2010 IEEE International Conference on Robotics and Automation
Anchorage Convention District
May 3-8, 2010, Anchorage, Alaska, USA

Robust Adaptive Formation Control of Fully Actuated Marine Vessels Using Local Potential Functions

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Abstract— **We study the problem of formation control and trajectory tracking for a group of fully actuated marine vehicles, in the presence of uncertainties and unknown disturbances. The objective is to achieve and maintain desired formation tracking, and guarantee no collision between the marine vehicles. The control development relies on existing potential functions which fall at a minimum value when the vehicles reach the desired formation, and blow up to infinity when the vehicles approach collision. The combination of the potential functions, backstepping and variable structure based design technique allows us to handle time varying disturbances by ensuring a stable formation. Using the sliding-Backstepping technique and Lyapunov synthesis, a stable coordination tracking controller is designed. Uniform boundedness of the closed loop signals system is achieved.**

I. Introduction

The use of marine vehicles for various missions has received growing attention in the last decade. Apart from the obvious advantage of not placing human life at risk, the lack of a human pilot enables significant weight savings and lower costs. Marine vehicles also provide an opportunity for new operational paradigms. To realize these advantages, this vehicles must have a high level of autonomy and preferably work cooperatively in groups. Exchanging information within these groups can greatly improve their capability. In this context, a concentrated research effort has been conducted in recent few years to develop novel cooperative control algorithms. The basic idea is that multi-marine vehicle systems can perform tasks more efficiently than a single vehicle or can accomplish tasks not executable by a single one, it can be a considered as a concept for the emergence of new capabilities.

A. Previous Work

Formation control is an important aspect in the coordination of multiple marine vehicles, it essentially involves two control problems: The Trajectory Tracking (TT) [3] or Path Following (PF) [4] and the formation maintaining problem. For the (TT) or (PF), one vehicle or a specific shape of the group is required to track desired locations relative to one or more reference points that can be either stationary or moving virtual marine vehicles. While for formation maintaining, the configuration of the group should converge at some desired geometric pattern, which either can be fixed by the relative positions among the the vehicles or maps to some values of a given functions (e.g. artificial potential functions) [18].

Several type of formation controllers have been suggested that enable a prescribed group behavior. Although the early focus was on centralized approaches, the emphasis today is on decentralized and distributed control to ensure computational efficiency, robustness to communication loss etc. Three prevailing approaches to formation control have been widespread used: leader-following, behavior and virtual structure approach. In the leader-following approach [22], [6], some vehicles are considered as leaders to follow, only those leaders are responsible for guiding the formation and are required to track a given trajectory or to follow a given path. The control objective behind this approach is to make the follower vehicles track the leaders with some prescribed offsets. In [14] the authors developed a new framework to leader-follower synchronization output feedback control scheme for the ship replenishment problem, Breivik et al [5], developed a guided formation control scheme for fully-actuated ships formation by means of a modular design procedure inspired by concepts from integrator backstepping and cascade theory.

Behavior-based approaches, have been widely studied for control of multiple vehicles [2], a number of desired behaviors like goal tracking and obstacle avoidance are assigned for each vehicle and the formation control is obtained from a weighted summation of each behavior output. In [1] an application of the Null-Space-Based behavioral control to a fleet of marine surface vessel was presented in which the vessels move in formation while avoiding collisions with environmental obstacles.

In the virtual structure approach [16], the motion of the vehicles in formation is treated as a rigid body that evolves in the workspace. The desired states of a single vehicle, may be specified such that the formation moves as a single structure. In this scheme it is easy to assign a certain behavior for the cooperating vehicles so that formation is kept maintained during the mission, given that the single vehicle is able to follow its trajectory. However, the extent of this approach, considerably limits the scope of application of the multi-vehicle formation, since the shape of the virtual structure cannot be changed or reconfigured. In [10] the authors applied the virtual structure approach to control a fleet of underactuated surface vessels, the conventional virtual structure approach is modified to make the formation shape varies during the manoeuvre.

Recently, there has been a surge of interest among control scientists in artificial potential functions as in [15]. The main

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978-1-4244-5040-4/10/$26.00 ©2010 IEEE

feature of such potential functions is that they are used to drive the vehicles to configurations away from the undesired space of disconnected networks, while avoiding collisions with each other. The solution does not mention how connectivity is preserved in the presence of obstacles. Tanner et al. [11], [12] present a formation using inter-vehicle potentials and detailed study on the resulting formation stability. A set of control laws is presented that give rise to formation behavior and provide a system theoretic justification, by combining results from classical control theory, mechanics and algebraical graph theory. Navigation in a space full of obstacles was not considered, nor the impact that any obstacle may have on the formation behavior. In [17] the authors proposed a schooling scheme for a group of underactuated AUVs based artificial potential function similar to [19] that guarantees local minimum of the vehicles formation, and the group's velocity and orientation matching in terms of polar coordinates, while keeping obstacle avoidance.

