# A vision-based method for estimating vibrations of a flexible arm using on-line sinusoidal regression

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*Abstract*— A vision-based vibration suppression scheme has previously been proposed to control the vibrational behaviour of long-reach arms operating in fusion reactors. In this paper we describe a new method to reconstruct the vibration using sinusoidal regression. This change makes the overall scheme more efficient since it enables the estimation of the tip oscillation whatever its origin may be. Both an exact solution and a simplified method are proposed to solve the regression problem. To limit the trade-off between good tracking capability and quality of the vibration reconstruction, these regression algorithms are performed over a variable-length sliding window. Consequently this paper also describes the change detection scheme used to automatically adjust the window length. Experimental results validate the proposed method.

#### I. CONTEXT OF THE STUDY

The International Thermonuclear Experimental Reactor (ITER) is an under construction experimental reactor aiming to demonstrate the scientific and technological feasibility of fusion energy. Inside the ITER torus, fusion reactions between Deuterium and Tritium isotopes will produce highenergy neutron fluxes that irradiate the structure. Because of this neutron activation, which forbids direct human access inside the reactor, the in-vessel plasma facing components will have to be inspected and maintained remotely. Due to the size and the arduous accessibility of the reactor the robotic arms designed for its maintenance will have to be long-reach arms, sometimes able to manipulate heavy loads and to bear high forces, but they will also have to be light and slender structures (see Fig.1).

The main difficulties when positioning such structures result from the vibrations due to their high flexibility. Consequently they need the integration of appropriate compensation schemes to complete the tasks within the requirements. The stimulation of the structural modes arises from:

- a critical trajectory imposed by the operator
- a collision or interaction with the environment (load transfer, e.g. during the installation of heavy modules)
- internal unmodelled dynamics (from carried processes, e.g. the rotating prism of a laser viewing system)

Input shaping techniques [1, 2] are very efficient to avoid critical trajectories by adjusting the actuators input in such a way that the natural modes are not excited. Considering the two other origins, the arm vibrational behaviour cannot be foreseen and it needs to be damped as soon as it occurs.

Usually additional sensors can be added to a system to control its flexible states. Unfortunately the ITER remote handling equipment will be subjected during a shutdown to a cumulated radiation dose in the order of several MGy. This constraint limits the use of dedicated on-the-shelf electronics such as accelerometers. In addition to the problems raised by radiations, the use of strain gauges would suffer from the inherent high noise due to electromagnetic interferences.

As a consequence the main idea behind our developments is to control the oscillatory behaviour of the flexible carriers without considering any extra sensor apart from the embedded rad-hardened vision processes inevitably used to provide real-time visual feedback to the operators.

In [3] an all-in-one method has been proposed to solve the problem of vibration suppression by using visual features without any markers nor a-priori knowledge on the environment. The tip displacement induced by vibrations is estimated exploiting a simple physical model of the manipulator. Thanks to the camera mounted in an eye-inhand configuration [4] this model is then readjusted using direct measurement of the oscillations with respect to the static environment. If this method provides successful results, it still has one main drawback: the Kalman filter used for the vibration estimator is based on a model whose input can be either the joint acceleration or the applied torque. As a consequence it raises an issue when the vibration is not due to the joint dynamics but to an embedded process. The camera may perceive a vibration, whereas the internal input-output model still believes the arm is stationary, which is detrimental to the accuracy of the estimation.

The primary contribution of our work is to remedy that problem by considering sinusoidal regression instead of a Kalman filter to reconstruct the vibration from visual data. In spite of the great flexibility of the solutions it brings, sine regression has hardly been used to address engineering problems such as vibration control. As the only hypothesis done here is that the vibration has a sinusoidal shape –which is verified if we only aspire to damp the fundamental– the proposed method is well adapted whatever the origin of the vibrational behaviour. We put forward two regression methods which are compared on the basis of their complexity and the results they yield. To limit the trade-off between good tracking capability and quality of the vibration reconstruction, these algorithms are performed over a variable-length sliding window. This paper also describes the change detection scheme used to automatically adjust the length of this window.

