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Quality Bottleneck Transitions in Flexible Manufacturing Systems

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Abstract— **In this paper, we introduce a Markov chain model to evaluate the quality performance in flexible manufacturing systems with batch productions. In such a model, the product quality is a function of the transition probabilities characterizing the changes among good and defective states (where good quality or defective parts are produced during a cycle, respectively). A transition that has the largest impact on quality, i.e., whose improvement will lead to the largest improvement in quality, is defined as the *quality bottleneck transition* (BN-t). Analytical expressions of sensitivity of quality with respect to transition probabilities are derived. Indicators to identify bottleneck transitions based on the data collected on the factory floor are developed. Numerical experiments show that such indicators have high accuracy in identifying the correct bottlenecks and can be used as an effective tool for quality improvement effort. Finally, a case study at an automotive paint shop to improve quality through quality bottleneck transition identification is introduced.**

Keywords: Quality, Markov chain, flexible manufacturing system, batch production, bottleneck transition, indicators.

I. Introduction

Empirical evidence and analytical studies have indicated that flexibility has a significant impact on quality [1]. In automotive paint shops, the number of available colors and paint quality are strongly correlated [2]. The paint quality may temporarily degrade after color change [3]. Similar example can be found in machining operations as well, while product change may result in quality declining due to reallocating flexible fixtures, since product quality is typically dominated by the location error of the fixtures. Therefore, batch operation is typically introduced to reduce frequent product changes in order to improve quality. In paint shops, vehicles with the same color are typically grouped into small batches. In powertrain manufacturing plants, different types of engines or transmissions are often assembled in batches and product changes occur on hourly basis.

Although flexible manufacturing systems have attracted substantial research attention (see books [4]-[6] and reviews [7]-[10]), the coupling between flexibility and quality is less studied. A typical assumption in flexibility studies is that the quality related issues have minimal impact [11]. In addition, batching in flexible manufacturing systems is studied in terms of batch size selection to minimize setup or flow times, while quality has not been addressed [12], [13]. Only limited studies investigate the interactions between quality and flexibility. For example, paper [14] introduces an empirical study on the issues of flexibility, productivity and quality to show that flexibility impacts quality. Paper [15] presents a fuzzy set method to model the flexibility where quality level is one of the elements. A Markov chain model is introduced in [16] to evaluate the quality in flexible manufacturing systems. It indicates that batch production may be an effective way to improve quality. Following this direction, Markov chain models for quality analysis in flexible systems with batch operations have been studied in [17] and [18], where quality evaluation, monotonic or non-monotonic properties and impact of product sequence on quality, are discussed.

In spite of these efforts, how to improve quality in a flexible manufacturing system with batch production has not been fully understood. It has been shown in [16]-[18] that quality is correlated to the transition probabilities that characterize the changes among good and defective states (where good quality or defective parts are produced during a cycle, respectively). Therefore, in this paper, we intend to develop methods to achieve higher quality through improving such transitions. Specifically, such improvement will be carried out through identification and mitigation of quality bottleneck transitions (BN-t), which are defined as the transitions that impede system quality in the strongest manner. In other words, improvement on these transitions will lead to the largest improvement in product quality comparing with improving other transitions.

Bottleneck identification and mitigation are essential enablers for continuous improvement in manufacturing operations. A system-theoretic method to identify throughput bottlenecks by measuring and comparing blockages and starvations has been developed and successfully applied on the factory floor (see monograph [19] and papers [20]-[25]). Paper [26] uses a similar method for due-time performance bottleneck analysis. Bottleneck sequence with respect to quality is introduced and an identification method using the collected data is proposed in paper [27]. Similar to these studies, in this paper, we intend to develop a quality bottleneck transition identification approach using the data available on the factory floor. Based on the collected data, we establish quality bottleneck transition indicators. Such indicators could lead to identification of bottleneck transitions without complicated calculations of quality performance.

The remainder of this paper is structured as follows: Section II introduces a Markov chain model to evaluate quality in flexible systems with batch operations. The quality

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bottleneck transitions are defined in Section III and analyzed in Sections IV and V. Numerical justifications of bottleneck transition indicators are presented in Section VI. Section VII introduces a case study of improving quality through identification and mitigation of bottleneck transitions at an automotive paint shop. Finally, the conclusions are formulated in Section VIII. Due to page limitation, all proofs are omitted and can be found in [28].

II. QUALITY EVALUATION MODEL

Consider a flexible manufacturing system capable of producing different types of products. The following assumptions address the flexible production system, product types, transition, and quality characteristics.

