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Pushing Motion Control of *n* Passive Off-hooked Trailers by a Car-Like Mobile Robot

Kwanghyun Yoo, Woojin Chung, *Member*, *IEEE*

***Abstract*—There are two different motions of a trailer system that consists of a car and *n* passive off-hooked trailers. When a car "pulls" *n* passive trailers, a trailer system moves forward. A trailer system moves backward when a car "pushes" *n* passive trailers. Backing up a trailer system is difficult because it is an open loop unstable problem. In this paper, we investigate the backward motion control problem of a car with *n* passive trailers. We have shown that *n* passive trailers can be successfully controlled by an omni-directional mobile robot in our prior works. Unlike an omni-directional robot, a car-like mobile robot has nonholonomic constraints and limitations of the steering angle. For these reasons, the backward motion control problem of a car-like mobile robot with *n* passive trailers is not trivial. In spite of difficulties, backing up a trailer system is useful for parking control. In this study, we proposed a mechanical alteration which is connecting *n* passive trailers to the front bumper of a car to improve the backward motion control performance. By adopting the new design, a car pushes *n* passive trailers by its forward motion. A practical trailer-pushing control algorithm was also proposed. Stability analysis of the controller under kinematic modeling error was presented. Theoretical verification and experimental results proved that the control strategy of pushing *n* passive trailers by forward motion of a car can be successfully implemented.**

I. INTRODUCTION

Fig. 1. Examples of passive trailer systems: Camping trailer(Courtesy of Airstream, INC., upper left), sightseeing vehicle(Courtesy of Seoul Grand Park, lower left), and Luggage trailers(Courtesy of Diamond trailers, INC., right)

Passive trailers increase transportation capacity. As shown in Fig. 1, multiple trailer trucks, articulated buses, boat and using paper) camping trailers, airport luggage carriers, sightseeing trailers, and passenger vehicles are good examples.

Nevertheless, control of these articulated systems is a highly nonlinear problem. In the field of nonlinear control, passive-trailer systems have been studied by many researchers [1-11]. Many soft computing techniques have also been developed for the motion control of a passive trailer system [12-14].

The backward-motion control of passive trailers by a car-like mobile robot has been studied by some researchers. Yi *et al.* proposed a backward-motion control strategy for truck-trailer systems using a fuzzy controller [14]. Matsushita *et al.* proposed a backward-motion controller based on the Lyapunov function for two passive trailers [15]. However, the backward-motion control of passive trailers using a general four-wheeled car is rarely investigated. In [13], Slagle *et al.* proposed a strategy for the backward-motion control of two passive trailers through a car-like mobile robot that uses a neural network. Low adaptability for new environments and system hardware changes is a major drawback of this approach. In [16], Ollero proposed a parallel-parking control method for a car with one trailer. Through switching strategies, Altafini *et al.* proposed a forward- and backward-motion controller for a four-wheeled tractor and a single trailer [17]. These investigations adopted a general car for backward-motion control. However only one passive trailer was considered in [16-17].

In this research, we defined two different motions of a trailer system that consists of a car and *n* passive off-hooked trailers. When a car "pulls" *n* passive trailers, a trailer system moves forward. Passive trailers move backward when a car "pushes" *n* passive trailers. The forward motion of a trailer system is open-loop stable and off-hooked trailers guarantee high trajectory-tracking performance. Therefore, pulling *n* passive off-hooked trailers by a car is advantageous in most cases. On the other hand, a backward motion of a trailer system is an open loop unstable problem. The backward motion by pushing is required for parking control.

In our prior work, we proposed the kinematic design of modular, off-hooked, passive trailers [18]. A kinematic design was established for achieving high performance in trajectory tracking. A practical solution for the backward-motion control of a holonomic, omni-directional, mobile robot with multiple passive off-hooked trailers was shown in our prior research [19]. However, passive trailers with a car-like mobile robot pose some difficulties. A car-like mobile robot has nonholonomic constraints and limitations with regard to the steering angle makes a control problem difficult. The relative joint angles between the car and the

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Kwanghyun Yoo is with the School of Mechanical Engineering, Korea University, Seoul, KOREA (e-mail: ykhhky@korea.ac.kr).