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Fig. 1. Examples of long-reach arms for operations in tokamaks: (a) deployment inside TORE SUPRA vacuum vessel of the 10m-long 8-DOF AIA equipped with its viewing system, (b) installation of the ITER-like ICRH antenna with the 12m-reach boom inside JET, (c) the in-vessel transporter (IVT) prototype able to handle 4T modules

The outline of this paper, which definitely puts the emphasis on robustness and adaptiveness, is organised as follows. After an overview of the constraints of the study in section I, section II gives an overall description of the vision-based vibration estimator. Section III presents the sine regression scheme used to solve this problem, introducing both an exact solution and a simplified method. At last, section IV validates the ability of this simplified, computationally light method to predict a time-varying vibration with good accuracy.

### II. OVERALL DESCRIPTION OF THE VISION-BASED VIBRATION ESTIMATION

In this section we consider the problem of designing an on-line vibration estimator using a camera and without any knowledge of the environment (see Fig. 2).

The first step consists in evaluating the speed of the environment in the camera basis. To that purpose we used the Lucas-Kanade-Tomasi (KLT) feature tracking algorithm [5], which extracts and tracks features from the camera images.



Fig. 2. Principle of the vibration estimator

We assume that no a-priori knowledge on the environment is available as well as no markers have been placed on it. As the manipulator moves, the tip camera moves and the image patterns change in a complex way. The goal of the tracking is then to select a pool of features and to determine the vector  $\delta X$  of their respective frame to frame displacements.

Features selection in image processing usually deals with extracting attributes resulting in some quantitative information of interest. A good feature is a textured patch with high intensity variation in both  $x$  and  $y$  directions, e.g. a corner or an edge. The particularity of the KLT algorithm lies in the fact it is designed to select features that are more than traditional "interest" measures. Its selection criterion is defined suitably with the tracking method and consequently it elects the features that make the tracker work best (see Fig.3). As a result, the selection criterion is optimal by construction and it makes KLT trackers extremely robust. When features are lost our algorithm replaces them by finding new features in the next image in order to keep a constant pool of features.

From this feature displacement vector  $\delta X$ , an M-estimator rejects outliers possibly resulting from the extraction noise. We chose to implement Tukey's influence function, which completely rejects outliers by giving them a zero weight and therefore gives a robust estimation of the environment overall displacement seen by the camera  $\xi$ . Then one can deduce the speed of the camera in the static environment basis.

Afterwards this signal is high-pass filtered and feeds the on-line sinusoidal regression algorithm, in order to predict the current deflection from the delayed vision-based measurements. Since this method is not based on a physical model of the arm, it is not necessary to express this deflection in the 3D world to perform a quality control. Consequently there is no more use of any interaction matrix, which linearly describes the relation between the motion of the image features to the camera motion, as it used to be in [3]. This greatly simplifies the implementation of the scheme.

At this point an estimation of the arm vibration  $\xi_{estim}$  is obtained in pixel unit and it is projected onto an orthogonal basis. Afterwards, as in [3], this estimation directly feeds a linear quadratic regulator (LQR) which makes the endpoint track the desired trajectory by using an inverse jacobian procedure, while  $\dot{\xi}_{estim}$  is brought to zero as fast as possible.



Fig. 3. Example of tracked features in an unknown and "untrimmed" environment (a tokamak vessel wall)

#### III. REAL-TIME SINUSOIDAL REGRESSION

To obtain a robust prediction of the vibration to be rejected, we chose to achieve a sinusoidal regression based on the data  $(t_k, y_k)$  received from the features tracker. Formally the following sinusoidal function is considered:

$$
f(t) = a + b\sin(\omega t) + c\cos(\omega t)
$$
 (1)

Our goal is to determine the values of the parameters  $a, b$ ,  $c$  and  $\omega$  that cause this function to best fit the observed data provided by the feature tracker.

