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Motion Tracking in Robotic Manipulators in Presence of Delay in Measurements

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Abstract— **Time-delay in sensor measurements can be a frequent cause of instability and performance degradation in a robotic system. In this paper, motion tracking of rigid manipulators in presence of constant and known delay in sensors is investigated. By using non-minimal model of a manipulator, a dynamically smooth controller based on the Linear Matrix Inequality (LMI) approach is proposed which guarantees asymptotic tracking of desired joint angles and velocities in presence of delayed measurements. For a given controller the maximum amount of delay that preserves system stability is computed by solving an LMI optimization and also by numerical simulations, and the results are compared. Finally, a simulation example is presented that illustrates the performance of the proposed controller in comparison with standard motion controllers.**

I. INTRODUCTION

Time-delay often appears in robot manipulators either in input control or in output measurements. Occurrence of delay in input control is mainly due to actuator dynamics, data processing used for generation of the control signal, or the existence of a distance between the place where control signal is generated and the place where control signal is applied to the robot manipulator. Delay in output measurements such as joint angle encoder readings can occur due to significant communication distance between the sensor and the controller.Time-delay can also appear due to malfunctioning of electronic interface devices or data acquisition or processing systems. In some cases delay is indirectly induced in the control system by a phase lag created by filtering out the noise components of velocity or force measurements.

Time-delay has been shown to be a frequent source of instability and performance deterioration in control systems. Stability analysis and controller design for linear time-delayed systems have been extensively studied in literature over the recent decade, see, e.g., [1], [2], [3], [4] and [5]. In case of robot manipulators with nonlinear dynamics, the presence of delay adds further challenges in control design. For example, in robotic tele-operation systems, the presence of delay in communication lines is usually considered as an important performance limiting factor, where the common assumption is that each robot (master or slave) has access to its own states without delay, [6], [7], [8], [9] and [10] .

In [11] the effect of computational time-delay on system performance was analyzed by using self-tuning predicted (STP) and PID controllers. Also, in [12] qualitative and quantitative analysis of the effect of computational delay on a robotic system were presented, and as a criteria for microprocessor selection, upper bounds for the maximum tolerable time-delay for preserving system stability were derived. The destabilizing effect of computational delay in the control law for a robot manipulator operating under a model-based PD controller was investigated in [13].

In [14], by considering time-delays in control signals and actuator dynamics, set-point *regulation* problem was considered and sufficient conditions for asymptotic stability of a selected operating point were established for a rigid robot under state and output feedback laws. In [15] set-point regulation problem for a flexible-joint robot with time-delay in the actuator input signals was investigated by using the output feedback controller of [16], and sufficient conditions for exponential stability of the selected operating point in the presence of single or multiple time-delays were established.

So far, *asymptotic* motion *tracking* problem for nonlinear dynamics of a robot manipulator with delayed measurements has not been fully considered in literature. The asymptotic nature of the tracking is important since unlike the controllers with ultimately bounded tracking property, in the asymptotic tracking, the controller gains are not required to be high and this in turn decreases system sensitivity to noise.

In this paper, we assume that joint angle and velocity measurements are subject to a constant and known time-delay. In Section II, non-minimal model of a rigid robot manipulator introduced in [17] is explained and Section III is devoted to the statement of the control problem. In Section IV, a dynamically smooth controller is proposed and sufficient conditions in the form of LMIs are formulated which ensure asymptotic convergence of motion tracking errors in the presence of measurement delays. In Section V, stability analysis for the proposed controller is presented. In Section VI, a simulation example is presented that illustrates the efficiency of proposed controller in comparison with standard controllers. The maximum amount of delay that preserves system stability is computed by solving an LMI optimization and also by numerical simulations. Comparison of the results gives a measure of conservatism in the LMI setups. Finally, conclusions are given in Section VII.

Notations. In the sequel, $\|\cdot\|$ denotes the Euclidean norm of vectors and the induced norm of matrices. $\mathcal{C}_{k,d} = \mathcal{C}([-d,0],\mathbb{R}^k)$ denotes the Banach space of continuous functions mapping the interval $[-d,0]$ into $\mathbb{R}^k$, with the topology of uniform convergence. $\mathbf{I}$ denotes the identity matrix with appropriate dimensions. $X > 0$ ($X \geq 0$) means that

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TABLE I

NOMENCLATURE

$q\in \mathbb{R}^n$	:	joint angle vector
$v\in \mathbb{R}^m$	:	generalized link velocity vector
$\pi\in \mathbb{R}^m$	:	generalized link coordinate vector
$p\in \mathbb{R}^{3n}$	:	link position vector
$\tau_m\in \mathbb{R}^n$	:	motor torque
$f\in \mathbb{R}^m$	:	link generalized forces

X is a real symmetric positive-definitive matrix (positive-semidefinite). $\star$ denotes the symmetric part.

