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Variable Stiffness Actuators: a Port-based Analysis and a Comparison of Energy Efficiency

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Abstract— **In this paper, a metric for comparing different designs of variable stiffness actuators is introduced. For the formulation of this metric, we focus on the energy efficiency of the actuators. In particular, we propose a metric that is a measure of how much energy is used by the actuator for changing the output stiffness. In order to facilitate the analysis of the energy usage, we present a port-based modeling framework, from which design criteria are derived for the optimization of the metric. Finally, the metric is interpreted in a comparison between existing actuators.**

I. INTRODUCTION

Variable stiffness actuators are capable of changing their apparent output stiffness independently of the output position. To achieve this, variable stiffness actuators consist of a number of internal springs and of internal degrees of freedom that determine how the springs are sensed at the output.

Variable stiffness actuators are suitable for a wide range of robotic applications and, in particular, for tasks in which robots work in a shared environment with humans. These actuators allow the robot to appear more or less compliant, depending on the task, and thus allow safe human-robot interaction [1]. In mobile robots, in particular walking robots, variable stiffness actuators can increase energy efficiency, due to the energy storing properties of the internal springs [2], and improve robustness. Several designs have been presented, including VSA [3], 'Jack Spring'™ [4], AMASC [5], VS-Joint [6] and MACCEPA [7].

The wide variety in designs makes it difficult to compare the actuators. Therefore, a set of measures should be formulated so to provide a metric for an objective comparison.

In this paper, we propose a metric that measures the performance of variable stiffness actuators in terms of energy efficiency. In particular, it measures how much energy is used by an actuator for changing its output stiffness. Both variable stiffness actuators and the metric are analyzed in a port-based setting, since it gives intuitions on energy flows. The metric is applied to different designs so to compare them and a design criterion for the optimization of the metric is derived.

The paper is organized as follows. In Sec. II, the metric is introduced by an intuitive argument and then it is mathematically defined. In Sec. III, a generic port-based model for variable stiffness actuators is derived and a relation between properties of the model and the metric is provided. Based on this analysis, in Sec. IV, an analysis of three distinct types of actuator designs is given. Conclusions and recommendations for future work are provided in Sec. V.

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II. DEFINITION OF THE METRIC

With the aim of comparing different actuators with similar physical specifications (e.g. range of stiffness, maximum output torque, etc.), we define a metric, which measures the performance. Since we are interested in the design of energy efficient variable stiffness actuators, we introduce a metric that relates energy usage to stiffness change.

In this work, we generalize the concept of variable stiffness actuators to make the analysis independent of the type of actuator, either linear or rotational. Therefore, we will denote a generalized output force by F, which is either a linear force in the case of linear actuators or a torque in the case of rotational actuators. Similarly, we denote a generalized displacement at the output by x, i.e. either a linear displacement or an angle. The stiffness felt at the output of the actuator is given by

$$K = \frac{\delta F}{\delta x} \tag{1}$$

where δF and δx denote infinitesimal changes in force and displacement.

Following the literature and without loss of generality, we consider a class of actuators in which the output stiffness is changed by changing the configuration of some internal springs. In particular, we assume the following:

- The energy stored in the internal springs is given by an energy function $H(s)$, where s denotes the state of the springs, i.e. their compression or elongation;
- There is a number of internal actuators that realize degrees of freedom that can be actuated through an input port. The configuration variables are denoted by $q \in \mathcal{Q}$, where $\mathcal{Q}$ is the space of all possible configurations;
- The configuration of the internal degrees of freedom determines the apparent output stiffness of the actuator.

