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A Globally Convergent Observer for Velocity Estimation in Robotic Manipulators with Uncertain Dynamics

Nima Lotfi and Mehrzad Namvar
Department of Electrical Engineering
Sharif University of Technology
P.O. Box 11155-8639, Tehran, Iran
namvar@sharif.ir

***Abstract*— We present a method for global estimation of joint velocities in robot manipulators. A non-minimal model of a robotic manipulator is used to design an adaptive observer capable of handling uncertainties in robot dynamics. Dimension of the proposed observer is shown to be at least $3n$ where n stands for the manipulator degrees of freedom. This number is less than the dimension of most of existing globally convergent adaptive observers. Global asymptotic convergence of system state estimates to their true values is achieved under no persistency of excitation condition. Smoothness of the dynamics of the proposed observer allows its easy implementation in comparison with non-smooth observers. Simulation results illustrate low noise sensitivity of the proposed observer in comparison with non-smooth observers.**

I. Introduction

Over the recent years, precise control of robot manipulators has been the subject of several industrial benchmarks, and to this end, one problem that has attracted a good deal of interest is the possibility of accessing full system states. In general, this issue arises in a number of applications from controller design to fault detection problems. In robotic systems, estimation of joint velocities which are considered as system states, is of practical importance since many commercially available robots are not commonly equipped with velocity sensors such as analogue tachometers. Even if such sensors are used, their output is usually contaminated with noise. For these reasons, in recent years a considerable amount of research has been devoted to the problem of estimating the velocity in mechanical systems. For example, it was experimentally shown in [1] that *ad hoc* numerical differentiation of joint positions is not a suitable method for generation of joint velocity especially in high and low velocities.

The existing methods for velocity estimation can be distinguished from different perspectives such as the extent of the use of manipulator model in the observer design. Non-model based filters such as high-gain differentiators used in [2] can provide model-free means of estimating velocity by approximating the behavior of a differentiator over a range of frequencies. However, they feature a so called peaking effect in high gains, in a sense that the amplitude of the estimated velocity during the transient period grows significantly as the filter gain become large [3]. On the other hand, model-based observers estimate the velocity by mimicking the manipulator dynamics, which is usually assumed to be exactly known. In [4], [5], [6], [7], [8], [9] model-based observers with smooth dynamics were introduced while [10] and [11] used model-based observers with non-smooth dynamics.

Unfortunately, in practical situations, exact knowledge of manipulator dynamics is rarely available. Unstructured uncertainty was studied in [12] for a class of nonlinear systems in observer forms. Also, several works in literature studied the design of observers being inherently robust against model uncertainty such as [13] and [14] for the case of smooth observers or as [15] using non-smooth observers. However, despite their simple structures, non-smooth observers often exhibit high-frequency chattering during real time implementation and have shown to be sensitive to measurement noise and this can potentially limit their application.

Another important factor in evaluation of an observer performance is the local or global convergence of its estimation error to zero. All previously mentioned observers, are locally or semi-globally convergent in a sense that their estimation error converges to zero if the initial guess for velocity stays within some neighborhood of the true velocity, called as the region of attraction. Despite this region can be expanded by choosing high observer gains, in practice, high gains result in higher computational burden and significant sensitivity to measurement noise. To achieve global convergence, [16] has proposed a sliding mode observer by using manipulator model to acquire finite-time convergence of the observation error. In [17] and [18] global convergence was achieved by using non-model based and non-smooth observers. However, despite the advantage of having simple structures, the observers in [17]-[18] still required a *priori* knowledge of manipulator model to compute the required upperbounds.

In [19] an observer was proposed with the advantage of being smooth and globally convergent. However, the dimension of the observer was $7n$ where n denotes the manipulator degrees of freedom and in addition the observer required full knowledge of manipulator model. Finally, in [20] a class of globally convergent velocity observers of dimension $3n$ with smooth dynamics was proposed. However, the uncertainty in manipulator model was assumed to depend only on positions and velocities. So far, no work has been done in design of smooth, adaptive and globally convergent observers which handles uncertainties in the entire manipulator model depending on accelerations, velocities and positions.

In this paper, we propose a globally convergent smooth

TABLE I

NOMENCLATURE

$q \in \mathbb{R}^n$	:	joint angle vector
$\tau_m \in \mathbb{R}^n$	:	motor torque
$p \in \mathbb{R}^{mn}$	:	link center of mass position
$v \in \mathbb{R}^{2mn}$	:	generalized link velocity
$v_p \in \mathbb{R}^{mn}$	:	link translational velocity
$\omega \in \mathbb{R}^{mn}$	:	link angular velocity
$F \in \mathbb{R}^{2mn}$	:	generalized force applied to links
$f_m \in \mathbb{R}^{mn}$	:	vector of link momentums
$f_f \in \mathbb{R}^{mn}$	:	vector of link forces
$\pi \in \mathbb{R}^n$	:	selected elements of p
$\nu \in \mathbb{R}^n$	:	selected elements of v_p
$\bar{f} \in \mathbb{R}^n$	:	selected elements of f_f
$\theta \in \mathbb{R}^r$	:	link parameter vector
L_∞	:	space of bounded signals
L_2	:	space of square integrable signals

observer for robot manipulators by making use of non-minimal model of a robotic system. We assume that all link masses and inertias are unknown. The proposed adaptive observer guarantees asymptotic convergence of the estimated velocities to their true values without any persistency of excitation condition. Besides, dimension of the adaptive observer is $2n+r$, where r stands for the number of unknown parameters related to manipulator dynamics. This number which can be reduced to $3n$, is less than the dimension of the existing globally convergent adaptive observers. Finally, performance of the proposed observer and its sensitivity to measurement noise is investigated and illustrated by simulations.

