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Adaptive Control of Robot Manipulators including Actuator Dynamics and without Joint Torque Measurement

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Abstract— **Ignoring actuator dynamics in control of rigid manipulators can in practice result in performance degradation or loss of system stability. However, consideration of actuator dynamics usually requires measurement of robot joint torques. This paper addresses motion tracking control of an n-DOF rigid robot by taking into account its actuator dynamics. Joint torque measurement is avoided by using an adaptive observer. The backstepping technique is adopted to develop a dynamically smooth adaptive nonlinear controller dealing with uncertainties in manipulator and actuator dynamics. Semi-global convergence of motion tracking errors as well as torque estimation error are proven without any persistency of excitation condition. Simulation examples demonstrate low noise sensitivity of the proposed method in comparison with those using torque measurement.**

I. Introduction

Most approaches to control of robotic manipulators usually neglect actuator dynamics and only take into account the manipulator rigid body dynamics. Examples are computed torque [1], passivity-based [2], [3], [4], adaptive [2], [5] and robust [6], [7], [8] control methods. However, the effect of actuator dynamics becomes important during fast robot motion and highly varying loads where the applied joint torque exhibits significant variation. Ignoring the actuator dynamics in control design can in practice result in performance degradation and loss of stability. However there are important factors in consideration of actuator dynamics in control design. The first is the possible need to *measure* joint torques or motor currents, in case of electric actuators, and the second, is the presence of *model uncertainty* in manipulator or actuator dynamics. To the aforementioned factors we should add the necessity of the *smoothness* property for controller dynamics which turns to be important in achieving high reliability and low noise sensitivity in real-time implementation.

Majority of works in the literature have applied the backstepping method to deal with actuator dynamics in robotic manipulators [9], [10], [11]. For example, [12] proposed two adaptive controllers including adaptive and robust observers to estimate the acceleration signal and achieve global asymptotic motion tracking. Also, [13] proposed an adaptive controller to ensure motion tracking for hydraulic robots. Using the backstepping method, [14] developed a non-smooth and semi-global adaptive controller to achieve position tracking for electrically driven robots in presence of uncertainties in robot inertia matrix and actuator parameters, and by measuring the armature current.

In [15], an adaptive non-smooth sliding mode controller was proposed for robots driven by brushless DC motors with uncertain manipulator dynamics and kinematics, to achieve semi-global position tracking where the armature current was assumed to be available. Also, in [16], adaptive and robust smooth controllers were presented to control a one-link robot with brush DC motors in presence of uncertainties in actuator and manipulator dynamics. In [17], an extended compensation adaptation law with pseudo-velocity filter was proposed which eliminated the need for velocity measurement and semi-global asymptotic position tracking was achieved in presence of uncertainties in system dynamics. Recently [18] has proposed a global adaptive position tracking controller for robots with uncertain kinematics and dynamics by assuming the actuator as a unknown constant gain. In [19] an adaptive controller using the visual task-space information was presented to deal with position tracking for rigid electrically driven robots with uncertainties in system kinematic and dynamics.

Also, in [20], a fuzzy-neural network controller was proposed to achieve global position tracking for a n-DOF robot with uncertainties in manipulator and actuator dynamics. In [21], a robust neural-fuzzy network control of a robot with uncertain manipulator dynamics was developed to deal with global tracking of a periodic position trajectory. Recently, [22] has proposed a non-smooth adaptive controller for electrically driven mobile robots via the backstepping and fuzzy approaches where all the system parameters were assumed to be unknown, and ultimately uniformly boundedness of all of error signals was achieved.

Most of the previously mentioned approaches use measurement of joint torques or armature currents in control design. However, apart from their costs, torque sensors are usually sensitive to environment factors such as temperature, and frequent calibration of torque sensors becomes necessary as they age. Moreover, torque sensor output is often contaminated by noise. These issues impose practical limitations on the use of torque sensors.

The objective of this paper is to develop a control law for motion tracking of rigid n-DOF manipulators with non-negligible actuator dynamics. The backstepping technique is adopted to develop an adaptive nonlinear controller taking into account uncertainties in manipulator and actuator dy-

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namics. An observer is designed to estimate the joint torque instead of measuring it. The dynamics of the entire controller is smooth which simplifies its real time implementation. Semi-global asymptotic convergence of motion tracking and torque estimation errors are proven.