In order to introduce the metric, we define a configuration r_0 in which the system is in a neutral externally unloaded equilibrium state, i.e.

$$r_0 = \{q \in \mathcal{Q} \mid F = 0, x = 0, \dot{x} = 0, H(s) = 0\} \tag{2}$$

and a parametrized path $r(t)$, $t \in [0, 1] \subseteq \mathbb{R}$, on $\mathcal{Q}$ along which the actuator remains in the equilibrium position $x = 0$ and there is no load at the output port

$$r(t) = \{q \in \mathcal{Q} \mid t \in [0, 1], F = 0, x = 0, \dot{x} = 0\} \tag{3}$$

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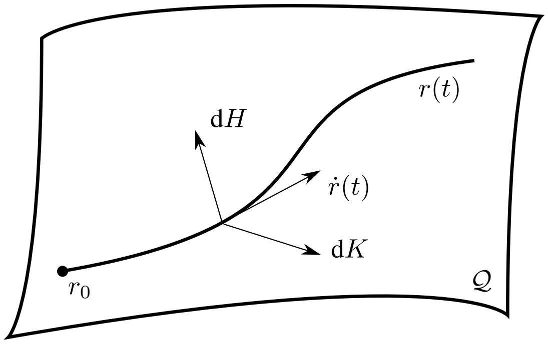

Fig. 1. Intuition of the metric - The configuration r_0 is such that the actuator is unloaded and in equilibrium. The path $r(t)$ through the configuration space Q is such that the output of the actuator is kept in equilibrium. The metric is the ratio between the increase of energy and the increase of stiffness along the path.

The total work done by the actuator along $r(t)$ is

$$\Delta E = \int_t \langle \mathrm{d}H | \dot{r} \rangle \, \mathrm{d}t \tag{4}$$

where $\mathrm{d}H$ is the differential of the energy function H, $\dot{r}(t)$ is the tangent vector to $r(t)$, and the dual product $\langle \mathrm{d}H|\dot{r}\rangle$ is in fact the Lie derivative of H along $\dot{r}$. Similarly, we may calculate the total change of stiffness along $r(t)$ as

$$\Delta K = \int_t \langle \mathrm{d}K | \dot{r} \rangle \, \mathrm{d}t \tag{5}$$

where $\mathrm{d}K$ is the differential of the output stiffness K and the dual product $\langle \mathrm{d}K|\dot{r}\rangle$ is the Lie derivative of K along $\dot{r}$. The scenario is depicted in Fig. 1. Based on the argument that, in the equilibrium configuration $x = 0$, the performance of a variable stiffness actuator can be considered high if a large change of stiffness is achieved with a small amount of energy, we introduce the following metric

$$\mu = \left| \frac{\Delta E}{\Delta K} \right| \qquad [\mathrm{J/Nm}] \tag{6}$$

in which the absolute value is taken so that $\mu = 0$ is the global minimum, i.e. the lower μ is, the better is the performance of the actuator. The units imply a rotational actuator, since a linear actuator can be converted into a rotational actuator by a simple transformation. Note that this metric is only valid to compare variable stiffness actuators with similar physical specifications in terms of range of output stiffness and range of deliverable output force.

III. PORT-BASED ANALYSIS OF VARIABLE STIFFNESS ACTUATORS

In this section, we present a port-based mathematical model for generic variable stiffness actuators. This modeling approach gives important intuitions on the energy flows and it helps both in the interpretation of the metric (6) and in the design criteria for new actuators, which optimize the metric.

The generalized bond graph representation of a variable stiffness actuator is depicted in Fig. 2, by using a Dirac structure. Each bond represents a power flow, defined to be positive in the direction of the half arrow. The power flow is

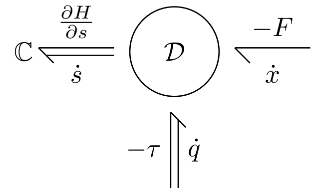

Fig. 2. Generalized representation of a variable stiffness actuator - The Dirac structure defines the interconnections between the different elements and, therefore, how power is distributed among the ports. The multi-bonds allow any number of springs, i.e. the $\mathbb{C}$-element, and any number of external inputs $(\tau, \dot{q})$. The output port $(F, \dot{x})$ is characterized by a single-bond.

characterized by two power conjugate variables: efforts and flows. If the linear space $\mathcal{F}$ is the space of admissible flows, then its dual space $\mathcal{E} := \mathcal{F}^*$ is the space of admissible efforts. The dual product $\langle e|f\rangle$ yields power, $\forall e \in \mathcal{E}$ and $\forall f \in \mathcal{F}$. In the mechanical domain, forces and torques are efforts, and velocities are flows.