II. System Description

We consider a rigid manipulator whose dynamics is given by [21]

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau_m \quad (1)$$

where $q \in \mathbb{R}^n$ is the vector of joint angles which is assumed to be measurable. The vector $\tau_m \in \mathbb{R}^n$ represents the effect of motor torques, friction torques and other nonlinearities such as backlash or joint flexibilities. Furthermore, the inertia matrix function is represented by $M(q) \in \mathbb{R}^{n\times n}$ which is bounded and positive definite. The vector $C(q,\dot{q})\dot{q} \in \mathbb{R}^n$ represents the Centrifugal and Coriolis forces and $g(q) \in \mathbb{R}^n$ denotes the vector of gravitational forces.

A. A Non-Minimal Model for Robotic System

Instead of the complex and nonlinear equation (1), in the sequel, we use a non-minimal set of equations to describe dynamics of a rigid manipulator. Let $p_i \in \mathbb{R}^m$ denote the position of the center of mass of the ith link expressed with respect to a fixed frame attached to the manipulator base where m is the dimension of the workspace. Let $\dot{p}_i$ and ω_i denote the linear and angular velocities of the ith link, respectively. We define m_i as the mass of the ith link and $I^i_{l_i}$ as the constant inertia tensor of the ith link relative to a frame attached to its center of mass. We also define the position and orientation Jacobians for the ith link as the maps, $J^i_p(q) : \mathbb{R}^n \to \mathbb{R}^{m\times n}$ and $J^i_o(q) : \mathbb{R}^n \to \mathbb{R}^{m\times n}$ such that $\dot{p}_i = J^i_p(q)\dot{q}$ and $\omega_i = J^i_o(q)\dot{q}$. We define the generalized link velocity vector by $v := \text{col}[\dot{p}_1, ..., \dot{p}_n, \omega^1_1, ..., \omega^n_n]$, where $\omega^i_i = R_i(q)^T\omega_i$ is the angular velocity of the ith link, expressed in the link frame and $R_i(q)$ is the rotation matrix of the ith link with respect to the base frame. Besides, manipulator link and joint velocities are related by

$$v = \mathcal{J}(q)\dot{q} \quad (2)$$

where $\mathcal{J}(q) \in \mathbb{R}^{(2mn)\times n}$ has the form

$$\mathcal{J}(q) := \left[J^1_p(q)^T \mid \cdots \mid J^n_p(q)^T \mid J^1_o(q)^T R_1 \mid \cdots \mid J^n_o(q)^T R_n \right]^T \quad (3)$$

As it was shown in [19], $\mathcal{J}(q)$ has full column rank for any $q \in \mathbb{R}^n$. The dynamics of the robotic system can then be expressed by

$$\mathcal{M}\dot{v} + h = F \quad (4)$$

where $\mathcal{M} := \text{diag}\{m_1\mathbf{I}_m, ..., m_n\mathbf{I}_m, I^1_{l_1}, ..., I^n_{l_n}\} \in \mathbb{R}^{(2mn)\times(2mn)}$ is the inertia matrix, which is assumed to be constant and positive-definite, and the vector $h \in \mathbb{R}^{2mn}$ has the form $h = [-m_1 g_o^T, \ldots, -m_n g_o^T, 0_{1\times nm}]^T$

where $g_o \in \mathbb{R}^m$ is the vector of gravity acceleration. In the equation (4), the vector $F \in \mathbb{R}^{2mn}$ is the vector of generalized forces given by $F = [f_f^T, f_m^T]^T$ where $f_{f_i} \in \mathbb{R}^m$ for $i = 1, ..n$ denotes the net force sensed at the center-of-mass of link i and is expressed with respect to the fixed frame.

Remark 1: Evidently, f_{f_i} includes the effect of the interaction of link i with other links. The moment $f_{m_i} \in \mathbb{R}^m$ for $i = 1, ..n$ is given by $-\omega^i_i \times I_i\omega^i_i + n_{m_i}$ where n_{m_i} is the net moment exerted to the link i. Note that in development of the observer; measurement of f_{m_i} will not be used and only n elements of $f_f \in \mathbb{R}^{mn}$ are needed for implementation of the observer. □

Remark 2: Differentiating (2) with respect to time and substituting for $\dot{v}$ in (4), yields

$$[\mathcal{J}^T\mathcal{M}\mathcal{J}](q)\ddot{q} + [\mathcal{J}^T\mathcal{M}\dot{\mathcal{J}}](q,\dot{q})\dot{q} + \mathcal{J}(q)^T h = \mathcal{J}(q)^T F \quad (5)$$

By virtue of (1) and (5) we infer that $M(q) := \mathcal{J}(q)^T\mathcal{M}\mathcal{J}(q) > 0$, $C(q,\dot{q}) := [\mathcal{J}^T\mathcal{M}\dot{\mathcal{J}}](q,\dot{q})$, and the gravity force vector is given by $g(q) := \mathcal{J}(q)^T h$.