(1) The flexible system can process n different types of products, denoted as 1, 2, ..., n. Each product type i is processed in a batch with batch size k_i, $k_i \geq 1$.

(2) The products flow into the system with a sequence $s = \{s_1, s_2, \ldots, s_n\}$, where s_m denotes the m-th product type in sequence s, $s_m \in \{1, 2, \ldots, n\}$.

(3) The flexible system will work on product type s_m for k_{s_m} parts before switching to product type s_{m+1}. It is assumed that product type s_1 is processed again after processing type s_n.

(4) The flexible system is in good state $g_{s_i,j}$, or defective state $d_{s_i,j}$, $i = 1, \ldots, n$, $j = 1, \ldots, k_{s_i}$, if it is processing the j-th part in the batch of the product type s_i with good quality, or with defects, respectively. Thus, there are $2K$, $K = \sum_{i=1}^{n} k_i$, states in the system, defined by the quality status, product type processed and its position within a batch.

(5) When the system is in good state $g_{s_i,j}$, $s_i = 1, \ldots, n$, $j = 1, \ldots, k_{s_i} - 1$, it has probabilities λ_{s_i,s_i} to transit to defective state $d_{s_i,j+1}$, and $1 - \lambda_{s_i,s_i}$ to good state $g_{s_i,j+1}$. Analogously, when the system is in defective state $d_{s_i,j}$, $s_i = 1, \ldots, n$, $j = 1, \ldots, k_{s_i} - 1$, it can transit to good state $g_{s_i,j+1}$ with probability μ_{s_i,s_i}, and to defective state $d_{s_i,j+1}$ with $1 - \mu_{s_i,s_i}$.

(6) When the system is processing the last part within a batch and in good state $g_{s_i,k_{s_i}}$, $i = 1, \ldots, n-1$, it has probabilities λ_{s_{i+1},s_i} and $1 - \lambda_{s_{i+1},s_i}$ to transit to states $d_{s_{i+1},1}$ and $g_{s_{i+1},1}$, respectively. Analogously, when the machine is in defective state $d_{s_i,k_{s_i}}$, it has probabilities μ_{s_{i+1},s_i} and $1 - \mu_{s_{i+1},s_i}$ to transit to states $g_{s_{i+1},1}$ and $d_{s_{i+1},1}$, respectively.

(7) When the system is in state $g_{s_n,k_{s_n}}$, it has probabilities λ_{s_1,s_n} and $1 - \lambda_{s_1,s_n}$ to transit to states $d_{s_1,1}$ and $g_{s_1,1}$, respectively. Analogously, when the system is in state $d_{s_n,k_{s_n}}$, it has probabilities μ_{s_1,s_n} and $1 - \mu_{s_1,s_n}$ to transit to states $g_{s_1,1}$ and $d_{s_1,1}$, respectively. Without loss of generality, we assume all $0 < \lambda_{ij} < 1$, $0 < \mu_{ij} < 1$, $\forall i, j$.

Remark 1: Probabilities λ_{s_i,s_i} and μ_{s_i,s_i}, $i = 1, \ldots, n$, are referred to as *quality failure and repair probabilities without product switch*, respectively [17]. Similarly, λ_{s_{j+1},s_j}, $j = 1, \ldots, n-1$ and λ_{s_1,s_n}, and μ_{s_{j+1},s_j}, $j = 1, \ldots, n-1$ and μ_{s_1,s_n} are the *quality failure and repair probabilities with product switch*, respectively. Moreover, we define *quality efficiency* with and without product switch as e_{s_i,s_j}, $j \neq i$, and e_{s_i,s_i}, respectively, where

$$e_{s_i,s_i} = \frac{\mu_{s_i,s_i}}{\lambda_{s_i,s_i} + \mu_{s_i,s_i}}, \quad e_{s_i,s_j} = \frac{\mu_{s_i,s_j}}{\lambda_{s_i,s_j} + \mu_{s_i,s_j}}, \quad i \neq j.$$