II. System Description

Consider an n-DOF rigid manipulator. Let $p_i \in \mathbb{R}^m$ denote the position of the center of mass of the ith link expressed with respect to a fixed frame attached to manipulator base. Let $\dot{p}_i$, denote the linear velocity of the center of mass of the ith link and ω_i its angular velocity expressed with respect to the base frame. Define m_i as the mass of the ith link and I_i^i the constant inertia tensor of the ith link relative to a frame attached to its center of mass. Define also, the position and orientation Jacobians of the ith link as the maps, $J_p^i : \mathbb{R}^n \to \mathbb{R}^{m\times n}$ and $J_o^i : \mathbb{R}^n \to \mathbb{R}^{m\times n}$ from joint space to the space of linear and angular velocities by

$$\dot{p}_i = J_p^i \dot{q} \tag{1}$$
$$\omega_i = J_o^i \dot{q} \tag{2}$$

where $q \in \mathbb{R}^n$ denotes the vector of robot joint angles. We define the generalized velocity vector by $v := \mathrm{col}[\dot{p}_1, \ldots, \dot{p}_n, \omega_1^1, \ldots, \omega_n^n]$ where $\omega_i^i = R_i(q)^T \omega_i$ is the angular velocity of the ith link, expressed in the link frame and $R_i(q)$ is the rotation matrix of the ith link with respect to the base frame. By defining a constant positive-definite inertia matrix $M = \mathrm{diag}\{m_1\mathbf{I_m}, \ldots, m_n\mathbf{I_m}, I_1^1, \ldots, I_n^n\}$, the manipulator dynamics can be expressed by

$$M\dot{v} + g = f \tag{3}$$

where $g \in \mathbb{R}^{2mn}$ is the vector of gravity forces defined by

$$g := \begin{bmatrix} -m_1 g_0^T & \ldots & -m_n g_0^T & 0_{1\times mn} \end{bmatrix}^T, \tag{4}$$

where $g_0 \in \mathbb{R}^m$ is the gravity acceleration vector. In the equation (3), $f \in \mathbb{R}^{2mn}$ is the generalized force vector representing the force and moments exerted to the links. Although the order of this model is more than the minimal model which is usually used for describing manipulator dynamics, the use of the proposed non-minimal model simplifies the adaptive controller design via backstepping method. This simplification is due to the simple and linear structure of non-minimal model. By virtue of (1) and (2), v and $\dot{q}$ are related by

$$v = J(q)\dot{q} \tag{5}$$

where

$$J(q) := \left[J_p^1(q)^T \ \ldots \ J_p^n(q)^T, \ J_o^1(q)^T R_1(q) \ \ldots \ J_o^n(q)^T R_n(q) \right]^T. \tag{6}$$

It can be shown that $J(q) \in \mathbb{R}^{2mn\times n}$ has full column rank and has the following properties [23]

Property 1: There exist some finite positive constants k_{Jm}, k_{JM} such that $k_{Jm} \leq \|J(q)\| \leq k_{JM} \quad \forall q \in \mathbb{R}^n$ Since $J(q)$ has full column rank its left pseudo-inverse $J^+(q) \in \mathbb{R}^{n\times 2mn}$ is given by $J^+(q) = \left[J(q)^T J(q)\right]^{-1} J(q)^T$ such that $J^+(q)J(q) \equiv \mathbf{I_n}$.

Property 2: $\frac{d}{dt}(J(q)) =: \dot{J}(q, \dot{q})$ is globally Lipschitz with respect to $\dot{q}$, i.e., $\exists l_J > 0$ such that $\left\|\dot{J}(q, x) - \dot{J}(q, y)\right\| \leq l_J \|x - y\|$.

We assume the manipulator actuator dynamics is expressed by

$$\dot{\tau} = A\tau + D(q, \dot{q}, t)u + h(q, \dot{q}, t) \tag{7}$$

where $u \in \mathbb{R}^k$ is the input control vector and $A \in \mathbb{R}^{n\times n}$ is assumed to be a Hurwitz stable matrix and $D(q, \dot{q}, t)$ and $h(q, \dot{q}, t)$ are assumed to be uncertain functions. In case of hydraulic actuators, D and h represent dynamics of chamber-valves [23], or in case of electric motors, h represents the back-EMF effect. We assume that the robot is fully actuated such that $D(q, \dot{q}, t)$ is invertible. Using the principle of virtual work, it can be shown that τ is the net mechanical torque applied to joints and is related to the generalized force f by $\tau = J^T(q)f$.

III. Problem Formulation and Solutions

The overall system dynamics can then be expressed by

$$M\dot{v} + g = f \tag{8}$$
$$\dot{\tau} = A\tau + D(q, \dot{q}, t)u + h(q, \dot{q}, t) \tag{9}$$
$$\dot{q} = J^+(q)v \tag{10}$$
$$\tau = J^T(q)f \tag{11}$$

where we assume that $M, g, D(q, \dot{q}, t)$ and $h(q, \dot{q}, t)$ are unknown, and the motor torque τ and f are not measurable but measurements of q and $\dot{q}$ are available. To deal with uncertainties in M and g, the linear parameterization of the manipulator dynamics is used as

$$M\dot{v} + g = Y(\dot{v})\theta \tag{12}$$

where $\theta \in \mathbb{R}^l$ is the vector of unknown manipulator dynamics parameters depending on the link mass m_i or inertia matrices I_i^i. Since $I_i^i \in \mathbb{R}^{m\times m}$ is symmetric, θ has the maximal dimension of $(2m + 1)n$. Also $Y(\cdot) : \mathbb{R}^{2mn} \to \mathbb{R}^{2mn\times l}$ is a known regressor function. To deal with uncertainties in $D(q, \dot{q}, t)$ and $h(q, \dot{q}, t)$, likewise, we use the linear parameterization of uncertain terms in actuator dynamics as

$$D(q, \dot{q}, t)u + h(q, \dot{q}, t) = Y_a(q, \dot{q}, t, u)\theta_a \tag{13}$$

where $\theta_a \in \mathbb{R}^{l_a}$ is the vector of unknown actuator dynamics parameters, and $Y_a(\cdot)$ is a known regressor function.