By the principle of virtual work [21], the relationship between the generalized force F and joint torque τ_m is also given by $\mathcal{J}^T(q)F = \tau_m$. In practice, the effect of complex nonlinear phenomena such as joint friction, backlash or flexibilities are reflected to F. Therefore, by measuring n elements of the joint force vector f_f, those hardly identifiable nonlinear effects are taken into account in the observer design. □

III. Problem Definition

The main purpose of this section is to design an observer for estimation of unmeasured velocity state $\dot{q}$, by assuming that link masses and inertias are unknown. For this purpose, we consider the non-minimal model (4) for robot manipulators. We select the first mn-elements of the generalized velocity v by

$$v_p := [\dot{p}_1^T, ..., \dot{p}_n^T]^T \in \mathbb{R}^{mn} \quad (6)$$

Therefore, by virtue of (4) we have

$$\dot{v}_p = -\mathcal{M}_p^{-1} h_p + \mathcal{M}_p^{-1} f_f \tag{7}$$

where $\mathcal{M}_p = \text{diag}\{m_1 \mathbf{I}_m, ..., m_n \mathbf{I}_m\}$ and $h_p = \Big[-m_1 g_o^T, \ldots, -m_n g_o^T\Big]^T$. From these definitions, it is obvious that the term $\mathcal{M}_p^{-1} h_p$ is a constant vector independent of system parameters (m_i) for $i = 1, ..., n$ and is given by $\mathcal{M}_p^{-1} h_p = [\underbrace{-g_o, ..., -g_o}_{n \text{ times}}]^T$. Also, from (2), (3) and (6), we have

$$v_p = \mathcal{J}_p(q)\dot{q} \tag{8}$$

where $\mathcal{J}_p(q) = \Big[J_p^1(q)^T, \cdots, J_p^n(q)^T\Big]^T$. Since the matrix $\mathcal{J}_p(q)$ belongs to $\mathbb{R}^{mn \times n}$ and has full-column rank, it is always possible to select n linearly-independent rows of this matrix. By virtue of (8), those elements of v_p corresponding to the selected n linearly-independent rows of $J_p(q)$, are denoted by $\nu = [\nu_1, \nu_2, ..., \nu_n]$. We also define the position vector π such that $\dot{\pi} = \nu$. Note that, elements of $\pi \in \mathbb{R}^n$ are not necessarily in the same sequence as in $p \in \mathbb{R}^{mn}$. Now if we arrange the linearly-independent rows of $\mathcal{J}_p(q)$ in a matrix called $\mathcal{J}_n(q) \in \mathbb{R}^{n \times n}$, then the constraint (8) will take the form

$$\nu = \mathcal{J}_n(q)\dot{q} \tag{9}$$

In light of (7) and diagonal nature of $\mathcal{M}_p$, we can write the following n-dimensional differential equations

$$\dot{\pi}_j = \nu_j \tag{10}$$
$$\dot{\nu}_j = a_j + m_j^{-1} f_{f_j} \tag{11}$$

where a_j is the corresponding jth element in the $\mathcal{M}_p^{-1} h_p$ vector for $j = 1, ..., n$. Also note that, we will assume m_j to be unknown for each link. For notational simplicity, we define $\theta_j := m_j^{-1}$. As we have assumed the measurability of joint angels q, we can calculate link positions vector p and π, by knowing the robot forward kinematics. Also, f_f is available from the measurements of force sensors installed on links.

IV. Observer Formulation

First, we introduce the following transformation

$$z(t) = [z_{j_1},\ z_{j_2}]^T = [\pi_j,\ \nu_j - \zeta_j(t)\theta_j]^T \in \mathbb{R}^2 \tag{12}$$

for $j = 1, ..., n$, where $\zeta_j(t)$ is given by the differential equation $\dot{\zeta}_j = -\alpha_j \zeta_j + f_{f_j}$ with the initial condition $\zeta_j(t_0) = 0$ where $\alpha_j > 0$ is a scalar gain. Differentiating (12) and using (10) and (11), yields $\dot{z}_{j_1} = z_{j_2} + \zeta_j \theta_j$ and $\dot{z}_{j_2} = a_j + \alpha_j \zeta_j \theta_j$ for $j = 1, ..., n$. Now, we propose an observer for estimation of z_{j_1} and z_{j_2} such that the estimated states $[\hat{z}_{j_1}, \hat{z}_{j_2}]^T$, converge asymptotically to $[z_{j_1}, z_{j_2}]^T$. This property will be shown to ensure asymptotic estimation of robot joint velocities. We propose the following observer

