2010 IEEE International Conference on Robotics and Automation
Anchorage Convention District
May 3-8, 2010, Anchorage, Alaska, USA

Location of optical mouse sensors on mobile robots for odometry

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Abstract— **Optical mouse sensors have been utilized recently to measure position of mobile robots. This work provides a systematic solution to the problem of locating N optical mouse sensors on a mobile robot with the aim of increasing the quality of the measurement. The developed analysis gives insights on how the selection of a particular configuration reflects on the quality of the measurement signal, and it allows to compare the effectiveness of different configurations. The set of all the optimal configurations is parameterized into two constraints. The results are derived from the analysis of the singular values of a particular matrix obtained by solving the sensor kinematics problem. Moreover, given any mobile robot platform, an end-user procedure is provided to select the best location for N optical mouse sensors on such a platform. The procedure consists of solving a feasible constrained optimization problem.**

I. INTRODUCTION

The problem of using optical mice to detect the position of a mobile robot has been recently addressed in literature. The use of optical mice as a dead-reckoning sensor is justified not only for their low cost and high resolution sensing capability, but mostly due to the advantage that the measurements are not dependent on the kinematics of the robot or on the rotation of the robot wheels [1]. Therefore, contrary to other dead-reckoning methods which use incremental encoders on the wheels or on the motor shafts, the measurements from optical mice are not affected by two of the most important sources of measurement errors: 1) slipping, which occurs when a rotation of the traction wheels does not generate a corresponding motion of the robot. Consequently, the measured robot displacement is higher than the actual displacement of the robot, and 2) crawling, which corresponds to a motion of the robot not measurable by incremental encoders.

However, like every dead-reckoning method, the position measurement based on optical mice can be affected by systematic and non-systematic errors [2]. In the case of incremental encoders, systematic errors can be caused by a miscalculation of wheel diameter and distance between wheels; in the case of optical mice, systematic errors could be caused by miscalculation of the exact location and orientation of the mice on the robot, and imprecise knowledge of mouse resolution. Causes of non-systematic measurement errors include imperfection of the ground surface which may result in incorrect measurement in the case of optical mice.

Even though the use of two optical mice to detect the position of the robot results in a significant reduction of measurement error with respect to the classic dead-reckoning method based on incremental encoders [3], several strategies to automatically detect and reduce systematic and non-systematic measurement errors have been developed in [2], [3] and [4]. In particular, the solutions provided in [3] and [4] are obtained with the use of a redundant number of optical mice. The measurements from the redundant set of mice are utilized to minimize certain cost functions formed from the holonomic constraints of the robot. As a result, it has been shown that the use of additional optical mice is beneficial to efficiently minimize measurement errors.

Considering that the location of the optical mice affects the measurement of the robot position [5], the questions that this work addresses are: 1) does the orientation of the mouse optical sensors affect the measurement quality? 2) what is the best location of the optical mice on the robot? and 3) is such a location unique? An attempt to answer these questions was considered in [5] for the case of only two optical mice. In particular, the notion of absolute deviation of a function is utilized in [5] to determine the best location of the optical mice on the robot. However, even in the case of just two optical mice, the high number of unknown parameters does not allow to express the solution in a closed form, and to compare different configurations. Moreover, the analysis in [5] does not allow to determine whether the orientation of the mice is important to minimize the measurement errors. One of the contributions of this work consists of providing a systematic analysis to answer the above questions, not just in the case of two mice, but also in the more general case of N optical mice (where N can be any positive integer). The effect of the orientation of the mice on the quality of the measurements is also determined. Moreover, the developed analysis, which makes use of singular values, provides insights on how the change in location of optical mice affects measurement quality. In this work, a systematic procedure is provided to locate N mice on any robot platform. The solution of this procedure can be obtained by solving a constrained optimization problem. The developed analysis can also be applied to other dead-reckoning sensors.

This work is organized as follows. The working principle of a laser mouse sensor is described in Section II. The sensor kinematics is discussed in Section III. In particular, the robot absolute position and orientation are expressed as functions of the measured variables from the mouse sensors. Section IV shows the singular value analysis and the procedure to obtain the best location of N sensors on a mobile robot platform. Conclusion and future work are given in Section V.