There are many popular parameter estimation algorithms, such as block/recursive least squares, instrumental variables, maximum likelihood and extended Kalman filter, among others. However, according to the best of our knowledge, none of them seems perfectly adapted to the vibration suppression problem. Indeed, here are the main applicationdriven requirements our algorithm has to respect. It must:

- be an on-line estimation process;
- track time-varying parameters, as the amplitude of the oscillation is likely to change in time;
- be extremely reactive to enable the controller to damp an abruptly occurring vibration as soon as it appears;
- be fitted to prediction purpose, due to the long processing time of visual data.

First of all, owing to their computational effectiveness and completeness, least-square (LS) regression techniques are preferred to other estimation processes. They provide good results with relatively small data sets. Moreover, to facilitate tracking of time-varying parameters, it is desirable to discard out-of-date data as new data are collected. This can be achieved by employing a weighting scheme that decreases the effect of old data exponentially, e.g. thanks to variable forgetting factors [6][7][8]. However, if such algorithms provide good results in some cases, their tracking capability remains limited because old data are never completely discarded. Then sliding windows are useful in the sense that they explicitly discard old data. Up to this point, any recursive LS method performed over a fixed-size sliding window is likely to fit most of our requirements. Nevertheless the statistical properties of these algorithms, which however

represent their main advantage in some cases, may not fit our needs for the estimation of abrupt changes of parameters.

Our objective in this section is to present a parameter estimation algorithm which is optimal and suitable for highly timevarying systems. The basic idea in achieving this objective is to use a sliding window blockwise least squares algorithm in which the window length is adjusted by a signal change detection algorithm. For this purpose, a new variable-length sliding window least-squares scheme has been developed to provide both:

- reactive parameter tracking during transients to enable quick damping of undesirable vibrations
- high quality estimation accuracy at the steady state to avoid soliciting the actuator in case of endurable minor oscillations

Consequently, the proposed scheme consists in solving online a non-linear LS problem, whose exact solution is given in section III-A. But accurate remote handling operations rely on good force feedback capabilities of the remote handling tools and the servo computational time is generally expected to be within 1 ms for stable and transparent interaction with the environment. Implementing a robust sinusoidal regression algorithm at such a high servo rate is far from trivial. As a consequence a much more easily implementable method is proposed in section III-B. The variable-length slidingwindow algorithm is described in section III-C, and its change detection algorithm follows in section III-D.

#### *A. Exact solution*

As we chose to estimate our set of parameters using least squares, the criterion to be minimized is the sum of the squares of the residuals:

$$
\epsilon_{a,b,c,\omega}^{2} = \sum_{k=1}^{n} (y_k - f(t_k))^{2}
$$
  
= 
$$
\sum_{k=1}^{n} (y_k - (a + b\sin(\omega t_k) + c\cos(\omega t_k)))^{2}
$$
 (2)

The best way to solve such a problem is to come down to a linear regression form. To that purpose we can use a differential equation whose solution is the considered sinusoidal function:

$$
f(t) = a - \frac{1}{\omega^2} \frac{d^2 f(t)}{dt^2}
$$
 (3)

The criterion (2) yields a linear system where the two unknowns are a and  $\nu = 1/\omega^2$ :

$$
\epsilon_{a,b,c,\omega}^2 = \sum_{k=1}^n (y_k - a + \nu y_k'')^2
$$
 (4)

Unfortunately this method is practically ineffective as the computation of the second derivative  $y_k''$  from the data  $(t_k, y_k)$  usually leads to large deviation. Conversely numerical computing of integrals is far less problematic. As a consequence one can use an integral equation instead of (3):

$$
f(t) = -\omega^2 G(t) + P(t) \tag{5}
$$

where  $G$  is the second antiderivative of  $f$  such as  $G'(t) = g(t) = F(t)$  and  $F'(t) = f(t)$ . P(t) is a secondorder polynomial depending on both the parameters  $a, b, c$ and  $\omega$ , and the arbitrary constants of integration  $C_1$  and  $C_2$ :

$$
P(t) = \frac{1}{2}a\omega^2 t^2 + C_1 \omega^2 t + a + C_2 \omega^2 = \beta t^2 + \gamma t + \delta
$$
 (6)