$$\begin{cases} \dot{\hat{z}}_{j_1} = -(\lambda_j + \alpha_j)\hat{z}_{j_1} + \hat{z}_{j_2} + \zeta_j(t)\hat{\theta}_j(t) + (\lambda_j + \alpha_j)\pi_j \\ \dot{\hat{z}}_{j_2} = -\lambda_j \alpha_j \hat{z}_{j_1} + a_j + \alpha_j \zeta_j(t)\hat{\theta}_j(t) + \lambda_j \alpha_j \pi_j \\ \dot{\zeta}_j = -\alpha_j \zeta_j + f_{f_j} \\ \dot{\hat{\theta}}_j = \gamma_j \zeta_j(t)(\pi_j - \hat{z}_{j_1}) \\ \hat{\pi}_j = \hat{z}_{j_1} \\ \hat{\nu}_j = \hat{z}_{j_2} + \zeta_j(t)\hat{\theta}_j(t) \end{cases} \tag{13}$$

for $j = 1, ..., n$, where $\hat{z} = [\hat{z}_{j_1}, \hat{z}_{j_2}]^T \in \mathbb{R}^2$, $\hat{\theta}_j \in \mathbb{R}$ and λ_j, γ_j are given positive scalar gains. Note that the states of the proposed observer are $[\hat{z}_{j_1}, \hat{z}_{j_2}, \zeta_j, \hat{\theta}_j]^T \in \mathbb{R}^4$ and its inputs are $[f_{f_j}, \pi_j]^T \in \mathbb{R}^2$. The observer outputs are $\hat{\pi}_j$ and $\hat{\nu}_j$. The initial conditions for the first three differential equations in (13) are given by $\hat{z}_{j_1}(t_0) = \hat{\pi}_j(t_0)$, $\hat{z}_{j_2}(t_0) = \hat{\nu}_j(t_0) - \zeta_j(t_0)\hat{\theta}_j(t_0)$, $\zeta_j(t_0) = 0$ and the initial condition $\hat{\theta}_j(t_0)$ is arbitrary.

Theorem 1: The observer introduced by equations (13) guarantees global asymptotic convergence of velocity $\hat{\nu}$ and position $\hat{\pi}$ estimates to their true values. □

Note that the convergence of position and velocity, as stated in Theorem 1, is achieved under no persistency of excitation condition. This is practically important because the Theorem does not require any constraint on robot motion. Another feature of the observer (13) is the smoothness of its dynamics with respect to its states and inputs which simplifies its numerical implementation in comparison to non-smooth observers.

Proof:

Error dynamics: First, for brevity in presentation, we define $x := [\pi_j, \nu_j]^T$. Then, the original system equations (10) and (11) transform into

$$\dot{x} = A_c x + \psi_j + \theta_j b f_{f_j} \tag{14}$$
$$y = C_c x \tag{15}$$

where $x_1 = \pi_j$, $x_2 = \nu_j$ and the vectors ψ_j and b are defined by $\psi_j = [0, a_j]^T, b = [0, 1]^T$ for $j = 1, ..., n$. Besides, the matrices A_c and C_c are given by $A_c = [0, 1; 0, 0]$ and $C_c = [1, 0]$. Now consider the transformation (12) and define $\beta_j(t) = [0, \zeta_j(t)]^T \in \mathbb{R}^2$, then the relationship between $x(t)$ and $z(t)$ will take the form $z(t) = x(t) - \beta_j(t)\theta_j$ that when applied to the equation (14) will result in the following transformed system equations with respect to z variables [12]

$$\dot{z} = A_c z + \psi_j + d_j \zeta_j(t)\theta_j \tag{16}$$

where $d_j = [1, \alpha_j]^T$ and α_j being as an arbitrary gain. Finally, we introduce the vector $k_j \in \mathbb{R}^2$ as $k_j = [\lambda_j + \alpha_j, \lambda_j \alpha_j]^T$. With these definitions, the observer equations (13) can be expressed by

$$\begin{cases} \dot{\hat{z}} = (A_c - k_j C_c)\hat{z} + \psi_j + d_j \zeta_j(t)\hat{\theta}_j(t) + k_j y \\ \dot{\hat{\theta}}_j = \gamma_j \zeta_j(t)(y - C_c \hat{z}) \\ \hat{x} = \hat{z}(t) + \beta_j(t)\hat{\theta}_j(t) \end{cases} \tag{17}$$

Now by defining $\tilde{z} = z - \hat{z}$, $\tilde{\theta}_j = \theta_j - \hat{\theta}_j$ and by virtue of (16) and (17), the estimation error dynamics become

$$\dot{\tilde{z}} = A\tilde{z} + d_j\zeta_j(t)\tilde{\theta}_j(t) \quad (18)$$

$$\dot{\tilde{\theta}}_j = -\gamma_j\zeta_j(t)C_c\tilde{z} \quad (19)$$

where $A = A_c - k_jC_c$.