The multi-dimensional $\mathbb{C}$-element represents the internal springs and it is characterized by an internal state, denoted by s, and by the energy function $H(s)$. The port behaviour is defined by the conjugate variables (e_s, f_s), given by

$$\begin{aligned} e_s &= \frac{\partial H}{\partial s} \\ f_s &= \dot{s} \end{aligned} \tag{7}$$

The port $(F, \dot{x})$ is the output port of the actuator. Recall that the force F can be either a linear force or a torque, depending on the type of the actuator. Likewise, $\dot{x}$ is either a linear or a rotational velocity. The multi-dimensional port $(\tau, \dot{q})$ is the port through which the internal degrees of freedom q are actuated. Depending on the actuator, τ denotes either generalized forces or torques and $\dot{q}$ generalized velocities, either translational or rotational.

The Dirac structure $\mathcal{D} \in \{\bar{\mathcal{D}}\}$, where $\{\bar{\mathcal{D}}\}$ is the complete set of allowable Dirac structures, defines how the power flows between the connected ports. The structure is power continuous, as follows from the definition in [8]

$$\{\bar{\mathcal{D}}\} = \{\bar{\mathcal{D}} \subset \mathcal{E} \times \mathcal{F} \mid \langle e|f\rangle = 0 \quad \forall (e,f) \in \bar{\mathcal{D}}\} \tag{8}$$

Note that the Dirac structure does not need to be constant. It may depend on the end effector position x and the configuration of the internal degrees of freedom q, i.e., it defines a constraint relation between the efforts and flows of the connected ports. This allows the following matrix representation

$$\begin{bmatrix} \dot{s} \\ \tau \\ F \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & A(q,x) & B(q,x) \\ -A(q,x)^T & 0 & C(q,x) \\ -B(q,x)^T & -C(q,x)^T & 0 \end{bmatrix}}_{D(q,x)} \begin{bmatrix} \frac{\partial H}{\partial s} \\ \dot{q} \\ \dot{x} \end{bmatrix} \tag{9}$$

where the skew-symmetric matrix $D(q,x)$ defines the Dirac structure. For simplicity, no friction or inertia are considered in the model, but the model can be easily extended to incorporate this.

From Eq. (9), some important observations may be derived. Since in the mechanical domain, a power continuous transformation between forces and velocities does not regularly exist, we assume $C(q,x) = 0$, without loss of generality.

The most important observation follows when we compute the energy stored in the system, i.e. in the springs. The variation of the energy stored in the system is given by

$$\begin{aligned}\frac{\mathrm{d}H}{\mathrm{d}t} &= \frac{\partial H}{\partial s}\frac{\mathrm{d}s}{\mathrm{d}t}\\ &= \frac{\partial H}{\partial s}(A(q,x)\dot{q} + B(q,x)\dot{x})\\ &= -\tau^T\dot{q} - F^T\dot{x}\end{aligned} \tag{10}$$

Note that the energy stored in the springs results from power supplied through the ports $(F, \dot{x})$ and $(\tau, \dot{q})$, which is consistent with the power continuity of the Dirac structure. It follows that, if the internal degrees of freedom q are changed via the port $(\tau, \dot{q})$, energy is added to or removed from the system unless $A(q,x)\dot{q} = 0$. This result is summarized in the following Lemmas.

Lemma 3.1: *Let the port-based representation of a variable stiffness actuator be*

$$\begin{bmatrix}\dot{s}\\ \tau\\ F\end{bmatrix} = \underbrace{\begin{bmatrix}0 & A(q,x) & B(q,x)\\ -A(q,x)^T & 0 & 0\\ -B(q,x)^T & 0 & 0\end{bmatrix}}_{D(q,x)}\begin{bmatrix}\frac{\partial H}{\partial s}\\ \dot{q}\\ \dot{x}\end{bmatrix} \tag{11}$$

