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Breathing motion compensation for robot assisted laser osteotomy

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Abstract— **This article proposes a control scheme for robot assisted laser osteotomy. Laser osteotomy consists in cutting a bone precisely with a laser. To achieve accurate cuts the laser has to be precisely sent to a desired pose with respect to the bone. In the proposed approach, a robot is used to position an end-effector with a scanhead, which deflects the cutting laser. The poses of the laser and of the bone are measured thanks to an optical tacking system which tracks the positions of optical markers placed on the bone and on the scanhead. The control of the robot is then performed thanks to a position-based visual servoing control scheme. In the case of bones affected by breathing motions, for instance the thorax bones, this control scheme is enhanced with a learning algorithm in order to compensate breathing motions. The main contribution of this paper is this control algorithm for motions compensation.**

I. INTRODUCTION

Laser osteotomy consists in accurately cutting or ablating bone tissues thanks to a laser. The fact of using a laser offers multiple advantages in comparison to the traditional mechanical tools (saws, drills or mills) currently used for osteotomy [1]. Indeed, the friction induced by mechanical tools can cause trauma and thermal damage to the surrounding tissues. These trauma are reduced when using a laser for osteotomy because there is no direct contact with the bone. Moreover, the use of a laser offers free cut geometries whereas the size and shape of traditional mechanical tools limits the cut geometry to straight cuts or large radii cuts. However, despite the advantages it offers, laser is not currently used for osteotomy. Indeed, the obtainable precision in cutting with a laser system can only be reached using means of computer and robot assisted surgery. Thus, for the practical medical adoption of the laser osteotomy, an online control system for the cut has to be developed.

This paper proposes a control scheme for robot assisted laser osteotomy. The setup that we use has been developed by the university of Karlsruhe [2]. It is composed of a CO2 laser and the KUKA lightweight robot. The laser beam is guided through a passive articulated mirror arm to a laser scanhead mounted on the robot end-effector. The objective of the control scheme proposed in this paper is to bring this laser scanhead in a desired situation (i.e. position and orientation) with respect to the bone to be cut. This desired situation has been defined during a preoperative planning step on CT images of the bone. During the operation an optical tracking system tracks the positions of optical markers placed on the bone and on the scanhead. Thanks to a registration process, performed during a preoperative step, between the robot, the optical tracking system and the CT images of the bone, the desired pose of the laser scanhead with respect to the bone can be expressed in terms of relative positions of the optical markers. The control of the robot is, then, performed thanks to a position-based visual servoing control scheme.

Two control schemes are proposed in this paper. The first control algorithm is devoted to static bones, for instance skull bones. Some bones, for instance the thorax bones, undergo motions due to respiration. In this case, for an accurate osteotomy, the control algorithm must reject the perturbation induced by breathing so that the robot performs motions synchronous with the bone. However, due to the limited bandwidth of the robot, the control algorithm proposed in the case of a static bone is not able to compensate breathing motion. Therefore, a second control scheme is proposed for breathing motion compensation.

Most of the contributions for physiological motions compensation use the repetitive property of these motions to predict and anticipate them. For instance, in [3], Riviere et al. have investigated the prediction of bodily motion due to respiration in order to actively compensate for these motions in a robot-assisted system for percutaneous kidney surgery. In [4], the compensation of respiratory motion is proposed in order to allow for an accurate treatment of tumors thanks to radiosurgery. This approach, which uses the measurements provided by infrared and X-ray imaging, is based on a model of the breathing motion. The approaches proposed in [5] are model based predictive methods for the compensation of cardiac motions using a robotic arm controlled by image-based visual servoing. Most of the existing approaches for physiological motions compensation are model based methods. The approach proposed in this paper for breathing motions compensation is based on a learning algorithm and is thus less sensitive to modelling errors.

This paper is organized as follows. Section II describes the principle of the approach and introduces the notations used in the paper. Section III presents the control algorithm developed for static bones. It is a classical position-based visual servoing algorithm with slight modifications. The main contribution of this paper is the learning algorithm proposed in section IV for bones undergoing breathing motions. Experimental results are presented in section V.

II. PRINCIPLE OF THE APPROACH

A. Description of the system and notations

As mentionned in the introduction, we use the laser osteotomy setup developed by the university of Karlsruhe [2]. The principle of this approach is depicted on Fig. 1. A rigid

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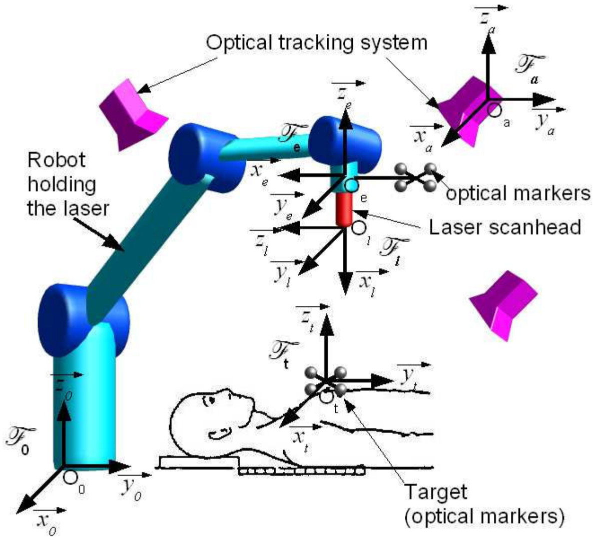

