2010 IEEE International Conference on Robotics and Automation
Anchorage Convention District
May 3-8, 2010, Anchorage, Alaska, USA

A Simple Nonlinear PID Control for Global Finite-Time Regulation of Robot Manipulators Without Velocity Measurements

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Abstract—**A simple nonlinear proportional-integral-derivative (PID) controller is proposed for global finite-time regulation of robot manipulators without velocity measurements. A Lyapunov-based stability argument is employ to prove global finite-time stabilization. The proposed control algorithm does not involve the model parameters in the control law formulation and the control gains can be explicitly determined based on some well-known bounds extracted from the robot dynamics, and thus permits easy implementation. Simulations are included to demonstrate the expected properties of the proposed approach.**

I. INTRODUCTION

In the past several years, robot manipulators have served as an interesting benchmark for the design and test of novel nonlinear control strategies. Regulation of robots maybe recognized as the simplest aim in robot control and at the same time finds its main application in the robotic field. Despite the success of modern control theory, robot manipulator controllers still commonly use classic proportional-derivative (PD) or proportional-integral-derivative (PID) algorithms [1], [2], [5], [17], [18], due mostly to their conceptual simplicity and explicit tuning procedures. Arimoto [3] first showed that a local and independent PID-type servo-loop replacing the linear position error term by a saturated position error, gives rise to global asymptotic stability of regulation control for nonlinear mechanical systems. Motivated by this seminal work, some nonlinear PID control schemes incorporating favorable saturation functions for global asymptotic regulation of robot manipulators were proposed [7], [8], [13], [14], [20], [21]. Other control schemes for global asymptotic regulation of robot manipulators with adaptive control technique can be found in [9], [25], and the references therein.

Most of the existing results so far on regulation of robot manipulators are achieved asymptotically [1]-[3], [7]-[9], [13]-[15], [17]-[21], [25]. Asymptotic stability implies that the system trajectories converge to the equilibrium as time goes to infinity. It is now known that finite-time stabilization of dynamical systems may give rise to a high-precision performance besides finite-time convergence to the equilibrium [6], [10]-[12].

To the best of our knowledge, the only previous work which targets at the finite-time regulation of robot manipulators is given in [12], [22], [23]. Specially, Hong *et al.* [12] formulated a PD plus gravity compensation scheme with a model-based observer and achieved local finite-time result. This result was later extended in Su *et al.* [22] by introducing the "dirty derivative" technique and obtained global finite-time stabilization. A drawback of both approaches is that the exact knowledge of the gravity term which depends on some parameters as mass of the payload, usually uncertain, has to be known. To overcome parametric uncertainties on the gravitational torque vector, Su *et al.* [23] proposed a simple continuous model-independent nonlinear PID control for finite-time regulation of robot manipulators. The control algorithm does not refer to the model parameters, and thus permits easy implementation in practice. The closed-loop system is shown to be semiglobal finite-time stable. One minor weakness for this scheme is that both position and velocity measurements are required.

The control of robot manipulators without velocity measurements is a topic that continuous to challenge control theoreticians. From a theoretical point of view, the challenge lies in the fact that, generally speaking, the separation principle does not hold true for such nonlinear dynamical systems. This topic is also of practical importance since many real-world robot manipulators are not commonly equipped with velocity sensors; hence, full access to the system states is impossible [4], [17], [21], [24].

In this paper the problem of designing a model-independent global finite-time controller for regulation of robot manipulators without velocity measurements is studied. The main contribution of this paper is twofold. A nonlinear filter is proposed to eliminate velocity measurements. A simple nonlinear PID control is developed for global finite-time regulation of robot manipulators with position measurements only. The closed-loop system formed by the nonlinear PID controller, filter and the robot system is shown to be global finite-time stable. The control algorithm does not utilize the

This work was supported in part by the National Natural Science Foundation of China under Grant 50675167, the Foundation for the Author of National Excellent Doctoral Dissertation of China under Grant 200535, NCET, and SRF for ROCS, SEM.

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model parameter in the controller formulation, and thus permits easy implementation in practice. To the best of our knowledge, the proposed approach yields the first model-independent, output feedback, global finite-time regulation for robot manipulators.