Problem 1: Let $q_d : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$ be a given twice continuously differentiable reference trajectory and assume that $q_d, \dot{q}_d$ and $\ddot{q}_d$ are bounded. Consider the system (8)-(11) and assume that only q and $\dot{q}$ are measurable. Under these conditions, find a dynamic controller $u(t, \xi) = \phi(t, \xi)$ where ξ contains all the measurable states and $\phi(t, \xi)$ is a continuously differentiable function with respect to its arguments, such that the tracking errors $e_q(t) := q(t) - q_d(t)$ and $\dot{e}_q := \dot{q}(t) - \dot{q}_d(t)$, converge uniformly and asymptotically to zero. □

A. Adaptive controller design without torque measurement

In this section we derive an adaptive state feedback controller for (8)-(11) in combination with an observer to estimate the joint torque, which render the closed-loop system asymptotically stable. In the sequel, for simplicity in presentation, the dependency of some functions to their arguments is occasionally neglected. We define the composite error s by

$$s := J(q)\dot{e}_q + \Lambda(q)e_q \tag{14}$$

where $\Lambda(q) := J(q)K_\Lambda$ and $K_\Lambda \in \mathbb{R}^{n\times n}$ is a constant positive-definite matrix gain. Also we define the estimated joint torque tracking error by

$$z_1 = \hat{\tau} - \tau_d \tag{15}$$

where τ_d is defined by

$$\begin{aligned}
\tau_d &= J^T f_d \\
f_d &= \hat{M}\left(\frac{\partial}{\partial q_d}(v_d - \Lambda e_q)\dot{q}_d + \frac{\partial}{\partial q}(v_d - \Lambda e_q)\dot{\hat{q}} + J\ddot{q}_d\right) \\
&\quad + \hat{g} - K_s(\hat{v} - v_d + \Lambda e_q),
\end{aligned} \tag{16}$$

in which $\hat{v}, \hat{\tau}$ and $\hat{q}$ are the estimates of v, τ and q, respectively. Also, $\hat{M}$ and $\hat{g}$ are the estimates of M and g and depend on the estimated parameter vector $\hat{\theta}$. The gain matrix K_s is constant positive definite. We consider the following adaptive observer

$$\hat{M}\dot{\hat{v}} + \hat{g} = J^{+T}\hat{\tau} + J^{+T}K_q\tilde{q} + K_{o2}\tilde{v} + K_s^T s \tag{17}$$

$$\dot{\hat{\tau}} = A\hat{\tau} + \hat{D}u + \hat{h} + P^{-1}J^+ s + P^{-1}J^+\tilde{v} \tag{18}$$

$$\dot{\hat{q}} = J^+\hat{v} + K_{o1}\tilde{q} \tag{19}$$

$$\dot{\hat{\theta}} = -\Gamma_1\left(Y^T(\rho)s + Y^T(\dot{\hat{v}})\tilde{v}\right) \tag{20}$$

$$\dot{\hat{\theta}}_a = \Gamma_2 Y_a^T(q,\dot{q},t,u)P\tilde{\tau} \tag{21}$$

where $\hat{D}$ and $\hat{h}$ are the estimates of the actuator dynamics elements and $K_{o1} \in \mathbb{R}^{n\times n}, K_{o2} \in \mathbb{R}^{2mn\times 2mn}, P \in \mathbb{R}^{n\times n}$, $K_q \in \mathbb{R}^{n\times n}$ and Γ_1, Γ_2 are constant positive definite observer gains. $Y(\rho)$ and $Y(\dot{\hat{v}})$ are the regressor matrices defined by

$$\begin{aligned}
M\rho + g &= Y(\rho)\theta \\
M\dot{\hat{v}} + g &= Y(\dot{\hat{v}})\theta
\end{aligned} \tag{22}$$

where ρ is given by $\rho := \frac{\partial}{\partial q}(v_d - \Lambda e_q)\dot{q} + \frac{\partial}{\partial q_d}(v_d - \Lambda e_q)\dot{q}_d + J\ddot{q}_d$, and $Y_a(q,\dot{q},t,u)$ is the regressor matrix defined in (13). Also Γ_1 and Γ_2 are positive matrix gains. We denote the joint angle estimation error by $\tilde{q} = q - \hat{q}$ and similarly, $\tilde{v} = v - \hat{v}$, $\tilde{\tau} = \tau - \hat{\tau}$. Note that the proposed observer (17)-(19) has a vector of $2(m+1)n$ states represented by $\zeta := \mathrm{col}[\hat{v}, \hat{\tau}, \hat{q}]$, and it uses $q, \dot{q}, \tilde{q}, \tilde{v}, s$ and u as the given inputs. The control signal is finally given by

$$u = \hat{D}^{-1}\left(-A\hat{\tau} - \hat{h} - P^{-1}J^+(s + \tilde{v}) - J^+ s + \dot{\tau}_d - c_1 z_1\right) \tag{23}$$

where c_1 is a constant positive-definite control gain. Note that we assume $\hat{D}$ is invertible.