Thus, by posing  $\alpha = -\omega^2$ , (5) can be re-written:

$$
f(t) = \alpha G(t) + \beta t^2 + \gamma t + \delta \tag{7}
$$

in which  $\alpha$ ,  $\beta$ ,  $\gamma$  and  $\delta$  are unknown but can be estimated thanks to linear regression. Indeed (2) yields this time to:

$$
\epsilon_{a,b,c,\omega}^2 = \sum_{k=1}^n \left( y_k - \left( \alpha G(t_k) + \beta t_k^2 + \gamma t_k + \delta \right) \right)^2 \tag{8}
$$

whose minimum can be found by setting its gradient to zero, provided that the vector  $G(t_k)$  has previously been computed. It can be done using usual numerical integration algorithms. From this point let's assume that  $F(t_k)$ and  $G(t_k)$  are computed according to the initial conditions  $F(0) = 0$  and  $G(0) = 0$ . The constants of integration  $C_1$  and  $C_2$  are now fully determined and can be related respectively to  $b$  and  $c$ :

$$
C_1 = \frac{b}{\omega} \qquad C_2 = \frac{c}{\omega^2} \qquad (9)
$$

Minimizing (8) leads to the linear system (16). Its solution can be written in the matrix form (17), where conventionally  $\Sigma = \sum^{n}$  $\sum_{k=1}$ . Then we can deduce  $\omega_0$ ,  $a_0$ ,  $b_0$  and  $c_0$ :

$$
\omega_0 = \sqrt{-\alpha_0} \tag{10}
$$

$$
a_0 = -\frac{2\beta_0}{\alpha_0} \tag{11}
$$

$$
b_0 = \frac{\gamma_0}{\sqrt{-\alpha_0}}
$$
 (12)

$$
c_0 = \frac{2\beta_0}{\alpha_0} + \delta_0 \tag{13}
$$

We now have an expression of the sinusoidal function that best fits the data received from the feature tracker over a

period of time. It is easy to predict the tip deflection until the next data reception, assuming that only slight changes affect the frequency and the amplitude of the oscillation.

## *B. Simplified method using M-estimator for frequency estimation*

As an estimation of the first vibrational modes of the robotic structure is often available, one can also consider the use of a simplified method having the advantage of a reduced computational cost.

Indeed, in the case where  $\omega$  can be considered as a known parameter, the optimisation only concerns the parameters  $a$ , b and c, and our problem is directly reduced to a linear least squares problem. As in III-A, the minimum of the sum of squares is found by setting its gradient to zero, which leads to the following system:

$$
\begin{cases}\n\sum_{k=1}^{n} (y_k - (a_1 + b_1 s_k + c_1 c_k)) = 0 \\
\sum_{k=1}^{n} (y_k - (a_1 + b_1 s_k + c_1 c_k)) s_k = 0 \\
\sum_{k=1}^{n} (y_k - (a_1 + b_1 s_k + c_1 c_k)) c_k = 0\n\end{cases}
$$
\n(14)

where, for simplicity of writing,  $s_k$  and  $c_k$  respectively refer to  $\sin(\omega_e t_k)$  and  $\cos(\omega_e t_k)$ .  $\omega_e$  is the estimated value of the vibration angular frequency.

The solution of system (14) can be written in the matrix form:

$$
\underbrace{\begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix}}_{X_1} = \underbrace{\begin{bmatrix} n & \Sigma s_k & \Sigma c_k \\ \Sigma s_k & \Sigma s_k^2 & \Sigma s_k c_k \\ \Sigma c_k & \Sigma s_k c_k & \Sigma c_k^2 \end{bmatrix}}_{M_1} - \underbrace{\begin{bmatrix} \Sigma y_k \\ \Sigma y_k s_k \\ \Sigma y_k c_k \end{bmatrix}}_{Y_1} \tag{15}
$$