II. Robot Manipulator Model and Properties

In the absence of disturbances, the dynamics of an n-DOF robot manipulator can be written as [2], [15]

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + D\dot{q} + g(q) = \tau \tag{1}$$

where $q, \dot{q}, \ddot{q} \in \Re^n$ denote the link position, velocity, and acceleration vectors, respectively, $M(q) \in \Re^{n\times n}$ represents the symmetric inertia matrix, $C(q,\dot{q}) \in \Re^{n\times n}$ denotes the centrifugal-Coriolis matrix, $D \in \Re^{n\times n}$ stands for the constant diagonal static friction matrix, $g(q) = \partial U(q)/\partial q \in \Re^n$ is a gravitational force, $U(q)$ is the potential energy due to gravitational force, and $\tau \in \Re^n$ denotes the torque input vector. Recalling that robot manipulators are being considered, the following properties can be established

Property 1 [2], [15]*:* The static friction matrix D is a diagonal positive definite matrix, i.e.

$$d_1 I \le D \le d_2 I \tag{2}$$

where d_1 and d_2 are known positive constants, and I denotes the approximate dimensional identity matrix.

Property 2 [2], [15]*:* The inertia matrix $M(q)$ is symmetric and positive definite and bounded by

$$0 < \lambda_m(M) \le \|M(q)\| \le \lambda_M(M) \tag{3}$$

where $\lambda_m(\cdot)$ and $\lambda_M(\cdot)$ denote the minimum and maximum eigenvalues of a matrix, respectively.

Property 3 [2], [15]*:* The matrix $\dot{M}(q) - 2C(q,\dot{q})$ is skew-symmetric, i.e.

$$\zeta^T\left(\dot{M}(q) - 2C(q,\dot{q})\right)\zeta = 0, \ \forall \zeta \in \Re^n \tag{4}$$

where $\dot{M}(q)$ is the time derivative of the inertia matrix $M(q)$. Equivalently, we have

$$\dot{M}(q) = C(q,\dot{q}) + C^T(q,\dot{q}) \tag{5}$$

Property 4 [2], [15]*:* The matrix $C(q,\dot{q})$ is bounded by

$$0 < C_m\|\dot{q}\|^2 \le \|C(q,\dot{q})\dot{q}\| \le C_M\|\dot{q}\|^2, \quad \forall\, q, \dot{q} \in \Re^n \tag{6}$$

where C_m and C_M are some known positive constants.

Property 5 [2]*:* There exists a constant positive definite diagonal matrix A such that the following two inequalities, with a specified constant $a > 0$, are satisfied simultaneously for any fixed q_d and any q

$$U(q) - U(q_d) - \Delta q^T g(q_d) + \frac{1}{2}\Delta q^T A \Delta q \ge a\|\Delta q\|^2 \tag{7}$$

$$\Delta q^T[g(q) - g(q_d)] + \Delta q^T A \Delta q \ge a\|\Delta q\|^2 \tag{8}$$

where $\Delta q = q - q_d$ denotes the position error, and q and q_d denote the actual and desired coordinates, respectively.

III. Preliminaries

Some concepts of finite-time stability and stabilization of nonlinear systems, and the properties of homogeneous systems are reviewed, following the treatment in [6], [11], [12].

Consider the system

$$\dot{\zeta} = f(\zeta), \quad f(0) = 0, \quad \zeta(0) = \zeta_0, \quad \zeta \in \Re^n \tag{9}$$

with $f: U_0 \to \Re^n$ continuous on an open neighborhood U_0 of the origin. Suppose that system (9) possesses unique solutions in forward time for all initial conditions. The equilibrium $\zeta = 0$ of system (9) is (locally) finite-time stable if it is Lyapunov stable and finite-time convergent in a neighborhood $U \subset U_0$ of the origin. The finite-time convergence means the existence of a function $T: U\setminus\{0\} \to (0,\infty)$, such that, $\forall\ \zeta_0 \in U \subset \Re^n$, the solution of (9) denoted by $s_t(\zeta_0)$ with ζ_0 as the initial condition is defined and $s_t(\zeta_0) \in U\setminus\{0\}$ for $t \in [0, T(\zeta_0))$, and $\lim_{t\to T(\zeta_0)} s_t(\zeta_0) = 0$ with $s_t(\zeta_0)$ for $t > T(\zeta_0)$. When $U = \Re^n$, we obtain the global finite-time stability.