Theorem 1: Consider an n-link robot manipulator including the actuator dynamics represented by (8)-(11). The control law (23) in combination with the adaptive observer (17)-(21), guarantees asymptotic convergence, in semi-global sense, of the position and velocity tracking error e_q, $\dot{e}_q$, and the torque estimation error $\tilde{\tau}$, to zero. □

In the proposed controller, we assume that the position of center mass for every link is known. The main advantage of the proposed controller is the use of the estimated torque in the control design, making the system insensitive to toque sensor noise. Persistency of excitation is not needed for motion tracking. Semi-global nature of the convergence indicates that by choosing high controller-observer gains, the region of attraction can be arbitrary enlarged. Smoothness of the controller-observer is inherently obtained during the backstepping design as shown in the proof of the theorem presented in the next section.

IV. Stability analysis

First we note that by virtue of (11), we can write f in terms of τ by

$$f = J^{+T}\tau + J^- f_0 \tag{24}$$

where f_0 is an arbitrary vector and the projection matrix J^- is defined by $J^- = I - (JJ^+)^T$; such that $J^T J^- = 0$. Similarly since by definition, $\tau_d = J^T f_d$, then

$$f_d = J^{+T}\tau_d + J^- f_{d0} \tag{25}$$

where f_{d0} is an arbitrary vector.

A. Error equations:

Subtracting the observer equations (17)-(19) from the system dynamics equations (8)-(11) and substituting for f from (24), yields

$$\begin{aligned}
M\dot{\tilde{v}} &= J^{+T}\tilde{\tau} + J^- f_0 - \tilde{g} - \tilde{M}\dot{\hat{v}} - J^{+T}K_q\tilde{q} \\
&\quad - K_{o2}\tilde{v} - K_s s
\end{aligned} \tag{26}$$

$$\dot{\tilde{\tau}} = A\tilde{\tau} + Y_a\tilde{\theta}_a - P^{-1}J^+ s - P^{-1}J^+\tilde{v} \tag{27}$$

$$\dot{\tilde{q}} = J^+\tilde{v} - K_{o1}\tilde{q}. \tag{28}$$

On the other hand, using (24) and (25) we can write (3) by

$$\begin{aligned}
M\dot{v} + g =& f_d + (J^{+T}\tau + J^- f_0) - J^{+T}\hat{\tau} + J^{+T}\hat{\tau} \\
&- (J^{+T}\tau_d + J^- f_{d0}).
\end{aligned} \tag{29}$$

Replacing for f_d from (16) in (29) and using definition of z_1, yields

$$\begin{aligned}
&M\dot{v} + g = \\
&\hat{M}\left(\frac{\partial}{\partial q}(v_d - \Lambda e_q)(\dot{q} - \dot{\tilde{q}}) + \frac{\partial}{\partial q_d}(v_d - \Lambda e_q)\dot{q}_d + J\ddot{q}_d\right) \\
&\quad + \hat{g} - K_s s + K_s\tilde{v} + J^{+T}\tilde{\tau} + J^{+T}z_1 + J^-(f_0 - f_{d0})
\end{aligned} \tag{30}$$

$$
H_1 = \begin{bmatrix} \underline{\lambda}(K_s) & -\frac{1}{2}k_{Jm}^{-1} & 0 & -\frac{1}{2}F_1\overline{\lambda}(K_{o1}) & -\frac{1}{2}F_1 k_{Jm}^{-1} & 0 \\ -\frac{1}{2}k_{Jm}^{-1} & \underline{\lambda}(K_\Lambda) & 0 & 0 & 0 & 0 \\ 0 & 0 & \underline{\lambda}(c_1) & 0 & 0 & 0 \\ -\frac{1}{2}F_1\overline{\lambda}(K_{o1}) & 0 & 0 & \underline{\lambda}(K_q K_{o1}) & 0 & 0 \\ -\frac{1}{2}F_1 k_{Jm}^{-1} & 0 & 0 & 0 & \underline{\lambda}(K_{o2}) & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2}\underline{\lambda}(Q) \end{bmatrix} \tag{32}
$$

In light of the the definition of s in (14), the last equality transforms into

$$
\begin{aligned} M\dot{s} + K_s s = &- \tilde{M}\rho - \tilde{g} + K_s\tilde{v} - \hat{M}\frac{\partial}{\partial q}\left(v_d - \Lambda e_q\right)\dot{\tilde{q}} \\ &+ J^{+T}\tilde{\tau} + J^{+T}z_1 + J^{-}(f_0 - f_{d0}). \end{aligned} \tag{31}
$$

where $\tilde{M} = M - \hat{M}$ and $\tilde{g} = g - \hat{g}$.