Here is a much simpler way to implement the on-line sinusoidal regression, provided that the parameter  $\omega_e$  can be evaluated apart. To that purpose it is assumed that an initial evaluation of  $\omega_e$  reasonably close to the real value is

$$
\begin{cases}\n\left(\frac{\partial \epsilon^{2}}{\partial \alpha}\right)_{(\alpha_{0},\beta_{0},\gamma_{0},\delta_{0})} = -2 \sum_{k=1}^{n} \left(y_{k} - (\alpha_{0}G(t_{k}) + \beta_{0}t_{k}^{2} + \gamma_{0}t_{k} + \delta_{0})\right) G(t_{k}) = 0 \\
\left(\frac{\partial \epsilon^{2}}{\partial \beta}\right)_{(\alpha_{0},\beta_{0},\gamma_{0},\delta_{0})} = -2 \sum_{k=1}^{n} \left(y_{k} - (\alpha_{0}G(t_{k}) + \beta_{0}t_{k}^{2} + \gamma_{0}t_{k} + \delta_{0})\right) t_{k}^{2} = 0 \\
\left(\frac{\partial \epsilon^{2}}{\partial \gamma}\right)_{(\alpha_{0},\beta_{0},\gamma_{0},\delta_{0})} = -2 \sum_{k=1}^{n} \left(y_{k} - (\alpha_{0}G(t_{k}) + \beta_{0}t_{k}^{2} + \gamma_{0}t_{k} + \delta_{0})\right) t_{k} = 0 \\
\left(\frac{\partial \epsilon^{2}}{\partial \delta}\right)_{(\alpha_{0},\beta_{0},\gamma_{0},\delta_{0})} = -2 \sum_{k=1}^{n} \left(y_{k} - (\alpha_{0}G(t_{k}) + \beta_{0}t_{k}^{2} + \gamma_{0}t_{k} + \delta_{0})\right) = 0 \\
\left(\frac{\partial \epsilon^{2}}{\partial \delta}\right)_{(\alpha_{0},\beta_{0},\gamma_{0},\delta_{0})} = -2 \sum_{k=1}^{n} \left(y_{k} - (\alpha_{0}G(t_{k}) + \beta_{0}t_{k}^{2} + \gamma_{0}t_{k} + \delta_{0})\right) = 0 \\
\frac{\left[\begin{matrix} \alpha_{0} \\ \beta_{0} \\ \gamma_{0} \\ \gamma_{0} \\ \delta_{0} \end{matrix}\right]}{=} \left[\begin{matrix} \Sigma G^{2}(t_{k}) & \Sigma t_{k}^{2} G(t_{k}) & \Sigma t_{k} G(t_{k}) & \Sigma G(t_{k}) \\ \Sigma t_{k}^{2} G(t_{k}) & \Sigma t_{k}^{2} & \Sigma t_{k}^{2} \\ \Sigma t_{k} G(t_{k}) & \Sigma t_{k}^{2} & \Sigma t_{k} \\ \Sigma G(t_{k}) & \Sigma t_{k}^{2} & \Sigma t_{k} \\ \Sigma t_{k} & \Sigma t_{k} \end{matrix}\right
$$

known. This can be obtained easily by computer-aided modal analysis being given that all the CAD models of the devices introduced inside ITER will be available.

As the quality of the vibration reconstruction is heavily based on the accuracy of the vibrational frequency evaluation, this estimation  $\omega_e$  is updated on-line by detecting the zero-crossing of the  $y_n$  which is supposed to happen every half-period. But because of the features extraction noise and potential temporary disturbances, multiple zerocrossings in short periods of time can corrupt these raw data. To minimize the influence of these outliers one can employ robust statistics, which makes it possible to recover the structure that best fits the majority of the computed values of  $\omega_e$  over a window while identifying and rejecting deviating substructures.

As in the feature tracker this is achieved with a robust Mestimator, which can be considered as a more general form of Maximum Likelihood Estimators (MLE) because it permits the use of different minimization functions not necessarily corresponding to normally distributed data.