B. Main Contribution

In this work we propose to extend the methodology of tracking a desired trajectory for fully actuated marine vehicles [21] to cooperative control of multiple marine vehicles, in the presence of uncertain and unknown disturbances to force a group of N marine vehicles to perform desired formation tracking, and guarantee no collisions between the vessels. Our goal is to combine local artificial potential function from [8] and an alternative backstepping technique in combination with variable structure control to derive decentralized control laws coordinating a group of marine vehicles subject to time-varying disturbances and which dynamics' parameters are uncertain.

The organization of the paper is as follows: Section II details the problem statement for the formation tracking control of a group of marine vehicles. Section III presents the proposed control considering the parameters' model uncertainties of the marine vehicles that are subject unknown environmental disturbances. Computer simulations of the proposed formation tracking control algorithm are shown in Section IV to demonstrate the effectiveness of our approach. Conclusion and future work are provided in Section V.

II. PROBLEM STATEMENT

A. Marine vehicle dynamics

We consider a group of N fully-actuated marine vehicles. The mathematical of each marine vehicle in the group moving in horizontal plan is described as [9]:

$$\begin{aligned} \dot{\eta}_i &= \mathbf{J}_i(\eta_i)\,\nu_i \\ \mathbf{M}_i\dot{\nu}_i + \mathbf{C}_i(\nu_i)\,\nu_i + \mathbf{D}_i(\nu_i)\,\nu_i &= \tau_i + \tau_{di}(\eta_i,\nu_i,t) \end{aligned} \quad (1)$$

where $i = 1,\ldots,N$, $\eta_i = [x_i, y_i, \psi_i]^T$ are the Earth-frame positions and heading respectively; $\nu_i = [u_i, v_i, r_i]^T$ are the i-th vessel-frame surge, sway, and yaw velocities, respectively; M_i is 3×3 inertia matrix, $C_i(\nu_i)$ is 3×3 matrix of centrifugal and Coriolis terms, $D_i(\nu_i)$ is 3×3 dissipative matrix of hydrodynamic damping terms, all these terms are unknown, $\tau_{di}(\eta_i,\nu_i,t) \in \mathbb{R}^3$ denotes the unknown disturbance from the environment, and $\tau \in \mathbb{R}^3$ is the vector of input signals.

Remark 1: Note that the vector disturbance term $\tau_{di}(\eta_i,\nu_i,t)$ is dependent of time and internal states of the ith vessels, ν_i, η_i. To simplify the control design and the stability analysis, the following assumption will be useful in the sequel analysis.

Assumption 1: Given a continuous function $\tau^k_{di}(\eta_i,\nu_i,t) : \mathbb{R}^3\times\mathbb{R}^3\times\mathbb{R}\to\mathbb{R}$, $k = 1,2,3$, there exist positive, smooth, nondecreasing functions $\chi^k_i(\eta_i,\nu_i) : \mathbb{R}^3\times\mathbb{R}^3\to\mathbb{R}^+$ such that

$$|\tau^k_{di}(\eta_i,\nu_i,t)| \le \chi^k_i(\eta_i,\nu_i)$$

B. Formation control objective

Our objective in this work is to make the entire group of marine vehicles move along a desired trajectory to form a desired formation shape while avoiding collision with all other marine vehicles in the group. We assume that at initial time $t_0 \ge 0$, each marine vehicle is positioned at a given location, the reference trajectory to be tracked by each marine vehicle is generated by

$$\begin{aligned} p_{di}(t) &= p_{od}(t) + l_i \\ \psi_{di} &= \arctan\left(\frac{y'_{od}}{x'_{od}}\right) \end{aligned} \quad (2)$$

where $(\bullet)'$ denotes the partial derivative of $(\bullet)$ with respect to the common trajectory parameter θ_{od} [8], ψ_{di} is the desired heading to be tracked by all the vehicles. $p_{id} = [x_{id}, y_{id}]^\top$ and $p_{od} = [x_{od}, y_{od}]^\top$ is referred to as the common reference trajectory such that there exists constants κ_1, κ_2, we have $\|\dot{p}_{od}(t)\| \le \kappa_1$, and $\|\ddot{p}_{od}(t)\| \le \kappa_2$, this means that the desired trajectory must be sufficiently smooth to avoid actuator saturation induced by the chattering of tracking error due to discontinuous command inputs. The parameter l_i is a constant vector that specifies the configuration of each marine vehicle in its group and satisfies:

$$\|l_i - l_j\| \ge \kappa_3, \qquad \forall (i,j) \in (1,2,\ldots,N), i \ne j \quad (3)$$

where κ_3 is a strictly positive constant. Design the control input τ_i for marine vehicle i such that each vehicle asymptotically converges to its trajectory η_{di} with a specified formation shape while avoiding collisions with all other vehicles in the group. Formally, this could be written as follows

$$\lim_{t\to\infty} \|\eta_i - \eta_{di}\| = 0, \quad \|\eta_i - \eta_j\| \ge \kappa_4 \quad (4)$$

III. FORMATION CONTROL DESIGN

In this section, we employ sliding-based adaptive backstepping of the marine vehicle dynamic to track adaptively a bounded reference signal η_{id}, which is smooth and has bounded derivatives as mentioned before, in the presence of unknown dynamic parameters and time varying disturbances $\tau_{di}(\eta_i,\nu_i,t)$.