where s is the state of the internal springs, q the configuration of the internal degrees of freedom, x the generalized output position and $D(q,x)$ the matrix representation of the Dirac structure connecting the ports. No energy is added to or removed from the system via the port $(\tau, \dot{q})$ if

$$\dot{q} \in \ker A(q,x) \quad \forall\, q, x \tag{12}$$

where ker *denotes the kernel.*

Let the stiffness of system (11) *be a function of the internal degrees of freedom $q \in \mathcal{Q}$*

$$K = K(q) \tag{13}$$

If condition (12) *is satisfied, the change of the stiffness requires no power.*

Proof: The proof follows from Eq. (10). If the stiffness at the output port depends on the configuration q, then the change of this configuration, i.e. $\dot{q} \neq 0$, while satisfying Eq. (12) results in the following energy balance

$$\frac{\mathrm{d}H}{\mathrm{d}t} = \frac{\partial H}{\partial s}(A(q,x)\dot{q} + B(q,x)\dot{x}) = -F^T\dot{x} \tag{14}$$

No power is supplied through the port $(\tau, \dot{q})$ and thus no power is required to change the stiffness. ■

Lemma 3.2: *Given the port-based representation of a variable stiffness actuator* (11)*. Consider the metric*

$$\mu = \left|\frac{\Delta E}{\Delta K}\right| \tag{15}$$

where ΔE is the amount of work done when the stiffness is changed of ΔK along a path defined in (3)*.*

If the actuator has internal degrees of freedom such that the relation $\dot{q} \in \ker A(q,x), \forall\, q, x$ is satisfied when changing the stiffness, then the metric is minimized, i.e. $\mu = 0$.

Proof: The proof consists of two parts. First, we prove that the path generated by $\dot{q} \in \ker A$ is a valid path for evaluating the metric. Then we will prove that this path yields $\mu = 0$.

Let $r_0 \in \mathcal{Q}$ be a configuration such that

$$r_0 = \{q \in \mathcal{Q} \mid F = 0, x = 0, \dot{x} = 0, H(s) = 0\} \tag{16}$$

Furthermore, let $\dot{q}(t)$ be a velocity vector satisfying $\dot{q} \in \ker A, \forall\, q, x$ for all t. Then, along the path

$$r(t) = \int \dot{q}(t)\mathrm{d}t, \qquad r(0) = r_0 \tag{17}$$

the variation of the stored energy is

$$\frac{\mathrm{d}H}{\mathrm{d}t} = -\tau^T\dot{q} = 0 \tag{18}$$

The first equality follows from the condition that no external load at the port $(F, \dot{x})$ is allowed. The second equality follows from the fact that $\dot{q} \in \ker A, \forall\, q, x$. Since the Dirac structure is power continuous and the initial conditions (16) require that no energy is present in the system at $t = 0$, the neutral equilibrium is maintained along the path and thus the path $r(t)$ in Eq. (17) is a valid path for the metric.

Since along the path defined in Eq. (17)

$$\mathrm{d}H = \frac{\partial H}{\partial q} = \frac{\partial H}{\partial s}\frac{\partial s}{\partial q} = \frac{\partial H}{\partial s}A(q,x) \tag{19}$$

it follows that, if $\dot{q} \in \ker A$,

$$\langle \mathrm{d}H | \dot{r}\rangle = \langle \mathrm{d}H | \dot{q}\rangle = 0 \tag{20}$$

and, therefore, $\mu = 0$. ■

IV. COMPARISON OF VARIABLE STIFFNESS ACTUATOR DESIGNS

In this section, we analyze three different types of variable stiffness actuators. For all three designs, it is possible to change the stiffness at the output port independently from the change of the joint position. To achieve this behaviour, all designs incorporate a nonlinear element. In order to analyze the three actuators from an energetic point of view and to compute the value of the metric, we model them in the framework presented in Sec. III.

A. Design I

The first design is depicted in Fig. 3. It consists of two series elastic actuators in an antagonistic setup. By operating the motors in common mode, the stiffness of the joint is changed. When the motors are operated in differential mode, the position of the joint is changed. A compact realization of this type of actuator is the VSA, presented in [3].