Convergence analysis: The triple C_c, A, d can be shown to be strictly positive real, so according to Meyer-Kalman-Yakubovic Lemma [12], there exists a diagonal positive definite matrix P satisfying

$$A^TP + PA = -l^Tl - \epsilon Q \quad (20)$$

$$Pd_j = C_c^T \quad (21)$$

for a positive real ϵ, a vector l, and a diagonal positive definite matrix Q. Now consider the Lyapanov function

$$V(\tilde{z}, \tilde{\theta}_j) = \frac{1}{2}\tilde{z}^TP\tilde{z} + \frac{1}{2}\tilde{\theta}_j^T\gamma^{-1}\tilde{\theta}_j \quad (22)$$

Computing dV/dt along the trajectories of (18) and (19) results in $\dot{V} = \frac{1}{2}\tilde{z}^T(PA + A^TP)\tilde{z}$ which in light of (20), can be rewritten as

$$\dot{V} \leq -\frac{\epsilon}{2}\tilde{z}^TQ\tilde{z} \leq 0 \quad (23)$$

Therefore, from the Lasalle-Yoshizawa Theorem [22], it is inferred that $\tilde{z}$ converges asymptotically and globally to zero. Next, we prove the convergence of $\tilde{x}$ to zero. To this end, note that from the Lasalle-Yoshizawa Theorem, it is concluded that $\tilde{z} \in L_\infty \cap L_2$ and $\tilde{\theta}_j \in L_\infty$. Now if we take the time derivative of the equation (18) and substitute from (18) and (19), we have

$$\begin{aligned}\ddot{\tilde{z}} &= A\dot{\tilde{z}} + d_j\dot{\zeta}_j\tilde{\theta}_j + d_j\zeta_j\dot{\tilde{\theta}}_j \\ &= A^2\tilde{z} + Ad_j\zeta_j\tilde{\theta}_j + d_j(-\alpha_j\zeta_j + f_{f_j})\tilde{\theta} + d_j\zeta_j(-\gamma_j\zeta_jC_c\tilde{z})\end{aligned} \quad (24)$$

Note that, since the differential equation $\dot{\zeta}_j = -\alpha_j\zeta_j + f_{f_j}$ is stable and has a bounded input by assumption, we can conclude that all terms in the right-hand side of (24) are bounded, so $\ddot{\tilde{z}}$ is bounded which implies that $\dot{\tilde{z}}$ is uniformly continuous. Since $\tilde{z}$ converges asymptotically to zero, and belongs to L_2, we conclude that $\dot{\tilde{z}} \to 0$. Now, from the equation (18), we infer that $d_j\zeta_j(t)\tilde{\theta}(t)$ converges asymptotically to zero. By virtue of the equation $\tilde{x} = \tilde{z} + \beta_j\tilde{\theta}_j = \tilde{z} + [0, \zeta_j(t)]^T\tilde{\theta}_j$, we conclude that $\tilde{x}$ converges to zero which is, as mentioned before, equivalent to asymptotic convergence of $[\tilde{\pi}_j, \tilde{\nu}_j]^T$ to zero, for all $j = 1, ..., n$. ■

By Theorem 1, we conclude the convergence of $\hat{\nu}_j$ to ν_j. Then, in light of (9), if we compute the estimated joint velocity by $\dot{\hat{q}} = \mathcal{J}_n(q)^{-1}\hat{\nu}$ then $\dot{\tilde{q}} = \mathcal{J}_n(q)^{-1}\tilde{\nu}$ where $\tilde{q} = q - \hat{q}$. This implies that the estimation error $\dot{\tilde{q}}$ converges globally and asymptotically to zero.

Remark 3: If center-of-mass of links is unknown, then there is uncertainty in evaluation of $\mathcal{J}_n(q)$. In this case, we can determine $\dot{\hat{q}}$ by

$$\dot{\hat{q}} = \hat{\mathcal{J}}_n^{-1}(q)\hat{\nu} + K\tilde{q}$$

where $K > 0$ is a positive definite matrix. Here $\hat{\mathcal{J}}_n^{-1}(q)$ is the estimate of $\mathcal{J}_n^{-1}(q)$. We assume that $\mathcal{J}_n^{-1}(q)\nu$ can be linearly parameterized by $\mathcal{J}_n^{-1}(q)\nu = Y(q,\nu)\rho$ where ρ is the vector of constant unknown parameters. As a result, $\dot{\hat{q}}$ is equivalently given by $\dot{\hat{q}} = Y(q,\hat{\nu})\hat{\rho} + K\tilde{q}$. Since $\dot{q} = \mathcal{J}_n^{-1}(q)\nu = Y(q,\nu)\rho$, it is straightforward to conclude that

$$\dot{\tilde{q}} = -K\tilde{q} - \tilde{\mathcal{J}}_n^{-1}(q)\hat{\nu} + \mathcal{J}_n^{-1}(q)\tilde{\nu} \quad (25)$$

$$= -K\tilde{q} - Y(q,\hat{\nu})\tilde{\rho} + d(t) \quad (26)$$

where the perturbation term $d(t) = \mathcal{J}_n^{-1}(q)\tilde{\nu}$ vanishes asymptotically (since $\tilde{\nu} \to 0$). Now, define $W = \frac{1}{2}\tilde{q}^T\tilde{q} + \frac{1}{2}\tilde{\rho}^T\Gamma_1^{-1}\tilde{\rho}$. Then, $\dot{W} = -\tilde{q}^TK\tilde{q} + \dot{\tilde{\rho}}^T\Gamma_1^{-1}\tilde{\rho} - \tilde{q}^TY(q,\hat{\nu})\tilde{\rho} + \tilde{q}^Td(t)$. Now if the parameter estimate is given by