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II. Working Principle of optical mice

The working principle of a laser mouse sensor is illustrated in Fig. 1. A laser diode (LD) projects a light source on the surface where the mouse is moving. The reflection and diffusion of the incident light by the surface generates a "speckle pattern" (see [6] for additional details). The sensor in the mouse detects the speckle pattern and stores the information in 2-D arrays of cells, usually referred to as frames. Consecutive frames, which are acquired at a fixed rate, are compared by the integrated DSP chip to determine the motion of the mouse and the corresponding direction. The sensor output consists of increments of position along the longitudinal and transverse directions. Therefore, any displacement due to a pure translational motion can be easily detected by the sensor. However, the optical mouse sensor is not able to detect pure rotations about the sensor itself. This is due to the fact that the displacement is computed by the DSP chip of the mouse by translation (not rotation) and superposition of the current frame with past frames. Therefore, as shown in the next Section, the use of more than one optical sensor has to be considered if it is desired to measure also the angular position.

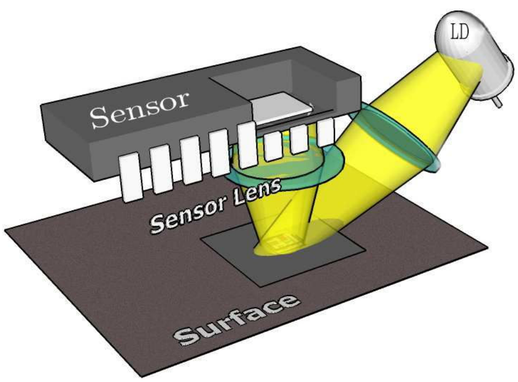

Fig. 1: Working principle of a generic optical mouse [7].

III. SENSOR KINEMATICS

Solving the sensor kinematics problem is important to determine the linear and angular absolute position of the robot with respect to the measurements from the mice. The kinematic equations obtained in this Section will be utilized in Section IV to determine the best location for N optical mouse sensors. To facilitate understanding, the sensor kinematics problem for only one mouse sensor is first discussed, followed by generalization to the N mice case.

Consider the robot shown in Fig. 2, where three coordinate frames are considered to describe the robot motion. In particular, the subscripts A, R and S are utilized to refer, respectively, to the absolute frame placed at the fixed point O_A, the robot frame placed at the geometric center O_R of the robot, and the sensor frame placed at a generic point O_S on the robot platform. In the following we will assume that the X_R axis is always aligned with the axis of the wheels, so that the Y_R axis always points toward the forward direction of robot motion. The orientation of the robot corresponds to the angle θ formed by the axes X_A and X_R, and positive rotations are given by the right-hand rule. We will denote with (r, ψ) the polar coordinates of the sensor position O_S with respect to the robot frame, and with the angle ϕ the fixed orientation of the sensor with respect to the robot frame (i.e., the angle formed by the axis X_R and X_S). In the following, the superscripts A, R and S will be utilized for each vector to denote the frame with respect to which the vector itself is expressed.

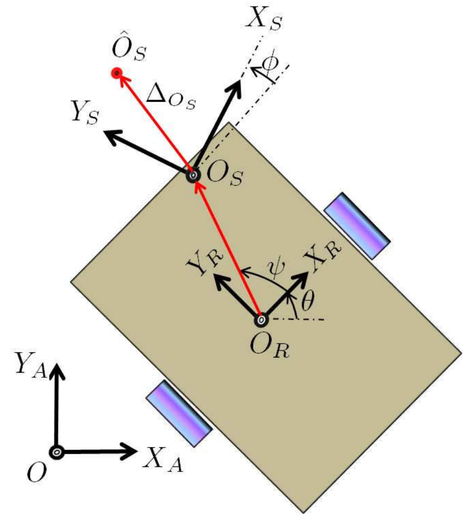

Fig. 2: Sensor kinematics: coordinate axis.