A scalar function $V(\zeta)$ is homogeneous of degree $\sigma \in \Re$ with $(r_1, \ldots, r_n), r_i > 0, i = 1, \ldots, n$, if for any given $\varepsilon > 0$,

$$V(\varepsilon^{r_1}\zeta_1, \ldots, \varepsilon^{r_n}\zeta_n) = \varepsilon^{\sigma} V(\zeta), \quad \forall \zeta \in \Re^n \tag{10}$$

A continuous vector field $f(\zeta) = [f_1(\zeta), \ldots, f_n(\zeta)]^T$ is homogeneous of degree $\kappa \in \Re$ with $r = (r_1, \ldots, r_n)$, if for any given $\varepsilon > 0$,

$$f_i(\varepsilon^{r_1}\zeta_1, \ldots, \varepsilon^{r_n}\zeta_n) = \varepsilon^{\kappa + r_i} f_i(\zeta), \ i = 1, \ldots, n, \forall \zeta \in \Re^n \tag{11}$$

System (9) is said to be homogeneous if $f(\zeta)$ is homogeneous. Some of results on finite-time stability of a nonlinear system in [11] that will be used in this paper are summarized by the following two lemmas.

Lemma 1: Consider the following system

$$\dot{\zeta} = f(\zeta) + \hat{f}(\zeta), \quad f(0) = 0, \quad \hat{f}(0) = 0, \quad \zeta \in \Re^n \tag{12}$$

where $f(\zeta)$ is a continuous homogeneous vector field of degree $\kappa < 0$ with respect to $(r_1, \ldots, r_n)$. Assume that $\zeta = 0$ is an asymptotically stable equilibrium of the system $\dot{\zeta} = f(\zeta)$. Then $\zeta = 0$ is a locally finite-time stable equilibrium of the system (12) if

$$\lim_{\varepsilon \to 0} \frac{\hat{f}_i(\varepsilon^{r_1}\zeta_1, \ldots, \varepsilon^{r_n}\zeta_n)}{\varepsilon^{\kappa + r_i}} = 0, \quad i = 1, \ldots, n, \forall \zeta \ne 0 \tag{13}$$

Lemma 2: Global asymptotic stability and local finite-time stability of the closed-loop system imply global finite-time stability.

IV. Control Design

A. Control Formulation

Given a desired constant position q_d for nonlinear robot manipulator (1), we consider the finite-time regulation control problem to design the input torque without velocity measurements and without reference to model parameters, such that $\Delta q(t) \to 0$ and $\dot{q}(t) \to 0$ in finite time for any initial state $(q(0), \dot{q}(0))$.

To aid the subsequent control design and analysis, we define the vectors $\mathrm{Tanh}(\cdot), \mathrm{Sig}(\cdot)^{\alpha} \in \Re^{n}$ and the diagonal matrix $\mathrm{Sech}(\cdot) \in \Re^{n\times n}$ as follows:

$$\mathrm{Tanh}(\xi) = [\tanh(\xi_1), \cdots, \tanh(\xi_n)]^T \tag{14}$$

$$\mathrm{Sig}(\xi)^{\alpha} = \left[|\xi_1|^{\alpha}\mathrm{sgn}(\xi_1), \cdots, |\xi_n|^{\alpha}\mathrm{sgn}(\xi_n)\right]^T \tag{15}$$

$$\mathrm{Sech}(\xi) = \mathrm{diag}(\mathrm{sech}(\xi_1), \cdots, \mathrm{sech}(\xi_n)) \tag{16}$$

where $\xi = [\xi_1, \cdots, \xi_n]^T \in \Re^n$, $0 < \alpha < 1$, $\tanh(\cdot)$ and $\mathrm{sech}(\cdot)$ being the standard hyperbolic tangent and secant functions, respectively, $\mathrm{sgn}(\cdot)$ the standard signum function, and $\mathrm{diag}(\cdot)$ denotes a diagonal matrix. Based on the definitions (14)-(16), it can easily be shown that the following expressions hold:

$$\xi^T\mathrm{Sig}(\xi)^{\alpha} \geq \mathrm{Tanh}^T(\xi)\mathrm{Sig}(\xi)^{\alpha} \geq \mathrm{Tanh}^T(\xi)\mathrm{Tanh}(\xi) \tag{17}$$