II. NON-MINIMAL MODEL OF AN n-DOF RIGID MANIPULATOR

We consider an n-DOF rigid robot manipulator. The generalized velocity vector $v \in \mathbb{R}^m$ is defined by $v := [\dot{p}_1, \cdots, \dot{p}_n, \omega_1^1, \cdots, \omega_n^n]$ where $\dot{p}_i \in \mathbb{R}^3$ denotes the translational velocity of the center of mass of the i-th link, expressed in the base fixed frame and $\omega_i^i := R_i^\top(q)\omega_i \in \mathbb{R}^3$ where $R_i(q) \in \mathbb{R}^{3\times3}$ is the rotation matrix of the i-th link frame with respect to the base fixed frame and ω_i denotes the angular velocity of the i-th link with respect to the base frame. Therefore, the generalized position vector $\pi \in \mathbb{R}^m$ is defined by $\pi := [p_1, \cdots, p_n, \alpha_1^1, \cdots, \alpha_n^n]$ where $p_i \in \mathbb{R}^3$ denotes the position of the center of mass of the i-th link and $\alpha_i^i \in \mathbb{R}^3$ is the integral of the vector ω_i^i with respect to the time. Note that α_i^i is a pseudo-angle and does not represent a physical angle.

The generalized velocity vector v is related to robot joint angles and velocities $(q, \dot{q}) \in \mathbb{R}^n \times \mathbb{R}^n$ by the following holonomic constraint

$$v = J(q)\dot{q} \tag{1}$$

where $J(q) \in \mathbb{R}^{m\times n}$ is the robot generalized Jacobian matrix.

Property 1: $J(q)$ is full-column rank for all $q \in \mathbb{R}^n$, and its pseudo-inverse is given by $J^\dagger(q) = (J^\top(q)J(q))^{-1}J^\top(q)$ with the property that $J^\dagger(q)J(q) = \mathbf{I}_{n\times n}$. Also, there exist finite-positive constants κ_1, κ_2, κ_3, κ_4 such that

$$\begin{cases} \kappa_1 \leq \|J(q)\| \leq \kappa_2 \\ \kappa_3 \leq \|J^\dagger(q)\| \leq \kappa_4 \qquad \forall q \in \mathbb{R}^n \end{cases} \tag{2}$$

□

A non-minimal dynamics of a manipulator is given by [17]

$$D\dot{v} + g_c = f \tag{3}$$

where $D \in \mathbb{R}^{m\times m}$ is the constant inertia matrix given by

$$D := diag\{m_1\mathbf{I_{3\times3}}, \cdots, m_n\mathbf{I_{3\times3}}, I_1^1, \cdots, I_n^n\}$$

where m_i denotes the mass of the i-th link and $I_i^i \in \mathbb{R}^{3\times3}$ denotes the constant inertia tensor of the i-th link relative to a frame attached to its center of mass. Also, $f \in \mathbb{R}^m$ is the link generalized force and $g_c \in \mathbb{R}^m$ is the constant link gravity vector defined by

$$g_c := \left[\frac{\partial U(p)}{\partial p}^\top, \quad \underbrace{0_{1\times3}, \cdots, 0_{1\times3}}_{\text{n times}} \right]^\top$$

where $U(p) := \sum_{i=1}^n m_i g_0^\top p_i$ is the potential energy of the manipulator and $g_0 \in \mathbb{R}^3$ is the vector of gravity acceleration.

Remark 1: By virtue of (3) and (1), the standard minimal dynamics of the manipulator can be rewritten by

$$J^\top(q)DJ(q)\ddot{q} + J^\top(q)D\dot{J}(q,\dot{q})\dot{q} + J^\top(q)g_c = J^\top(q)f \tag{4}$$

where $M(q) = J^\top(q)DJ(q) \in \mathbb{R}^{n\times n}$ is the robot inertia matrix and $C(q,\dot{q})\dot{q} = J^\top(q)D\dot{J}(q,\dot{q})\dot{q} \in \mathbb{R}^n$ is the vector of Coriolis and Centrifugal forces. The gravity force vector is given by $g(q) = J^\top(q)g_c \in \mathbb{R}^n$. The actuator torque input is presented by $\tau_m = J^\top(q)f \in \mathbb{R}^n$. □

Remark 2: By integrating (1), we obtain a general $\mathcal{C}^1$ forward kinematics map $q \longmapsto p$ defined by $p := \varphi_p(q)$. We assume that this map is uniquely invertible which means that by specifying the positions of the center of mass of *all* links, its joint angles are uniquely determined. It can be verified that for a serial manipulator with fixed base, this assumption holds. Consequently, by this assumption if p tracks a reference trajectory defined by $p_r := \varphi_p(q_r)$, then the robot joint angle vector q tracks also the reference trajectory q_r. □

Property 2: $J(q)$ is Lipschitz in q, i.e., $\exists l_j > 0$ such that

$$\|J(x) - J(y)\| \leq l_j\|x - y\| \tag{5}$$

Moreover, $\varphi_p^{-1}(p)$ is Lipschitz in p, i.e., $\exists l_\varphi > 0$ such that

$$\|\varphi_p^{-1}(x) - \varphi_p^{-1}(y)\| \leq l_\varphi\|x - y\| \tag{6}$$

where $\varphi_p^{-1}(.)$ is the unique and continuous inverse of $\varphi_p(.)$.