Fig. 1. Principle of the osteotomy system

body constituted by optical markers is attached to the bone to be cut. In the following, we will refer to this rigid body by the name target. Another rigid body constituted by optical markers is attached to the robot's end-effector. The positions of the markers are measured by an optical tracking system. In this approach, a multi camera optical tracking system from ART with six cameras is used. The multi camera approach helps to avoid occlusion of the optical markers.

We introduce the following frames:

- $\mathcal{F}_0 : (O_0; \overrightarrow{x}_0; \overrightarrow{y}_0; \overrightarrow{z}_0)$ is the base frame of the robot. This frame is static.
- $\mathcal{F}_a : (O_a; \overrightarrow{x}_a; \overrightarrow{y}_a; \overrightarrow{z}_a)$ is the frame attached to the optical tracking system. This frame is static. The coordinates of the optical markers measured by the optical tracking system are expressed in $\mathcal{F}_a$.
- $\mathcal{F}_t : (O_t; \overrightarrow{x}_t; \overrightarrow{y}_t; \overrightarrow{z}_t)$ is the frame attached to the target.
- $\mathcal{F}_e : (O_e; \overrightarrow{x}_e; \overrightarrow{y}_e; \overrightarrow{z}_e)$ is the frame of the robot's end-effector.
- $\mathcal{F}_l : (O_l; \overrightarrow{x}_l; \overrightarrow{y}_l; \overrightarrow{z}_l)$ is the frame attached to the laser scanhead. As the laser scanhead is rigidly linked to the robot's end-effector, the frame $\mathcal{F}_l$ is static with respect to $\mathcal{F}_e$.

In the sequel, we note $\overrightarrow{O_iO_j}$ the intrinsic expression of the vector from point O_i to point O_j and $\left[\overrightarrow{O_iO_j}\right]_{\mathcal{F}_k}$ the expression of $\overrightarrow{O_iO_j}$ in the basis of frame $\mathcal{F}_k$. The pose of frame $\mathcal{F}_i$ with respect to frame $\mathcal{F}_j$ is noted:

$$\chi_{ij} = \begin{bmatrix} \left[\overrightarrow{O_jO_i}\right]_{\mathcal{F}_j} \\ q_{ij} \end{bmatrix} \in \mathbb{R}^7$$

where $\left[\overrightarrow{O_jO_i}\right]_{\mathcal{F}_j} \in \mathbb{R}^3$ is the position and the quaternion $q_{ij} \in \mathbb{R}^4$ expresses the orientation of $\mathcal{F}_i$ with respect to $\mathcal{F}_j$. We note R_{ji} the rotation matrix from $\mathcal{F}_i$ to frame $\mathcal{F}_j$ i.e. we have:

$$[\overrightarrow{u}]_{\mathcal{F}_j} = R_{ji}\ [\overrightarrow{u}]_{\mathcal{F}_i}\,, \qquad \forall \overrightarrow{u}$$

This rotation matrix can easily be computed from quaternion q_{ij}.

We note the velocity screw of a frame $\mathcal{F}_i$ with respect to a frame $\mathcal{F}_j$ and expressed in the basis of frame $\mathcal{F}_k$:

$$[T_{ij}]_{\mathcal{F}_k} = \begin{bmatrix} [V_{ij}]_{\mathcal{F}_k} \\ [\omega_{ij}]_{\mathcal{F}_k} \end{bmatrix} \in \mathbb{R}^6$$

where $[V_{ij}]_{\mathcal{F}_k} = \left[\left.\frac{\mathrm{d}\overrightarrow{O_jO_i}}{\mathrm{d}t}\right|_{\mathcal{F}_j} \right]_{\mathcal{F}_k} \in \mathbb{R}^3$ is the translational velocity and $[\omega_{ij}]_{\mathcal{F}_k} \in \mathbb{R}^3$ is the angular velocity.

B. Objectives of the control law

The laser scanhead mounted on the robot's end-effector is composed of two galvanometric mirrors, which deflect the pulsed laser beam onto the tissue. During the osteotomy process, the scanhead moved by the robot has to be sent to a desired pose (i.e. position and orientation) with respect to the bone. When this desired pose is reached, the mirrors deflect the laser beam in order to perform the first part of the planned cut geometry. Then, the scanhead is moved to another desired pose with respect to the bone in order to perform the second part of the cut trajectory, and so on. This paper concerns the control of the robot in order to send the scanhead in the desired poses with respect to the bone.