The absolute position of the robot, O_R^A, can be expressed as a function of the absolute position of the sensor, O_S^A, by the expression

$$O_S^A = O_R^A + \mathcal{R}(\theta) O_S^R \tag{1}$$

where $\mathcal{R}(\theta)$ is an isomorphic transformation that rotates vectors in the $X_A - Y_A$ plane counterclockwise by an angle of θ. By taking the time-derivative of (1), and considering that the position of the sensor with respect to the robot does not change (i.e., $O_S^R = [r\cos(\psi), r\sin(\psi)]^T$ is constant), the absolute velocity vector of the robot, $v_{O_R}^A$, and of the sensor, $v_{O_S}^A$, can be related by the expression

$$\begin{aligned} v_{O_S}^A &= v_{O_R}^A + \mathcal{S}(\omega)\mathcal{R}(\theta) O_S^R \\ &= v_{O_R}^A + \|\omega\|_2 \begin{bmatrix} -r\cos(\theta+\psi) \\ r\sin(\theta+\psi) \end{bmatrix} \end{aligned} \tag{2}$$

where $\|\omega\|_2 = d\theta/dt$, and $\mathcal{S}(\omega)$ is a 2×2 skew-symmetric matrix such that $d\mathcal{R}(\theta)/dt = \mathcal{S}(\omega)\mathcal{R}(\theta)$. Considering that the output of the optical sensor updates at a certain rate depending on the sensor characteristics, we can rewrite (2) in the form

$$\Delta_{O_S}^A = \Delta_{O_R}^A + \Delta\theta \begin{bmatrix} -r\sin(\theta+\psi) \\ r\cos(\theta+\psi) \end{bmatrix} \tag{3}$$

where $\Delta\theta$ denotes the increment of angular position of the robot, and the symbol $\Delta_{\mathcal{P}}^{\mathcal{F}}$ denotes the increment of motion of the point $\mathcal{P}$ with respect to the coordinate frame $\mathcal{F}$.

Considering that $\Delta^A_{O_S} = \mathcal{R}(\theta+\phi)\Delta^S_{O_S}$, where $\Delta^S_{O_S}$ is the measurement from the sensor, (3) can be rewritten as

$$\mathcal{R}(\theta+\phi)\Delta^S_{O_S} = \Delta^A_{O_R} + \Delta\theta \begin{bmatrix} -r\sin(\theta+\psi) \\ r\cos(\theta+\psi) \end{bmatrix} \tag{4}$$

Equation (4) relates the robot linear and angular absolute motion (i.e., $\Delta^A_{O_R}$ and $\Delta\theta$) to the sensor measurements $\Delta^S_{O_S}$. Since it is desired to express the absolute motion of the robot as function of the sensor measurements, equation (4) can be rewritten in the more convenient form

$$F(r,\theta,\psi)u = b(\theta,\phi,\Delta^S_{O_S}) \tag{5}$$

where

$$F \triangleq \begin{bmatrix} 1 & 0 & -r\sin(\theta+\psi) \\ 0 & 1 & r\cos(\theta+\psi) \end{bmatrix}, \quad u \triangleq \begin{bmatrix} \Delta^A_{O_R,x} \\ \Delta^A_{O_R,y} \\ \Delta\theta \end{bmatrix},$$

$$b \triangleq \mathcal{R}(\theta+\phi)\Delta^S_{O_S}$$

From (5) it is clear that to obtain a unique solution u for a given value of $\left(r,\theta,\psi,\phi,\Delta^S_{O_S}\right)$, the matrix F must have a left pseudo-inverse $F^+ \triangleq (F^TF)^{-1}F^T$, i.e., F must have full column rank for every value of r, θ and ψ. However, this is not possible because of the dimension of F. Therefore, the use of a second mouse sensor is strictly necessary. If two mouse sensors are utilized, equation (5) can be rewritten with F and b given by

$$F \triangleq \begin{bmatrix} 1 & 0 & -r_1\sin(\theta+\psi_1) \\ 0 & 1 & r_1\cos(\theta+\psi_1) \\ 1 & 0 & -r_2\sin(\theta+\psi_2) \\ 0 & 1 & r_2\cos(\theta+\psi_2) \end{bmatrix} \tag{6}$$

$$b \triangleq \begin{bmatrix} \mathcal{R}(\theta+\phi_1) & 0 \\ 0 & \mathcal{R}(\theta+\phi_2) \end{bmatrix} \begin{bmatrix} \Delta^{S_1}_{O_{S_1}} \\ \Delta^{S_2}_{O_{S_2}} \end{bmatrix}$$

where the subscript $i = 1, 2$ refer to the first and second mouse sensor. It is evident that the matrix F, in the case of two sensors, has full column rank for every value of (r_i, θ_i, ψ_i), $i = 1, 2$, as long as the two mice are not located at the same point.