Woojin Chung is with the School of Mechanical Engineering, Korea University, Seoul, KOREA (Phone/Fax of the corresponding author: +822-3290-3375; e-mail: smartrobot@ korea.ac.kr).titute for Metals, Tsukuba, Japan (e-mail: author@nrim.go.jp).

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respective passive trailers are also limited. The combination of instability and input and state saturations results in the so-called 'jack-knife phenomenon'. Therefore, pushing n passive trailers by a car-like mobile robot remains as a challenging problem. In spite of the input saturation and nonholonomic constraints of a car, backward motion of a trailer system is useful for parking control.

In this paper, it is shown how n passive trailers are controlled for backward motion by a car-like mobile robot. To achieve successful trailer-pushing control, kinematic analysis was carried out for a car with n passive trailers. We proposed a mechanical alteration which is connecting n passive trailers to the front bumper of a car to improve the backward motion performance. A practical trailer-pushing control strategy by forward motion of a car was also proposed. Stability analysis of the controller under kinematic modeling error was presented. Theoretical verification and experimental results proved that the control strategy of pushing n passive trailers by forward motion of a car can be successfully implemented.

This paper is organized as follows. Section II shows the kinematic model of a trailer system. The trailer-pushing control strategy is proposed in section III. The stability analysis of the trailer-pushing controller under kinematic modeling error was presented. Section IV presents the experimental results. Some concluding remarks are given in Section V.

II. Kinematic model

A. The kinematic model of a car with off-hooked trailers

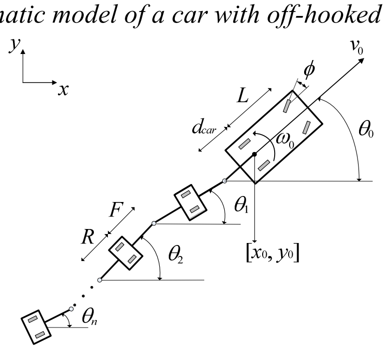


Fig. 2. Passive trailers are connected at the rear bumper of the car.

The kinematic model of the off-hooked trailer system in [18] is presented in Fig. 2. The car-like mobile robot's pose, $[x_0, y_0, \theta_0]$, is defined at the center of the rear axle of the robot. The kinematic model shown in Fig. 2 can be represented as follows.

$$\begin{bmatrix} v_0 \\ \omega_0 \end{bmatrix} = \begin{bmatrix} \cos\psi_n & (-1)^{n-1} D \sin\psi_n \\ \sin\psi_n / D & -(-1)^{n-1}\cos\psi_n \end{bmatrix} \begin{bmatrix} v_n \\ \omega_n \end{bmatrix}. \tag{1}$$

$$\psi_i = \sum_{k=1}^{i} (-1)^{k-1}(\theta_{k-1} - \theta_k),\ i = 1,\ldots,n$$

In [18], it is shown that the trajectory tracking error can be minimized when the link length, D satisfies a following condition.

A1) $D = F = R$.

The kinematic model of off-hooked trailers is simple and it can be easily extended to n trailers.

B. The feasible-velocity region

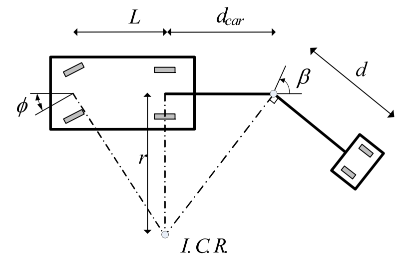


Fig. 3. Illustration of the feasible-velocity region when a car pushes a passive trailer.

TABLE I
Notations for Deriving Feasible Velocity Region

Notations	Meaning
L	Length of the wheelbase of the car.
$dcar$	Length of the connecting link of the car.
ϕ_{max}	Maximum steering angle of the car.
r	Turning radius of a car and a passive trailer with respect to the center of rear wheel axle of a car.
d	Length of the connecting link of the passive trailer.
$I.C.R.$	Instantaneous center of rotation.