The desired poses of the laser scanhead $\mathcal{F}_l$ with respect to the bone are defined by the preoperative surgical planning. A registration is performed during a preoperative step (see [6]) beween the model of the bone and the target attached to the bone so that the desired pose of $\mathcal{F}_l$ with respect to the bone can be expressed in terms of a desired pose of $\mathcal{F}_l$ with respect to $\mathcal{F}_t$. More precisely, as far as the relative position is concerned, we note $\left[\overrightarrow{O_tO_l}\right]^d_{\mathcal{F}_t}$ the desired value for $\left[\overrightarrow{O_tO_l}\right]_{\mathcal{F}_t}$. As far as the orientation is concerned, it is required that the laser beam which is collinear to axis $\overrightarrow{x}_l$ points towards a point of the bone that we call P_l. This point is fixed with respect to the target's frame and its coordinates in $\mathcal{F}_t$, $\left[\overrightarrow{O_tP_l}\right]_{\mathcal{F}_t}$, have been determined during the preoperative registration process.

The Optical Tracking System (OTS) provides with the coordinates of the target's points in $\mathcal{F}_a$. From these measures, we can compute the pose of the target with respect to the OTS frame: χ_{ta}. The OTS also provides with the coordinates, in $\mathcal{F}_a$, of the points of the markers attached to the robot's end-effector. From these measures and thanks to a preoperative registration process (see [6]) we can compute the pose of the laser with respect to the OTS: χ_{la}. Therefore, the measures provided to the controller are χ_{ta} and χ_{la}.

In the considered control algorithm the data provided to the controller come from visual sensors and the error is defined in 3D (task space) coordinates. Such an approach is called position-based visual servoing.

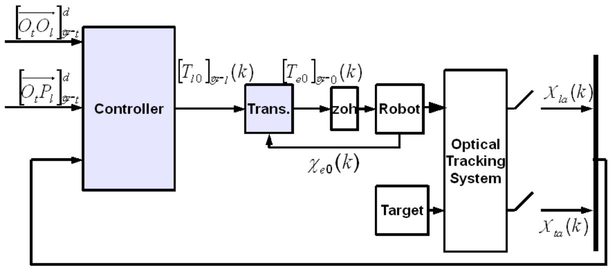

Fig. 2. Proposed position-based control scheme in the case of a static bone

At a lower level, a Cartesian velocity controller is used to drive the manipulator in Cartesian space. The reference signal required by this Cartesian velocity controller is the desired velocity screw of the robot's end-effector with respect to the base frame $\mathcal{F}_0$, expressed in $\mathcal{F}_0$: $[T_{e0}]_{\mathcal{F}_0}$. Thus, the output of the position-based controller must be $[T_{e0}]_{\mathcal{F}_0}$. The low level Cartesian velocity control loop, whose settling time is much lower than the one of the position-based visual servoing control loop, won't be represented on the control schemes depicted in the following sections.

III. CASE OF A STATIC BONE

The control algorithm proposed in this section is a classical position-based visual servoing scheme inspired from [7]. It is slightly modified as far as the control of the orientation is concerned. Indeed, in [7] the desired orientation of the end-effector is fully specified. In the case of laser osteotomy, the desired orientation constrains only one degree of freedom of the end-effector. Among the different possible final orientations, we have chosen the shortest rotation from the current to the desired end-effector orientation.

The proposed control scheme is depicted on figure 2. This is a numeric control loop where the sampling period is noted T_e and the sampling instant k refers to time $t = kT_e$. The first part of the controller computes the desired velocity screw for the laser:

$$[T_{l0}]_{\mathcal{F}_l}(k) = \begin{bmatrix} [V_{l0}]_{\mathcal{F}_l}(k) \\ [\omega_{l0}]_{\mathcal{F}_l}(k) \end{bmatrix} \in \mathbb{R}^6$$

In order to compute this screw, we first express the desired position of the laser, $\left[\overrightarrow{O_tO_l}\right]^d_{\mathcal{F}_t}$, in the static frame $\mathcal{F}_a$:

$$\left[\overrightarrow{O_aO_l}\right]^d_{\mathcal{F}_a}(k) = R_{at}(k)\left[\overrightarrow{O_tO_l}\right]^d_{\mathcal{F}_t} + \left[\overrightarrow{O_aO_t}\right]_{\mathcal{F}_a}(k)$$