For the general case of N mice, the matrices F and b take the form

$$F \triangleq \begin{bmatrix} 1 & 0 & -r_1\sin(\theta+\psi_1) \\ 0 & 1 & r_1\cos(\theta+\psi_1) \\ \vdots & \vdots & \vdots \\ 1 & 0 & -r_N\sin(\theta+\psi_N) \\ 0 & 1 & r_N\cos(\theta+\psi_N) \end{bmatrix} \tag{7}$$

$$b \triangleq diag\left\{\mathcal{R}(\theta+\phi_1),\ldots,\mathcal{R}(\theta+\phi_N)\right\} \begin{bmatrix} \Delta^{S_1}_{O_{S_1}} \\ \vdots \\ \Delta^{S_N}_{O_{S_N}} \end{bmatrix}$$

IV. SENSOR PLACEMENT

The selection of the location of the optical sensors on the robot is a very important step in the process of building a mobile robot. An incorrect location of the sensors can result in partial or total inability to detect robot displacements. For instance, if two mouse sensors are utilized and placed in the vicinity of the robot geometrical center, even though the matrix F in (5) has full column rank, a large rotation of the robot about its center results in a small (in a 2-norm sense) output from the sensors. If the measurements from the sensors are also affected by systematic or non-systematic errors, the magnitude of the measurement error would become comparable to the measured robot displacement, causing loss of information and inability to measure the actual robot position. Therefore, it is important to maximize the quality of the feedback measurements by properly placing the sensors in such a way that any small motion of the robot results in a large measurement signal from the sensors.

The criterion utilized in this section to determine the best location for mouse sensors on the robot is based on a singular value analysis. Given the system $Fu = b$, like the one in (5), the singular values of the real matrix F correspond to the length of the semi-axes of the hyperellipsoid E defined by

$$E \triangleq \left\{b : b = Fu, u \in \mathbb{C}^3, \|u\| = 1\right\}$$

Considering that

$$\|b\| = \left\| \begin{bmatrix} \Delta^{S_1}_{O_{S_1}} \\ \vdots \\ \Delta^{S_N}_{O_{S_N}} \end{bmatrix} \right\| \tag{8}$$

the singular values of F can be utilized as an index of how large the measurements from the sensors are for a small displacement of the robot. Therefore, the problem of maximizing the magnitude of the sensor measurements can be converted into the problem of maximizing the smallest singular value of F. The variables of the minimization problem are the polar coordinates (r_i, ψ_i) of the i^{th} sensor, $i = 1, \ldots, N$. It is important to point out that, since the matrix F in (7) does not depend on sensor orientation ϕ_i, $i = 1, \ldots, N$, the magnitude of the measurements is not affected by sensor orientation. Therefore, the simplest choice, $\phi_i = 0$, $i = 1, \ldots, N$, can be made.

In the following we develop the singular value analysis for the matrix F which will provide criteria for selecting the location of the sensors on the robot platform. The two mice case is considered first to highlight the ideas followed by the general case of N mice.

A. Two-mice case

In the case of two optical mouse sensors, the matrix F is given by (6). The square of the singular values of F correspond to the eigenvalues of the square matrix

$$F^TF = \begin{bmatrix} 2 & 0 & p \\ 0 & 2 & q \\ p & q & r_1^2 + r_2^2 \end{bmatrix}$$

where $p \triangleq -r_1\sin(\theta+\psi_1) - r_2\sin(\theta+\psi_2)$ and $q \triangleq r_1\cos(\theta+\psi_1) + r_2\cos(\theta+\psi_2)$. Therefore, the singular values of F, denoted as σ_i ($i = 1, 2, 3$), are given by

$$\sigma_1^2 = \frac{1}{2}\left(r_1^2 + r_2^2 + 2\right) + \frac{1}{2}\sqrt{\left(r_1^2 + r_2^2\right)^2 + 4 + 8 r_1 r_2 \cos(\tilde{\psi})}$$
$$\sigma_2^2 = 2$$
$$\sigma_3^2 = \frac{1}{2}\left(r_1^2 + r_2^2 + 2\right) - \frac{1}{2}\sqrt{\left(r_1^2 + r_2^2\right)^2 + 4 + 8 r_1 r_2 \cos(\tilde{\psi})}$$