Note that this is a rotational actuator, therefore the generalized output force F is identified with a torque T and the generalized output joint position x with an angle θ.

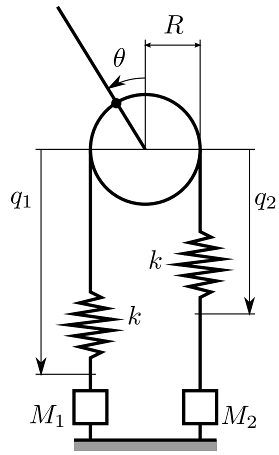

Fig. 3. Design I - This design is based on two series elastic actuators in an antagonistic setup. The linear motors M_1 and M_2 generate linear displacements q_1 and q_2. The nonlinear quadratic springs with fixed elastic constant k generate the output torque. By operating the motors in common mode, the output joint stiffness changes, while by operating the motors in differential mode the equilibrium of the output joint position θ changes.

The two motors M_1 and M_2 generate linear displacements q_1 and q_2. The state of the springs, i.e. their elongation, is

$$\begin{aligned} s_1 &= q_1 - R\theta \\ s_2 &= q_2 + R\theta \end{aligned} \tag{21}$$

where R is the radius of the pulley and θ the output joint position. Assuming that the two springs are nonlinear quadratic springs with fixed elastic constant k, the forces that they exert, i.e. the efforts, are

$$\begin{aligned} e_{s_1} &= \frac{\partial H}{\partial s_1} = k s_1^2 = k(q_1 - R\theta)^2 \\ e_{s_2} &= \frac{\partial H}{\partial s_2} = k s_2^2 = k(q_2 + R\theta)^2 \end{aligned} \tag{22}$$

where $H(s_1, s_2) = H_1(s_1) + H_2(s_2)$ is the total energy function. Since the motors are in series with the springs, their generalized forces are equal to the forces of the springs, i.e. $\tau_i = -e_{s_i}$, with $i = 1, 2$.

Since the radius R of the pulley is constant, the torque generated at the output port is

$$T = R(e_{s_1} - e_{s_2}) = kR(q_1^2 - q_2^2 - 2R(q_1 + q_2)\theta) \tag{23}$$

By taking the time derivative of Eq. (21) and by using Eqs. (22), (23), we can model this actuator in the port-based setting through a Dirac structure of the form of Eq. (9), i.e.

$$\begin{bmatrix} \dot{s}_1 \\ \dot{s}_2 \\ \tau_1 \\ \tau_2 \\ T \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 & -R \\ 0 & 0 & 0 & 1 & R \\ -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ R & -R & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial s_1} \\ \frac{\partial H}{\partial s_2} \\ \dot{q}_1 \\ \dot{q}_2 \\ \dot{\theta} \end{bmatrix} \tag{24}$$

The output joint stiffness is given by

$$K = \frac{\partial T}{\partial \theta} = -2kR^2(q_1 + q_2) \tag{25}$$

From Eq. (24) it follows that, for this type of actuator, $A(q, x) = I_2$, i.e. the 2×2 identity matrix. Since matrix A has no kernel, we expect to find $\mu > 0$.

From Eqs. (21), (23) it follows that the initial conditions given in (2) are satisfied for $(q_1, q_2) = (0, 0)$. Since the equilibrium does not change when q_1 and q_2 are operated in common mode, i.e. $\dot{q}_1 = \dot{q}_2$, we can take for the path $r(t)$

$$r(t) = \begin{bmatrix} \bar{q}t \\ \bar{q}t \end{bmatrix}, \qquad t \in [0, 1] \tag{26}$$

with $q_1 \in [0, \bar{q}]$ and $q_2 \in [0, \bar{q}]$, where $\bar{q}$ is the maximum allowed value for the configuration variables.