B. Convergence analysis

Now we define the Lyapunov function candidate by

$$
\begin{aligned} V := \frac{1}{2}\big(&s^T M s + z_1^T z_1 + \tilde{q}^T K_q \tilde{q} + \tilde{\tau}^T P \tilde{\tau} \\ &+ \tilde{v}^T M \tilde{v} + e_q^T e_q + \tilde{\theta}^T\Gamma_1^{-1}\tilde{\theta} + \tilde{\theta}_a^T\Gamma_2^{-1}\tilde{\theta}_a\big) \end{aligned} \tag{33}
$$

where P, Γ_1, Γ_2 are positive definite matrices and $\tilde{\theta} = \theta - \hat{\theta}$ and $\tilde{\theta}_a = \theta_a - \hat{\theta}_a$.

By taking time-derivative of V along (31) and using (15), (26)-(28) and (14), we have

$$
\begin{aligned} \dot{V} = &s^T\Big(- \tilde{M}\rho - \tilde{g} - K_s s + K_s\tilde{v} - \hat{M}\frac{\partial}{\partial q}\left(v_d - \Lambda e_q\right)\dot{\tilde{q}} \\ &+ J^{+T}\tilde{\tau} + J^{+T}z_1 + J^{-}(f_0 - f_{d0})\Big) \\ &+ z_1^T(A\hat{\tau} + \hat{D}u + \hat{h} + P^{-1}J^{+}s + P^{-1}J^{+}\tilde{v} - \dot{\tau}_d) \\ &+ \tilde{q}^T K_q(J^{+}\tilde{v} - K_{o1}\tilde{q}) + \tilde{v}^T\Big(J^{+T}\tilde{\tau} + J^{-}f_0 - \tilde{M}\dot{\tilde{v}} \\ &- \tilde{g} - J^{+T}K_q\tilde{q} - K_{o2}\tilde{v} - K_s^T s\Big) + e_q^T\big(J^{+}s \\ &- K_\Lambda e_q\big) + \tilde{\tau}^T P(Y_a\tilde{\theta}_a - P^{-1}J^{+}s - P^{-1}J^{+}\tilde{v}) \\ &+ \frac{1}{2}\tilde{\tau}^T(PA + A^T P)\tilde{\tau} + \dot{\tilde{\theta}}^T\Gamma_1^{-1}\tilde{\theta} + \dot{\tilde{\theta}}_a^T\Gamma_2^{-1}\tilde{\theta}_a \end{aligned} \tag{34}
$$

Substituting for u from (23) and straightforward simplifications, yield

$$
\begin{aligned} \dot{V} = &-s^T Y(\rho)\tilde{\theta} - s^T K_s s - s^T\hat{M}\frac{\partial}{\partial q}\left(v_d - \Lambda e_q\right)\dot{\tilde{q}} \\ &+ s^T J^{-}(f_0 - f_{d0}) - z_1^T c_1 z_1 - \tilde{q}^T K_q K_{o1}\tilde{q} + \tilde{v}^T J^{-} f_0 \\ &- \tilde{v}^T Y(\dot{\tilde{v}})\tilde{\theta} - \tilde{v}^T K_{o2}\tilde{v} + e_q^T J^{+}s - e_q^T K_\Lambda e_q + \tilde{\tau}^T P Y_a\tilde{\theta}_a \\ &+ \frac{1}{2}\tilde{\tau}^T(PA + A^T P)\tilde{\tau} - \dot{\hat{\theta}}^T\Gamma_1^{-1}\tilde{\theta} - \dot{\hat{\theta}}_a^T\Gamma_2^{-1}\tilde{\theta}_a. \end{aligned} \tag{35}
$$

Since $J^T J^{-} = 0$ then $s^T J^{-} = 0$ and $\tilde{v}^T J^{-} = 0$. Now, substituting for $\dot{\hat{\theta}}$ and $\dot{\hat{\theta}}_a$ from (20) and (21) in (35), yields

$$
\begin{aligned} \dot{V} = &- s^T K_s s - s^T\hat{M}\frac{\partial}{\partial q}\left(v_d - \Lambda e_q\right)(J^{+}\tilde{v} - K_{o1}\tilde{q}) \\ &- z_1^T c_1 z_1 - \tilde{q}^T K_q K_{o1}\tilde{q} - \frac{1}{2}\tilde{\tau}^T Q\tilde{\tau} - \tilde{v}^T K_{o2}\tilde{v} \\ &+ e_q^T J^{+}s - e_q^T K_\Lambda e_q. \end{aligned} \tag{36}
$$

where $Q > 0$ is such that $PA + A^T P = -Q$. Note that the existence of $Q > 0$ is guaranteed due to the fact that A is Hurwitz stable [11].