where $\tilde{\psi} \triangleq \psi_1 - \psi_2$. As expected, the singular values of F do not depend on the robot orientation θ. It is important to notice that the singular values of F also do not depend on the individual polar coordinates ψ_i of the sensors, but just on their relative angular position $\tilde{\psi}$. It is straightforward to show that $0 \leq \sigma_3^2 \leq \sigma_2^2 \leq \sigma_1^2$, and therefore, σ_3 is the smallest singular value. In fact, by inspection

$$\min_{r_1, r_2, \tilde{\psi}} \sigma_1^2 = 2, \quad \max_{r_1, r_2, \tilde{\psi}} \sigma_1^2 = \infty$$
$$\min_{r_1, r_2, \tilde{\psi}} \sigma_3^2 = 0, \quad \max_{r_1, r_2, \tilde{\psi}} \sigma_3^2 = 2$$

The problem consists of choosing $(r_1, r_2, \tilde{\psi})$ to maximize σ_3^2 and possibly also σ_1^2. It is straightforward to show that σ_3^2 attains its maximum value for $r_1 = r_2$ and $\psi = \pi$. Moreover, it is possible to see by inspection that σ_1^2 is a monotonically increasing function of the variables r_1 and r_2. Therefore, the solution of the optimization problem is obtained by selecting $r_1 = r_2$ as large as possible (depending on the shape of the robot platform) and $\tilde{\psi} = \pi$. In other words, the two sensors have to be placed on diametrically opposite sides of the robot with respect to its center, and as far as possible from the center of the robot.

B. *N-mice case*

The strategy adopted to solve the problem in the N-mice case consists of first finding closed-form expressions for the singular values of F, and then in determining the properties of those singular values. The results obtained in this Section can be utilized to prove what has been shown in the two-mice case (i.e., for $N = 2$).

Lemma 4.1: In the N-mice case, the singular values, σ_i ($i = 1, 2, 3$), of the matrix F are given by

$$\sigma_{1,3}^2 = \frac{1}{2}\left(N + \sum_{i=1}^{N} r_i^2\right) \pm \frac{1}{2}\left(\left(N - \sum_{i=1}^{N} r_i^2\right)^2 + 4\sum_{i=1}^{N} r_i^2 + 8\sum_{i=1}^{N}\sum_{j=i+1}^{N} r_i r_j \cos(\tilde{\psi}_{i,j})\right)^{1/2} \tag{9}$$

$$\sigma_2^2 = N \tag{10}$$

where $\tilde{\psi}_{i,j} \triangleq \psi_i - \psi_j$.

Proof: In the N-mice case, the matrix F is given by (7). Therefore, $F^T F$ has the form

$$F^T F = \begin{bmatrix} N & 0 & p_N \\ 0 & N & q_N \\ p_N & q_N & \sum_{i=1}^{N} r_i^2 \end{bmatrix}$$

where p_N and q_N are given by

$$p_N \triangleq -\sum_{i=1}^{N} r_i \sin(\theta + \psi_i) \tag{11}$$

$$q_N \triangleq \sum_{i=1}^{N} r_i \cos(\theta + \psi_i) \tag{12}$$

The characteristic polynomial of $F^T F$ is given by

$$(\lambda - N)\left((\lambda - N)\left(\lambda - \sum_{i=1}^{N} r_i^2\right) - q_N^2 - p_N^2\right) = 0$$

Therefore, the value of one of the singular values of F is identically equal to $\sqrt{N}$. We will refer to that singular value as σ_2. The remaining two eigenvalues of the matrix $F^T F$ are given by

$$\lambda_{1,3} = \frac{1}{2}\left(N + \sum_{i=1}^{N} r_i^2\right) \pm \frac{1}{2}\left(\left(N + \sum_{i=1}^{N} r_i^2\right)^2 + 4p_N^2 + 4q_N^2 - 4N\sum_{i=1}^{N} r_i^2\right)^{1/2}$$