By observing that for this design $\frac{\partial H}{\partial q_i} = \frac{\partial H}{\partial s_i}\frac{\partial s_i}{\partial q_i}$, it follows that ΔE is calculated as

$$\Delta E = \int_t \langle \mathrm{d}H | \dot{r} \rangle \, \mathrm{d}t = \frac{2}{3} k \bar{q}^3 \tag{27}$$

Along the same path $r(t)$, using Eq. (25), we calculate ΔK

$$\Delta K = \int_t \langle \mathrm{d}K | \dot{r} \rangle \, \mathrm{d}t = -4kR^2 \bar{q} \tag{28}$$

It follows that, for this particular actuator design

$$\mu = \frac{\bar{q}^2}{6R^2} \tag{29}$$

B. Design II

The second design is depicted in Fig. 4. It consists of two actuators: one actuator is used to change the stiffness and the other is used to changed the output joint position. In general, the actuator used to change the stiffness can be much smaller, allowing a smaller actuator realization. The VS-Joint presented in [6] belongs to this category.

Note that this is a rotational actuator, therefore the generalized output force F is identified with a torque T and the generalized output position x with an angle θ.

The state of the springs, i.e. their elongation, is

$$\begin{aligned} s_1 &= q_2 - R\alpha \\ s_2 &= q_2 + R\alpha \end{aligned} \tag{30}$$

where R is the radius of the pulley and $\alpha = \theta - q_1 + \frac{\pi}{2}$, with θ the output joint position.

Assuming that the two springs are nonlinear quadratic springs with fixed elastic constant k, the forces that they exert, i.e. the efforts, are

$$\begin{aligned} e_{s_1} &= \frac{\partial H}{\partial s_1} = k s_1^2 = k(q_2 - R\alpha)^2 \\ e_{s_2} &= \frac{\partial H}{\partial s_2} = k s_2^2 = k(q_2 + R\alpha)^2 \end{aligned} \tag{31}$$

In this design, the actuators are not in series with the springs and their generalized forces are

$$\begin{aligned} \tau_1 &= -R(e_{s_1} - e_{s_2}) \\ \tau_2 &= -(e_{s_1} + e_{s_2}) \end{aligned} \tag{32}$$

Since the radius R of the pulley is constant, the torque generated at the output port is

$$T = R(e_{s_1} - e_{s_2}) = -4kR^2\left(\theta - q_1 + \frac{\pi}{2}\right) q_2 \tag{33}$$

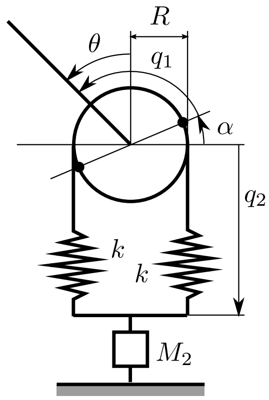

Fig. 4. Design II - In this design, the change of the stiffness and the output joint position θ is decoupled. The linear motor M_2 generates a linear displacement q_2 and is used for changing the output joint stiffness. The nonlinear quadratic springs with fixed elastic constant k generate the output torque. The equilibrium of the output joint position θ is determined by q_1. Note that the end effector can rotate independently from the pulley.

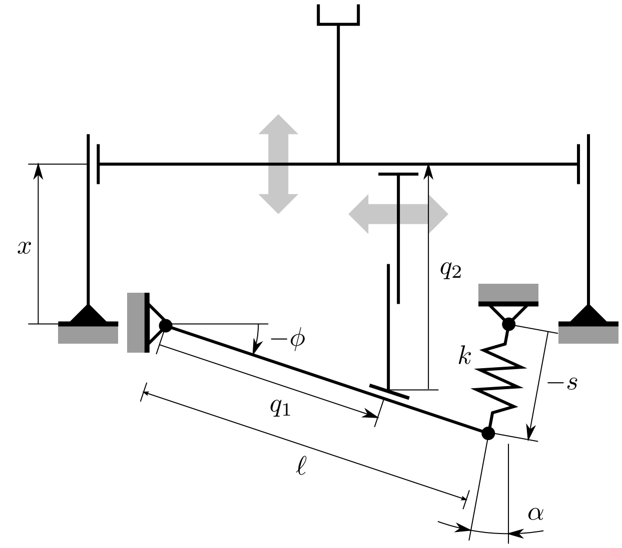

Fig. 5. Design III - This design is based on a lever arm with variable effective length. The effective length of the lever is determined by the linear motor q_1 and determines how the stiffness of the linear spring with fixed stiffness k is felt at the output. The degree of freedom given by the linear motor q_2 controls the equilibrium of the output joint position x.

