \times S_J\right] \text{ - ISO procedure}$$

$$REP_{intr} = \frac{1}{\sqrt{2}}\overline{J} \text{ - ANSI procedure}$$

3) Case of a simulated straight trajectory: To illustrate the relevance of this new procedure, a 3D random normal trajectory of size 300 is simulated on fig.5. It is then possible from this trajectory to estimate ISO and ANSI repeatability indices and compare them with the corresponding intrinsic repeatability indices.

For the ISO procedure, 10 different repeatability indices can be computed taking into account sample 1: attempts 1-30, sample 2: attempts 31-60,... These indices are plot on fig.6. It is clear that the variability of the indices are important, so which one will be chosen ? On the same figure, we display the mean of ISO repeatability indices computed on the first k samples: $rep\_progressive\_mean(k) = \sum_{i=1}^{k} rep(i)$ and the intrinsic repeatability computed on the $k \times 30$

first position values: $REP_{intr}\_progressive\_value(k) = REP_{intr}(1->30 \times k)$. Several simulations showed that in the end repeatability indices progressive mean and intrinsic repeatability progressive value are very close and converge to the final value in the same pace, which are very interesting properties.

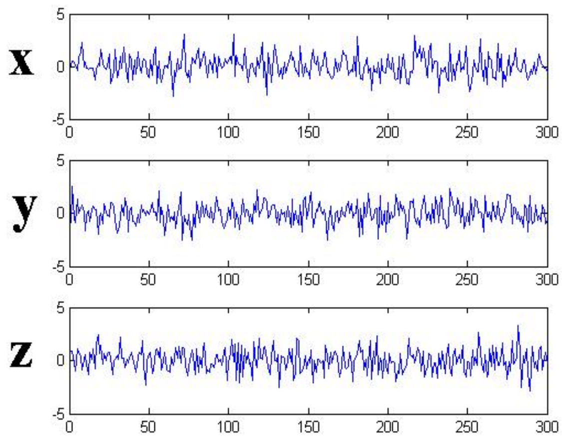

Fig. 5. (x,y,z) position simulation in μm

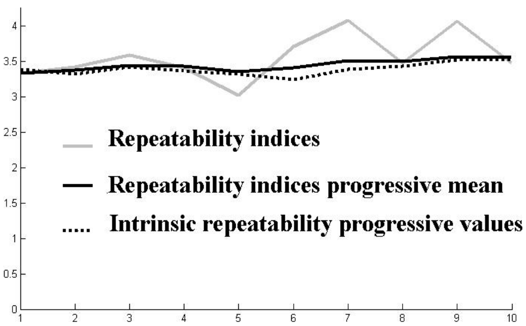

Fig. 6. Comparison of ISO repeatability and intrinsic repeatability

For the ANSI procedure, we display on fig.7 the ANSI and intrinsic repeatability indices computed on the first k attempts. The values are very close and it shows that the intrinsic repeatability and ANSI repeatability have the same value when the position trajectory is straight, meaning with a first order stationarity.

4) Case of a simulated drifting trajectory: Now the simulated trajectory of fig.8 is affected by a drift that could represent the thermic dilatation effects during the warm-up period. This was modelled by the sum of a Gaussian distribution added with a sinusoid signal.

We used the same methodology as in the previous subsection and the results are displayed in fig.9. It is clear that the different repeatability indices still keep their variability but the mean of these indices and the intrinsic repeatability converge nearly to the same value with a difference below 6%.

On fig.10 the results for the ANSI procedure. At the beginning, both values are very close but on the long time period, the ANSI repeatability is severely affected by the drift. On the contrary the ANSI intrinsic repeatability remains stable.