The result is proved by considering that

$$p_N^2 + q_N^2 = \sum_{i=1}^{N} r_i^2 + 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} r_i r_j \cos(\tilde{\psi}_{i,j}) \tag{13}$$

Equation (13) can be proved by induction. The case of $N = 2$ can be easily verified. Assuming that (13) is true, in the $(N+1)$-case equation (13) becomes:

$$\begin{aligned} p_{N+1}^2 + q_{N+1}^2 &= \sum_{i=1}^{N+1} r_i^2 + 2\sum_{i=1}^{N}\sum_{j=i+1}^{N+1} r_i r_j \cos(\tilde{\psi}_{i,j}) \\ &= \sum_{i=1}^{N} r_i^2 + 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} r_i r_j \cos(\tilde{\psi}_{i,j}) \\ &\quad + r_{N+1}^2 + 2\sum_{i=1}^{N} r_i r_{N+1} \cos(\tilde{\psi}_{i,N+1}) \end{aligned} \tag{14}$$

Therefore, by subtracting (13) from (14), we get

$$(p_{N+1}^2 + q_{N+1}^2) - (p_N^2 + q_N^2) = r_{N+1}^2 + 2\sum_{i=1}^{N} r_i r_{N+1} \cos(\tilde{\psi}_{i,N+1}) \tag{15}$$

By using the definitions (11) and (12) of p_N and q_N, respectively, it is possible to verify that

$$\begin{aligned}(p_{N+1}^2 + q_{N+1}^2) &- (p_N^2 + q_N^2)\\ &= r_{N+1}^2 \sin^2(\theta + \psi_{N+1}) + r_{N+1}^2 \cos^2(\theta + \psi_{N+1})\\ &\quad + 2\sum_{i=1}^{N} r_i r_{N+1} \sin(\theta + \psi_{N+1}) \sin(\theta + \psi_i)\\ &\quad + 2\sum_{i=1}^{N} r_i r_{N+1} \cos(\theta + \psi_{N+1}) \cos(\theta + \psi_i)\\ &= r_{N+1}^2 + 2\sum_{i=1}^{N} r_i r_{N+1} \cos(\tilde{\psi}_{i,N+1}) \quad (16)\end{aligned}$$

Since (15) equals (16), the validity of (13) is proved. ∎

The properties of the singular values of F are given in the following theorem. These properties will be utilized to determine the best location of N mice on the robot platform.

Theorem 4.1: The following statements hold

(a) $\min \sigma_1^2 = \max \sigma_3^2 = N$.

(b) The necessary and sufficient conditions for achieving the maximum value of σ_3^2 are that the parameters r_i and $\tilde{\psi}_{i,j}$ (with $i, j = 1, \ldots, N$) satisfy the following two expressions:

$$\sum_{i=1}^{N} r_i^2 \geq N \quad (17)$$

$$\sum_{i=1}^{N} r_i^2 = -2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} r_i r_j \cos(\tilde{\psi}_{i,j}) \quad (18)$$

(c) A particular solution of (17) and (18) is given by $r_i = r_j \geq 1$, and $\tilde{\psi}_{i,j} = (j - i)2\pi/N$ (with $i, j = 1, \ldots, N$).

(d) If $\sigma_3^2 = N$, σ_1^2 is a monotonically increasing function of the variables r_i, $i = 1, \ldots, N$.

Proof: Statement (a). By inspection, and considering that by definition of singular values both σ_1^2 and σ_3^2 must be real quantities, it is clear from (9) that $\sigma_1^2 \geq \sigma_3^2$. It is also possible to verify that $\sigma_1^2 = N$ if $r_i = 0\ \forall i = 1, \ldots, N$. Therefore, to prove the statement (a) it is sufficient to show the existence of a set of parameters $(r_i^\star, \tilde{\psi}_{i,j}^\star)$, with $i, j = 1, \ldots, N$, such that $\sigma_3^2 = N$.