By taking the time derivative of Eq. (30) and by using Eqs. (32), (33), we can model this actuator in the port-based setting through a Dirac structure of the form of Eq. (9), i.e.

$$\begin{bmatrix} \dot{s}_1 \\ \dot{s}_2 \\ \tau_1 \\ \tau_2 \\ T \end{bmatrix} = \begin{bmatrix} 0 & 0 & R & 1 & -R \\ 0 & 0 & -R & 1 & R \\ -R & R & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 & 0 \\ R & -R & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial s_1} \\ \frac{\partial H}{\partial s_2} \\ \dot{q}_1 \\ \dot{q}_2 \\ \dot{\theta} \end{bmatrix} \tag{34}$$

The output joint stiffness is given by

$$K = \frac{\partial T}{\partial \theta} = -4kR^2 q_2 \tag{35}$$

From Eq. (34), it follows that, for this type of actuator, the matrix $A(q, x)$ has full rank and, thus, it has no kernel and we expect to find $\mu > 0$.

From Eqs. (30), (33), it follows that the initial conditions given in Eq. (2) are satisfied for $(q_1, q_2) = (\frac{\pi}{2}, 0)$. In order to keep the equilibrium, only q_2 is allowed to change, therefore $r(t)$ is given by

$$r(t) = \begin{bmatrix} \frac{\pi}{2} \\ \bar{q}t \end{bmatrix}, \qquad t \in [0, 1] \tag{36}$$

with $q_2 \in [0, \bar{q}]$, where $\bar{q}$ is the maximum allowed value for this configuration variable. Then, ΔE and ΔK are calculated as

$$\Delta E = \frac{2}{3} k \bar{q}^3 \tag{37}$$

$$\Delta K = -4kR^2 \bar{q} \tag{38}$$

For this design we therefore also obtain

$$\mu = \frac{\bar{q}^2}{6R^2} \tag{39}$$

C. Design III

The third design is a conceptual actuator, extensively described in [9] and depicted in Fig. 5. The design is based on the insights gained from the analysis in Sec. III. In particular, a decoupling of position and stiffness control is achieved on a mechanical level, so that Eq. (12) is satisfied. The mechanism relies on a lever arm with variable effective length. This effective length depends only on the degree of freedom given by the linear motor q_1 and determines how the stiffness of the linear spring is felt at the output. Note that $0 < q_1 \leq \ell$, since $q_1 = 0$ is a singular configuration. The degree of freedom given by the linear motor q_2 controls the output.

For simplicity, we assume that the lever length ℓ is large compared to the displacement s, and thus that we may assume $\alpha = 0$.

The state s of the linear spring is

$$s = \ell \sin \phi = \ell \frac{x - q_2}{q_1} \tag{40}$$

The linear spring has fixed elastic constant k and energy function $H(s) = \frac{1}{2}ks^2$. The force exerted by the spring, i.e. the effort, is

$$e_s = \frac{\partial H}{\partial s} = ks = k\ell \frac{x - q_2}{q_1} \tag{41}$$

It has been shown in [9] that the generalized forces τ_1 and τ_2 are

$$\begin{aligned} \tau_1 &= \frac{\ell}{q_1} \sin(\phi) e_s \\ \tau_2 &= \frac{\ell}{q_1} e_s \end{aligned} \tag{42}$$

Since the end effector is actuated by q_2, the output force $F = -\tau_2$.