III. PROBLEM STATEMENT

Consider the equations (3) and (1) and let $q_r : \mathbb{R}_{\geq0} \to \mathbb{R}^n$ be a given twice continuously differentiable reference trajectory for the joint angles. Define the desired generalized velocity by $v_r := J(q_r)\dot{q}_r$. Define $\pi_d := [p_{r1}, ...p_{rn}, \alpha_{r1}^1, ...\alpha_{rn}^n]$ where $\pi_r(t) = \int_0^t v_r(\sigma)d\sigma$. Evidently by this definition, we have $p_r = \varphi_p(q_r)$. Assume that measurement of robot joint angles and velocities are available with a constant and known time-delay $d > 0$. Note that if delay is time-varying, the notation d in this paper can be treated as an upper bound for the true time-varying delay. Under these conditions, we propose a control law $\tau_m(t) = \psi(q_r, \dot{q}_r, \ddot{q}_r, q(t-d), \dot{q}(t-d))$ such that the position and velocity tracking errors defined by $\tilde{\pi}(t) = \pi(t) - \pi_r(t)$, $\tilde{v}(t) = v(t) - v_r(t)$, $\tilde{q}(t) = q(t) - q_r(t)$ and $\dot{\tilde{q}}(t) = \dot{q}(t) - \dot{q}_r(t)$, converge to zero, asymptotically.

In the sequel, for brevity in notation wherever $q(t)$ or $\dot{q}(t)$ appear as arguments of a function, they are simply written as $q, \dot{q}$.

IV. CONTROL DESIGN

We propose the control law given by

$$\begin{cases} \tau_m(t) = J^\top(q_r(t))f_d(t) \\ f_d(t) = D\dot{v}_r(t) + Du(t) + g_c \end{cases} \tag{7}$$

where $u(t) \in \mathbb{R}^m$ is in the form of a delayed smooth state-feedback as

$$u(t) = Ke(t-d) \tag{8}$$

$$\begin{bmatrix} \Gamma_{11} & \Gamma_{12} & \Gamma_{13} & Z & 0 & 0 & 0 & 0 & 0 & M & d\beta_1 X_1^\top \\ \Gamma_{12}^\top & -d^{-1}(\beta_1-1)Z & \Gamma_{23} & 0 & Z & 0 & M & 0 & 0 & 0 & 0 \\ \Gamma_{13}^\top & \Gamma_{23}^\top & -W+dE_{33} & 0 & 0 & Z & 0 & \delta_1 X^\top N^\top & \delta_2 X^\top N^\top & 0 & 0 \\ Z & 0 & 0 & -\gamma\mathbf{I} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & Z & 0 & 0 & -\gamma\mathbf{I} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & Z & 0 & 0 & -\gamma\mathbf{I} & 0 & 0 & 0 & 0 & 0 \\ 0 & M^\top & 0 & 0 & 0 & 0 & -\delta_2\mathbf{I} & 0 & 0 & 0 & 0 \\ 0 & 0 & \delta_1 NX & 0 & 0 & 0 & 0 & -\delta_1\mathbf{I} & 0 & 0 & 0 \\ 0 & 0 & \delta_2 NX & 0 & 0 & 0 & 0 & 0 & -\delta_2\mathbf{I} & 0 & 0 \\ M^\top & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\delta_1\mathbf{I} & 0 \\ d\beta_1 X_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -d\beta_1 Z \end{bmatrix} < 0 \tag{9}$$

$$-Z + E_{22} < 0 \tag{10}$$

$$-Z + \beta_2\gamma\mathbf{I} < 0 \tag{11}$$

$$\begin{bmatrix} -Z & dX_1^\top \\ dX_1 & -Z \end{bmatrix} < 0 \tag{12}$$

$$E = \begin{bmatrix} E_{11} & E_{12} & E_{13} \\ E_{12}^\top & E_{22} & E_{23} \\ E_{13}^\top & E_{23}^\top & E_{33} \end{bmatrix} > 0 \tag{13}$$

with K as a constant matrix and $e(t-d) = \begin{bmatrix} \tilde{\pi}(t-d)^\top & \tilde{v}(t-d)^\top \end{bmatrix}^\top$.