Under assumptions (1)-(7), the system in consideration can be described by an ergodic Markov chain. Referring to its steady state, let $P(g_{s_i,j})$ and $P(d_{s_i,j})$, $i = 1, \ldots, n$, $j = 1, \ldots, k_{s_i}$, be the probabilities that the system is in states $g_{s_i,j}$ or $d_{s_i,j}$ (i.e., producing a good or a defective job for the j-th part in the batch of product type s_i), respectively. Then,

$$P(g_{bt}) = \sum_{s_i=1}^{n} \sum_{j=1}^{k_{s_i}} P(g_{s_i,j}) \quad (1)$$

$$\left(\text{respectively, } P(d_{bt}) = \sum_{s_i=1}^{n} \sum_{j=1}^{k_{s_i}} P(d_{s_i,j})\right) \quad (2)$$

defines the overall quality performance of the flexible system for a given sequence s, i.e., the probability to produce a good (or, a defective) part in batch production. Then we obtain [17]:

Proposition 1: Under assumptions (1)-(7), the probability of good parts $P(g_{bt})$ is calculated by

$$P(g_{bt}) = \sum_{i=1}^{K} x_i, \quad (3)$$

where $K = \sum_{i=1}^{n} k_{s_i}$, x_i can be solved from

$$X = \frac{1}{K} \cdot \frac{\sum_{i=1}^{K} \Gamma^{i-1}}{\det(I - \Gamma)} \Phi. \quad (4)$$

Here vectors X, Φ and matrix Γ are defined as (see next page for Γ)

$$X = [P(g_{s_1,1}), \ldots, P(g_{s_1,k_{s_1}}), P(g_{s_2,1}), \ldots, P(g_{s_2,k_{s_2}}), \ldots, P(g_{s_n,1}), \ldots, P(g_{s_n,k_{s_n}})]^T, \quad (5)$$

$$\Phi = [\mu_{s_1,s_n}, \mu_{s_1,s_1}, \ldots, \mu_{s_1,s_1}, \mu_{s_2,s_1}, \mu_{s_2,s_2}, \ldots, \mu_{s_n,s_{n-1}}, \mu_{s_n,s_n}, \ldots, \mu_{s_n,s_n}]^T, \quad (6)$$

and

$$\delta_{s_i,j} = 1 - \lambda_{s_i,j} - \mu_{s_i,j}, \quad i = 1, \ldots, n, j = 1, \ldots, k_{s_i}, \quad (8)$$

When part type changes every cycle, i.e., batch size $k_i = 1, \forall i$, we obtain a policy referred to as strictly sequencing, the above results can be simplified. The product quality, denoted as $P(g_{ss})$, can be calculated as

$$P(g_{ss}) = \sum_{i=1}^{n} x_i, \quad (9)$$

where

$$X = [P(g_{s_1,1}), P(g_{s_2,1}), \ldots, P(g_{s_n,1})]^T, \quad (10)$$

$$\Phi = [\mu_{s_1,s_n}, \mu_{s_2,s_1}, \ldots, \mu_{s_n,s_{n-1}}]^T, \quad (11)$$

$$\Gamma = \begin{pmatrix} 0 & \cdots & 0 & \delta_{s_1,s_n} \\ \delta_{s_2,s_1} & \cdots & 0 & 0 \\ 0 & \ddots & 0 & 0 \\ 0 & \cdots & \delta_{s_n,s_{n-1}} & 0 \end{pmatrix}. \quad (12)$$

$$\Gamma = \begin{pmatrix} 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & \delta_{s_1,s_n} \\ \delta_{s_1,s_1} & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 \\ 0 & \ddots & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 \\ 0 & \cdots & \delta_{s_1,s_1} & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 \\ 0 & \cdots & 0 & \delta_{s_2,s_1} & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 \\ 0 & \cdots & 0 & 0 & \delta_{s_2,s_2} & \cdots & 0 & 0 & \cdots & 0 & 0 \\ 0 & \cdots & 0 & 0 & 0 & \ddots & 0 & 0 & \cdots & 0 & 0 \\ 0 & \cdots & 0 & 0 & 0 & \cdots & \delta_{s_n,s_{n-1}} & 0 & \cdots & 0 & 0 \\ 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \delta_{s_n,s_n} & \cdots & 0 & 0 \\ 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & \ddots & 0 & 0 \\ 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & \delta_{s_n,s_n} & 0 \end{pmatrix}, \quad (7)$$

III. Quality Bottleneck Transitions

Using the method introduced above, we can evaluate the quality of a given flexible manufacturing system. As shown in [16]-[18], the system quality is a function of the transition probabilities, such as λ_{ij} and μ_{ij}. Thus, improving these transition probabilities could lead to improvement of system quality. The question is, which transition should we focus on? To improve system quality more efficiently, the transition, whose improvement will lead to the largest improvement in system quality comparing with improving all other transitions, should be the one to emphasize. Such a transition is referred to as *quality bottleneck transition*, which impedes the system quality in the strongest manner. Therefore, the bottleneck transition has the largest impact on quality, and is defined as follows:

Definition 1: Under assumptions (1)-(7), transition $j \rightarrow i$ is the *negative bottleneck transition* (n-BN-t) with respect to quality if it satisfies

$$\frac{\partial P(g)}{\partial \lambda_{ij}} < \frac{\partial P(g)}{\partial \lambda_{mk}}, \quad \forall mk \neq ij. \quad (13)$$

Definition 2: Under assumptions (1)-(7), transition $j \rightarrow i$ is the *positive bottleneck transition* (p-BN-t) with respect to quality if it satisfies

$$\frac{\partial P(g)}{\partial \mu_{ij}} > \frac{\partial P(g)}{\partial \mu_{mk}}, \quad \forall mk \neq ij. \quad (14)$$

Although Definitions 1 and 2 provide a characterization of quality bottleneck transitions, it is difficult to implement on the factory floor since, first, such derivatives are not measurable on the factory floor; second, the closed form calculation formula for these derivatives may not be available. Moreover, even if we can use $\Delta\lambda_{ij}$ or $\Delta\mu_{ij}$ to calculate corresponding $\Delta P(g)$ to approximate such derivatives, the computation effort is intensive. Therefore, it is necessary to develop bottleneck transition indicators for n-BN-t and p-BN-t based on the available data on the factory floor. The goal of this paper is devoted to such development.

To do this, we first start with strictly sequencing policy, then extend the results to more general cases.

IV. Bottleneck Transitions in Strictly Sequencing Policy

We begin with the simplest case, three-product case. Then we generalize to $n > 3$-product case.

A. Three-product case

Assume the product sequence is $1 \rightarrow 2 \rightarrow 3 \rightarrow 1$. Then the transition equations can be written as follows:

$$\begin{aligned} P(g_{11}) &= (1-\lambda_{13})P(g_{31}) + \mu_{13}P(d_{31}), \\ P(g_{21}) &= (1-\lambda_{21})P(g_{11}) + \mu_{21}P(d_{11}), \\ P(g_{31}) &= (1-\lambda_{32})P(g_{21}) + \mu_{32}P(d_{21}), \\ P(d_{11}) &= (1-\mu_{13})P(d_{31}) + \lambda_{13}P(g_{31}), \\ P(d_{21}) &= (1-\mu_{21})P(d_{11}) + \lambda_{21}P(g_{11}), \\ P(d_{31}) &= (1-\mu_{32})P(d_{21}) + \lambda_{32}P(g_{21}). \end{aligned}$$

In this case, it is possible to derive a closed formula for the partial derivatives, $\frac{\partial P(g_{ss})}{\partial \lambda_{21}}$ and $\frac{\partial P(g_{ss})}{\partial \mu_{21}}$.

Proposition 2: Under assumptions (1)-(7),

$$\begin{aligned} \frac{\partial P(g_{ss})}{\partial \lambda_{21}} &= -\frac{1}{3(1-\delta_{21}\delta_{13}\delta_{32})^2}[e_{13} + e_{13}\delta_{32} \\ &\quad +(e_{32}-e_{13})\delta_{13} + e_{21}\delta_{13}\delta_{32} - e_{21}\delta_{13}\delta_{32}\delta_{21} \\ &\quad -e_{32}\delta_{13}\delta_{32}^2 + e_{21}\delta_{13}\delta_{32}^2 - e_{21}\delta_{13}\delta_{32}^2\delta_{21} \\ &\quad -e_{13}\delta_{13}^2\delta_{32} + e_{32}\delta_{13}^2\delta_{32} - e_{32}\delta_{13}^2\delta_{32}^2 \\ &\quad +e_{21}\delta_{13}^2\delta_{32}^2 - e_{21}\delta_{13}^2\delta_{32}^2\delta_{21}], \quad (15) \\ \frac{\partial P(g_{ss})}{\partial \mu_{21}} &= \frac{\partial P(g_{ss})}{\partial \lambda_{21}} + \frac{1+\delta_{32}+\delta_{13}\delta_{32}}{3(1-\delta_{21}\delta_{13}\delta_{32})}. \quad (16) \end{aligned}$$

Introduce

$$\delta_{max} = \max_{ij} |\delta_{ij}|, \quad i = 1, \ldots, n, j = 1, \ldots, k_i.$$