$$|\xi_i|^{\alpha+1} \geq \tanh^2(\xi_i) \tag{18}$$

$$\lambda_M(\mathrm{Sech}^2(\xi)) = 1 \tag{19}$$

The proposed output feedback nonlinear PID (ONPID) controller is formulated as

$$\tau = -K_p\mathrm{Sig}(\Delta q)^{\alpha_1} - K_i z - K_d\mathrm{Sig}(\upsilon)^{\alpha_2} \tag{20}$$

$$z = \Delta q + \varepsilon\int_0^t \mathrm{Tanh}(\Delta q(\sigma))d\sigma \tag{21}$$

$$\dot{q}_c = -A\mathrm{Sig}(q_c + Bq)^{\alpha_2} \tag{22}$$

$$\upsilon = q_c + Bq \tag{23}$$

where K_p, K_i, K_d, A and B are positive definite constant diagonal control proportional, integral, and derivative matrices, respectively, $0 < \alpha_1 < 1$, $\alpha_2 = 2\alpha_1/(\alpha_1+1)$, and ε is a small positive constant and will be subsequently defined in (27).

Introducing the following vector

$$\phi = z - K_i^{-1}g(q_d) \tag{24}$$

Then substituting (20) and (24) into (1) and using (22) and (23), the closed-loop dynamics is given by

$$\begin{aligned} &M(q)\ddot{q} + C(q,\dot{q})\dot{q} + D\dot{q} + g(q) - g(q_d) \\ &\quad + K_p\mathrm{Sig}(\Delta q)^{\alpha_1} + K_d\mathrm{Sig}(\upsilon)^{\alpha_2} + K_i\phi = 0 \end{aligned} \tag{25}$$

$$\dot{\upsilon} = -A\mathrm{Sig}(\upsilon)^{\alpha_2} + B\dot{q} \tag{26}$$

whose origin $\left[\Delta q^T\ \dot{q}^T\ \upsilon^T\ \phi^T\right]^T = 0 \in \Re^{4n}$ is the unique equilibrium.

B. Stability Analysis

Theorem 1: Given the robotic system (1), with the proposed output feedback nonlinear PID controller (20)-(23), the closed-loop system (25) and (26) is globally finite-time stable, provided the control gains are chosen as follows:

$$\varepsilon \leq \frac{d_1}{\sqrt{n}C_M + \lambda_M(M)} \tag{27}$$

$$k_{pi} > 2(\alpha_1+1)\varepsilon^2\lambda_M(M) \tag{28}$$

$$\begin{aligned} &U(q) - U(q_d) - \Delta q^T g(q_d) \\ &\quad + \frac{1}{2(\alpha_1+1)}\sum_{i=1}^{n}k_{pi}|\Delta q_i|^{\alpha_1+1} > a\|\mathrm{Tanh}(\Delta q)\|^2 \end{aligned} \tag{29}$$

$$\begin{aligned} &\mathrm{Tanh}^T(\Delta q)(g(q) - g(q_d)) + \mathrm{Tanh}^T(\Delta q)K_p\mathrm{Sig}(\Delta q)^{\alpha_1} \\ &\quad > \left(a + \frac{1}{2}\lambda_M(K_d)\right)\|\mathrm{Tanh}(\Delta q)\|^2 \end{aligned} \tag{30}$$

$$\lambda_m(K_d AB^{-1}) > \frac{1}{2}\varepsilon\lambda_M(K_d) \tag{31}$$

where k_{pi} denotes the ith diagonal elements of matrix K_p, a is a small positive constant.

Remark 1: Condition (27) in Theorem 1 is not excessively restrictive and limitative, due to the fact that there always exists friction in practical robot and the friction coefficient matrix D is diagonal positive definite [2], [15], which guarantees that ε always exists and can be selected as so small [21], [23].

Remark 2: Note that inequalities (29) and (30) correspond to inequalities (7) and (8) of Property 5, respectively, and the existence of such a matrix K_p is confirmed by the same argument given in proposing (7), (8), (17), and (18) [2], [23].

Proof: The proof proceeds in the following two steps. First, the semiglobal asymptotic stability is proved with Lyapunov's direct method and LaSalle's invariance principle. Second the finite-time stability is shown using Lemma 1. Finally Lemma 2 is involved to guarantee the semiglobal finite-time stability.