Theorem 1: Consider the control law (7) and (8). For some given scalars $h > 0$, $\beta_1 > 1$, β_2, δ_1 and δ_2, the error signal $e(t)$ is asymptotically stable if we select the control gain as $K = XZ^{-1}$, and if there exist positive-definite matrices $Z, W, E_{11}, E_{22}, E_{33}$, positive scaler γ and any matrices $X, X_1, E_{12}, E_{13}, E_{23}$ satisfying the LMIs shown in (9)-(13), where $M = BD^{-1}$, $N = h^{-1}D$, $B = \begin{bmatrix} 0 & \mathbf{I} \end{bmatrix}_{m\times k}^\top$ ($k = 2m$) and

$\Gamma_{11} = AZ + ZA^\top + X_1 + X_1^\top + W + dE_{11}$
$\Gamma_{12} = ZA^\top + X_1^\top + E_{12}$
$\Gamma_{13} = BX - X_1 + dE_{13}$
$\Gamma_{23} = BX - X_1 + E_{23}$

□

This Theorem provides delay-dependent sufficient conditions for asymptotic stability of link position and velocity tracking error $\tilde{\pi}$ and $\tilde{v}$ in a rigid robot manipulator. In light of Remark 2, the joint angle tracking error is given by $\tilde{q} = \varphi_p^{-1}(p) - \varphi_p^{-1}(p_r)$ where $\varphi_p^{-1}(.)$ is the unique and continuous inverse of $\varphi_p(.)$. Therefore, joint angle tracking error $\tilde{q}$ uniformly asymptotically converges to zero. Also, link velocity tracking error $\tilde{v}$ is given by $\tilde{v} = J(q)\dot{q} - J(q_r)\dot{q}_r$, which implies that

$$J(q)\dot{\tilde{q}} = \tilde{v} + J(q_r)\dot{q}_r - J(q)\dot{q}_r$$

Since by Property 1, the Jacobian matrix $J(q)$ has full-column rank and bounded for all q, we conclude that joint angle velocity tracking error $\dot{\tilde{q}}$ converges to zero, asymptotically.

Proof: Proof of Theorem 1 is presented in the next section.

V. Stability analysis

A. Error dynamics

Replacing for the torque control signal $\tau_m(t)$ in the manipulator minimal dynamics (4) from (7), yields

$$J^\top(q)DJ(q)\ddot{q} + J^\top(q)D\dot{J}(q,\dot{q})\dot{q} + J^\top(q)g_c = J^\top(q_r(t))f_d \tag{14}$$

The right-hand side of this equation can be expanded by

$$\begin{aligned} J^\top(q)DJ(q)\ddot{q} + J^\top(q)D\dot{J}(q,\dot{q})\dot{q} + J^\top(q)g_c &= J^\top(q)f_d \\ &+ \big(J^\top(q_r(t)) - J^\top(q)\big)f_d \\ &= J^\top(q)\Big(f_d + J^{\dagger\top}(q)\big(J^\top(q_r(t)) - J^\top(q)\big)f_d\Big) \end{aligned} \tag{15}$$

where we use the fact that $J^\top(q)J^{\dagger\top}(q) = \mathbf{I}_{n\times n}$. From the RHS of (15) we conclude that the application of the torque $\tau_m = J^\top(q_r(t))f_d$ to the manipulator joints, generates the generalized force $f_d + J^{\dagger\top}(q)(\Delta J)^\top f_d$ in the manipulator links, where $\Delta J = J(q_r(t)) - J(q)$. Therefore, by virtue of (3) we have

$$D\dot{v} + g_c = f_d + J^{\dagger\top}(q)(\Delta J)^\top f_d \tag{16}$$

By replacing for f_d and u in the equation (16) from (7) and (8) , and since $\dot{\pi}_r(t) = v_r(t)$, the error dynamics can be expressed by

$$\dot{e}(t) = Ae(t) + (B + \Delta B)Ke(t-d) + Bw(t) \tag{17}$$

where

$$A = \begin{bmatrix} 0 & \mathbf{I} \\ 0 & 0 \end{bmatrix}_{k\times k}, \qquad B = \begin{bmatrix} 0 \\ \mathbf{I} \end{bmatrix}_{k\times m}$$

$$w(t) = D^{-1}J^{\dagger\top}(q)(\Delta J)^\top(D\dot{v}_r(t) + g_c) \tag{18}$$

and ΔB is in the form

$$\Delta B = BD^{-1}J^{\dagger\top}(q)(\Delta J)^\top D = MF(t)N \tag{19}$$

where $M = BD^{-1}$, $N = h^{-1}D$ and $F(t) = hJ^{\dagger\top}(q)(\Delta J)^\top$.