$$\dot{\hat{\rho}} = \Gamma_1Y^T(q,\hat{\nu})\tilde{q} \quad (27)$$

we obtain $\dot{W} = -\tilde{q}^TK\tilde{q} - \tilde{q}^Td(t)$. Since $d(t)$ is bounded and converges asymptotically to zero; and the unperturbed system (26)-(27), when $d \equiv 0$, is asymptotically stable, we conclude that $\tilde{q}(t)$ converges to zero, asymptotically [23]. □ Note that the dimension of the overall observer (13) is $2n+r$ where r stands for the number of unknown parameters to be identified. To avoid poor rank conditioning of $\mathcal{J}_n(q)$ for some configurations, the number of linearly-independent rows in $\mathcal{J}_n(q)$ can be increased. This implies that the dimension of the observer will be increased, proportionally. Note also that, in this case the $\mathcal{J}_n(q)$ will no longer be square and therefore its right-pseudoinverse should replace its inverse.

A. *Effect of noise in position and force measurement*

Suppose that the measurements of q and f_f are contaminated with noise in a sense that the sensor outputs q_s and f_{f_s} are of the form $q_s = q + n_q$ and $f_{f_s} = f_f + n_f$, respectively, where n_q and n_f are the corresponding measurement noises with $\|n_q(t)\| \leq c_q, \|n_f(t)\| \leq c_f$ for $t \geq 0$ for some known constants c_q and c_f. Link position vector π can be expressed by forward kinematics as $\pi = \varphi(q)$ where $\varphi(q)$ is assumed to be Lipschitz continuous. Now if we define the computed link position by $\pi_s := \varphi(q_s)$, then the error in computation of link position is given by $n_p := \pi_s - \pi$. An upper bound for n_p can be calculated as

$$\|n_p\| \leq \| \varphi(q_s) - \varphi(q) \| \leq \delta \| q_s - q \| \leq \delta c_q =: c_p \quad (28)$$

So, c_p is the upper bound for the error in computation of π in the sense that $\| \pi_s - \pi \| \leq c_p$. Now, as the measurements of force and position are subject to noise, we use π_s and f_{f_s} in the observer (13) instead of π and f_f. In this case, it can be shown that the error dynamics changes into

$$\begin{cases} \dot{\tilde{z}}_{j_1} = (-\lambda_j - \alpha_j)\tilde{z}_{j_1} + \tilde{z}_{j_2} + \zeta_j(t)\tilde{\theta}_j(t) - (\lambda_j + \alpha_j)n_{p_j} \\ \dot{\tilde{z}}_{j_2} = (-\lambda_j\alpha_j)\tilde{z}_{j_1} + \alpha_j\zeta_j(t)\tilde{\theta}_j(t) - (\lambda_j\alpha_j)n_{p_j} - n_{f_j}\theta \\ \dot{\tilde{\theta}}_j = \gamma_j\zeta_j(t)\tilde{z}_{j_1} - \gamma_j\zeta_j(t)n_{p_j} \end{cases} \quad (29)$$

where $\tilde{z}_{j_1} = z_{j_1} - \hat{z}_{j_1}$, $\tilde{z}_{j_2} = z_{j_2} - \hat{z}_{j_2}$ and $\tilde{\theta}_j = \theta_j - \hat{\theta}_j$. To perform the stability analysis of the error dynamics equation (29), we express it in the closed form as

$$\begin{cases} \dot{\tilde{z}} = A\tilde{z} + B_1\tilde{\theta}_j + C_1n_j(t) \\ \dot{\tilde{\theta}} = D_1\tilde{z} + E_1n_j(t) \end{cases} \quad (30)$$

TABLE II

DESIGN PARAMETERS AND INITIAL CONDITIONS FOR THE OBSERVERS

	α_j	λ_j	γ_j	$\hat{z}(t_0)$	$\hat{\theta}_j(t_0)$	$\zeta_j(t_0)$
$j=1$	30	12	200	$[0,0]^T$	3	0
$j=2$	30	12	250	$[0,0]^T$	3	0
$j=3$	30	12	250	$[0,0]^T$	0.5	0
$j=4$	30	12	100	$[0,0]^T$	0.5	0

where $n_j(t)$ vector includes the measurement noises of both position and force in the form $n_j(t) = [n_{p_j}, n_{f_j}]T$ and therefore, is bounded. Other matrices introduced in (30) are as follows $B_1 = [\zeta_j(t),\ \alpha_j\zeta_j(t)]^T$ and

$$C_1 = \begin{bmatrix} -(\lambda_j+\alpha_j) & 0 \\ -(\lambda_j\alpha_j) & -\theta_j \end{bmatrix}$$