By taking the time derivative of Eq. (40) and by using Eq. (42) we can model this actuator in the port-based setting through a Dirac structure of the form of Eq. (9), i.e.

$$\begin{bmatrix} \dot{s} \\ \tau \\ F \end{bmatrix} = \begin{bmatrix} 0 & A(q,x) & \frac{\ell}{q_1} \\ -A(q,x)^T & 0 & 0 \\ -\frac{\ell}{q_1} & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial s} \\ \dot{q} \\ \dot{x} \end{bmatrix} \tag{43}$$

with

$$A(q,x) = -\frac{\ell}{q_1}\begin{bmatrix} \sin\phi & 1 \end{bmatrix} \tag{44}$$

From Eq. (43), it follows that, for this type of actuator, the matrix A has a kernel and therefore, from Lemma 3.2, $\mu = 0$.

From Eqs. (40), (42) it follows that the initial conditions given in Eq. (2) are satisfied for $(q_1, q_2) = (q_1, 0)$. Observe that any $q_1 > 0$ is allowed. Hence, the path $r(t)$ can be

$$r(t) = \begin{bmatrix} \ell t \\ 0 \end{bmatrix}, \qquad t \in [0,1] \tag{45}$$

Since in along the path $\sin\phi = 0$, it is easily seen that the tangent vector $\dot{r} \in \ker A$.

The differential of H is given by

$$\left(\frac{\partial H}{\partial s}\frac{\partial s}{\partial q_1}, \frac{\partial H}{\partial s}\frac{\partial s}{\partial q_2}\right) = \left(-\frac{k\ell^2}{q_1}\sin^2\phi, -\frac{k\ell^2}{q_1}\sin\phi\right) \tag{46}$$

Since $\sin\phi = 0$ along the trajectory, the metric $\mu = 0$. For completeness, the stiffness K may be calculated as

$$K = \frac{\ell^2}{q_1^2}k \tag{47}$$

This shows that along the chosen curve, the stiffness is indeed changed without using energy.

D. Design comparison

The metric calculated for Design I and II depends on the design parameters R and $\bar{q}$. Fig. 6 shows how the metric is influenced for a range of values of R and $\bar{q}$. Also, since the metric is not zero, it can be deduced that the change of stiffness for these types of actuators is never energy efficient, which is in accordance with previous studies [10].

Design III is optimal with respect to the metric. This is achieved by decoupling the change of the output actuator position and the change of stiffness according to the design guidelines obtained from the analysis of the port-based model. This results in an energy efficient design for which the stiffness can be changed without using energy, while the system is kept in an equilibrium configuration.

V. CONCLUSIONS AND FUTURE WORK

In this work, we introduced a metric, which allows the comparison of different designs of variable stiffness actuators in terms of energy efficiency. The metric expresses how much energy is used to change stiffness while the actuator is kept in an equilibrium configuration. The metric is related to a port-based mathematical model for variable stiffness actuators and, by analysis of this model, design guidelines for energy efficient actuators were derived. In particular, it was shown that the metric can be optimized by following the design guidelines.

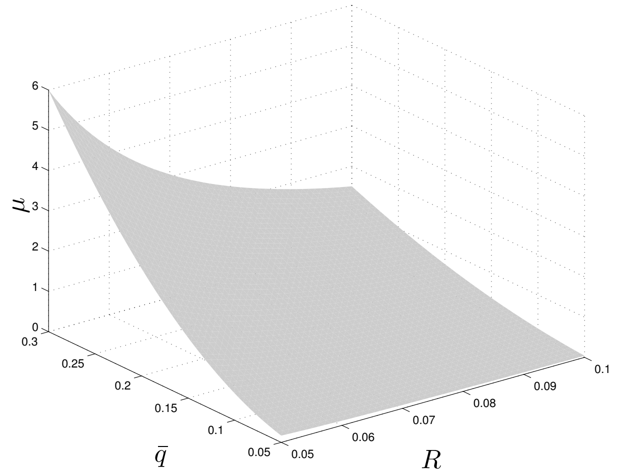

Fig. 6. Influence of design parameters R and $\bar{q}$ on the metric - For Design I and II, the metric is dependent on the actual dimensions of the actuator, i.e. R and $\bar{q}$.

Future work will focus on expanding the metric to reflect energy efficiency in dynamic conditions and under arbitrary loads.

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