Since by Property 1, $J^\dagger(q)$ and $J(q)$ are bounded, so $F(t)$ and $w(t)$ are bounded. Positive scaler h is selected such that matrix $F(t)$ satisfies $F^\top(t)F(t) \le \mathbf{I}$.

B. Asymptotic stability of tracking errors

Before presenting the proof of the convergence, we define the operator $\mathcal{D}(e_t) : \mathcal{C}_{k,d} \to \mathbb{R}^k$ by

$$\mathcal{D}(e_t) = e(t) + H\int_{t-d}^{t} e(\varsigma)\mathrm{d}\varsigma \tag{20}$$

where $e_t = e(t+\varsigma), \varsigma \in [-d, 0]$ and $H \in \mathbb{R}^{k\times k}$ is a constant matrix which will be specified later on.

Differentiating (20) with respect to time and replacing for $\dot{e}(t)$ from (17), the transformed closed-loop system is

$$\dot{\mathcal{D}}(e_t) = (A+H)e(t) + \Big((B+\Delta B)K - H\Big)e(t-d) + Bw(t) \tag{21}$$

This transformation is similar to the so called parameterized neutral model transformation introduced in [18].

Next, we remind the following technical results which will be used in the stability analysis.

Fact 1: [19] For a given scaler $\delta > 0$, matrix $F(t)$ with $F^\top(t)F(t) \le \mathbf{I}$, and any constant matrices M, N, the inequality $MF(t)N + N^\top F^\top(t)M^\top \le \delta MM^\top + \delta^{-1}N^\top N$ is always satisfied.

Lemma 1: [20] For any positive symmetric matrix $\Omega \in \mathbb{R}^{k\times k}$, a scaler $\rho > 0$, and any integrable vector function $z : [0, \rho] \to \mathbb{R}^k$, the following inequality holds

$$(\int_0^\rho z(\varsigma)\mathrm{d}\varsigma)^\top \Omega (\int_0^\rho z(\varsigma)\mathrm{d}\varsigma) \le \rho \int_0^\rho z(\varsigma)^\top \Omega z(\varsigma)\mathrm{d}\varsigma.$$

Lemma 2: [21] Consider the operator $\mathcal{D}(.) : \mathcal{C}_{k,d} \to \mathbb{R}^k$ defined by $\mathcal{D}(e_t) = e(t) + \hat{B}\int_{t-d}^{t} e(\varsigma)\mathrm{d}\varsigma$ where $e(t) \in \mathbb{R}^k$ and $\hat{B} \in \mathbb{R}^{k\times k}$. For a given scaler μ, with $0 < \mu < 1$, if there exists a positive-definite matrix S such that

$$\begin{bmatrix} -\mu S & d\hat{B}^\top S \\ dS\hat{B} & -S \end{bmatrix} < 0$$

then, the operator $\mathcal{D}(e_t)$ is stable.

We start the stability analysis by considering the following Lyapunov functional candidate

$$V = V_1 + V_2 + V_3 + V_4$$

where

$$V_1 = \mathcal{D}^\top(e_t)P\mathcal{D}(e_t) \tag{22}$$

$$V_2 = \beta_1 \int_{t-d}^{t}\int_{\varsigma}^{t} e^\top(s)H^\top PHe(s)\mathrm{d}s\mathrm{d}\varsigma \tag{23}$$

$$V_3 = \int_{t-d}^{t} e^\top(\varsigma)Qe(\varsigma)\mathrm{d}\varsigma \tag{24}$$

$$V_4 = \int_0^t \int_{\varsigma-d}^{\varsigma} \xi^\top \Sigma E \Sigma \xi \mathrm{d}s\mathrm{d}\varsigma \tag{25}$$

and $\xi = \begin{bmatrix} e^\top(\varsigma) & e^\top(s)H^\top & e^\top(\varsigma-d) \end{bmatrix}^\top$, $P > 0$, $\Sigma = \mathrm{diag}\{P, P, P\}$ and $Q > 0$.

Defining $y(t) = \int_{t-d}^{t} He(\varsigma)\mathrm{d}\varsigma$ and differentiating V with respect to time, yields

$$\begin{aligned}\dot{V} &= \dot{V}_1 + \dot{V}_2 + \dot{V}_3 + \dot{V}_4\\ \dot{V}_1 &= 2\mathcal{D}^\top(e_t)P\dot{\mathcal{D}}(e_t)\end{aligned}$$