Moreover, $D_1 = E_1 = [-\gamma_j\zeta_j(t), 0]$. As it is clear from the definition of these matrices, all of them are continuous and uniformly bounded. Now, if we take the time derivative of the Lyapanov function (22) along the trajectories of system (30) and use a projection, as introduced in [24], for the estimation of $\hat{\theta}_j$ such that $\|\tilde{\theta}_j\| \le \eta_j$, then it can be easily seen that time derivative of the Lyapanov function can be written as

$$\frac{dV}{dt} \le -c_1\|\tilde{z}\|^2 + c_2\|\tilde{z}\| + c_3 \tag{31}$$

where $c_1 = \frac{1}{2}\underline{\lambda}(l^T l + \epsilon Q)$, $c_2 = \sqrt{c_{p_j}^2 + c_{f_j}^2}\|C_1\|_2\sigma_{max}(P)$, $c_3 = c_{p_j}\rho_j \mathrm{sup}_t|\bar{f}_{s_j}|/\alpha_j$. Note that, the term $\mathrm{sup}_t|\bar{f}_{s_j}|$ always exists and is bounded because we have assumed the closed-loop robotic system is stable. As it is clear form the inequality (31), there always exists a lower bound for $\|\tilde{z}\|$ like β such that if $\|\tilde{z}\| \ge \beta$, then $\dot{V}(t) \le 0$. The value of β is the solution of equation $-c_1\|\tilde{z}\|^2 + c_2\|\tilde{z}\| + c_3 = 0$ given by $\beta = (c_2 + \sqrt{c_2^2 + 4c_1c_3})/(2c_1)$. The condition $\dot{V}(t) \le 0$ for $\|\tilde{z}\| \ge \beta$ implies that $\tilde{z}$ will be ultimately bounded. To calculate this ultimate bound of $\|\tilde{z}\|$, consider the Lyapanov function introduced by (22). From the positive-definiteness of this Lyapanov function, it can be inferred that

$$\frac{1}{2}\underline{\lambda}(P)\|\tilde{z}\|^2 + \frac{1}{2}\gamma^{-1}\|\tilde{\theta}_j\|^2 \le V \le \frac{1}{2}\bar{\lambda}(P)\|\tilde{z}\|^2 + \frac{1}{2}\gamma^{-1}\|\tilde{\theta}_j\|^2 \tag{32}$$

From the equation (32), it is possible to calculate the ultimate bound on $\|\tilde{z}\|$ as

$$\|\tilde{z}\|^2 \le \frac{\bar{\lambda}(P)}{\underline{\lambda}(P)}\beta^2 + \frac{\gamma^{-1}}{\underline{\lambda}(P)}\eta^2 \tag{33}$$

It is observed that by an increase in the measurement noise (increase in n_{p_j} and n_{f_j}), the constants c_2 and c_3 increase and as a result, η will also increase. From the equation (33), higher values of η correspond to larger ultimate bound for the estimation error $\|\tilde{z}\|$.

V. SIMULATION RESULTS

A. Observer Performance

In this section, we implement the proposed observer on a two-link planar manipulator [21]. The nominal values of the manipulator parameters are $m_1 = 2$kg, $m_2 = 1$ kg, $l_1 = 0.4$ m, $l_2 = 0.2$m, $l_{c_1} = 0.2$m, $l_{c_2} = 0.1$m where m_1, m_2 denote link masses, l_1, l_2 denote the length of links, and l_{c_1}, l_{c_2} are the distance of the centers of mass of each link to the starting joints. The gravity acceleration vector is given by $g_o = [0, -9.81]^T$. We consider the manipulator in a closed loop motion control such that its joint angles track the reference trajectory $q_d(t) = [\sin(10t),\ \cos(10t)]^T$. The position Jacobian matrix $\mathcal{J}_p(q)$ introduced in (8) is given by

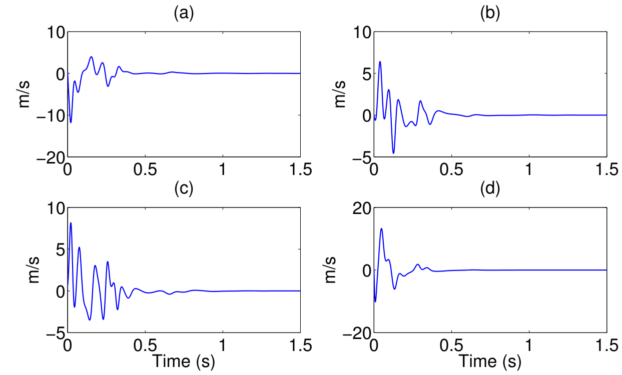

Fig. 1. Link velocity estimation error $\tilde{\nu}_j$ for (a): $j=1$, (b): $j=2$, (c): $j=3$, (d): $j=4$.

$$\mathcal{J}_p(q) = \begin{bmatrix} -l_{c_1}\mathrm{s}_1 & l_{c_1}\mathrm{c}_1 & -l_1\mathrm{s}_1 - l_{c_2}\mathrm{s}_{12} & l_1\mathrm{c}_1 + l_{c_2}\mathrm{c}_{12} \\ 0 & 0 & -l_{c_2}\mathrm{s}_{12} & l_{c_2}\mathrm{c}_{12} \end{bmatrix}^T \tag{34}$$

where $\mathrm{s}_1 = \sin(q_1)$, $\mathrm{c}_1 = \cos(q_1)$, $\mathrm{s}_{12} = \sin(q_1+q_2)$ and $\mathrm{c}_{12} = \cos(q_1+q_2)$. As it is clear from the structure of $\mathcal{J}_p(q)$, it is always possible to select $n = 2$ linearly-independent rows of the matrix to produce the matrix $\mathcal{J}_n(q) \in \mathbb{R}^{2\times 2}$ in the equation (9). However, to avoid $\mathcal{J}_n(q)$ become ill conditioned for some configurations, we consider two more rows in $\mathcal{J}_n(q)$ and hence, $\mathcal{J}_n(q) = \mathcal{J}_p(q)$.