Such an estimator can be written:

$$
\hat{\omega} = \underset{\omega}{\text{argmin}} \left[ \sum_{i=1}^{n} \lambda(\omega_{e,i}, \omega) \right] \tag{18}
$$

where  $\lambda$  is an influence function (Tukey's, Huber's,...). It can be chosen in such a way to provide the estimator desirable properties, in terms of bias and efficiency. As a consequence four M-estimators have been compared to each other in order to obtain the most appropriate estimation  $\hat{\omega}$  of  $\omega_e$ .

1) The Huber estimator asymptotically reduces the influence of outliers toward zero. Its influence function is given by:

$$
\lambda(u_i) = \begin{cases} \frac{1}{2}u_i^2 & \text{if } |u_i| \le a \\ a|u_i| - \frac{1}{2}a^2 & \text{if } |u_i| > a \end{cases}
$$
 (19)

where  $u_i = \frac{\omega_{e,i} - \omega}{MAD}$ . MAD represents the Median Absolute Deviation estimator. With  $a = 1.345$  this estimator assumes that all values within the bounds of 95% of the data are 100% correct and gradually reduces the probability of features outside this region.

2) Tukey's estimator completely rejects outliers by giving them a zero weight. Its influence function is:

$$
\lambda(u_i) = \begin{cases} \frac{1}{6} [1 - (1 - u_i^2)^3] & \text{if } |u_i| \le 1\\ \frac{1}{6} & \text{if } |u_i| > 1 \end{cases} \tag{20}
$$

with, this time,  $u_i = \frac{\omega_{e,i} - \omega}{c \times MAD}$ . *MAD* still represents the MAD estimator and  $c$  is a potentiometer that adjusts the asymptotic efficiency of the obtained M-estimator. The value  $c = 4.6851$  gives 95% efficiency on the standard normal distribution.

3) The Cauchy robust estimator provides a gradual attenuation of the outliers such as:

$$
\lambda(u_i) = \frac{c^2}{2} ln[1 + u_i^2]
$$
 (21)

where  $u_i$  is defined the same way as for the previous estimator. The 95% asymptotic efficiency on the standard normal distribution is obtained with  $c = 2.3849$ .

4) At last the Geman-McClure influence function tends to further reduce the effect of large errors such as:

$$
\lambda(u_i) = \frac{u_i^2/2}{1 + u_i^2}
$$
 (22)

with  $u_i$  defined as for Huber's estimator.

In many practical situations, the choice of the influence function is not critical to obtain a good robust estimate, and different choices will give similar results in terms of improvement over classical estimation techniques. Section IV includes a critical analysis showing that Tukey's and Cauchy's functions both fit our needs.

#### *C. Variable-length sliding window*

Performance of the two above-described algorithms obviously depends on the window length. The longer the window, the higher the estimation accuracy. On the other hand, the shorter the window, the more responsive the estimation. As a consequence one main feature of the proposed approach lies in its ability to quickly adapt the window length as soon as a change in system parameters is detected, in order to achieve the best performance in both transient and steady states.

In the case of a sudden change, the window will be shrunk to a minimal size rudely. Then it will progressively expand until it returns to its original length in order to maintain steady-state performance. In case of a continuous signal change, the window will be shrunk/expanded progressively



Fig. 4. Window length adjustment strategy

depending on the rate of change until the end of the change is detected. This algorithm is illustrated by Fig.4.

As long as no change is detected the sliding window keeps a size of  $N_{max}$  values.  $N_{max}$  is chosen to provide the best estimation accuracy as possible. From experience it implies that the window entails about one period of the vibration, which results in:

$$
N_{max} = k \left[ \frac{1}{2\pi\omega_e \Delta t} \right] \tag{23}
$$

where  $k$  is an adjusting factor. When a sudden change is detected the window size is set to  $N_{min}$  which has to be adjusted experimentally to obtain the desired responsivity. Finally, when a progressive change is detected, the window size is intermediate and linearly varies with the normalized change rate  $\rho$ :

$$
N = (1 - \rho)N_{max} + \rho N_{min} \tag{24}
$$

#### *D. Change detection mechanism*

To automatically initiate and complete the window length adjustment, a change detection scheme must be used. This change detection mechanism must also distinguish the sudden emergence of a vibration from the progressive growth of a once negligible vibration.