As we know, typically $|\delta_{ij}| \ll 1$, due to that λ_{ij} is usually small and μ_{ij} is often close to 1. Then we obtain

Corollary 1: Under assumptions (1)-(7),

$$\begin{aligned} \frac{\partial P(g_{ss})}{\partial \lambda_{21}} &= -\frac{1}{3}[e_{13}(1+\delta_{32}) + \delta_{13}(e_{32}-e_{13})] \\ &\quad +o(\delta_{max}^2), \quad (17) \\ \frac{\partial P(g_{ss})}{\partial \mu_{21}} &= \frac{1}{3}[(1-e_{13})(1+\delta_{32}) + \delta_{13}(e_{13}-e_{32})] \\ &\quad +o(\delta_{max}^2). \quad (18) \end{aligned}$$

Thus, by ignoring the higher order terms of δ_{max}^2, which is small when δ_{max} is small, we obtain the estimates of $\frac{\partial P(g_{ss})}{\partial \lambda_{21}}$ and $\frac{\partial P(g_{ss})}{\partial \mu_{21}}$. When the difference between e_{13} and e_{32} is small, e_{13} and δ_{32} will be the dominating factors. Thus, they can be used for BN-t identification.

In addition, as we can see, $\frac{\partial P(g_{ss})}{\partial \lambda_{21}}$ is mainly dependent on immediate upstream transition (characterized by e_{13}, δ_{13}), and immediate downstream transition (characterized by δ_{32}, e_{32}), but not on the current transition (λ_{21}). This may be due to that e_{13} determines the possibility that system can stay in good state g_{11}, and δ_{32} may affect the possibility at defective state d_{21}. Then such possibilities will affect the impact of changing λ_{21}. Similar results are observed for $\frac{\partial P(g_{ss})}{\partial \mu_{21}}$ as well.

Analogously, we obtain

Corollary 2: Under assumptions (1)-(7),

$$\begin{aligned}\frac{\partial P(g_{ss})}{\partial \lambda_{32}} &= -\frac{1}{3}[e_{21}(1+\delta_{13})+\delta_{21}(e_{13}-e_{21})] \\ &\quad +o(\delta_{max}^2), \qquad (19)\\ \frac{\partial P(g_{ss})}{\partial \mu_{32}} &= \frac{1}{3}[(1-e_{21})(1+\delta_{13})+\delta_{21}(e_{21}-e_{13})] \\ &\quad +o(\delta_{max}^2), \qquad (20)\\ \frac{\partial P(g_{ss})}{\partial \lambda_{13}} &= -\frac{1}{3}[e_{32}(1+\delta_{21})+\delta_{32}(e_{21}-e_{32})] \\ &\quad +o(\delta_{max}^2), \qquad (21)\\ \frac{\partial P(g_{ss})}{\partial \mu_{13}} &= \frac{1}{3}[(1-e_{32})(1+\delta_{21})+\delta_{32}(e_{32}-e_{21})] \\ &\quad +o(\delta_{max}^2). \qquad (22)\end{aligned}$$

B. $n > 3$-product case

Similar to three-product case, the probability of good states can be described by:

$$P(g_{s_{i+1},1}) = \delta_{s_{i+1},s_i} P(g_{s_i,1}) + \frac{\mu_{s_{i+1},s_i}}{n}, \ i=1,\ldots,n. \quad (23)$$

Remark 2: In equation (23), we allow subscripts go beyond n to simplify expressions. This implies that s_{n+1} represents s_1. Similarly, we may also use s_0 to represent s_n, and s_{-1} for s_{n-1}. Similar notations are used throughout the paper.

Again, estimates of partial derivatives of $P(g_{ss})$ with respect to λ_{ij} and μ_{ij} can be obtained.

Proposition 3: Under assumptions (1)-(7),

$$\begin{aligned}\frac{\partial P(g_{ss})}{\partial \lambda_{s_{i+1},s_i}} &= \frac{-1}{n}\cdot\Big[e_{s_i,s_{i-1}}(1+\delta_{s_{i+2},s_{i+1}}) \\ &\quad +(e_{s_{i-1},s_1}-e_{s_i,s_2})\delta_{s_i,s_{i-1}}\Big] \\ &\quad +o(\delta_{max}^2), \quad i=1,\ldots,n, \qquad (24)\\ \frac{\partial P(g_{ss})}{\partial \mu_{s_{i+1},s_i}} &= \frac{-1}{n}\cdot\Big[(e_{s_i,s_{i-1}}-1)(1+\delta_{s_{i+2},s_{i+1}}) \\ &\quad +(e_{s_{i-1},s_{i-2}}-e_{s_i,s_{i-1}})\delta_{s_i,s_{i-1}}\Big] \\ &\quad +o(\delta_{max}^2), \quad i=1,\ldots,n. \qquad (25)\end{aligned}$$