We recall that the rotation matrix $R_{at}(k)$ is computed from the quaternion $q_{ta}(k)$ and that $q_{ta}(k)$ and $\left[\overrightarrow{O_aO_t}\right]_{\mathcal{F}_a}(k)$ compose the measured pose $\chi_{ta}(k)$. Then, the desired translational velocity of the laser is given by:

$$[V_{l0}]_{\mathcal{F}_l}(k) = R_{la}(k)\,[V_{l0}]_{\mathcal{F}_a}(k) \quad \text{with}$$
$$[V_{l0}]_{\mathcal{F}_a}(k) = \lambda_v\left(\left[\overrightarrow{O_aO_l}\right]^d_{\mathcal{F}_a}(k) - \left[\overrightarrow{O_aO_l}\right]_{\mathcal{F}_a}(k)\right) \quad (1)$$

where $\lambda_v \in \mathbb{R}^+$ is a positive gain. Note that, as $\mathcal{F}_a$ and $\mathcal{F}_0$ are static frames, a velocity computed with respect to $\mathcal{F}_a$ has the same value as this velocity computed with respect to $\mathcal{F}_0$. In order to compute the angular velocity, $[\omega_{l0}]_{\mathcal{F}_l}(k)$, we first determine the coordinates of point P_l in frame $\mathcal{F}_a$:

$$\left[\overrightarrow{O_aP_l}\right]_{\mathcal{F}_a}(k) = R_{at}(k)\left[\overrightarrow{O_tP_l}\right]_{\mathcal{F}_t} + \left[\overrightarrow{O_aO_t}\right]_{\mathcal{F}_a}(k)$$

Then, we deduce the desired orientation of the laser when it is in the desired position:

$$[\overrightarrow{x_l}]^d_{\mathcal{F}_a}(k) = \frac{\left[\overrightarrow{O_aP_l}\right]_{\mathcal{F}_a}(k) - \left[\overrightarrow{O_aO_l}\right]^d_{\mathcal{F}_a}(k)}{\left\|\left[\overrightarrow{O_aP_l}\right]_{\mathcal{F}_a}(k) - \left[\overrightarrow{O_aO_l}\right]^d_{\mathcal{F}_a}(k)\right\|}$$

The current expression of $[\overrightarrow{x_l}]_{\mathcal{F}_a}(k)$ is given by the first column of the rotation matrix $R_{al}(k)$. Then, we compute the vector $[\overrightarrow{u}]_{\mathcal{F}_a}(k)$ and the angle $\theta(k)$ of the shortest rotation that brings $[\overrightarrow{x_l}]_{\mathcal{F}_a}(k)$ on the desired value $[\overrightarrow{x_l}]^d_{\mathcal{F}_a}(k)$:

$$[\overrightarrow{u}]_{\mathcal{F}_a}(k) = \frac{[\overrightarrow{x_l}]_{\mathcal{F}_a}(k) \times [\overrightarrow{x_l}]^d_{\mathcal{F}_a}(k)}{\left\|[\overrightarrow{x_l}]_{\mathcal{F}_a}(k) \times [\overrightarrow{x_l}]^d_{\mathcal{F}_a}(k)\right\|}$$
$$S_\theta = \left\|[\overrightarrow{x_l}]_{\mathcal{F}_a}(k) \times [\overrightarrow{x_l}]^d_{\mathcal{F}_a}(k)\right\|$$
$$C_\theta = \left([\overrightarrow{x_l}]_{\mathcal{F}_a}(k)\right)^T\,[\overrightarrow{x_l}]^d_{\mathcal{F}_a}(k)$$
$$\theta(k) = \arctan 2(S_\theta\,,\,C_\theta)$$

where $\times$ is the cross product and the superscript T is the transpose operator. The desired angular velocity of the laser is:

$$[\omega_{l0}]_{\mathcal{F}_l}(k) = R_{la}(k)\,[\omega_{l0}]_{\mathcal{F}_a}(k)$$
$$\text{with} \quad [\omega_{l0}]_{\mathcal{F}_a}(k) = \lambda_\omega\,\theta(k)\,[\overrightarrow{u}]_{\mathcal{F}_a}(k) \quad (2)$$

where $\lambda_\omega \in \mathbb{R}^+$ is a positive gain. The next step of the control algorithm (noted "Trans." on figure 2) consists in deducing $[T_{e0}]_{\mathcal{F}_0}(k)$ from the value of $[T_{l0}]_{\mathcal{F}_l}(k)$. Thanks to the preoperative registration process (see [6]) the relative pose of $\mathcal{F}_l$ with respect to $\mathcal{F}_e$ can be estimated i.e. we can estimate R_{el} and $\left[\overrightarrow{O_eO_l}\right]_{\mathcal{F}_e}$. Thus, as $\mathcal{F}_l$ is rigidly linked to $\mathcal{F}_e$, it yields:

$$[\omega_{e0}]_{\mathcal{F}_e}(k) = R_{el}\,[\omega_{l0}]_{\mathcal{F}_l}(k)$$
$$[V_{e0}]_{\mathcal{F}_e}(k) = R_{el}(k)\,[V_{l0}]_{\mathcal{F}_l}(k) + \left[\overrightarrow{O_eO_l}\right]_{\mathcal{F}_e} \times [\omega_{e0}]_{\mathcal{F}_e}(k)$$