Step 1) Define the error variables $z_{1i} = \eta_i - \eta_{di}$ and $z_{2i} = \nu_i - \alpha_{1i}$, and consider the Lyapunov function candidate

$V_1 = U_{z_1} + U_{ob}$, where U_{z_1} is the attractive potential between the marine vehicles and their trajectory, written as

$$U_{z_1} = 0.5 \sum_{i=1}^{N} z_{1i}^\top z_{1i} \tag{5}$$

U_{ob} reflects the collision avoidance behavior, and should be chosen such that it is equal to infinity whenever any vehicle comes in contact with another vehicle and becomes minimum when vehicle i approaches its trajectory with respect to other group members belong to N_i, where N_i is the set of all vehicles in the group that does not contain vehicle i. One example of such potential function is as given in [8]

$$\begin{aligned} U_{ob} &= 0.5 \sum_{i=1}^{N} \sum_{j \in N_i} U_{ob,ij} \\ U_{ob,ij} &= \frac{U_{ij}}{U_{ijl}^2} + \frac{1}{U_{ij}} \end{aligned} \tag{6}$$

U_{ij} and U_{ijl} are collision and desired collision functions chosen as

$$U_{ij} = 0.5(\eta_i - \eta_j)^\top (\eta_i - \eta_j), \qquad U_{ijl} = 0.5\|l_i - l_j\|^2$$

At each time instant, each marine vehicle moves along the gradient of the potential function V_1 given as

$$\begin{aligned} \dot{V}_1 &= \sum_{i=1}^{N} z_{1i}^\top \Big(\mathbf{J}_i(\eta_i)(z_{2i} + \alpha_{1i}) - \dot{\eta}_{di} \Big) \\ &\quad + \sum_{i=1}^{N} \sum_{j \in N_i} U_{ij}^{'} (\eta_i - \eta_j)^\top \Big(\mathbf{J}_i(\eta_i)(z_{2i} + \alpha_{1i}) \\ &\quad - \dot{\eta}_{di} - \Big(\mathbf{J}_j(\eta_j)(z_{2j} + \alpha_{1j}) - \dot{\eta}_{dj} \Big) \Big) \\ &= \sum_{i=1}^{N} \Big(z_{1i}^\top + \sum_{j \in N_i} U_{ob,ij}^{'} \eta_{ij}^\top \Big) \Big(\mathbf{J}_i(\eta_i)(z_{2i} + \alpha_{1i}) - \dot{\eta}_{di} \Big) \\ &= \sum_{i=1}^{N} \Psi_i^\top \Big(\mathbf{J}_i(\eta_i)(z_{2i} + \alpha_{1i}) - \dot{\eta}_{di} \Big) \end{aligned} \tag{7}$$

where $\eta_{ij} = \eta_i - \eta_j$, $U_{ob,ij}^{'} = \frac{\partial U_{ob,ij}}{\partial \eta_{ij}}$ and Ψ_i is defined as

$$\Psi_i = z_{1i}^\top + \sum_{j \in N_i} U_{ob,ij}^{'} \eta_{ij}^\top \tag{8}$$

Noting the property $\mathbf{J}_i(\eta_i)\mathbf{J}_i(\eta_i)^\top = \mathbf{I}$, leads to the choice of the virtual control as

$$\alpha_{1i} = \mathbf{J}_i(\eta_i)^\top (-K_1 \Psi_i + \dot{\eta}_{di}) \tag{9}$$

Now substituting (9) into (7) results in

$$\dot{V}_1 = -\sum_{i=1}^{N} \Psi_i^\top K_1 \Psi_i + \sum_{i=1}^{N} \Psi_i^\top \mathbf{J}_i(\eta_i) z_{2i} \tag{10}$$

where $K_1 = K_1^\top > 0$. The first term on the right is stable, and the second term $\Psi_i^\top \mathbf{J}(\eta_i) z_{2i}$ will be addressed in the next step of the backstepping procedure. The η_i-dynamic in closed loop system with the virtual control α_{1i} being chosen as in (9), is given as

$$\dot{\eta}_i = -K_1 \Psi_i + \dot{\eta}_{di} + \mathbf{J}_i(\eta_i) z_{2i} \tag{11}$$

The closed loop subsystem (11) will be used in the stability analysis in next section.