In Fig. 3, the feasible-velocity region at the hinge is presented when a car-like mobile robot pushes a passive trailer. Fig. 3 describes a car-like mobile robot and a passive trailer that moves backward to the right. The notation in Fig. 3 is explained in Table I. β denotes the admissible velocity direction at the hinge point. Since the steering angle ϕ is limited, β is limited to $[-\beta_{max}, \beta_{max}]$, where β_{max} can be derived by a following equation.

$$\beta_{max} = \tan^{-1}\left(\frac{d_{car}}{L}\tan\phi_{\max}\right). \tag{2}$$

$[-\beta_{max}, \beta_{max}]$ signifies the input saturation. From eq. (2), it follows that β_{max} is a function of the wheelbase of the car L, the maximum steering angle ϕ_{max}, and the length of the rear link d_{car}. In order to avoid input saturation, larger β_{max} is preferred. We have three alternatives for increasing the $\beta_{rear,max}$.

B1) Increasing the maximum steering angle of the car, ϕ_{max}.

B2) Decreasing the length of the wheelbase of the car, L.

B3) Increasing the length of the rear link, d_{car}.

Since the above alternative *B3)* violates the condition *A1)* in section II-A, the trajectory-tracking error increases. Therefore, *B3)* is unacceptable. *B1)* and *B2)* can increase the β_{max}. However, the change of vehicle parameters is difficult in practical applications. Therefore, we consider mechanical alteration as illustrated in the following section.

III. Pushing Trailers by a Forward Motion of a Car

A. The kinematic model for pushing trailers by a forward motion of a car

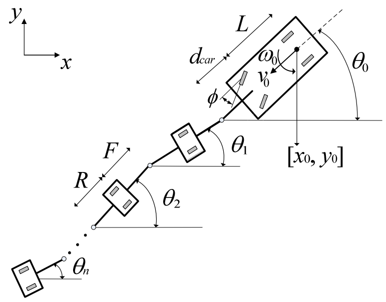


Fig. 4. Passive trailers are connected at the front bumper of the car.

As shown in fig. 4, the main idea is to connect passive trailers to the front bumper of a car when a backward motion for a trailer system is required. Please note that the velocity mapping between the last trailer and the first trailer is completely identical to the mapping in eq. (1). On the other hand, the velocity mapping between the first trailer's velocities and the car-like mobile robot's input velocities is different from eq. (1). The kinematic equation between the first trailer and the car-like mobile robot in Fig. 4 is as follows.

$$\begin{bmatrix} v_0 \\ \omega_0 \end{bmatrix} = \begin{bmatrix} \cos(\theta_0-\theta_1) & (d_{car}+L)\cdot\sin(\theta_0-\theta_1) \\ \sin(\theta_0-\theta_1)/D & -\cos(\theta_0-\theta_1)\cdot(d_{car}+L)/D \end{bmatrix}^{-1} \begin{bmatrix} v_1 \\ \omega_1 \end{bmatrix}. \quad (3)$$

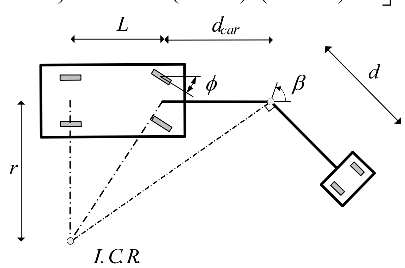


Fig. 5. Illustration of the feasible velocity region during a car pushes a trailer by its forward motion.

The β of the proposed approach, which is shown in Fig. 5, can be obtained by a following equation.

$$\beta_{max} = \tan^{-1}((1+\frac{d_{car}}{L})\tan\phi_{\max}) \cdot \quad (4)$$

Mathematically, β of the proposed approach, which is derived from eq. (4), is always larger than β of the trailers connected to the rear bumper of a car. We consider a commercialized mid-sized car, where ϕ_{max}=30°, L=2.78m, d_{car}=1.25m. The links of the passive trailers are $D = F = R =$ 1.25m. When trailers are connected to the rear bumper of a car, $\beta_{\max}$ = 14.6°. On the other hand, $\beta_{\max}$ =39.9° when the proposed design in Fig. 4 is adopted. The possibility to meet input saturation can be radically decreased.