The robot's low level controll provides with $R_{0e}(k)$. So, we get:

$$[T_{e0}]_{\mathcal{F}_0}(k) = \begin{bmatrix} R_{0e}(k)\,[V_{e0}]_{\mathcal{F}_e}(k) \\ R_{0e}(k)\,[\omega_{e0}]_{\mathcal{F}_e}(k) \end{bmatrix}$$

IV. BREATHING MOTION COMPENSATION

Some bones, for instance, the thorax bones, move because of breathing. In this case, for an accurate osteotomy, the control algorithm must reject the perturbation induced by breathing. However, due to the limited bandwidth of the robot, the control algorithm proposed in section III is not able to compensate breathing motion. The proposed solution for breathing motion compensation consists in enhancing the control algorithm proposed in section III with a learning

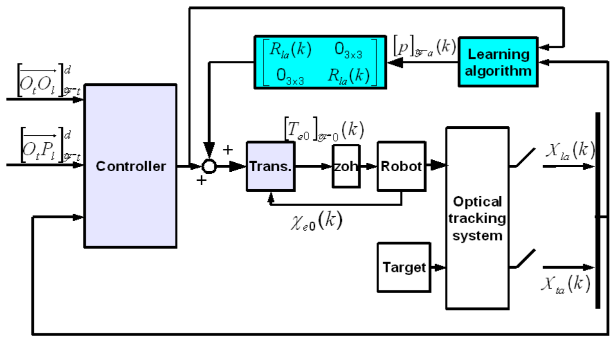

Fig. 3. Proposed position-based control scheme for breathing motion compensation

algorithm (see figure 3). This technique for motions compensation is the main contribution of this paper.

The objective of the learning algorithm is to cancel the relative velocity of the robot with respect to the target due to breathing so that the laser performs motion synchronous with the bone. Meanwhile, the controller presented in section III brings the relative pose of the laser with respect to the target to its desired value. The learning algorithm adds on the control signal computed by the controller presented in section III a correcting signal for motions compensation. The computation of this correcting signal is based on the repetitive properties of the breathing signal. Indeed, assume that, at the previous breathing cycle, the relative velocity of the laser with respect to the target was not zero i.e. the breathing motion was not compensated. If, at the current cycle, we take the same value of the correcting signal as in the previous cycle, due to the repetitive properties of breathing, the relative velocity of the laser with respect to the target will have nearly the same value. Therefore, the learning algorithm proposes to compute the current value of the correcting signal by adding to its value at the previous cycle a term proportional to the relative velocity of the laser with respect to the target at the previous breathing cycle. By this technique, cycle after cycle, the algorithm adjusts the value of the correcting signal until the relative velocity of the laser with respect to the target is cancelled. First, an originality of this learning algorithm is that, unlike most of the controllers used for motions compensation (e.g. MPC controllers), it does not require any model of the system to be controlled. Therefore, it will be more robust to modelling errors. The only assumption we make on breathing motions is that they are periodic and that their period, T_p is known. This is not a very strong hypothesis since, usually, during the osteotomy of the thorax bones the respiration of the patient is assisted by a breathing machine. Moreover, we assume that the period of the respiration, T_p, is a multiple of the sampling period, T_e, i.e.

$$T_p = m\,T_e \quad \text{with } m \text{ a positive integer}$$

Let's mention that except its periodicity, we do not make any assumption on the shape of the perturbation induced by breathing. Another originality of the proposed control algorithm is that, unlike most of the algorithms based on the repetitive properties of a system (e.g. Iterative Learning Controllers), it does not require that the reference signal be periodic. This freedom on the choice of the reference comes from the structure of the proposed control scheme : the control signal is the addition of two terms, one computed by the controller of section III which aims at reaching the reference and another term computed by the learning algorithm in order to reject the repetitive perturbation induced by breathing.

The learning algorithm adds to the control signal computed by the controller proposed in section III, the signal:

$$\begin{bmatrix} R_{la} & 0_{3\times3} \\ 0_{3\times3} & R_{la} \end{bmatrix} [p]_{\mathcal{F}_a}(k) \in \mathbb{R}^6$$

where $[p]_{\mathcal{F}_a}(k) \in \mathbb{R}^6$ is such that:

$$[p]_{\mathcal{F}_a}(k) = [p]_{\mathcal{F}_a}(k-m) - \beta\,[e]_{\mathcal{F}_a}(k-m) + [h]_{\mathcal{F}_a}(k)$$