Step 2) Differentiating z_{2i} with respect to time yields

$$\begin{aligned} \dot{z}_{2i} &= \dot{\nu}_i - \dot{\alpha}_{1i} = \mathbf{M}_i^{-1}(-C_i(\nu_i)\nu_i - D(\nu_i)\nu_i + \tau_i \\ &\quad + \tau_{di}(\nu_i, \eta_i, t)) - \dot{\alpha}_{1i} \end{aligned} \tag{12}$$

where $\dot{\alpha}_{1i} = \dot{\mathbf{J}}_i^\top(\eta_i)(-K_1\Psi_i + \dot{\eta}_{di}) + \mathbf{J}_i^\top(\eta_i)(\ddot{\eta}_{di} - K_1(\frac{\partial \Psi_i}{\partial \eta_i}\dot{\eta}_i + \sum_{j \in N_i} \frac{\partial \Psi_i}{\partial \eta_{ij}} \dot{\eta}_{ij}))$. Consider the following Lyapunov function candidate:

$$V_2^* = V_1 + 0.5 \sum_{i=1}^{N} \sigma_i^\top \mathbf{M}_i \sigma_i \tag{13}$$

where the sliding surface is defined as

$$\sigma_i = \delta_i \Psi_i + z_{2i} \tag{14}$$

where $\delta_i > 0$ and $\sigma_i = [\sigma_{1i}, \sigma_{2i}, \sigma_{3i}]^\top$. The time derivative of V_2^* is given by:

$$\dot{V}_2^* = -\sum_{i=1}^{N} \Psi_i^\top K_1 \Psi_i + \sum_{i=1}^{N} \Psi_i^\top \mathbf{J}_i(\eta_i) z_{2i} + \sum_{i=1}^{N} \sigma_i^\top \mathbf{M}_i \dot{\sigma}_i \tag{15}$$

Considering (10), (12) and Assumption 1, substituting (14) into (15) yields

$$\begin{aligned} \dot{V}_2^* &= -\sum_{i=1}^{N} \Psi_i^\top \Big(K_1 + \delta_i \mathbf{J}_i^\top(\eta_i) \Big) \Psi_i + \sum_{i=1}^{N} \Psi_i^\top \mathbf{J}_i(\eta_i) \sigma_i \\ &\quad + \sum_{i=1}^{N} \sigma_i^\top \mathbf{M}_i \Big(\delta_i (\frac{\partial \Psi_i}{\partial \eta_i} \dot{\eta}_i + \frac{\partial \Psi_i}{\partial \eta_{di}} \dot{\eta}_{di} \\ &\quad + \sum_{j \in N_i} \frac{\partial \Psi_i}{\partial \eta_{ij}} \dot{\eta}_{ij}) + \dot{z}_{2i} \Big) \\ &\leq -\sum_{i=1}^{N} \Psi_i^\top \Big(K_1 + \delta_i \mathbf{J}_i^\top(\eta_i) \Big) \Psi_i + \sum_{i=1}^{N} \sigma_i^\top \Big(\mathbf{J}_i(\eta_i)^\top \\ &\quad \times \Psi_i + \delta_i \mathbf{M}_i (\frac{\partial \Psi_i}{\partial \eta_i} \dot{\eta}_i + \frac{\partial \Psi_i}{\partial \eta_{di}} \dot{\eta}_{di} + \sum_{j \in N_i} \frac{\partial \Psi_i}{\partial \eta_{ij}} \dot{\eta}_{ij}) \\ &\quad - \mathbf{C}_i(\nu_i)\nu_i - \mathbf{D}(\nu_i) + \tau_i + \chi_i(\nu_i, \eta_i) - \mathbf{M}_i \dot{\alpha}_{1i} \Big) \end{aligned} \tag{16}$$

To make the time derivative of the candidate Lyapunov function V_2^* negative definite, it is easy to choose a control input τ_i, such that the second right hand side term is negative. However since $\mathbf{M}_i, \mathbf{C}_i(\nu_i), \mathbf{D}_i(\nu_i)$ and $\chi_i(\nu_i, \eta_i)$ are all unknown, a full state feedback control cannot be directly designed. To solve the formation control problem in the presence of parametric modeling uncertainty, we assume that the terms $\mathbf{M}_i, \mathbf{C}_i(\nu_i), \mathbf{D}_i(\nu_i)$ are linear in their parameters. We let $\Phi_i(\eta_i, \nu_i, \dot{\eta}_i)$ a known regressor matrix and $\Theta_i \in \mathbb{R}^{n_\Theta}$

be the vector that contains all the unknown parameters of the unknown term $\rho_i(\eta_i, \nu_i, \dot{\eta}_i)$ defined as

$$\begin{aligned}\rho_i(\eta_i, \nu_i, \dot{\eta}_i) &= \delta_i \mathbf{M}_i(\frac{\partial \Psi_i}{\partial \eta_i}\dot{\eta}_i + \frac{\partial \Psi_i}{\partial \eta_{di}}\dot{\eta}_{di} + \sum_{j\in N_i}\frac{\partial \Psi_i}{\partial \eta_{ij}}\dot{\eta}_{ij}) \\ &\quad -\mathbf{C}_i(\nu_i)\nu_i - \mathbf{D}(\nu_i) + \tau_i - \mathbf{M}_i\dot{\alpha}_{1i} \\ &= \Phi_i(\eta_i, \nu_i, \dot{\eta}_i)\Theta_i \qquad (17)\end{aligned}$$