B. Trailer-pushing control algorithm for passive trailers by the forward motion of a car

In [19], we proposed a strategy for the backward-motion control of a holonomic omni-directional mobile robot with passive trailers. However, for the case of a car-like towing vehicle, control input can be saturated by $\beta_{\max}$. This fact is a significant difference between a holonomic mobile robot and a car-like mobile robot. For holonomic robots, inputs v_0 and ω_0 can be independently given. However, the car-like mobile robot should satisfy a following constraint.

$$\phi = \arctan\left(\frac{L\omega_0}{v_0}\right). \quad (5)$$

From eqs. (1) and (3), a direct velocity mapping is always possible between the input velocities of the last trailer and the car-like mobile robot. This fact implies that the last trailer can be controlled as an active mobile robot. Therefore, any existing controllers for two-wheeled mobile robots can be employed. We exploit the tracking controller, which was developed by Kanayama *et al.* [20]. In [20], the error vector and the control scheme are specified in line with the following equations. K_x, K_y, and K_θ are the control gains, which are positive constants.

$$\mathbf{p}_e = \begin{pmatrix} x_e \\ y_e \\ \theta_e \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c & 0 \\ -\sin\theta_c & \cos\theta_c & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_r - x_c \\ y_r - y_c \\ \theta_r - \theta_c \end{pmatrix}. \quad (6)$$

$$\mathbf{q}_n = \begin{pmatrix} v_n \\ \omega_n \end{pmatrix} = \begin{pmatrix} v_r\cos\theta_e + K_x x_e \\ \omega_r + v_r(K_y y_e + K_\theta \sin\theta_e) \end{pmatrix}. \quad (7)$$

Our control strategy for trailer-pushing by a forward motion of a car can be summarized as follows.

Algorithm I. Trailer-pushing Control for Passive Trailers by the Forward Motion of a Car

G ← The Total Number of Time-increments
X_r, Y_r, θ_r ← Pose of the Reference Trajectory of the Last Trailer
X_c, Y_c, θ_c ← Actual Pose of the Last Trailer
θ_c, θ_1, θ_2 ← Actual Orientations of the Car, 1st Trailer, and 2nd Trailer
K_x, K_y, K_θ ← Control Gains of the Tracking Controller
v_r, ω_r ← Reference Velocity Inputs of the Last Trailer
v_n, ω_n ← Velocity Inputs of the Last Trailer
v_0, ω_0 ← Velocity Inputs of the Car-like Vehicle
ϕ ← Steering Angle of the Car-like Vehicle
D ← Length of the Trailers' Links
L ← Length of the Wheelbase of the Car-like Vehicle

TRAILERPUSHINGCONTROL ( )
do X_r, Y_r, θ_r ← Reference-trajectory generation for the last trailer
while(Timestep < G)
{
do v_n, ω_n ← Compute the last trailer's motion by using the error vector and the control scheme as per eqs. (6) and (7):
(X_r, Y_r, θ_r, X_c, Y_c, θ_c, K_x, K_y, K_θ).
do v_0, ω_0 ← Compute the control inputs of the car using eqs. (1) and (3):
(v_n, ω_n, θ_c, θ_1, θ_a).
do ϕ ← Convert ω_0 to the steering angle of the car by using eq. (5).
(v_0, ω_0, L)
}

Stability analysis for the trailer-pushing control algorithm of passive trailers by a car's forward motion

To investigate the control stability of the proposed trailer-pushing controller, stability analysis was carried out. If there is no error in the kinematic model in eqs. (1) and (3), the control stability can be easily proven according to the analysis in [20].