Assuming that

$$\sum_{i=1}^{N} r_i^2 \geq N \quad (19)$$

it is possible to show from (9) that the following statements are equivalent:

$$\sigma_3^2 = N \quad (20)$$

$$\left(N - \sum_{i=1}^{N} r_i^2\right)^2 = \left(N - \sum_{i=1}^{N} r_i^2\right)^2 + 4\left(p_N^2 + q_N^2\right) \quad (21)$$

$$\sum_{i=1}^{N} r_i^2 = -2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} r_i r_j \cos(\tilde{\psi}_{i,j}) \quad (22)$$

This equivalence is given by the fact that, in order for σ_3^2 to be equal to N, the second term of (9) must be equal to $-(N - \sum_{i=1}^{N} r_i^2)$. In turn, this requires (19) to be satisfied in order to have $\sigma_1^2 \geq \sigma_3^2$. Therefore, if the square root term in (9) has to be equal to $-(N - \sum_{i=1}^{N} r_i^2)$, the quantity inside the square root must be equal to $(N - \sum_{i=1}^{N} r_i^2)^2$. Hence (21) follows. The equivalence between (21) and (22) is obtained by using (13).

It is now possible to show that a particular solution of (22) is given by $r_i = r_j$ and $\tilde{\psi}_{i,j} = (j - i)2\pi/N$ (with $i, j = 1, \ldots, N$). To do that, it is necessary and sufficient to prove that such a particular solution satisfies

$$\sum_{i=1}^{N} r_i^2 + 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} r_i r_j \cos(\tilde{\psi}_{i,j}) = 0 \quad (23)$$

By substituting $r_i = r_j$ and $\tilde{\psi}_{i,j} = (j - i)2\pi/N$ in (23), the problem can be reduced to showing that

$$\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} \cos((j - i)2\pi/N) = -N/2 \quad (24)$$

Since the terms involved in the above summation correspond to all the non-zero entries of the $N \times N$ matrix

$$\begin{bmatrix} 0 & \cos(\frac{2\pi}{N}) & \cos(\frac{4\pi}{N}) & \cos(\frac{6\pi}{N}) & \ldots & \cos((N-1)\frac{2\pi}{N}) \\ 0 & 0 & \cos(\frac{2\pi}{N}) & \cos(\frac{4\pi}{N}) & \ldots & \cos((N-2)\frac{2\pi}{N}) \\ 0 & 0 & 0 & \cos(\frac{2\pi}{N}) & \ldots & \cos((N-3)\frac{2\pi}{N}) \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & 0 & \ldots & \cos(\frac{2\pi}{N}) \\ 0 & 0 & 0 & 0 & \ldots & 0 \end{bmatrix}$$

equation (24) is equivalent to

$$\sum_{i=1}^{N-1} (N - i)\cos(2\pi i/N) = -N/2 \quad (25)$$

Equation (25) can be proved by considering that $\sum_{i=0}^{N-1} \cos(\frac{2\pi i}{N}) = 0$ and $\sum_{i=0}^{N-1} i\cos(\frac{2\pi i}{N}) = -N/2$. Since r_i (with $i = 1, \ldots, N$) can be chosen in order to satisfy the initial assumption (19), $r_i = r_j$ and $\tilde{\psi}_{i,j} = (j - i)2\pi/N$ (with $i, j = 1, \ldots, N$) is a solution of (22), and this ends the proof of statement (a). Therefore, in the following we will refer to σ_1 and σ_3, respectively, as the largest and the smallest singular value of F.

Statement (c). Follows from the proof of statement (a).

Statement (b). Sufficiency. If (17) and (18) are satisfied, the maximum singular value σ_1 and the minimum singular value σ_3 have the form

$$\sigma_1^2 = \frac{1}{2}\left(N + \sum_{i=1}^{N} r_i^2\right) - \frac{1}{2}\left(N - \sum_{i=1}^{N} r_i^2\right) = \sum_{i=1}^{N} r_i^2 \geq N$$

$$\sigma_3^2 = \frac{1}{2}\left(N + \sum_{i=1}^{N} r_i^2\right) + \frac{1}{2}\left(N - \sum_{i=1}^{N} r_i^2\right) = N$$

Statement (b). Necessity. Let us assume, by contradiction, that there exists a set of parameters (r_i, ψ_i), $i = 1, \ldots, N$, such that $\sigma_3^2 = N$ under the hypothesis that (17) is not satisfied and (18) is satisfied. If (18) is satisfied $p_N^2 + q_N^2 = 0$, and the maximum and minimum singular values in (9) become

$$\sigma_1^2 = \frac{1}{2}\left(N + \sum_{i=1}^{N} r_i^2\right) + \frac{1}{2}\left(N - \sum_{i=1}^{N} r_i^2\right) = N$$
$$\sigma_3^2 = \frac{1}{2}\left(N + \sum_{i=1}^{N} r_i^2\right) - \frac{1}{2}\left(N - \sum_{i=1}^{N} r_i^2\right) = \sum_{i=1}^{N} r_i^2$$

Therefore, $\sigma_3^2 < N$, which contradicts the initial statement.