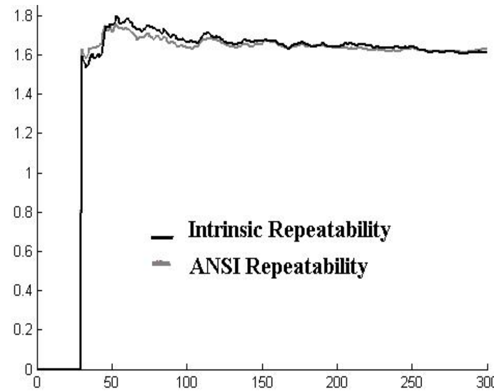

Fig. 7. ANSI and intrinsic repeatability for the straight trajectory

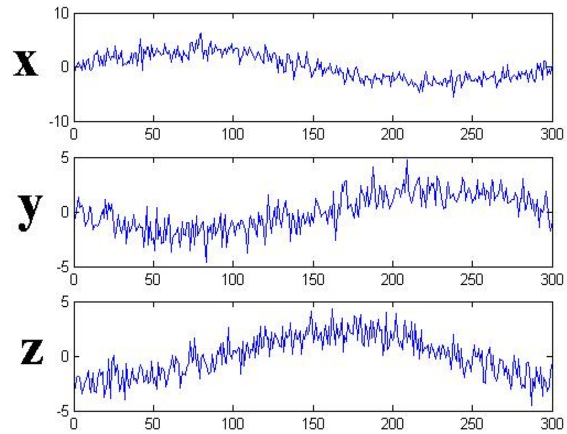

Fig. 8. A simulated drifting trajectory

The conclusions are clear. When the trajectory is straight, intrinsic repeatability gives the same results as ISO or ANSI indices. It is better than the ISO because it converges whereas the ISO different indices vary. When the trajectory is affected by a drift, intrinsic repeatability is close to ISO repeatability progressive mean. On the contrary, ANSI repeatability indices overestimates the process variability. So intrinsic repeatability is a convenient estimator because it converges in $\frac{1}{\sqrt{N}}$ and can eliminate the long-period drift effect. Consequently, the measures can be taken without waiting for the warm-up delay.

At this stage, we can notice that the slope of the drift depends on several factors among then the cycle time, the load and posture which have an influence on the motor torques, the current intensity, the Joule effects, the thermic equilibrium delay... That is the reason why we want to eliminate this drift because it depends partly on the user and is not an intrinsic characterisation of the manufacturer control and architecture.

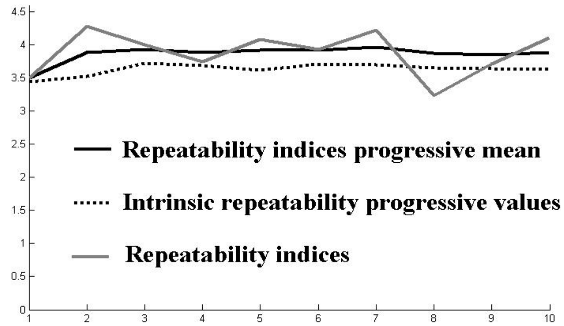

Fig. 9. ISO and intrinsic repeatability - case of the drift

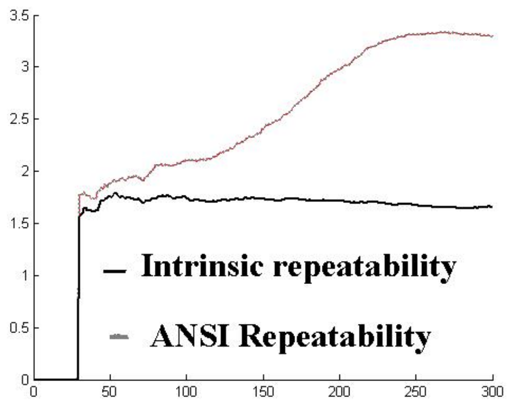

Fig. 10. ANSI and intrinsic repeatability for the drifting trajectory

IV. COMPUTING INTRINSIC REPEATABILITY FROM THE COVARIANCE MATRIX

We built a new intrinsic repeatability index that has a lot of advantages. But this index has the same disadvantage as the traditional ones because it does not describe the 3D-spatial distribution of the repeatability phenomenon. On the contrary, the stochastic ellipsoid approach can give precise information about this spatial distribution. In this section, we just present a simple experimental procedure to estimate the covariance matrix and then explain how to compute directly intrinsic repeatability from this covariance matrix.