To design the actual control input vector τ_i we take the Lyapunov function

$$V_2 = V_2^* + 0.5\sum_{i=1}^{N}\tilde{\Theta}_i^\top \Gamma_i^{-1}\tilde{\Theta}_i + \sum_{i=1}^{N}\frac{\varepsilon_i}{\alpha_i}e^{-\alpha_i t} \qquad (18)$$

where $\tilde{\Theta}_i = \Theta_i - \hat{\Theta}_i$ and $\hat{\Theta}_i$ is an estimate of Θ_i, and Γ_i is a symmetric positive definite matrix, ε_i, α_i are some positive constants. Differentiating both sides of (18) along the solutions of (16) yields:

$$\begin{aligned}\dot{V}_2 &\le -\sum_{i=1}^{N}\Psi_i^\top\Big(K_1 + \delta_i\mathbf{J}_i^\top(\eta_i)\Big)\Psi_i + \sum_{i=1}^{N}\sigma_i^\top\Big(\mathbf{J}_i(\eta_i)^\top\Psi_i \\ &\quad \Phi_i(\eta_i, \nu_i, \dot{\eta}_i)\Theta_i + \tau_i + \chi_i(\nu_i, \eta_i)\Big) - \sum_{i=1}^{N}\tilde{\Theta}_i^\top\Gamma_i^{-1}\dot{\hat{\Theta}}_i \\ &\quad -\sum_{i=1}^{N}\varepsilon_i e^{-\alpha_i t} \qquad (19)\end{aligned}$$

which suggests after completing the square that we choose the control law (20), shown at the top of this page, where W_i and K_{σ_i} are symmetric positive definite matrices, must be chosen in a way to reduce the chattering obtained from the discontinuous term, they should be tuned so that the desired performances are attained. Notice that the control τ_i and the update $\dot{\hat{\Theta}}_i$ given in (20) of the marine vehicle i contain only the state and reference trajectory of vessel i and the states of the neighbor vessel j. Now substituting (20) into (19) results in

$$\begin{aligned}\dot{V}_2 &\le -\sum_{i=1}^{N}\Psi_i^\top\Big(K_1 + \delta_i\mathbf{J}_i^\top(\eta_i)\Big)\Psi_i - \sum_{i=1}^{N}\sigma_i^\top W_i\sigma_i \\ &\quad -\sum_{i=1}^{N}\sigma_i^\top K_{\sigma_i}\sigma_i\tanh(\sigma_i) \qquad (21)\end{aligned}$$

With the control law τ_i and the update law (20), we write the closed loop system that comprises equation (11), the dynamic of σ_i and the second equation of (20) as follows:

$$\begin{aligned}\dot{\eta}_i &= -K_1\Psi_i + \dot{\eta}_{di} + \mathbf{J}_i(\eta_i)z_{2i} \\ \mathbf{M}_i\dot{\sigma}_i &= -W_i\sigma_i - K_{\sigma_i}\tanh(\sigma_i) - \mathbf{J}_i(\eta_i)^\top\Psi_i + \Phi_i\tilde{\Theta}_i \\ &\quad +\tau_{di}(\eta_i, \nu_i, t) - \frac{1}{4\varepsilon_i}\chi_i\chi_i^\top\sigma_i e^{\alpha_i t} \\ \dot{\hat{\Theta}}_i &= -\Gamma_i\Phi_i^\top\sigma_i \qquad (22)\end{aligned}$$

We now state the main result of this paper in the following theorem.

Theorem 1: Under Assumption 1, the control τ_i and the parameter update law $\dot{\hat{\Theta}}_i$ given in (20) for the i-th marine vehicle solve the formation control objective. In particular, no collision between any vehicles can take place for all $t \ge t_0 > 0$, the position and orientation of each marine vehicle track their desired reference trajectories asymptotically.

Proof: The proof of the theorem follows the same line as in [7] and [8]. The proof unfolds in two steps. At the first step, we show that there is no collision between marine vehicles and that the closed loop system (22) is forward complete. At the second step we prove that the equilibrium point of the inter-vessels dynamics closed loop system (22), at which $\eta_i - \eta_j = 0$ is asymptotically stable and show that the position and orientation of the marine vehicles asymptotically converge to their reference trajectories.

- *Proof of No collision and forward completeness:* From (21) it is clear that $\dot{V}_2 \le 0$ which implies that for all $t \ge t_0 \ge 0$, we have $V_2(t) \le V_2(t_0)$, with the definition of the potential function V_2 in (18), we have

$$\begin{aligned}&\sum_{i=1}^{N}\Big(U_{z_1}(t) + 0.5\sum_{j\in N_i}U_{ob,ij}(t) + 0.5\sigma_i(t)^\top\mathbf{M}_i\sigma_i(t) \\ &\quad + 0.5(\tilde{\Theta}_i(t) - \hat{\Theta}_i(t))^\top\Gamma_i^{-1}(\tilde{\Theta}_i(t) - \hat{\Theta}_i(t)) \\ &\quad + \frac{\varepsilon_i}{\alpha_i}e^{-\alpha_i t}\Big) \le \Big(U_{z_1}(t_0) + 0.5\sum_{j\in N_i}U_{ob,ij}(t_0) \\ &\quad + 0.5\sigma_i(t_0)^\top\mathbf{M}_i\sigma_i(t_0) + 0.5(\tilde{\Theta}_i(t_0) - \hat{\Theta}_i(t_0))^\top\Gamma_i^{-1} \\ &\quad \times(\tilde{\Theta}_i(t_0) - \hat{\Theta}_i(t_0)) + \frac{\varepsilon_i}{\alpha_i}e^{-\alpha_i t_0}\Big) \qquad (23)\end{aligned}$$