1) Semiglobal asymptotic stability: To this end, we propose the following Lyapunov-like function candidate

$$\begin{aligned} V = &\frac{1}{2}\dot{q}^T M(q)\dot{q} + \varepsilon\mathrm{Tanh}^T(\Delta q)M(q)\dot{q} + U(q) - U(q_d) \\ &- \Delta q^T g(q_d) + \frac{1}{\alpha_1+1}\sum_{i=1}^{n}k_{pi}|\Delta q_i|^{\alpha_1+1} + \varepsilon\sum_{i=1}^{n}d_i\ln(\cosh(\Delta q_i)) \\ &+ \frac{1}{\alpha_2+1}\sum_{i=1}^{n}k_{di}b_i^{-1}|\upsilon_i|^{\alpha_2+1} + \frac{1}{2}\phi^T K_i\phi \end{aligned} \tag{32}$$

where d_i, k_{di}, and b_i denote the ith diagonal elements of matrices D, K_d, and B, respectively. It should be noted that the seventh term of (32) is motivated by the work given in [16].

We first consider the following

$$\begin{aligned} &\frac{1}{4}\dot{q}^T M(q)\dot{q} + \frac{1}{2(\alpha_1+1)}\sum_{i=1}^{n}k_{pi}|\Delta q_i|^{\alpha_1+1} + \varepsilon\mathrm{Tanh}^T(\Delta q)M(q)\dot{q} \\ &= \frac{1}{4}(\dot{q} + 2\varepsilon\mathrm{Tanh}(\Delta q))^T M(q)(\dot{q} + 2\varepsilon\mathrm{Tanh}(\Delta q)) \\ &\quad - \varepsilon^2\mathrm{Tanh}^T(\Delta q)M(q)\mathrm{Tanh}(\Delta q) + \frac{1}{2(\alpha_1+1)}\sum_{i=1}^{n}k_{pi}|\Delta q_i|^{\alpha_1+1} \\ &\geq \frac{1}{2(\alpha_1+1)}\sum_{i=1}^{n}k_{pi}|\Delta q_i|^{\alpha_1+1} - \varepsilon^2\mathrm{Tanh}^T(\Delta q)M(q)\mathrm{Tanh}(\Delta q) \\ &\geq \sum_{i=1}^{n}\left[\frac{k_{pi}}{2(\alpha_1+1)} - \varepsilon^2\lambda_M(M)\right]\tanh^2(\Delta q_i) \end{aligned} \tag{33}$$

where (2) of Property 2 and (18) have been used.

Substituting (33) into (32), we have

$$V \geq \frac{1}{4}\dot{q}^T M(q)\dot{q} + \sum_{i=1}^{n}\left[\frac{k_{pi}}{2(\alpha_1+1)} - \varepsilon^2 \lambda_M(M)\right]\tanh^2(\Delta q_i)$$
$$+ U(q) - U(q_d) - \Delta q^T g(q_d) + \frac{1}{2(\alpha_1+1)}\sum_{i=1}^{n} k_{pi}|\Delta q_i|^{\alpha_1+1}$$
$$+ \varepsilon \sum_{i=1}^{n} d_i \ln(\cosh(\Delta q_i)) + \frac{1}{\alpha_2+1}\sum_{i=1}^{n} k_{di} b_i^{-1}|\upsilon_i|^{\alpha_2+1} + \frac{1}{2}\phi^T K_i \phi \tag{34}$$

From (34), (28), and (29), we get

$$V \geq \frac{1}{4}\dot{q}^T M(q)\dot{q} + a\|\mathrm{Tanh}(\Delta q)\|^2 + \varepsilon \sum_{i=1}^{n} d_i \ln(\cosh(\Delta q_i))$$
$$+ \frac{1}{\alpha_2+1}\sum_{i=1}^{n} k_{di} b_i^{-1}|\upsilon_i|^{\alpha_2+1} + \frac{1}{2}\phi^T K_i \phi > 0 \tag{35}$$

for $\left[\Delta q^T\ \dot{q}^T\ \ \upsilon^T\ \ \phi^T\right]^T \neq 0$.

Hence, we can conclude that V is a positive definite Lyapunov function with respect to $\Delta q, \dot{q}, \upsilon, \phi$.