Angular covariance matrix. For the angular covariance matrix estimation, we just use one micrometer and move one axis at a time, the other axes are blocked by the brakes. For each axis, at least 200 cycles are commanded. From the statistical series, we estimate the standard deviation and at this stage, it is better to use the jump process. This procedure was completed for a KUKA IR384 and a SAMSUNG Faraman, for a medium load and a high load. Here is for instance the covariance matrix D_0 for the Kuka robot loaded with 3.5 kg:

$$D_0 = diag\,[28.9; 20.6; 67.1; 219; 130; 291] \ \ (10^{-12}\ rad^2)$$

Influence factors for repeatability. Many factors have been suspected to influence repeatability, as speed, load, workspace location, backlash, temperature,... But there are not so many statistical work performed to discriminate which factor was the most influent. Riemer and Edan were interested in workspace location influence [6], Offodile and Ugwu in load and speed influence [7].

It is possible to evaluate the load influence on repeatability considering the differences in the covariance matrix for medium and high load. For instance, for the Kuka robot with $8.5kg$, the covariance matrix D_1 is:

$$D_1 = diag\,[34.4; 24.5; 64.7; 166; 1710; 685] \ \ (10^{-12}\ rad^2)$$

It is clear than the load has a very important influence on the variance for the 5th and 6th axes. For the other axes, the estimations are statistically the same. Certainly, it is possible to improve the design of the 5th axis control. It is now interesting to know if these differences have important consequences on the value of intrinsic repeatability. For this, we must take into account the lever arm lengths and spatial combination of all these uncertainty sources.

Computation of intrinsic repeatability from the covariance matrix is easier when the density is isotropic as the density follows the Maxwell function. In other cases, the computation is numerical with the following steps.

1. The mean computation $\overline{L}$. Let σ_x^2, σ_y^2, σ_z^2 be the eigenvalues of the covariance matrix in a given location, $L = \|X\|_2$ the distance from final point $X = (x, y, z)$ to the barycentre. The mean of the distance is computed from the following integral:

$$\overline{L}=\frac{(2\pi)^{-\frac{3}{2}}}{\sigma_x\sigma_y\sigma_z}\iiint \|X\|_2 \exp\left[-\frac{1}{2}\left(\frac{x^2}{\sigma_x^2}+\frac{y^2}{\sigma_y^2}+\frac{z^2}{\sigma_z^2}\right)\right]dX$$

Let $C_\theta = \cos\theta$; $S_\theta = \sin\theta$. Using ellipsoidal coordinates $x = r\sigma_x C_\varphi C_\theta$; $y = r\sigma_y C_\varphi S_\theta$; $z = r\sigma_z S_\varphi$, we obtain:

$$\overline{L}=\alpha\iint\sqrt{(C_\varphi C_\theta\sigma_x)^2+(C_\varphi S_\theta\sigma_y)^2+(S_\varphi\sigma_z)^2}C_\varphi d\theta d\varphi$$

where $\alpha = 2\times(2\pi)^{-\frac{3}{2}}$. This integral has to be evaluated numerically.

2. The variance computation S_L From the formula: $S_L^2 = Var(L) = \overline{L^2} - \overline{L}^2$. The mean of L^2 is easier to compute and leads to the simple result: $\overline{L^2} = \sigma_x^2+\sigma_y^2+\sigma_z^2 = tr(C) = tr(D)$. So the final expression of the intrinsic repeatability index is $REP_{ISO} = \overline{L}+3\sqrt{tr(D)-\overline{L}^2}$ in the ISO approach and $REP_{ANSI} = \overline{L}$ in the ANSI approach..

CONCLUSIONS

In this paper, we presented a new index for repeatability characterization: intrinsic repeatability. Intrinsic repeatability has the following interesting properties: it has no-bias ; it converges in $\frac{1}{\sqrt{N}}$ more quickly than ISO or ANSI indices ; it can be evaluated without waiting for warm-up delay. For all these reasons, this index is wellsuited for industrial applications. It could replace the ISO or ANSI indices. Moreover, it is possible to compute intrinsic repeatability from the angular covariance matrix, which can be estimated experimentally using only one micrometer. The influence of load on repeatability can be estimated from the differences in the covariance matrices. Workspace location influence can also be evaluated through computation of intrinsic repeatability and this simplifies the test procedure. There is no more need to specifiy "a standard test path" for instance. Finally, the spatial distribution of the cloud can be determined from the covariance matrix giving a reliable 3-D confidence set for the robot endpoint.

Because of all these advantages, we suggest that robot manufacturers should provide covariance matrices corresponding to medium and nominal load and should adopt the concept of intrinsic repeatability.

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