As a consequence, the key features of this scheme are:

- 1) to detect the onset of a change
- 2) to distinguish a gradual change from an abrupt one
- 3) to estimate the change rate in case of a gradual one
- 4) to detect the termination of a change

There are several different ways to detect parameter changes in a system [9]. To be as reactive as possible we chose to use a change detection scheme based on the last received measurement rather than the last estimated set of parameters or the last prediction  $f(t_n)$  which are inherently averaged over a window. Assuming that the variations of  $\omega$  are correctly evaluated by the M-estimator described in section III-B, a change in the vibrational behaviour of the arm will only affect the amplitude of the oscillation. Such a change can be detected by monitoring changes in the signal variance.

The problem of testing for change in the variance has received considerable interest in recent years, with applications in various fields such as economics [10], engineering [11] or health sciences [12]. In our case, it can be considered that there is no time dependence between the observations. We want to monitor the stability of the variance of the time series  $(y, t)$  defined by the independent sequence  $\{y_k\}_{k\geq 1}$ of normal random variables with mean  $\overline{y}_k$  and variance  $\sigma_k^2$ .

First let's assume that  $\sigma_1^2 \simeq \cdots \simeq \sigma_m^2 \simeq \sigma_0^2$ . Then, observing the data the goal is to detect if a change occurs in the variance by testing the hypothesis:

$$
H_0: \sigma_1^2 \simeq \cdots \simeq \sigma_n^2 \simeq \sigma^2 \tag{25}
$$

against the alternative:

$$
H_1: \sigma_1^2 \simeq \cdots \simeq \sigma_{n-1}^2 \neq \sigma_n^2 = \sigma_*^2 \tag{26}
$$

In our case  $\sigma_*$  is considered unknown whereas  $\sigma$  is assumed to be computed at every cycle and known.



Fig. 5. Principle of the on-line sine regression (simplified method)

To that purpose one can use the statistical test defined by:

$$
B = \sum_{k=1}^{n} \frac{(k-1)(y_k - \overline{y})^2}{(n-1)\sum_{i=1}^{n} (y_i - \overline{y})^2}
$$
(27)

It is derived from the Bayesian test proposed by [13] assuming that both the initial level of variance and the mean are computed and known under  $H_0$ . It yields a value comprised between 0 and 1 which is symmetrically distributed around the mean 0.5. Then the kind of signal change is reported if B exceeds conveniently pre-set thresholds:

$$
B = \begin{cases} \n\geq \Gamma_a^+ & \text{abrupt change} \\ \n\Gamma_g^+ \leq \ldots < \Gamma_a^+ \\ \n\Gamma_g^- < \ldots < \Gamma_g^+ \\ \n\varGamma_a^- \ldots < \Gamma_g^- \n\end{cases} \n\text{no change} \n\tag{28}
$$
\n
$$
\leq \Gamma_a^- \qquad \text{abrupt change} \n\leq \Gamma_g^-
$$

The careful choice of the thresholds is imperative as it directly impacts the probability of missed detections and false alarms. Ultimately we obtain a simplified on-line vibration predictor based on sinusoidal regression (Fig. 5).

#### IV. EXPERIMENTAL RESULTS

A validation campaign has been carried out on the experimental mock-up shown in Fig.6 [14]. It consists of:

- an actuated joint (capacity  $\simeq 1000$  N.m) driven by a motor through an Harmonic Drive based speed reducer
- a 3m-long circular beam with a calibrated tip mass
- a 5000 cpr optical encoder to measure the joint position
- a tip-mounted industrial camera IDS uEye UI-122xLE (resolution:  $640 \times 480$ )

The controller runs on the real-time OS VxWorks at a sampling time of 1 ms. The overall vision-based application is based on the ViSP software [15] and runs at around  $60-70Hz$ . The joint friction and gravity torques applied on the beam have been compensated considering measurements from a rigid bar of the same weight. To avoid jitter effect on the actuator the vibration control is only performed if the oscillation amplitude exceeds a threshold set to 5 pixels.