Following the procedure presented in the Section III, we select four elements of v_p corresponding to these rows and put them in the vector $\nu \in \mathbb{R}^4$. The observer gains and the initial conditions used in the observer (13) are reported in Table II. The estimation errors for linear velocities $\tilde{\nu}_j$ for $j = 1, ..., 4$, are shown in Fig. 1. Also, Fig. 2, demonstrates evolution of parameter estimates $\hat{\theta}_j$ with respect to time.

B. Comparison results

For comparison purpose, we implemented a non-smooth and globally convergent observer [18]

$$\ddot{\hat{q}} = -K_0\mathrm{sgn}(\hat{q}-q) - (K_1 + \mathbf{I}_n)\dot{\hat{q}} - K_2(\hat{q}-q) \tag{35}$$

with the initial conditions $\hat{q}(t_0) = [-1,-1]^T$ rad, $\dot{\hat{q}}(t_0) = [2,2]^T$rad/s. As explained in [18], to ensure asymptotic convergence in absence of measurement noise, the design parameter K_0 should be kept larger than a specific value. For our manipulator, this value was found to be $K_0 = 500\mathbf{I}_2$. For comparison purpose we set $K_1 = 10\mathbf{I}_2$ and $K_2 = 2\times10^3\mathbf{I}_2$.

Moreover, we considered about thirty percent white noise in joint angle measurements in both observers. Transient and steady-state response of the adaptive smooth observer (13) versus the non-smooth observer (35) are illustrated in

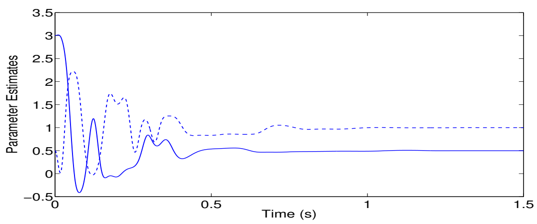

Fig. 2. Parameter Estimates $\hat{\theta}_1$(solid) and $\hat{\theta}_2$(dash), as they converge to their true values $\theta_1 = 0.5$ and $\theta_2 = 1$, respectively.

Fig. 3 and Fig. 4, respectively. As expected, smoothness of the adaptive observer (13) is an important factor in achieving fast convergence rate with low noise sensitivity. Due to smoothness of the proposed observer; the estimation error shown in Figs. 3 and 4 contains less noise and has faster convergence rate than the non-smooth observer. The convergence rate and the overshoot in the estimation error in the proposed observer can be improved by proper choice of parameter update gains γ_j, and observer gains λ_j and α_j.

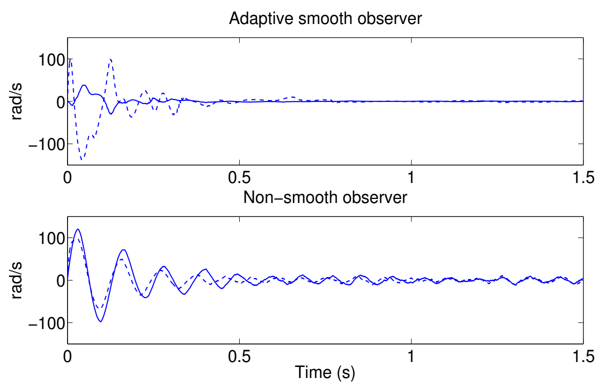

Fig. 3. Joint velocity estimation error $\dot{\tilde{q}}_1$(solid) and $\dot{\tilde{q}}_2$(dash) in the presence of measurement noise.

VI. Conclusion

We have presented an adaptive observer for the estimation of joint velocities in robotic manipulators capable of handling uncertainties in the robot dynamics. Freedom in choosing the number of linearly-independent rows of $\mathcal{J}_p(q)$ matrix along with various gains such as γ_j, λ_j and α_j in the structure of the observer, can significantly affect the observer performance, noise sensitivity and computational complexity. The proposed observer shows low noise sensitivity in comparison with non-smooth observers.

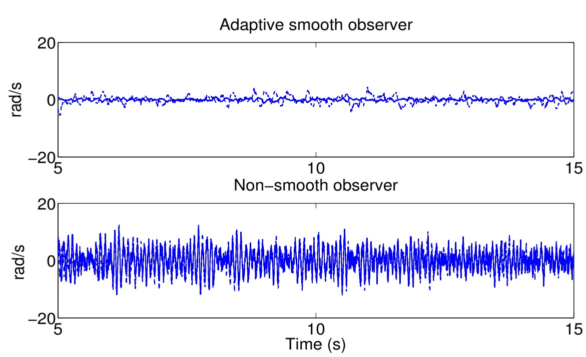

Fig. 4. Steady-state response of $\dot{\tilde{q}}_1$(solid) and $\dot{\tilde{q}}_2$(dash) in the presence of measurement noise.

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