Differentiating V with respect to time, it follows that

$$\dot{V} = \frac{1}{2}\dot{q}^T \dot{M}(q)\dot{q} + \dot{q}^T M(q)\ddot{q} + \varepsilon(\mathrm{Sech}^2(\Delta q)\Delta\dot{q})^T M(q)\dot{q}$$
$$+ \varepsilon \mathrm{Tanh}^T(\Delta q)\dot{M}(q)\dot{q} + \varepsilon \mathrm{Tanh}^T(\Delta q) M(q)\ddot{q}$$
$$+ \dot{q}^T g(q) - \Delta\dot{q}^T g(q_d) + \Delta\dot{q}^T K_p \mathrm{Sig}(\Delta q)^{\alpha_1}$$
$$+ \varepsilon \mathrm{Tanh}^T(\Delta q) D\dot{q} + \dot{\upsilon}^T K_p \mathrm{Sig}(\upsilon)^{\alpha_2} + \dot{\phi}^T K_i \phi \tag{36}$$

where we have utilized the fact that $\Delta\dot{q} = \dot{q}$ from (8) and the regulation control.

Substituting $M(q)\ddot{q}$ from (25) and $\dot{\phi} = \dot{z} = \Delta\dot{q}(t) + \varepsilon \mathrm{Tanh}(\Delta q)$ from (24), and (26) into (33), and using Property 3, we have

$$\dot{V} = -\mathrm{Sig}^T(\upsilon)^{\alpha_2} K_d A B^{-1} \mathrm{Sig}(\dot{q})^{\alpha_2} - \varepsilon \mathrm{Tanh}^T(\Delta q) K_d \mathrm{Sig}(\upsilon)^{\alpha_2}$$
$$- \dot{q}^T D\dot{q} + \varepsilon\left[\mathrm{Tanh}^T(\Delta q) C^T(q,\dot{q})\dot{q} + (\mathrm{Sech}^2(\Delta q)\Delta\dot{q})^T M(q)\dot{q}\right]$$
$$- \varepsilon\left[\mathrm{Tanh}^T(\Delta q)(g(q) - g(q_d)) + \mathrm{Tanh}^T(\Delta q) K_p \mathrm{Sig}(\Delta q)^{\alpha_1}\right] \tag{37}$$

Using (3) of Property 2 and (6) of Property 4 and (19), the fourth term of the right-hand side of (37) can be upper bounded by

$$\varepsilon\left[\mathrm{Tanh}^T(\Delta q) C^T(q,\dot{q})\dot{q} + (\mathrm{Sech}^2(\Delta q)\Delta\dot{q})^T M(q)\dot{q}\right]$$
$$\leq \varepsilon(\sqrt{n} C_M + \lambda_M(M))\|\dot{q}\|^2 \tag{38}$$

Note that in the derivation of the first term of (38) we utilized $\|\mathrm{Tanh}(\Delta q)\| \leq \sqrt{n}$ according to (14) and $|\tanh(\Delta q)| \leq 1$.

Substituting (30) and (38) into (37), we have

$$\dot{V} \leq -\dot{q}^T D\dot{q} + \varepsilon \lambda_M(K_d)\|\mathrm{Tanh}(\Delta q)\|\left\|\mathrm{Sig}(\dot{q})^{\alpha_2}\right\|$$
$$- \dot{q}^T K_d \mathrm{Sig}(\dot{q})^{\alpha_2} + \varepsilon(\sqrt{n} C_M + \lambda_M(M))\|\dot{q}\|^2$$
$$- \varepsilon\left(a + \frac{1}{2}\lambda_M(K_d)\right)\|\mathrm{Tanh}(\Delta q)\|^2 \tag{39}$$

Applying triangle inequality $bc \leq \frac{1}{2}(b^2 + c^2)$ with $b = \|\mathrm{Tanh}(\Delta q)\|$ and $c = \left\|\mathrm{Sig}(\dot{q})^{\alpha_2}\right\|$ to (39), it follows that

$$\dot{V} \leq -\left[d_1 - \varepsilon(\sqrt{n} C_M + \lambda_M(M))\right]\|\dot{q}\|^2 - \varepsilon a\|\mathrm{Tanh}(\Delta q)\|^2$$
$$- \frac{1}{2}\left[2\lambda_m(K_d A B^{-1}) - \varepsilon \lambda_M(K_d)\right]\left\|\mathrm{Sig}(\upsilon)^{\alpha_2}\right\|^2 \tag{40}$$