Fig.7(a) illustrates the ability of our algorithm to predict the vibration with a pretty good accuracy in presence of both sudden and progressive amplitude change. In this experiment  $N_{min}$  and  $N_{max}$  have respectively been set to 3 and 20. As depicted by Fig.7(b) and Fig.7(c) an abrupt variance change is first detected around  $t = 2.7s$ . The window size is shrunk directly to  $N_{min}$  and then expanded quickly back to  $N_{max}$ before the progressive damping of the vibration makes the window size decrease again to about  $N = 10$ . Once this progressive variance change stops, around  $t = 12s$ , the window expands stepwise to its full length. To distinguish a gradual change from a sudden one,  $\Gamma_g^-, \Gamma_g^+, \Gamma_a^-$  and  $\Gamma_a^+$  have respectively been set to  $0.45$ ,  $0.55$ ,  $0.3$  and  $0.7$ . The chosen example also illustrates the response of our algorithm to two other gradual changes around  $t = 17s$  and  $t = 25s$ .

Then Fig.7(d) highlights the benefit of using a variablesized window over a fixed-sized window. In this case of abrupt change, the vibration prediction is very reactive and yields very accurate results whereas a proper estimation is only obtained after one period with N fixed to  $N_{max}$ .

This quality vibration reconstruction relies on a good estimation of the vibrational frequency. Fig.7(e) compares the frequency evaluation performed with the four robust estimators described in III-B with both theoretical values and the results obtained by a classical trimmed mean. Around  $t = 20s$  the tip payload has been suddenly changed to alter the oscillation frequency. Whereas the trimmed mean is clearly corrupted by outliers the M-estimators yields robust estimation of  $\hat{\omega}$ . Because it does not completely discard the outliers, Huber's estimator performs less accurate results than the others. On the contrary, since it also decreases the influence of correct data, the Geman-McClure estimator turns to be too sensitive to noise. Tukey's and Cauchy's influence functions provide quite similar and good results. Concretely, the raw frequency estimation based on zerocrossing yields a mean error, a maximum error and a relative standard deviation respectively in the order of  $\bar{\epsilon} = 185\%,$  $\epsilon_{max} = 1679\%$  and  $\sigma = 449\%$ . Using a classic trimmed



Fig. 6. Experimental mock-up at CEA List site in Fontenay-aux-Roses

mean yields  $\bar{\epsilon} = 37.76\%, \epsilon_{max} = 361.0\%$  and  $\sigma = 91.42\%.$ In comparison, the same estimation made by the Tukey Mestimator yields  $\bar{\epsilon} = 1.59\%$ ,  $\epsilon_{max} = 6.29\%$  and  $\sigma = 1.71\%$ .

#### V. CONCLUSION AND FUTURE WORKS

In this paper an on-line sinusoidal regression algorithm has been described. It enables the vision-based vibration control of long-reach flexible arms regardless the origin of their vibrational behaviour. To obtain both good tracking capability and estimation accuracy our method includes the use of a variable-sized window coupled to a signal change detector. The whole control scheme is validated on a singlejoint flexible mock-up until the availability of a robotic arm destined for ITER makes possible to implement it on a complete remote handling system.

Up to this point one limitation of this algorithm is the computation of an environment overall displacement on the basis of randomly distributed features, which is strictly correct only if the camera observe a plane normally. Indeed points at different distances from the camera currently result in features having different displacements in the image. In future works we will try to extend these results to the observation of complex and not necessarily normal planes by considering a field of displacement and not only an estimation of the overall displacement.

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Fig. 7. (a) Normalized predicted amplitude, (b) Bayesian test B(t), (c) Window size N(t), (d) Tracking of a vibration w/wo variable-length sliding window, (e) Frequency robust estimation (real-time recorded data)