However, practical control performances are affected by various error sources. As pointed out in [21], control robustness with respect to uncertainties and disturbances is open and challenging issue in the nonholonomic system control. Although it is difficult to analyze the general trailer-pushing control properties, it is possible to investigate the effect of error for a specific case. For simplicity, following assumptions are made.

C1) The reference trajectory is a straight line.

C2) A robot with one trailer is considered.

C3) Local configuration around $\theta \approx 0$, $\theta_e \approx 0$ is considered.

C4) Translational velocity is perfectly controlled ($x_e \approx 0$).

We concentrate on measurement error of joint angles. In the fabricated prototype, joint measurement errors can be assumed to be constants because of the difficulty of wheel alignment of multiple trailers. Since the structure of trailers is a serial chain, the joint angle error directly affects the control performance. In [22], Chung established the sensitivity to errors of the serial-chained system under nonholonomic constraints. It is advantageous to mechanically allow relative motion between trailers to cope with uneven ground conditions. Otherwise, some trailers possibly encounter "wheel floating" under irregular ground conditions, as pointed out by Nakamura *et al.* in [23]. The relative motion between trailers may result in joint measurement error.

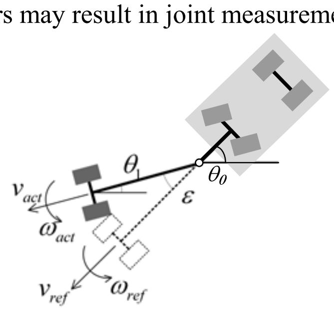


Fig. 6. A mobile robot with a trailer under the joint measurement error ε.

Since the trailer-pushing is feedback controlled by using the proposed scheme in section III-B, some errors can be compensated. The above assumptions *C3* and *C4* become feasible owing to the robustness of the feedback controller. Fig. 6 shows the robot with one trailer in trailer-pushing control. The solid line shows the actual configuration. The dashed line represents the configuration that contains the joint measurement error ε. The required velocities of the trailer $[v_n, \omega_n]^T$ are computed from the trajectory-tracking controller. The control inputs of the car-like mobile robot $[v_0, \omega_0]^T$ is determined by eqs. (1) and (3). The measurement error ε is included as follows.

$$\begin{pmatrix} v_0 \\ \omega_0 \end{pmatrix} = \begin{bmatrix} \cos(\theta_0 - \theta_1 - \varepsilon) & D\sin(\theta_0 - \theta_1 - \varepsilon) \\ \sin(\theta_0 - \theta_1 - \varepsilon)/D & -\cos(\theta_0 - \theta_1 - \varepsilon) \end{bmatrix} \begin{pmatrix} v_n \\ \omega_n \end{pmatrix} \tag{8}$$

Then, the resultant velocity of the real trailer, $[v_{act}, \omega_{act}]^T$, can be obtained as follows.

$$\begin{aligned} \begin{pmatrix} v_{act} \\ \omega_{act} \end{pmatrix} &= \begin{bmatrix} \cos(\theta_0 - \theta_1) & D\sin(\theta_0 - \theta_1) \\ \sin(\theta_0 - \theta_1)/D & -\cos(\theta_0 - \theta_1) \end{bmatrix} \begin{pmatrix} v_0 \\ \omega_0 \end{pmatrix} \\ &= \begin{bmatrix} \cos\varepsilon & -D\sin\varepsilon \\ \sin\varepsilon/D & \cos\varepsilon \end{bmatrix} \begin{pmatrix} v_n \\ \omega_n \end{pmatrix} \end{aligned} \tag{9}$$

From eq. (9), we can find the relationship between the reference velocities and the actual velocities under joint errors. It is clear that $[v_{act}, \omega_{act}]^T = [v_n, \omega_n]^T$, when $\varepsilon = 0$.

From eq. (7), it is clear that $\omega_n = 0$ when there is no tracking error. If there is no joint measurement error, then eq. (9) implies that $\omega_{act} = 0$. Accordingly, the robot and the trailer head in a constant direction by following a straight line. Similarly, we assume the configuration which results in $\omega_{act} = 0$ under the joint measurement error. Fig. 7 shows this assumption when the target trajectory is $y = 0$ and the robot is moving to the right.