Similar to three-product case, we observe that the immediate upstream and downstream transitions have dominant impact on the partial derivatives with respect to current transition. By ignoring the smaller terms, equations (24) and (25) provide estimates of partial derivatives of $P(g_{ss})$ with respect to λ_{ij} and μ_{ij}. Therefore, by comparing these estimates, we can find the largest $\left|\frac{\partial P(g_{ss})}{\partial \lambda_{s_{i+1},s_i}}\right|$ as the negative bottleneck transition (n-BN-t), and the largest $\frac{\partial P(g_{ss})}{\partial \mu_{s_{i+1},s_i}}$ as the positive bottleneck transition (p-BN-t). Since typically $e_{i,i-1}-e_{i-1,i-2}$ is small, therefore, we can ignore the impact of second term in the bracket. The dominant factor will be $e_{s_i,s_{i-1}}(1+\delta_{s_{i+2},s_{i+1}})$ and $(e_{s_i,s_{i-1}}-1)(1+\delta_{s_{i+2},s_{i+1}})$ for $\frac{\partial P(g_{ss})}{\partial \lambda_{s_{i+1},s_i}}$ and $\frac{\partial P(g_{ss})}{\partial \mu_{s_{i+1},s_i}}$, respectively. Such information can be collected directly from the data measured on the factory floor without intensive calculations. Thus, they can be used as bottleneck identification indicators.

Then a bottleneck transition identification method is introduced as follows:

n-BN-t Indicator: A transition is the negative bottleneck transition (n-BN-t) with respect to quality in strictly sequencing policy if it satisfies

$$\max_i e_{s_{i-1},s_{i-2}}(1+\delta_{s_{i+1},s_i}), \quad i\in\{1,2,\ldots,n\}. \quad (26)$$

p-BN-t Indicator: A transition is the positive bottleneck transition (p-BN-t) with respect to quality in strictly sequencing policy if it satisfies

$$\max_i (1-e_{s_{i-1},s_{i-2}})(1+\delta_{s_{i+1},s_i}), \quad i\in\{1,2,\ldots,n\}. \quad (27)$$

V. Bottleneck Transitions in Batch Policy

We again start with two-product types, then extend to more general cases.

A. Two-product types

Assume there are two types of products, 1 and 2, each with batch size three. The product sequence will be $1\to1\to1\to2\to2\to2\to1$. Similar to the strictly sequencing case, the balance equations of good states can be written as follows:

$$\begin{aligned}P(g_{11}) &= \delta_{12}P(g_{23})+\frac{1}{6}\mu_{12},\\ P(g_{12}) &= \delta_{11}P(g_{11})+\frac{1}{6}\mu_{11},\\ P(g_{13}) &= \delta_{11}P(g_{12})+\frac{1}{6}\mu_{11},\\ P(g_{21}) &= \delta_{21}P(g_{13})+\frac{1}{6}\mu_{21},\\ P(g_{22}) &= \delta_{22}P(g_{21})+\frac{1}{6}\mu_{22},\\ P(g_{23}) &= \delta_{22}P(g_{22})+\frac{1}{6}\mu_{22}.\end{aligned}$$

We consider the transition from product 1 to product 2 (i.e., between batches) first and then study the transition within the batch of product 1. It can be shown that

Proposition 4: Under assumptions (1)-(7),

$$\frac{\partial P(g_{bt})}{\partial \lambda_{21}} = -\frac{1}{6}e_{11}(1+\delta_{22}) + o(\delta_{max}), \quad (28)$$

$$\frac{\partial P(g_{bt})}{\partial \mu_{21}} = \frac{1}{6}(1-e_{11})(1+\delta_{22}) + o(\delta_{max}). \quad (29)$$

Similarly,

$$\frac{\partial P(g_{bt})}{\partial \lambda_{12}} = -\frac{1}{6}e_{22}(1+\delta_{11}) + o(\delta_{max}), \quad (30)$$

$$\frac{\partial P(g_{bt})}{\partial \mu_{12}} = \frac{1}{6}(1-e_{22})(1+\delta_{11}) + o(\delta_{max}). \quad (31)$$

As one can see, the results are similar to the strictly sequencing case, where the partial derivatives are mainly determined by the transitions immediately upstream and downstream.