To prove the stability of the system we need to determine under which conditions $\dot{V} \leq 0$. We consider the sign indefinite terms in $\dot{V}$ and derive an upper bound for it. To this end, we note that

$$
\begin{aligned} \frac{\partial}{\partial q}\left(v_d - \Lambda(q)e_q\right) &= \frac{\partial}{\partial q}\left(J(q)(\dot{q}_d - K_\Lambda(q - q_d))\right) \\ &= -J(q)K_\Lambda + \left[\ \tfrac{\partial J(q)}{\partial q_1}\dot{q}_d \ |\ \ldots\ |\ \tfrac{\partial J(q)}{\partial q_n}\dot{q}_d\ \right] \\ &\quad - \left[\ \tfrac{\partial J(q)}{\partial q_1}K_\Lambda e_q \ |\ \ldots\ |\ \tfrac{\partial J(q)}{\partial q_n}K_\Lambda e_q\ \right] \end{aligned} \tag{37}
$$

Hence,

$$
\left\|\frac{\partial}{\partial q}\left(v_d - \Lambda(q)e_q\right)\right\| \leq k_{jM}\left(\|K_\Lambda\| + \|\dot{q}_d\| + \|K_\Lambda\|\,\|e_q\|\right). \tag{38}
$$

Then, we have

$$
\begin{aligned} \dot{V} \leq &- \underline{\lambda}(K_s)\,\|s\|^2 + \|s\|\, k_{JM}\overline{\lambda}(\hat{M})\Big(\overline{\lambda}(K_\Lambda) + \|\dot{q}_d\| \\ &+ \overline{\lambda}(K_\Lambda)\,\|e_q\|\Big)\left(k_{Jm}^{-1}\,\|\tilde{v}\| + \overline{\lambda}(K_{o1})\,\|\tilde{q}\|\right) \\ &+ k_{Jm}^{-1}\,\|s\|\,\|e_q\| - \underline{\lambda}(c_1)\,\|z_1\|^2 - \underline{\lambda}(K_q K_{o1})\,\|\tilde{q}\|^2 \\ &- \frac{1}{2}\underline{\lambda}(Q)\,\|\tilde{\tau}\|^2 - \underline{\lambda}(K_{o2})\,\|\tilde{v}\|^2 - \underline{\lambda}(K_\Lambda)\,\|e_q\|^2, \end{aligned} \tag{39}
$$

where we have used the fact that $\|J(q)\|, \left\|\dot{J}(q)\right\|, q_d$ and $\dot{q}_d$ are bounded. Here, $\underline{\lambda}(\cdot)$ and $\overline{\lambda}(\cdot)$ represent the minimum and the maximum eigenvalues of a matrix. Next, we define the error vector χ by

$$
\chi^T := \left[\ \|s\|,\quad \|e_q\|,\quad \|z_1\|,\quad \|\tilde{q}\|,\quad \|\tilde{v}\|,\quad \|\tilde{\tau}\|\ \right], \tag{40}
$$

and $F := F_1 + F_2$ where

$$
F_1 := k_{JM}\overline{\lambda}(\hat{M})\left(\overline{\lambda}(K_\Lambda) + \|\dot{q}_d\|\right) \tag{41}
$$

$$
F_2 := k_{JM}\overline{\lambda}(\hat{M})\overline{\lambda}(K_\Lambda)\,\|e_q\|. \tag{42}
$$

Then, it can be inferred that

$$
\dot{V} \leq -\chi^T H\chi \tag{43}
$$

where $H = H_1 + H_2$, in which H_1 is a constant matrix defined by (32) and H_2 is a matrix depending on the tracking error $\|e_q\|$, and is given by $H_2 = H_2^T = [H_{2_{ij}}]$ where

$$
H_{2_{ij}} = \begin{cases} -\frac{1}{2}F_2\overline{\lambda}(K_{o1}) & \text{if } (i,j) = (1,4) \\ -\frac{1}{2}F_2 k_{Jm}^{-1} & \text{if } (i,j) = (1,5) \\ 0 & \text{otherwise.} \end{cases} \tag{44}
$$

$$J(q)^T = \begin{bmatrix} l_{c_1}c_1 & l_{c_1}s_1 & 0 & l_1c_1 + l_{c_2}c_{12} & l_1s_1 + l_{c_2}s_{12} & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & l_{c_2}c_{12} & l_{c_2}s_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{45}$$

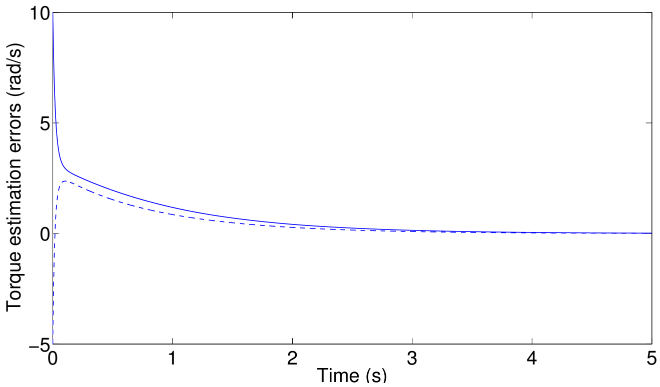

Fig. 1: Joint torque estimation errors for the controller (a). τ_1(solid) and τ_2(dash).