The signal p at current sampling instant, $[p]_{\mathcal{F}_a}(k)$, is this signal at the previous breathing cycle, $[p]_{\mathcal{F}_a}(k-m)$, corrected by $[e]_{\mathcal{F}_a}(k-m)$ which is the velocity of $\mathcal{F}_l$ with respect to $\mathcal{F}_t$ due to the breathing motion of $\mathcal{F}_t$ and expressed in $\mathcal{F}_a$. The learning gain β is positive. Changes in the reference signal induce a difference between the pose of the laser with respect to the bone at the current instant, $\chi_{lt}(k)$, and the value of this pose at the previous breathing cycle, $\chi_{lt}(k-m)$. Because of this difference, the velocity of the laser required to compensate breathing motions at time k may be different from the velocity required at time $k-m$. Therefore, if the learning algorithm were $[p]_{\mathcal{F}_a}(k) = [p]_{\mathcal{F}_a}(k-m) - \beta\,[e]_{\mathcal{F}_a}(k-m)$, after changes in the reference signal, this algorithm would have to learn again the velocity required by the laser to compensate breathing motions. Such a new learning phase would increase the duration of the osteotomy operation. The objective of the term $[h]_{\mathcal{F}_a}(k)$ is to take into account the consequences of the difference between $\chi_{lt}(k-m)$ and $\chi_{lt}(k)$ on the velocity of the laser required to compensate breathing motions, in order to avoid any new learning phase when the reference signal changes.

We split $[p]_{\mathcal{F}_a}(k)$, $[e]_{\mathcal{F}_a}(k-m)$ and $[h]_{\mathcal{F}_a}(k)$ as follows:

$$[p]_{\mathcal{F}_a}(k) = \begin{bmatrix} [V_p]_{\mathcal{F}_a}(k) \\ [\omega_p]_{\mathcal{F}_a}(k) \end{bmatrix},\ [h]_{\mathcal{F}_a}(k) = \begin{bmatrix} [V_h]_{\mathcal{F}_a}(k) \\ 0_{3\times1} \end{bmatrix},$$

$$[e]_{\mathcal{F}_a}(k-m) = \begin{bmatrix} [V_e]_{\mathcal{F}_a}(k-m) \\ [\omega_e]_{\mathcal{F}_a}(k-m) \end{bmatrix},$$

where $[V_p]_{\mathcal{F}_a}(k) \in \mathbb{R}^3$, $[V_h]_{\mathcal{F}_a}(k) \in \mathbb{R}^3$ and $[V_e]_{\mathcal{F}_a}(k-m) \in \mathbb{R}^3$ affect the translational velocity and where $[\omega_p]_{\mathcal{F}_a}(k) \in \mathbb{R}^3$ and $[\omega_e]_{\mathcal{F}_a}(k-m) \in \mathbb{R}^3$ affect the angular velocity.

In order to compute $[V_p]_{\mathcal{F}_a}(k) \in \mathbb{R}^3$, $[V_h]_{\mathcal{F}_a}(k) \in \mathbb{R}^3$ and $[V_e]_{\mathcal{F}_a}(k-m) \in \mathbb{R}^3$, we, first, deduce the position of the laser in the target's from the measures of χ_{ta} and χ_{la}:

$$\left[\overrightarrow{O_tO_l}\right]_{\mathcal{F}_t}(k) = R_{tl}(k)\left(\left[\overrightarrow{O_aO_l}\right]_{\mathcal{F}_a}(k) - \left[\overrightarrow{O_aO_t}\right]_{\mathcal{F}_a}(k)\right)$$

We record the values of $\left[\overrightarrow{O_tO_l}\right]_{\mathcal{F}_t}$ and proceed to a numerical differentiation of this signal in order to get an approximation of the relative velocity of the laser with respect to the target at the previous breathing cycle, $[V_{lt}]_{\mathcal{F}_t}(k-m)$. Then, we express the approximation of this velocity in $\mathcal{F}_a$: $[V_{lt}]_{\mathcal{F}_a}(k-m) = R_{at}(k-m)\,[V_{lt}]_{\mathcal{F}_t}(k-m)$. This velocity is due to the motion of the target induced by breathing and to the motion of the laser induced by the controller proposed in section III in order to reach the reference. The velocity induced by the controller proposed in section III is $[V_{l0}]_{\mathcal{F}_a}(k-m)$ (see equation (1)). It yields:

$$[V_e]_{\mathcal{F}_a}(k-m) = [V_{lt}]_{\mathcal{F}_a}(k-m) - [V_{l0}]_{\mathcal{F}_a}(k-m)$$

As the motion of the target is supposed to be periodic, we have $[\omega_{t0}]_{\mathcal{F}_a}(k-m) = [\omega_{t0}]_{\mathcal{F}_a}(k)$. Moreover, as $\mathcal{F}_a$ and $\mathcal{F}_0$ are static frames, $[\omega_{t0}]_{\mathcal{F}_a} = [\omega_{ta}]_{\mathcal{F}_a}$. So, we get:

$$\begin{aligned} [V_h]_{\mathcal{F}_a}(k) &= \left[\overrightarrow{O_l(k)O_l(k-m)}\right]_{\mathcal{F}_a} \times [\omega_{t0}]_{\mathcal{F}_a}(k-m) \\ \text{where} \quad & \left[\overrightarrow{O_l(k)O_l(k-m)}\right]_{\mathcal{F}_a} = \\ & \left[\overrightarrow{O_aO_l}\right]_{\mathcal{F}_a}(k-m) - \left[\overrightarrow{O_aO_l}\right]_{\mathcal{F}_a}(k) \end{aligned}$$