Next we consider the transitions within the batch of product 1, and similar results are obtained.

Proposition 5: Under assumptions (1)-(7),

$$\frac{\partial P(g_{bt})}{\partial \lambda_{11}} = -\frac{1}{6}e_{12}(1+\delta_{11}) - \frac{1}{6}e_{11}(1+\delta_{21}) + o(\delta_{max}), \quad (32)$$

$$\frac{\partial P(g_{bt})}{\partial \mu_{11}} = \frac{1}{6}(1-e_{12})(1+\delta_{11}) + \frac{1}{6}(1-e_{11})(1+\delta_{21}) + o(\delta_{max}). \quad (33)$$

Similarly,

$$\frac{\partial P(g_{bt})}{\partial \lambda_{22}} = -\frac{1}{6}e_{21}(1+\delta_{22}) - \frac{1}{6}e_{22}(1+\delta_{12}) + o(\delta_{max}), \quad (34)$$

$$\frac{\partial P(g_{bt})}{\partial \mu_{22}} = \frac{1}{6}(1-e_{21})(1+\delta_{22}) + \frac{1}{6}(1-e_{22})(1+\delta_{12}) + o(\delta_{max}). \quad (35)$$

B. $n > 2$-product case

Proposition 6: Under assumptions (1)-(7),

$$\frac{\partial P(g_{bt})}{\partial \lambda_{s_i,s_{i-1}}} = -\frac{1}{K}e_{s_{i-1},s_{i-1}}(1+\delta_{s_i,s_i}) + o(\delta_{max}), \quad i = 1,\ldots,n, \quad (36)$$

$$\frac{\partial P(g_{bt})}{\partial \mu_{s_i,s_{i-1}}} = \frac{1}{K}(1-e_{s_{i-1},s_{i-1}})(1+\delta_{s_i,s_i}) + o(\delta_{max}), \quad i = 1,\ldots,n. \quad (37)$$

When $k_{s_i} \geq 3$,

$$\frac{\partial P(g_{bt})}{\partial \lambda_{s_i,s_i}} = -\frac{1}{K}[e_{s_i,s_{i-1}}(1+\delta_{s_i,s_i}) + (k_{s_i}-3)e_{s_i,s_i}(1+\delta_{s_i,s_i}) + e_{s_i,s_i}(1+\delta_{s_{i+1},s_i})] + o(\delta_{max}), \quad (38)$$

$$\frac{\partial P(g_{bt})}{\partial \mu_{s_i,s_i}} = \frac{1}{K}[(1-e_{s_i,s_{i-1}})(1+\delta_{s_i,s_i}) + (k_{s_i}-3)(1-e_{s_i,s_i})(1+\delta_{s_i,s_i}) + (1-e_{s_i,s_i})(1+\delta_{s_{i+1},s_i})] + o(\delta_{max}). \quad (39)$$

When $k_{s_i} = 2$,

$$\frac{\partial P(g_{bt})}{\partial \lambda_{s_i,s_i}} = -\frac{1}{K}e_{s_i,s_{i-1}}(1+\delta_{s_{i+1},s_i}) + o(\delta_{max}), \quad (40)$$

$$\frac{\partial P(g_{bt})}{\partial \mu_{s_i,s_i}} = \frac{1}{K}(1-e_{s_i,s_{i-1}})(1+\delta_{s_{i+1},s_i}) + o(\delta_{max}). \quad (41)$$

Again, the partial derivatives with respect to current transitions only depend on the immediate upstream and downstream transitions. Therefore, by ignoring the smaller terms, we obtain the bottleneck transition indicators as:

n-BN-t Indicator 3: Given a sequence s, a transition is the negative bottleneck transition (n-BN-t) with respect to quality in batch policy if it satisfies

$$\max_{i=1,\ldots,n} \tau_i,$$

where

$$\tau_i = \begin{cases} \max\{e_{s_{i-1},s_{i-1}}(1+\delta_{s_i,s_i}), e_{s_i,s_{i-1}}(1+\delta_{s_i,s_i}) \\ \quad +(k_{s_i}-3)e_{s_i,s_i}(1+\delta_{s_i,s_i}) \\ \quad +e_{s_i,s_i}(1+\delta_{s_{i+1},s_i})\}, \quad k_{s_i} \geq 3, \\ \max_i\{e_{s_{i-1},s_{i-1}}(1+\delta_{s_i,s_i}), \\ \quad e_{s_i,s_{i-1}}(1+\delta_{s_{i+1},s_i})\}, \quad k_{s_i} = 2. \end{cases} \quad (42)$$