  We force each marine vehicle to start at $t = t_0$ at different locations, this implies that there exists a positive constant κ_5 such that $\|\eta_i(t_0) - \eta_j(t_0)\| \ge \kappa_5$, and therefore $\sum_{j\in N_i}U_{ob,ij}(t_0)$ is smaller than a positive constant. With the definition of σ_i, the right hand side of (23) is bounded, the boundedness of (23) also implies that of the left hand side of (23), as a result $U_{ob,ij}(t)$ is smaller than a positive constant that depends on the initial conditions for all $t \ge t_0 \ge 0$, therefore, there exists a positive constant κ_4 such that the second condition of (4) is satisfied, this means that there is no collisions between marine vehicles for all $t \ge t_0 \ge 0$. Boundedness of the left hand side of (23) also implies that of $\eta_i - \eta_{di}, \sigma_i, \hat{\Theta}_i$ for all $t \ge t_0 \ge 0$. This implies that η_i, ν_i do not escape to infinity in finite time. Consequently the closed loop system (22) is forward complete.
- *Equilibrium points*: We have shown that the closed loop system (22) is forward complete and that the states $\eta_i - \eta_{di}, \sigma_i$ and Θ_i are bounded, since V_2 is a continuous differentiable function and its differentiation along the the solutions of the closed loop system (22) is negative. Then an application of Theorem 8.4 in [13] to (21)

$$\tau_i = -W_i\sigma_i - K_{\sigma_i}\tanh(\sigma_i) - \mathbf{J}_i(\eta_i)^\top\Psi_i - \Phi_i\hat{\Theta}_i - \frac{1}{4\varepsilon_i}\chi_i\chi_i^\top\sigma_i e^{\alpha_i t}$$
$$\dot{\hat{\Theta}}_i = \Gamma_i\Phi_i^\top\sigma_i \quad (20)$$

yields

$$\lim_{t\to\infty} \Big(\sum_{i=1}^{N}\Psi_i^\top\Big(K_1+\delta_i\mathbf{J}_i^\top(\eta_i)\Big)\Psi_i + \sum_{i=1}^{N}\sigma_i^\top W_i\sigma_i + \sum_{i=1}^{N}\sigma_i^\top K_{\sigma_i}\tanh(\sigma_i)\Big) = 0 \quad (24)$$

This implies that

$$\lim_{t\to\infty}\Psi_i = 0, \qquad \lim_{t\to\infty}\sigma_i = 0, \qquad \lim_{t\to\infty} z_{2i} = 0 \quad (25)$$

From the definition of ψ, the first limit in (25) means

$$\lim_{t\to\infty}\Big((\eta_i(t)-\eta_{di}(t))^\top + \sum_{j\in N_i} U'_{ob,ij}\eta_{ij}^\top\Big) = 0 \quad (26)$$

It has to be noted that when η_i and η_j converge to their trajectories (i.e., $p_i = p_{od}+l_i$ and $p_j = p_{od}+l_j$) the term $U'_{ob,ij} = 0$, therefore the limit equation (25) implies that $\bar{\eta}_i = \eta_i - \eta_{di}$ may converges to zero or to some other limit $\bar{l}_i$ as time goes to infinity. Let us denote by $\eta = [\eta_1^\top,\ldots,\eta_i^\top,\ldots,\eta_N^\top]^\top$, $\eta_d = [\eta_{d1}^\top,\ldots,\eta_{di}^\top,\ldots,\eta_{dN}^\top]^\top$ and by $\bar{l} = [\bar{l}_1^\top,\ldots,\bar{l}_i^\top,\ldots,\bar{l}_N^\top]^\top$, the vector $\eta(t)$ can tend either to η_d or to $\bar{l}$ as time goes to infinity. To analyze the nature of the equilibrium η_d and $\bar{l}$, we follow [8] through analyzing the first equation of the closed loop system (22), which in a vector form can be written as

$$\dot{\eta} = -\Lambda\Psi(\eta,\eta_d) + \dot{\eta}_d + \Omega \quad (27)$$

where $\Lambda = \mathrm{diag}(K_1,\ldots,K_1), \Omega = [\mathbf{J}_1(\eta_1)z_{21},\ldots,\mathbf{J}_i(\eta_i)z_{2i},\ldots,\mathbf{J}_N(\eta_N)z_{2N}]^\top$. Near an equilibrium point η_e which can be either η_d or $\bar{l}$, we have

$$\dot{\eta} = -\Lambda\frac{\partial\Psi}{\partial\eta}\mid_{\eta=\eta_e}(\eta-\eta_e) + \dot{\eta}_d + \Omega \quad (28)$$