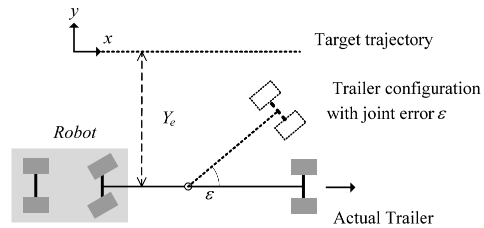


Fig. 7. The robot pushes the trailer to the right under the joint measurement error ε. The robot moves straight with the steady state error Y_e.

From eq. (9), the following condition is derived when $\omega_{act}=0$.

$$\frac{\omega_n}{v_n} = -\frac{1}{D}\tan\varepsilon \tag{10}$$

From eqs. (6), (7), and (10), and assumption *C4,* the steady-state error, Y_e, can be obtained as a following equation.

$$Y_e = -\frac{1}{D \cdot K_y}\tan\varepsilon \tag{11}$$

The above condition provides an equilibrium point of convergence. In addition, it can be easily checked that the equilibrium point is stable. For example, if $y_e < Y_e$ ($y_e > Y_e$), y_e increases (decreases) to Y_e because ω_{act} is positive (negative). Therefore, we can conclude that the trailer locally converges to a straight line with the steady-state error Y_e, if there is joint measurement error ε. It is assumed that ε is small ($\varepsilon \approx 0$). This result can be iteratively extended to the n trailer problem. It is assumed that the resultant tracking error is the sum of the individual tracking errors when there are multiple joint errors.

IV. Experimental Results

A. Experimental setup

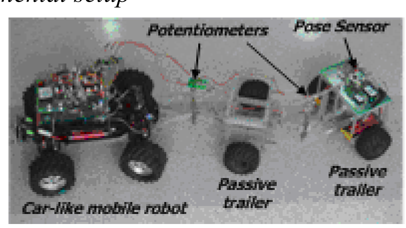


Fig. 8. Experimental setup: a car-like mobile robot and two passive trailers.

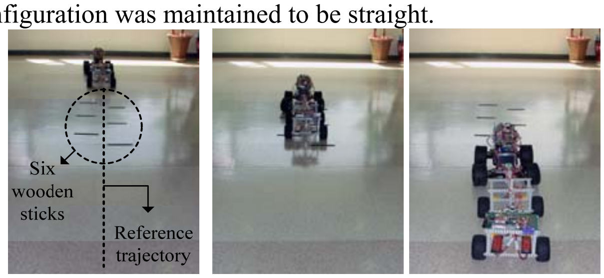
Fig. 8 shows our experimental system. The active robot is a nonholonomic car-like mobile robot. This car-like mobile robot has a same kinematic structure with the commercially available automobiles. Two passive off-hooked trailers are connected to the front bumper of the car-like mobile robot. Wheel diameters of the car-like mobile robot and passive trailers are 14.8cm and 11cm, respectively. The pose of the last trailer can be monitored by a commercially available pose sensor STARGAZER. The poses of the remaining vehicles are computed from joint angle measurements and link parameters. The joint angles are measured by potentiometers at the joints. The maximum steering angle and the length of the wheelbase of the car-like robot are 20.5° and 0.315m, respectively.

B. Trailer-pushing control using a car-like mobile robot with passive off-hooked trailers

Experimental verifications are carried out for two reference trajectories, which are a straight line and a circle. The reference trajectories are predefined on global Cartesian coordinate. The first experiment is carried out with a straight trajectory.