p-BN-t Indicator 4: Given a sequence s, a transition is the positive bottleneck transition (p-BN-t) with respect to quality in batch policy if satisfies

$$\max_{i=1,\ldots,n} \tau_i,$$

where

$$\tau_i = \begin{cases} \max_i\{(1-e_{s_{i-1},s_{i-1}})(1+\delta_{s_i,s_i}), \quad (1+\delta_{s_i,s_i}) \\ \quad (1-e_{s_i,s_{i-1}})(k_{s_i}-3)(1-e_{s_i,s_i}) \\ \quad +(1-e_{s_i,s_i})(1+\delta_{s_{i+1},s_i})\}, \quad k_{s_i} \geq 3, \\ \max_i\{(1-e_{s_{i-1},s_{i-1}})(1+\delta_{s_i,s_i}), \\ \quad (1-e_{s_i,s_{i-1}})(1+\delta_{s_{i+1},s_i})\}, \quad k_{s_i} = 2. \end{cases} \quad (43)$$

VI. Numerical Justification

Numerical experiments have been carried out to justify the bottleneck indicators introduced above. More than 10,000 cases are generated and tested. The system parameters are randomly selected, where $\delta_{max} = 0.2$. The n-BN-t and p-BN-t identified using Indicators 1-4 are compared with the results obtained by computing and selecting the largest $\Delta P(g)/\Delta\lambda_{ij}$ and $\Delta P(g)/\Delta\mu_{ij}$, where $\Delta = 0.001$. When both methods result in same conclusion, a correct identification is obtained. The ratios of correct bottleneck identification using Indicators 1-4 are summarized in Tables I and II for strictly sequencing and batch policies, respectively.

It is shown that the above bottleneck indicators have resulted in high accuracy to identify the bottlenecks, with more than 92% correctness in strictly sequencing policy and 97% in batch policy. Examining the cases corresponding to incorrect identifications, typically the BN-t indicators can identify the transitions whose partial derivatives are close

TABLE I

CORRECTNESS OF N-BN-T AND P-BN-T IN STRICTLY SEQUENCING POLICY

N	2	3	4	5	6
n-BN-t	99.20%	96.13%	95.73%	94.31%	94.19%
p-BN-t	99.90%	98.01%	95.26%	94.10%	92.58%

TABLE II

CORRECTNESS OF N-BN-T AND P-BN-T IN BATCH POLICY

N	2	3	4	5	6
n-BN-t	99.66%	99.45%	99.28%	99.34%	99.11%
p-BN-t	97.99%	97.64%	97.41%	97.24%	97.50%

to the largest one. Therefore, we conclude that these bottleneck indicators can be used for identifying the bottleneck transitions for quality improvement.

VII. CASE STUDY

A case study at an automotive paint shop has been carried out to study the quality bottleneck transition. First, based on the data collected on the factory floor, transition probabilities λ_{ij} and μ_{ij} are estimated and the product quality is calculated as 0.8278. Comparing with the collected data on paint quality, 0.8261, the difference is extremely small, only 0.21%. Thus, the model is validated.

Next, the bottleneck transitions are identified to improve product quality. Using the n-BN-t and p-BN-t indicators introduced in this paper, we identify λ_{22} and μ_{22} as the negative and positive bottleneck transitions, respectively. This result is also verified by calculating $\Delta P(g)/\Delta\lambda_{ij}$ and $\Delta P(g)/\Delta\mu_{ij}$ numerically.

Then, improving λ_{22} from 0.2160 to 0.2, the paint quality is increased to 0.8427. Or improving μ_{22} from 0.7215 to 0.75, the product quality is upgraded to 0.8394. Therefore, quality improvement can be achieved by focusing on mitigating the bottleneck transitions.

VIII. CONCLUSIONS

Identifying the bottleneck is an effective way to improve the quality. In this paper, we define a quality bottleneck transition as the transition that impedes quality performance in the strongest manner. A method to evaluate the sensitivity of quality performance with respect to its transition probabilities is presented and quality bottleneck transition indicators based on the data collected on the factory floor are proposed. Using these indicators, negative and positive quality bottleneck transitions can be identified effectively. A quality bottleneck transition detection study at an automotive paint shop illustrates the applicability of the results. Such methods provide a quantitative and practical tool for production engineers and managers to improve quality in flexible manufacturing systems.

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