Because all the elements in H_1 are known or they are arbitrary control parameters, it can be shown that there always exists a set of parameters such that $H_1 > 0$. Besides, It can be shown that there exists a finite positive constant β such that $\|H_2\| \leq \beta \|e_q\|$, which implies that

$$\begin{aligned} \dot{V} &\leq -\underline{\lambda}(H_1)\|\chi\|^2 + \beta\|\chi\|^2\|e_q\| \\ &\leq -\underline{\lambda}(H_1)\|\chi\|^2 + \beta\|\chi\|^3. \end{aligned} \tag{46}$$

Therefore, if χ belongs to the set $\Omega_1 := \left\{\chi \in \mathbb{R}^6 \middle| \|\chi\| \leq \frac{\underline{\lambda}(H_1)}{\beta}\right\}$, then $\dot{V} \leq 0$. On the other hand, by virtue of the definition of V in (33) there exist $\gamma_1, \gamma_2, \gamma_3 > 0$ such that

$$\gamma_1\|\chi\|^2 \leq V \leq \gamma_2\|\chi\|^2 + \gamma_3\|\tilde{\theta}\|^2 \tag{47}$$

which implies $\|\chi(t)\| \leq \sqrt{\frac{V(t)}{\gamma_1}}$ for all $t \geq 0$. Therefore if $\chi(0) \in \Omega_1$ then $\dot{V}(0) \leq 0$ and consequently $V(t) \leq V(0)$ and hence

$$\|\chi\| \leq \sqrt{\frac{V(t)}{\gamma_1}} \leq \sqrt{\frac{V(0)}{\gamma_1}} \tag{48}$$

So, if $\chi(0) \in \Omega_1$ then $\chi(t)$ is bounded for all $t > 0$, and its upper bound is $\sqrt{\frac{V(0)}{\gamma_1}}$. Note that $V(0)$ depends on the initial values of the motion tracking errors, torque estimation error and dynamics parameter errors. As a result if controller-observer gains are chosen such that $\frac{\gamma_1}{\beta^2}\underline{\lambda}(H_1)^2 \geq V(0)$ then χ remains always in Ω_1 and consequently converges asymptotically to zero.

An inner approximation of the region of attraction is given by those $\chi(0)$ and $\hat{\theta}(0)$ belonging to the set

$$\Omega = \left\{(\chi(0), \hat{\theta}(0)) \middle| \gamma_2\|\chi(0)\|^2 + \gamma_3\|\hat{\theta}(0)\|^2 \leq \frac{\gamma_1\underline{\lambda}(H_1)^2}{\beta^2}\right\} \tag{49}$$

V. Simulation Results

We consider a 2-DOF planar robot with the parameters given by $l_1 = 0.45$m, $l_2 = 0.45$m, $l_{c_1} = 0.091$m, $l_{c_2} = 0.048$m, $m_1 = 1$kg, $m_2 = 1$kg, $I_1 = 0.5\text{diag}\{1, 1, 0.1\}\text{kgm}^2$, $I_2 = 0.5\text{diag}\{1, 1, 0.1\}\text{kgm}^2$ where l_i denotes the length of the i-th link and l_{c_i} denotes the distance between the center of mass of the link i and its starting joint. The vector of manipulator parameters is given by $\theta = [m_1 \;\; m_2]^T$. Actuator dynamics is also given by $D = 100\mathbf{I_2}$, $h = 0$, $A = \text{diag}\{-20\pi, -20\pi\}$. The gravity vector is then represented by [24]

$$g = [0 \;\; 9.81m_1 \;\; 0 \;\; 0 \;\; 9.81m_2 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0]^T \tag{50}$$

The Jacobian matrix is given by (45), where c_i, s_i, c_{ij} and s_{ij} denote $\cos(q_i), \sin(q_i), \cos(q_i + q_j)$ and $\sin(q_i + q_j)$, respectively. The desired joint angle trajectory is defined by

$$q_d(t) = \begin{bmatrix} 1 + 0.1\sin(t)\sin(2\pi t) \\ 0.5 + 0.1\cos(t)\cos(2\pi t) \end{bmatrix} \text{rad}. \tag{51}$$

The initial conditions are also given by $q(0) = [1.6 - \pi/4 \;\; 1 - \pi/6]^T$rad, $v(0) = 0$, $\tau(0) = [0 \;\; 0]^T$Nm , $\hat{q}(0) = [2 \;\; 2]^T$rad, $\hat{v}_i(0) = 1$ for $i = 1, ..., 12$, $\hat{\tau} = [-10 \;\; 5]$Nm and $\hat{\theta}(0) = [1.5 \;\; 1.3]^T$. Gains of the controller and the adaptive observer were chosen as $K_\Lambda = 150\mathbf{I_2}, K_s = 5\mathbf{I_{12}}, c_1 = \mathbf{I_{12}}, K_{o1} =$