It can be checked that (see [8] for more details on calculations of those terms)

$$\frac{\partial\Psi_i}{\partial\eta_i}\mid_{\eta=\eta_d} = I_n + \sum_{j\in N_i} U''_{ob,ijd}\eta_{ijd}\eta_{ijd}^\top$$
$$\frac{\partial\Psi_j}{\partial\eta_j}\mid_{\eta=\eta_d} = -U''_{ob,ijd}\eta_{ijd}\eta_{ijd}^\top \quad (29)$$

where I_n is the identity matrix of dimension n, $U''_{ob,ijd} = U''_{ob,ij}\mid_{\eta_{ij}=\eta_i-\eta_{di}-\eta_j+\eta_{dj}}$. To instigate the properties of the equilibrium $\eta_e = \eta_d$, consider the following Lyapunov function candidate

$$V_{deq} = 0.5(\eta-\eta_e)^\top(\eta-\eta_e) \quad (30)$$

whose derivative along the solutions of (28) satisfies

$$\dot{V}_{deq} = -\sum_{i=1}^{N}(\eta_i-\eta_{di})^\top K_1(\eta_i-\eta_{di}) - \sum_{N_i} U''_{ob,ijd}\eta_{ijd}^\top(\eta_{ij}-\eta_{ijd})^\top(\eta_{ij}-\eta_{ijd})\eta_{ijd} + \sum_{i=1}^{N}(\eta_i-\eta_{di})^\top\mathbf{J}_i(\eta_i)z_{2i} \quad (31)$$

It can be checked that $U''_{ob,ijd} \geq 0$, using the Young's inequality we obtain

$$\dot{V}_{deq} \leq -\sum_{i=1}^{N}(\eta_i-\eta_{di})^\top(K_1-\rho_i I_n)(\eta_i-\eta_{di}) + \sum_{i=1}^{N}\frac{1}{4\rho_i}\|\Delta_i\|^2 \quad (32)$$

where $\Delta_i = \mathbf{J}_i(\eta_i)z_{2i}$ and ρ_i is a positive constant such that $K_1-\rho_i I_n > 0$. Since we have already shown that z_{2i} converges to zero as time goes to infinity so does Δ_i, therefore η_d is asymptotically stable. Next, we will show that the remaining equilibrium points $\bar{l}$ of the subsystem, first equation of (22) are unstable equilibrium points. Define $\Psi_{ij\bar{l}} = \Psi_{i\bar{l}} - \Psi_{j\bar{l}}, , \forall(i,j)\in\{1,\ldots,N\}$ where $\Psi_{i\bar{l}} = \Psi_i \mid \eta_i = \bar{l} = 0$, therefore $\Psi_{ij\bar{l}} = 0$. Consequently we have

$$\sum_{i,j\in\mathbb{N}^*}\eta_{ij\bar{l}}\Psi_{ij\bar{l}} = 0$$
$$\Rightarrow \sum_{i,j\in\mathbb{N}^*}\Big(\eta_{ij\bar{l}}^\top(\bar{l}_{ij}-\eta_{ijd}) + NU'_{ob,ij\bar{l}}\bar{l}_{ij}^\top\bar{l}_{ij}\Big) = 0$$
$$\Rightarrow \sum_{i,j\in\mathbb{N}^*}(1+NU'_{ob,ij\bar{l}})\bar{l}_{ij}^\top\bar{l}_{ij} = \sum_{i,j\in\mathbb{N}^*}\bar{l}_{ij}\eta_{ijd}$$

The term $\sum_{i,j\in\mathbb{N}^*}\bar{l}_{ij}\eta_{ijd}$ is strictly negative, since at $\eta_{ij} = \bar{l}_{ij}$, vehicle i and j are lying along a straight line between η_{id} and η_{jd}. That is the point $\eta_{ij} = 0$ is in between η_{ijd} and $\bar{l}_{ij}$ such that the three points are collinear. Thus there exists a strictly positive constant β such that $\sum_{i,j\in\mathbb{N}^*}\bar{l}_{ij}\eta_{ijd} \leq -\beta$. Since the term $\bar{l}_{ij}^\top\bar{l}_{ij} > 0, \forall(i,j)$, then there exists at least one pair (i,j) denoted (i^*,j^*) such that there exists a strictly positive constant $\bar{\beta}$ such that

$$(1+NU'_{ob,i^*j^*\bar{l}}) \leq -\bar{\beta} \quad (33)$$

In the subsequent analysis, we will consider the following Lyapunov function candidate

$$V_{\bar{l}eq} = 0.5(\bar{\eta}-\bar{\mathbf{l}})^\top(\bar{\eta}-\bar{\mathbf{l}}) \quad (34)$$

where we define $\bar{\eta} = [\eta_{12}^\top, \eta_{13} \ldots, \eta_{ij}^\top, \ldots, \eta_{N-1,N}^\top]^\top$ and $\bar{\mathbf{l}} = [\bar{l}_{12}^\top, \bar{l}_{13} \ldots, \bar{l}_{ij}^\top, \ldots, \bar{l}_{N-1,N}^\top]^\top$. The time derivative of (34) along the solutions of (27) satisfies