Substituting for $\mathcal{D}^\top(e_t)$ and $\dot{\mathcal{D}}(e_t)$ from (20) and (21), yields

$$\begin{aligned}\dot{V}_1 &= 2\Big\{e(t) + H\int_{t-d}^{t} e(\varsigma)\mathrm{d}\varsigma\Big\}^\top P\Big\{(A+H)e(t)\\ &\quad + \Big((B+\Delta B)K - H\Big)e(t-d) + Bw(t)\Big\}\\ &= e^\top(t)\Big\{P(A+H) + (A+H)^\top P\Big\}e(t)\\ &\quad + 2e^\top(t)P(BK-H)e(t-d)\\ &\quad + 2y^\top(t)P(A+H)e(t)\\ &\quad + 2y^\top(t)P(BK-H)e(t-d) + 2e^\top(t)PBw(t)\\ &\quad + 2y^\top(t)PBw(t) + 2e^\top(t)P\Delta BKe(t-d)\\ &\quad + 2y^\top(t)P\Delta BKe(t-d)\end{aligned} \tag{26}$$

$$\dot{V}_2 = d\beta_1 e^\top(t)H^\top PHe(t) - \beta_1\int_{t-d}^{t} e^\top(\varsigma)H^\top PHe(\varsigma)\mathrm{d}\varsigma$$

$$\begin{aligned}\dot{V}_2 &\le d\beta_1 e^\top(t)H^\top PHe(t) - \int_{t-d}^{t} e^\top(\varsigma)H^\top PHe(\varsigma)\mathrm{d}\varsigma\\ &\quad - d^{-1}(\beta_1 - 1)y^\top(t)Py(t)\end{aligned} \tag{27}$$

where Lemma 1 was utilized in deriving the bound of $\dot{V}_2$. Moreover,

$$\dot{V}_3 = e^\top(t)Qe(t) - e^\top(t-d)Qe(t-d) \tag{28}$$

$$\begin{aligned}\dot{V}_4 &= de^\top(t)PE_{11}Pe(t) + 2e^\top(t)PE_{12}P\int_{t-d}^{t} He(\varsigma)\mathrm{d}\varsigma\\ &\quad + \int_{t-d}^{t} e^\top(\varsigma)H^\top PE_{22}PHe(\varsigma)\mathrm{d}\varsigma\\ &\quad + 2de^\top(t)PE_{13}Pe(t-d) + 2y^\top(t)PE_{23}Pe(t-d)\\ &\quad + de^\top(t-d)PE_{33}Pe(t-d)\end{aligned} \tag{29}$$

On the other hand, in light of Fact 1 we have

$$\begin{aligned}2e^\top(t)P\Delta BKe(t-d) &\le \delta_1^{-1}e^\top(t)PMM^\top Pe(t)\\ &\quad + \delta_1 e^\top(t-d)K^T N^T NKe(t-d)\\ 2y^\top(t)P\Delta BKe(t-d) &\le \delta_2^{-1}y^\top(t)PMM^\top Py(t)\\ &\quad + \delta_2 e^\top(t-d)K^\top N^\top NKe(t-d)\end{aligned} \tag{30}$$

Substituting (30) into (26), $\dot{V}$ can be bounded by

$$\begin{aligned}\dot{V} &\le \eta^\top\Psi\eta + \int_{t-d}^{t} e^\top(\varsigma)H^\top(-P + PE_{22}P)He(\varsigma)\mathrm{d}\varsigma\\ &\quad + 2y^\top(t)PBw(t) + 2e^\top(t)PBw(t)\\ &\le \eta^\top\Psi\eta + \int_{t-d}^{t} e^\top(\varsigma)H^\top(-P + PE_{22}P)He(\varsigma)\mathrm{d}\varsigma\\ &\quad + 2\|\eta\|\|P\|\|B\|\|w(t)\|\end{aligned} \tag{31}$$

where $\eta = \left[\begin{array}{ccc} e^\top(t) & y^\top(t) & e^\top(t-d) \end{array} \right]^\top$, and Ψ is given by

$$\left[\begin{array}{ccc} \Lambda_1 & \Lambda_{12} & \Lambda_{13} \\ \star & \Lambda_{22} & \Lambda_{23} \\ \star & \star & \Lambda_{33} \end{array} \right] \quad (32)$$

where, for simplicity in notation we have defined Λ_{ij}s as follows

$$\begin{aligned} \Lambda_{11} &= P(A+H) + (A+H)^\top P + \beta_1 d H^\top P H + Q \\ &\quad + dPE_{11}P + \delta_1^{-1} PMM^\top P \\ \Lambda_{12} &= (A+H)^\top P + PE_{12}P \\ \Lambda_{13} &= P(BK-H) + dPE_{13}P \\ \Lambda_{22} &= -d^{-1}(\beta_1 - 1)P + \delta_2^{-1} PMM^\top P \\ \Lambda_{23} &= P(BK-H) + PE_{23}P \\ \Lambda_{33} &= dPE_{33}P - Q + \delta_2 K^\top N^\top NK + \delta_1 K^\top N^\top NK \end{aligned}$$