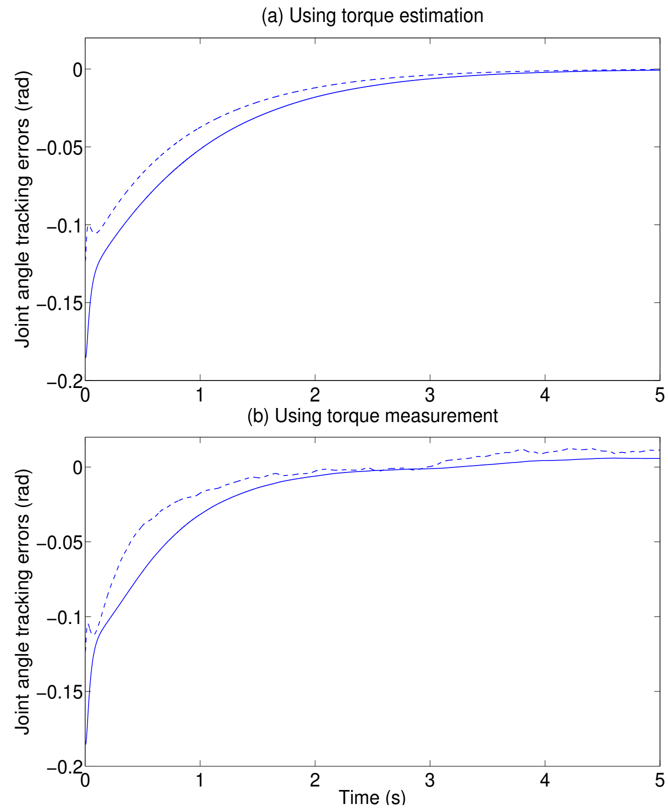

Fig. 2: Joint angle tracking errors, e_{q_1}(solid) and e_{q_2}(dash), for two controllers. (a):using torque estimates and (b):using torque measurement. Torque measurement is contaminated by an additive noise and a step-wise signal initiated at $t =$ 3sec.

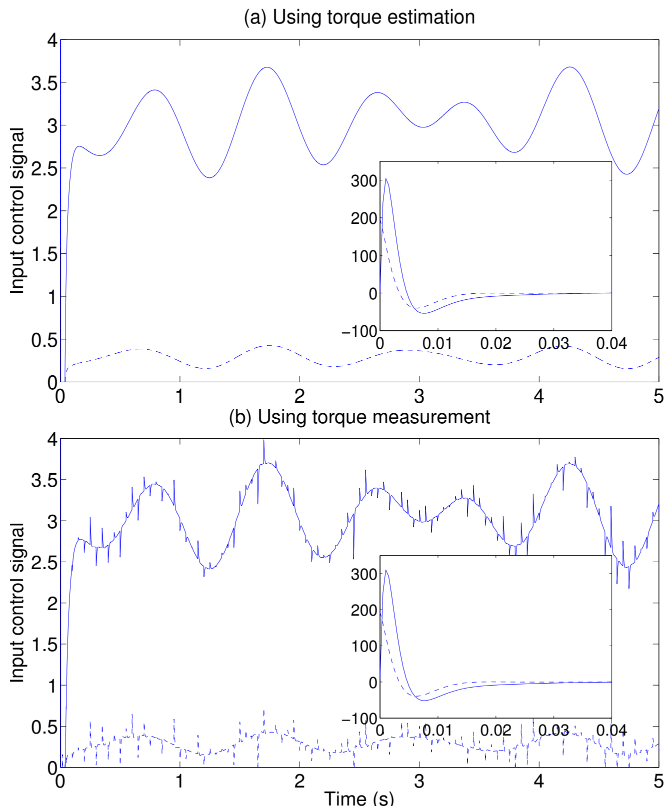

Fig. 3: Input control signal, u_1(solid) and u_2(dash), for the two controllers.

$0.1\mathbf{I_2}, K_{o2} = 100\mathbf{I_2}, K_q = 0.1\mathbf{I_2}, P = 0.1\mathbf{I_2}$ and $\Gamma_1 = 10^{-3}\text{diag}\{10, 5\}\mathbf{I_2}$.

To demonstrate the performance of the proposed controller using torque estimation (controller a), we implemented another controller with a similar structure which used torque measurement (controller b). In both cases the measurements of joint torques were contaminated by an additive zero-mean Gaussian noise with the variance of 0.05Nm. Moreover, to represent a possible fault in torque sensors, a step-wise signal with amplitude of 0.5Nm was added at $t = 3$sec to torque sensor outputs.

Torque estimation error for the controller (a) is shown in Fig. 1. The joint position tracking errors for both controllers are demonstrated in Figs. 2. Moreover, input control signals are shown and compared in Fig. 3. As expected, since the controller (a) does not use torque measurement, its performance is insensitive to torque sensor noise and fault. Practically, this property has important implications in enhancing system reliability and in reducing actuator fatigue and link vibration.

VI. Conclusion

The use of torque estimator instead of joint torque sensor reduces significantly system sensitivity to noise and possible sensor failure. The proposed smooth adaptive controller is capable of handling uncertainty in manipulator and actuator dynamics and ensures semi-global asymptotic motion tracking without any persistency of excitation condition.

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