We use for $[\omega_{t0}]_{\mathcal{F}_a}(k-m)$ the following numerical approximation:

$$[\omega_{t0}]_{\mathcal{F}_a}(k-m) = [\omega_{ta}]_{\mathcal{F}_a}(k-m) \simeq \frac{1}{T_e}\,\alpha\,[\overrightarrow{v}]_{\mathcal{F}_a}$$

where α and $[\overrightarrow{v}]_{\mathcal{F}_a}$ are respectively the angle and the axis of the rotation from the target's frame at instant $k-m$ to the target's frame at instant $k-m+1$, computed thanks to the records of $\chi_{ta}(k-m)$ and $\chi_{ta}(k-m+1)$.

The relative angular velocity of $\overrightarrow{x_l}$ with respect to the target due to breathing motion at the time instant $k-m$ is:

$$[\omega_e]_{\mathcal{F}_a}(k-m) = [\omega_{lt}]_{\mathcal{F}_a}(k-m) - [\omega_{l0}]_{\mathcal{F}_a}(k-m)$$

where $[\omega_{l0}]_{\mathcal{F}_a}(k-m)$ is the angular velocity of $\overrightarrow{x_l}$ with respect to the base frame of the robot, induced by the control algorithm proposed in section III (see equation (2)). We compute a numerical approximation of $[\omega_{lt}]_{\mathcal{F}_a}(k-m)$, the angular velocity of $\overrightarrow{x_l}$ with respect to $\mathcal{F}_t$, as follows. First, we compute the rotation matrices $R_{tl}(k-m) = R_{ta}(k-m)\,R_{al}(k-m)$ and $R_{tl}(k-m+1) = R_{ta}(k-m+1)\,R_{al}(k-m+1)$. The first row of $R_{tl}(k-m)$ and of $R_{tl}(k-m+1)$ give respectively the vectors $[\overrightarrow{x_l}]_{\mathcal{F}_t}(k-m)$ and $[\overrightarrow{x_l}]_{\mathcal{F}_t}(k-m+1)$. Then, we compute the angle φ and the vector $[\overrightarrow{w}]_{\mathcal{F}_a}$ of the minimal rotation from $[\overrightarrow{x_l}]_{\mathcal{F}_t}(k-m)$ to $[\overrightarrow{x_l}]_{\mathcal{F}_t}(k-m+1)$ and we get the approximation :

$$[\omega_{lt}]_{\mathcal{F}_a}(k-m) \simeq \frac{1}{T_e}\,\varphi\,[\overrightarrow{w}]_{\mathcal{F}_a}$$

Simulations performed without any delay in the control loop lead to a satisfying compensation. However, when introducing, in the simulations, a delay on the control input in order to take into account the computational time required by the controller, the closed-loop becomes unstable. A solution in order to circumvent this problem consists in introducing an anticipation $\gamma > 0$ in the learning algorithm:

$$[p]_{\mathcal{F}_a}(k) = [p]_{\mathcal{F}_a}(k-m) - \beta\,[e]_{\mathcal{F}_a}(k-m+\gamma) + [h]_{\mathcal{F}_a}(k)$$

Simulations performed with $\gamma = 1$ and a delay of $T_e/3$ on the control signal are shown on Fig. 4 and Fig. 5. The sampling period is $T_e = 0.025$ second and the period of the breathing motion is $T_p = 4$ seconds. Thus, we have $m = 160$. The parameters of the controller are $\lambda_v = \lambda_\omega = 1.5$ and $\beta = 0.2$. In this simulation a change in the reference $\left[\overrightarrow{O_tO_l}\right]^d_{\mathcal{F}_t}$ occurs at time 200 seconds and a change in the reference $\overrightarrow{O_lP_l}$ occurs at time 300 seconds. We can see that the closed-loop is stable and that the perturbation induced by breathing is rejected. The oscillations at the beginning of the simulation are due to the learning phase of the algorithm. After the algorithm has learned the adequate control signal for the breathing motions compensation, there are no more oscillations, even when the reference changes. The learning phase is rather long (around 70 seconds) but it has to be performed only once, at the beginning of the operation.