Fig. 9 shows the experimental motion when the reference trajectory is a straight line. As shown in Fig. 9 (a), six wooden sticks are installed in front of the trailer system. These wooden sticks that have 0.5cm height are intentionally installed on the floor to investigate the effect of disturbances. Fig. 10 shows the resultant experimental trajectory. The car-like mobile robot and two trailers are moving. A desired trajectory is a straight line from (-1.3, 0) to (3, 0). Dotted line represents the last trailer's actual path during the trailer-pushing control. In spite of uneven floor condition and the last trailer's initial pose error (0.04m, 0.04m, 2°), the car-like mobile robot and passive trailers converge to the desired trajectory, successfully. Control gains in eq. (7) are K_x=0.3, K_y=0.0001, and K_θ=0.08. Fig. 11 shows two control inputs v_0 and ω_0, which do not exceed ±0.1m/s and ±0.2rad/s. In order to avoid dynamic effects, the trailer speed was sufficiently low.

Since the reference trajectory is straight, the angular velocity of the car-like mobile robot has to converge to zero, in an ideal case. From Fig. 11, it is clear that the angular velocities of a car were small. A little oscillation was possibly caused by the potentiometer noise, backlashes at each connecting joint, and unevenness by wooden sticks on the floor. Steering input for the car-like mobile robot is also presented in Fig. 11. It is clear that the steering input is bounded from -20.5° to 20.5°. In the ideal case, the steering angle should converge to zero. The resultant steering angle was a little oscillatory in order to compensate various errors during the motion. Fig. 12 shows relative joint angles during the trailer-pushing control. It is clear that the joint angles converge to zero. This result implies that the trailer configuration was maintained to be straight.



(a) (b) (c)

Fig. 9. A car-like mobile robot pushes two off-hooked trailers by its forward motion for tracking a straight reference trajectory.

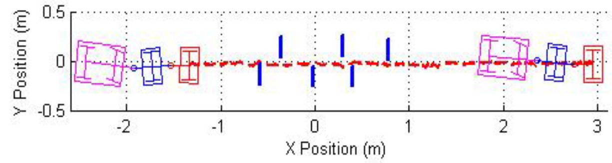


Fig. 10. Experimental result: tracking a straight reference- trajectory.

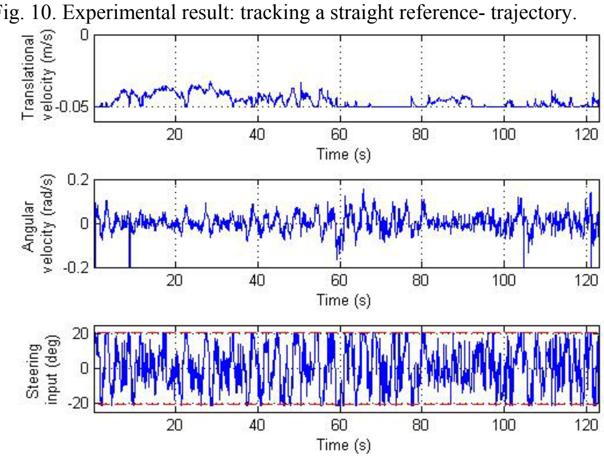


Fig. 11. The translational velocity (top), angular velocity (middle), and steering-angle input (bottom) of a car-like mobile robot. (straight path)

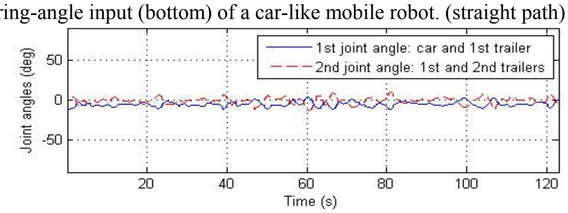


Fig. 12. Joint angles during trailer-pushing control process. (straight reference trajectory)

The second reference trajectory is a circle. Although the control stability was proven only for a straight reference trajectory in section III-C, following a circular trajectory is possible owing to the robustness of the controller. A radius of the circular path is 1.5m and the center of the circle is (-0.3, -0.1). Fig. 13 shows the experimental motion when the reference trajectory is a circle. Fig. 14 presents the trailer system following the circular trajectory. It is evident that the trailers are successfully controlled by a car-like mobile robot. Control gains in eq. (7) were K_x=0.06, K_y=0.0001, and K_θ=0.08. The velocity inputs for the car-like mobile robot are shown in Fig. 15. Steering input is also presented in the lowest figure. The control inputs are slightly oscillatory because the same error sources in the first experiment.