Let us assume, yet by contradiction, that there exists a set of parameters (r_i, ψ_i), $i = 1, \dots, N$, such that $\sigma_3^2 = N$ under the hypothesis that (17) is satisfied and (18) is not satisfied. This implies that $p_N^2 + q_N^2 = c$, where $c \in \mathbb{R}$ and $c > 0$. Moreover, it is possible to rewrite (17) as

$$\sum_{i=1}^{N} r_i^2 = N + d, \quad d \geq 0$$

Therefore, the maximum and minimum singular values in (9) can be rewritten as

$$\sigma_1^2 = N + \frac{1}{2}\left(d + \sqrt{d^2 + c}\right) > N$$
$$\sigma_3^2 = N + \frac{1}{2}\left(d - \sqrt{d^2 + c}\right) < N$$

which contradicts the hypothesis that σ_3^2 can attain the value N. Therefore, (17) and (18) are necessary to guarantee that σ_3^2 can achieve a value equal to N.

Statement (d). Follows from the proof of the sufficiency of statement (b). ■

Theorem 4.1 provides all the information needed to determine the optimal location of the optical mouse sensors on the robot platform. The best location of the sensors is considered as the one that first maximizes the minimum singular value, σ_3, of the matrix F, and then maximizes the largest singular value σ_1. The results obtained so far are summarized in the following optimization problem.

Theorem 4.2: Given a robot platform with contour represented by the function $f(r, \psi) = 0$, expressed in the polar coordinate (r, ψ) with respect to the robot frame, such that every point $(r^\star, \psi^\star)$ on the platform satisfies the inequality $f(r^\star, \psi^\star) \leq 0$, the best location for the optical sensors on the robot platform can be found by solving the optimization problem

$$\max_{(r_1, \psi_1, \dots, r_N, \psi_N)} \sum_{i=1}^{N} r_i^2 \tag{26}$$

subject to the constraints

$$\sum_{i=1}^{N} r_i^2 \geq N$$
$$\sum_{i=1}^{N} r_i^2 = -2 \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} r_i r_j \cos(\psi_i - \psi_j)$$
$$f(r_i, \psi_i) \leq 0, \quad i = 1, \dots, N$$

Proof: Follows from Theorem 4.1. ■

Remark 4.1: It is possible to show, with the use of Theorem 4.2, that statement (c) and (d) of Theorem 4.1 provide the solution to the problem of locating N sensors if the robot platform has a circular shape with radius $r \geq 1$. In fact, in such a case, the best location of the mice is given by $r_i = r$ and $\tilde{\psi}_{i,j} = (j - i)2\pi/N$ (with $i, j = 1, \dots, N$). This corresponds to locating all the sensors on the outer radius, r, of the platform, each separated by an angle of $2\pi/N$ from the next one.

Remark 4.2: As a consequence of Theorem 4.1, the values of the three singular values of the matrix F increase with the number of sensors on the robot platform. Therefore, the quality of the measurements of the robot displacements also increase with the number of sensors utilized as long as the constraints in Theorem 4.2 are satisfied.

V. CONCLUSIONS AND FUTURE WORK

In this paper we have developed systematic criteria for the placement of optical mouse sensors on mobile robot platforms. The development was facilitated by singular value analysis of a particular matrix which is obtained from the sensor kinematics. We provided a systematic procedure to properly locate N sensors on the robot platform. This procedure requires solving a constrained optimization problem where the constraints force the solution to maximize the smallest singular value of F. The cost function to maximize is related to the value of the largest singular value of F, and corresponds to the distance of each of the sensors from the geometrical center of the robot. Additional constraints are utilized to force the solution to stay inside the perimeter of the robot platform. Further, experimental validation is also being considered for multiple mobile robot coordination problems.

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