Since by Property 1, $J^\dagger(q)$ is bounded, so we have

$$2\|B\|\|w(t)\| < b\|\Delta J\|$$

also in light of Property 2 we have

$$\|\Delta J\| \leq l_j \|q - q_r\|$$

and

$$\|q - q_r\| = \|\varphi_p^{-1}(p) - \varphi_p^{-1}(p_r)\| \leq l_\varphi \|p - p_r\|$$

Hence,

$$2\|B\|\|w(t)\| < bl_j l_\varphi \|p - p_r\| < \beta_2 \|\eta\| \quad (33)$$

Now, if $\Psi < -\varepsilon \mathbf{I}$ and $-P + PE_{22}P < 0$ where ε is a positive scaler, then by virtue of (31) and (33), $\dot{V}$ can be bounded by

$$\dot{V} \leq -\varepsilon \|\eta\|^2 + \beta_2 \|P\| \|\eta\|^2 \quad (34)$$

Hence, if $-\varepsilon + \beta_2 \|P\| < 0$, then there exists a positive scalar θ satisfying $\dot{V} < -\theta \|\eta\|^2$ which implies η converges asymptotically to zero.

Next, we investigate under which conditions the inequalities $\Psi < -\varepsilon \mathbf{I}$, $-P + PE_{22}P < 0$ and $-\varepsilon + \beta_2 \|P\| < 0$ can be satisfied. We define $\gamma = \varepsilon^{-1}$, $Z = P^{-1}$, $W = ZQZ$, $X = KZ$, $X_1 = HZ$, and pre and post multiply the inequalities $\Psi < -\varepsilon \mathbf{I}$ and $-P + PE_{22}P < 0$ by $diag\{Z, Z, Z\}$ and Z, respectively. Finally, we apply the Schur Complement in [18] to the resulting inequalities and obtain the inequalities (9) and (10). On the other hand, if inequality (11) holds, then $\|Z\| > \beta_2 \gamma$. Since $\|Z\| = \frac{1}{\|P\|}$ and $\gamma = \varepsilon^{-1}$, this is equivalent to $\|P\| < \beta_2^{-1} \varepsilon$. Therefore, we conclude that if inequality (11) holds, then $-\varepsilon + \beta_2 \|P\| < 0$ is satisfied.

Also, pre and post multiplying the inequality (12) by $diag\{Z^{-1}, Z^{-1}\}$, yields,

$$\left[\begin{array}{cc} -P & dH^\top P \\ dPH & -P \end{array} \right] < 0 \quad (35)$$

According to matrix theory, if (35) holds, then it can be proven that a positive scalar $\mu < 1$ exists such that

$$\left[\begin{array}{cc} -\mu P & dH^\top P \\ dPH & -P \end{array} \right] < 0 \quad (36)$$

Therefore, by Lemma 2, (36) implies that the operator $\mathcal{D}(e_t)$ is stable.

Finally, we note that the inequality (13) ensures that V_4 is positive definite. According to Theorem 1, we conclude that if matrix inequalities (9)-(13) hold, then tracking error $e(t)$ converges asymptotically to zero. ■

VI. Simulation results

We consider a planar 2-DOF revolute-joint manipulator of [22] moving on the $x - y$ plane. Link lengths are given by $l_1 = 0.38$m, $l_2 = 0.38$m, and the distance between the center of mass of each link to its starting joint is given by $l_{c1} = 0.19$m, $l_{c2} = 0.19$m. Also, link mass and inertias are given by $m_1 = 2.24$kg, $m_2 = 1$kg, $I_1 = 0.81$kgm^2, $I_2 = 0.4$kgm^2. The vector of gravity acceleration is represented by $g_0 = [0, 9.81, 0]^\top$. The reference trajectory is chosen as $q_r(t) = [\sin(t), \cos(2t)]^\top$rad. The initial conditions are selected by $q(0) = [0, 0]^\top$rad, $\dot{q}(0) = [0, 0]^\top$rad/sec, $\pi_i(0) = 0$ for $i \neq 1, 4$, and $\pi_1(0) = l_{c1}$, $\pi_4(0) = l_1 + l_{c1}$ and $v_i(0) = 0\ \forall i$.

Design parameters for the proposed controller are set to $\beta_1 = 1.7$, $\beta_2 = 260$, $\delta_1 = 100$, $\delta_2 = 100$ and $h = 0.1$. Fig. 1 illustrates the performance of the proposed controller (7) in presence of the time-delay $d = 0.43$sec in measurements. Fig 2 demonstrates evolution of actuator torques for the proposed controller.

For comparison purpose, the performance of a standard controller is illustrated in Fig. 3 , where the time-delay of $d = 0.1$sec is considered in sensor measurements. The maximum time-delay that the system under the standard controller can tolerate before becoming unstable was calculated through simulation as being $d = 0.15$sec. Comparing Fig. 1 and Fig. 3 shows that since the proposed controller takes into account the presence of delay in its design procedure, it demonstrates a superior performance in face of measurement delay.