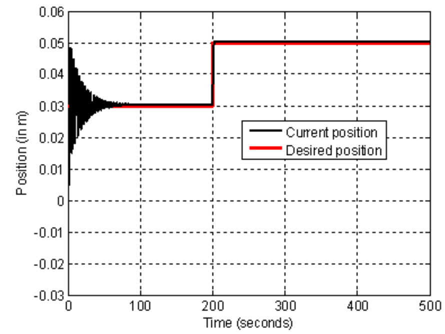

Fig. 4. Evolution of the projection of $[O_tO_l]_{\mathcal{F}_t}$ on y_t in the case of a bone affected by breathing motion

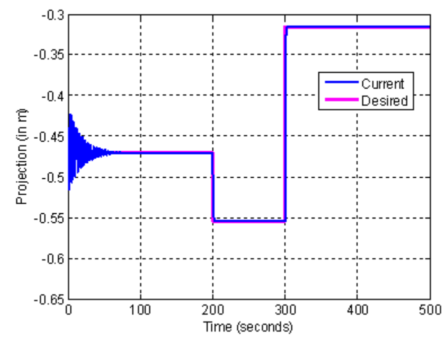

Fig. 5. Evolution of the projection of x_l on y_t in the case of a bone affected by breathing motion

V. EXPERIMENTAL RESULTS

The experiments presented in this section have been performed at the university of Karlsruhe. A Stäubli RX90 robot is used to move the bone in order to simulate physiological motions (see Fig. 6). The lwr robot is used to position

Fig. 6. Initial (left) and first of the desired (right) situations of the robot with respect to the target

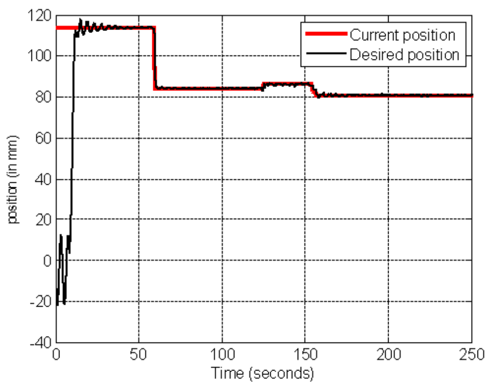

Fig. 7. Experimental results: evolution of the projection of $[O_tO_l]_{\mathcal{F}_t}$ on z_t

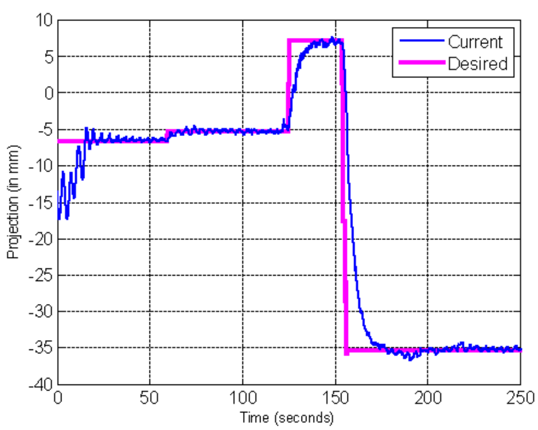

Fig. 8. Experimental results: evolution of the projection of x_l on y_t

the laser equipment and is controlled via visual servoing as explained in the sections above. The target undergoes a sinusoidal translational motion of period $T_p = 4$ seconds and of amplitude 20mm along the vertical axis $\overrightarrow{z}$ represented in Fig. 6 and a sinusoidal rotational motion of period $T_p = 4$ seconds and of amplitude 1 degree along the axis $\overrightarrow{x}$ represented in Fig. 6. The parameters of the control algorithm are the same as in section IV. The evolution of the projection of $\left[\overrightarrow{O_tO_l}\right]_{\mathcal{F}_t}$ on $\overrightarrow{z_t}$ is shown on figure 7. The evolutions of the positions of the laser on the other axis of $\mathcal{F}_t$ are similar and are thus not drawn in this paper. For an evaluation of the orientation error, figure 8 shows the evolution of $\overrightarrow{x_l}$ on $\overrightarrow{y_t}$. The evolutions of the projections of $\overrightarrow{x_l}$ on the other axis of $\mathcal{F}_t$ are similar and are thus not drawn in this paper. We can see that the closed-loop is stable, that the perturbation induced by breathing is rejected and that there is no steady-state error. Moreover, the control algorithm is such that changes in the reference do not require any new learning phase of the perturbation induced by breathing.

VI. CONCLUSIONS AND FUTURE WORKS

This paper has proposed a control algorithm for breathing motions compensation during a robot assisted laser osteotomy. It is composed of a classical position-based visual servoing controller and of a learning algorithm. Simulations and experimental results have shown that this control loop is stable and that there is no steady-state error. The learning phase is a little bit long (there are oscillations during around 70 seconds) but it has to be performed only once, at the beginning of the operation. No other learning phase is required during the operation, even when the reference changes. In the simulations and in the experiments the motion due to breathing is sinusoidal. The proposed learning algorithm makes no assumption on the shape of this motion. Thus, in theory, it must be hable to reject more complex signals, with more high and low frequency components. Future experiments will aim at checking this point. This paper has focused on control and did not explained the registration method. Of course, the registration influences much on the precision of the osteotomy. This influence has to be quantified experimentally. The control algorithm has only be evaluated in laboratory tests and further work is required to show its validity in clinical situation.

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