$$
\begin{aligned}
\dot{V}_{\bar{l}eq} =& - \sum_{i,j \in \mathbb{N}^*} (\eta_{ij} - \bar{l}_{ij})^\top K_1 \Big(1 + NU'_{ob,ij\bar{l}} + NU''_{ob,ij\bar{l}} \\
& \times \eta_{ij\bar{l}}^\top \eta_{ij\bar{l}}\Big)(\eta_{ij} - \bar{l}_{ij}) + \sum_{i,j \in \mathbb{N}^*} (\eta_{ij} - \bar{l}_{ij})^\top (\Omega_i - \Omega_j) \\
& \geq \lambda_{\min}(K_1)(\eta_{i^*j^*} - \bar{l}_{i^*j^*})^\top (\eta_{i^*j^*} - \bar{l}_{i^*j^*}) \\
& - \sum_{i \neq i^*, j \neq j^*} (\eta_{ij} - \bar{l}_{ij})^\top K_1 \Big(1 + NU'_{ob,ij\bar{l}}\Big)(\eta_{ij} - \bar{l}_{ij}) \\
& - \sum_{i \neq i^*, j \neq j^*} (\eta_{ij} - \bar{l}_{ij})^\top \Big(NU''_{ob,ij\bar{l}} \bar{l}_{ij}^\top \bar{l}_{ij}\Big)(\eta_{ij} - \bar{l}_{ij}) \\
& + \sum_{i,j \in \mathbb{N}^*} (\eta_{ij} - \bar{l}_{ij})^\top (\Omega_i - \Omega_j) \qquad (35)
\end{aligned}
$$

Define a subspace such that $\eta_{ij} = \eta_{ij\bar{l}}, \forall (i,j) \in \{1, \ldots, N\}, (i,j) \neq (i^*, j^*)$ and $(\eta_{ij} - \bar{l}_{ij})^\top \bar{l}_{ij}^\top \bar{l}_{ij} (\eta_{ij} - \bar{l}_{ij}) = 0, \forall (i,j) \in \{1, \ldots, N\}$. In this subset, the following holds

$$
\begin{aligned}
V_{\bar{l}eq} &= 0.5(\eta_{i^*j^*} - \bar{l}_{i^*j^*})^\top (\eta_{i^*j^*} - \bar{l}_{i^*j^*}) \\
\dot{V}_{\bar{l}eq} &\geq 2\lambda_{\min}(K_1) V_{\bar{l}eq} \qquad (36)
\end{aligned}
$$

form (36), it is clear that $\dot{V}_{\bar{l}eq}$ will diverge and consequently shows that $\bar{l}$ is unstable. This completes the proof.

■

IV. Simulation results

In this section, we carry out computer simulations to demonstrate the performance of our robust formation control based potential functions. Simulations are performed on four i.e $N = 4$ identical models of Cybership-II with parameters obtained from [20]. The disturbances τ_{di} are time varying forces and moments given as function of η_i and ν_i as:

$$
\begin{aligned}
\tau_{di} &= \mathbf{J}_i^\top (\eta_i) f(t, \eta_i, \nu_i) \\
f(t, \eta_i, \nu_i) &= \Big[\sum_{k=1}^{3} b_k + a_k \sin(c_k t), 0, 0\Big]^\top \qquad (37)
\end{aligned}
$$

with $b_1 = 4, b_2 = b_3 = 0, a_k = 0.5$ and $c_k = 0.2, \forall k =\in$

Fig. 1. 2D animation of the position synchronization

$\{1, 2, 3\}$. The common reference trajectory is taken as $p_{od} = [0.1\sqrt{2}t, 10\sin(0.1t)]^\top$. The desired heading of each marine vehicle to be tracked is $\psi_{di} = \tan^{-1}\left(\frac{0.1\sqrt{2}}{\cos(0.1t)}\right)$ and the desired formation configuration is a parallelepiped. The control gains are $K_1 = \text{diag}(25), W_i = \text{diag}(50), K_{\sigma_i} = \text{diag}(0.5)$ and $\Gamma_i = \text{diag}(10)$. Simulation result for the formation tracking is plotted in Fig.1. It is seen that the marine vehicles nicely track their reference trajectories.For clarity only the tracking error states of the first marine vehicle are plotted in Fig.2 it clear how the error tracking states tend to zero asymptotically. Fig. 3 plots the velocity norm of each vessel which clearly converges to each other when the formation is achieved. The distance between the first marine vehicle and other vessels are plotted in Fig.4, from this figure we conclude that there is no collision may take place with marine vehicle 1.

Fig. 2. Tracking error of the first marine vehicle in formation

Fig. 3. Plot of ν_i over time for four marine vessels

Fig. 4. Distance between the first marine vehicle and the other vessels

V. Conclusion

In this paper, formation tracking control has been designed for a team of surface vessels in the presence of time-varying environmental disturbances, unmodled dynamics. The control law is a combination of sliding mode and local potential function taken from [8], it ensures that all marine vehicles asymptotically approach their desired trajectories, collision between marine vehicles is also ensured. Simulation results have demonstrated that the formation of a team of surface vessels is achieved satisfactorily. Further work is required to extend the methodology proposed to address the problems of robustness against lack of communications and the cost of exchanging information.

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