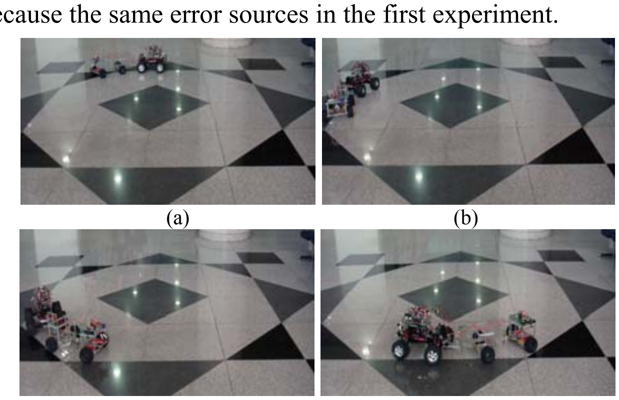

(a) (b) (c) (d)

Fig. 13. Motion of a car-like mobile robot with 2 off-hooked trailers for a circular reference trajectory.

Fig. 16 shows the joint angles during the second experiment. In an ideal case, when the radius of the circle is 1.5m, the first and second relative joint angles should converge to 27° and 15°. However, the first and the second joint angles converge to 34° and 19° respectively. The error was possibly caused by the potentiometer and external sensor noise, and backlashes at each connecting joint.

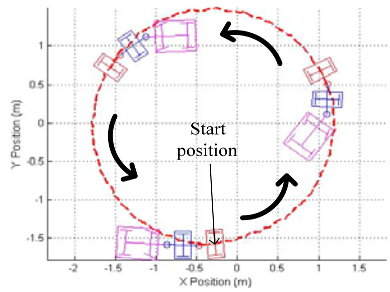


Fig. 14. Experimental result: tracking a circular reference-trajectory.

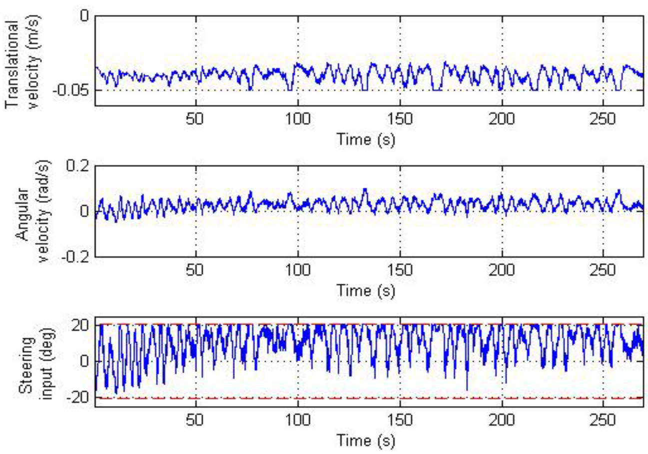


Fig. 15. The translational velocity (top), angular velocity (middle), and steering-angle input (bottom) of a car-like mobile robot. (circular path)

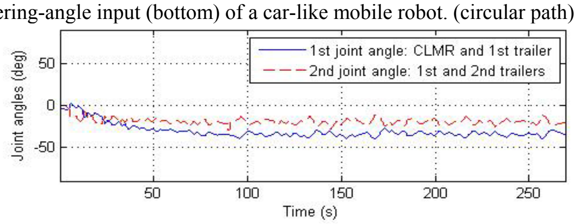


Fig. 16. Joint angles during trailer-pushing control process. (circular path)

V. CONCLUSION

In this study, we proposed the control strategy for the car with *n* passive trailer system. It was shown that the control performance can be improved by connecting trailers to the front bumper of a car. The control strategy of trailer-pushing by a forward motion of a car-like mobile robot was established. The stability analysis of the controller under kinematic modeling error was presented. Experimental verification illustrated the successful implementation of our trailer-pushing control algorithm.

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