To evaluate the degree of conservatism encountered by using the LMI optimization, the maximum time-delay for feasibility of the LMIs (9)-(13) was analytically computed by $\bar{d}_{ana} = 0.43$sec. Also, the maximum time-delay that the robot under the proposed controller can actually tolerate and remain stable, was computed through simulation as being $\bar{d}_{sim} = 0.51$sec. Obviously, the difference between $\bar{d}_{ana}$ and $\bar{d}_{sim}$ can be reduced by using less conservative LMIs in the control design.

VII. Conclusion

By making use of a non-minimal model of a rigid manipulator, a dynamically smooth controller has been proposed which guarantees robot stability in the presence of a constant and known time-delay in sensor measurements. Sufficient conditions in the form of LMIs have been formulated which ensure asymptotic convergence of the motion tracking errors. The proposed controller is in form of a smooth static feedback law.

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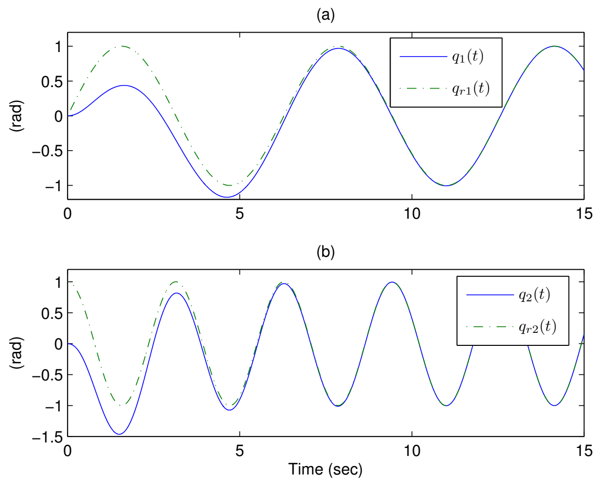

Fig. 1. (a): First joint angle $q_1(t)$ versus its desired trajectory $q_{r1}(t)$. (b): Second joint angle $q_2(t)$ versus its desired trajectory $q_{r2}(t)$, in presence of time-delay $d = 0.43$sec for the proposed controller (7). System remains stable up to $d = 0.51$sec

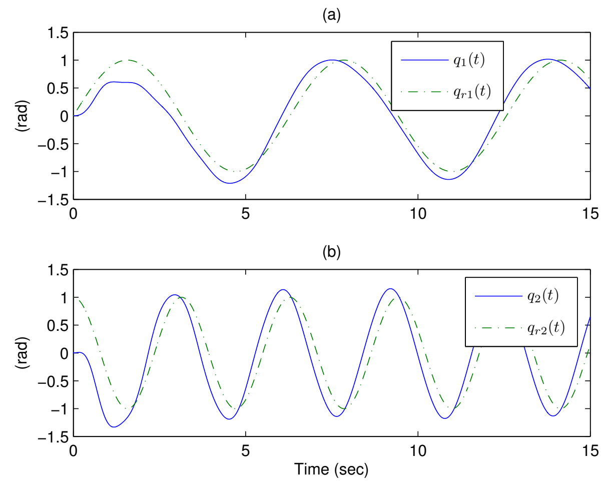

Fig. 3. (a): First joint angle $q_1(t)$ versus its desired trajectory $q_{r1}(t)$. (b): Second joint angle $q_2(t)$ versus its desired trajectory $q_{r2}(t)$, in presence of time-delay $d = 0.1$sec for the standard controller [23].

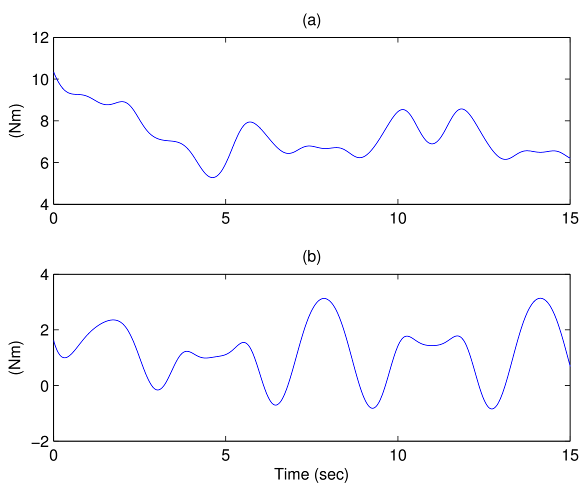

Fig. 2. Joint torques, (a): τ_{m1} and (b): τ_{m2} for the proposed controller (7) in presence of time-delay $d = 0.43$ sec.

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