From (27), (31), and the fact that a is a small positive constant, we conclude that $\dot{V} \leq 0$. Since $\dot{V} \equiv 0$ means $\mathrm{Tanh}(\Delta q) \equiv 0$, $\dot{q} \equiv 0$ and $\upsilon \equiv 0$. From the definition of hyperbolic tangent function, we have $\Delta q \equiv 0$. Therefore, by LaSalle's invariance theorem [19], we have $\Delta q(t) \to 0$, $\dot{q}(t) \to 0$, $\upsilon(t) \to 0$, and $\phi(t) \to 0$, as $t \to \infty$ for any initial state $(q(0), \dot{q}(0))$. Hence, we have global asymptotic stability about $(\Delta q = 0, \dot{q} = 0, \upsilon = 0, \phi = 0)$.

2) Finite-time stability: Following the idea presented in [12], the local finite-time stability is proved using Lemma 1. To do so, let $x_1 = \Delta q$, $x_2 = \dot{x}_1 = \dot{q}$, $x_3 = \upsilon$, and $x = (x_1^T, x_2^T, x_3^T)^T$. The state equation of the closed-loop system (25) and (26) can be rewritten as

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -M^{-1}(x_1 + q_d)\big[C(x_1 + q_d, x_2)x_2 + Dx_2 + g(x_1 + q_d) \\ \qquad - g(q_d) + K_p \mathrm{Sig}(x_1)^{\alpha_1} + K_d \mathrm{Sig}(x_3)^{\alpha_2} + K_i z(x_1)\big] \\ \dot{x}_3 = -A\mathrm{Sig}(x_3)^{\alpha_2} + Bx_2 \end{cases} \tag{41}$$

Clearly, $x = 0$ is the equilibrium of (41). It can be seen that the closed-loop system (41) is not homogeneous. To use Lemma 1, we rewrite (41) as follows:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -M^{-1}(q_d) K_p \mathrm{Sig}(x_1)^{\alpha_1} + \hat{f}_2(x) \\ \dot{x}_3 = -A\mathrm{Sig}(x_3)^{\alpha_2} + \hat{f}_3(x) \end{cases} \tag{42}$$

with

$$\hat{f}_2 = -M^{-1}(x_1 + q_d)\big[C(x_1 + q_d, x_2)x_2 + Dx_2 + g(x_1 + q_d) - g(q_d)$$
$$+ K_i z(x_1)\big] - \widetilde{M}(x_1, q_d) K_p \mathrm{Sig}(x_1)^{\alpha_1} - M^{-1}(x_1 + q_d) K_d \mathrm{Sig}(x_2)^{\alpha_2} \tag{43}$$

$$\hat{f}_3 = Bx_2 \tag{44}$$

and

$$\widetilde{M}(x_1, q_d) = M^{-1}(x_1 + q_d) - M^{-1}(q_d) \tag{45}$$

It can be easily verified that the following system

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -M^{-1}(q_d) K_p \mathrm{Sig}(x_1)^{\alpha_1} \\ \dot{x}_3 = -A\mathrm{Sig}(x_3)^{\alpha_2} \end{cases} \tag{46}$$

is homogeneous of degree $\kappa = \alpha_2 - 1 < 0$ with respect to $(r_{11}, r_{12}, \ldots, r_{1n}, r_{21}, r_{22}, \ldots, r_{2n}, r_{31}, r_{32}, \ldots, r_{3n})$ with $r_{1i} = r_1 = 2/(\alpha_1 + 1)$, $r_{2i} = r_2 = 1$, and $r_{3i} = r_2 = 1$. Note that $f(0) = 0$ and $\hat{f}(0) = 0$ from (46), and (43) and (44), respectively.

Now we will use Lemma 1 to show the finite-time stability of the closed-loop system (42). To this end, we should show $x = 0$

is the asymptotic equilibrium of (46). Consider the system (46) and take a nonnegative Lyapunov function candidate as follows:

$$V_* = \frac{1}{\alpha_1+1}\sum_{i=1}^{n} k_{pi}\left|x_{1i}\right|^{\alpha_1+1} + \frac{1}{2}x_2^T M(q_d)x_2 + \frac{1}{2}x_3^T x_3 \qquad (47)$$

where x_{1i} denotes the ith component of vector x_1.

After taking the time derivative of (47) along (46), we have

$$\dot{V}_*\big|_{(46)} = \dot{x}_1^T K_p \mathrm{Sig}(x_1)^{\alpha_1} + x_2^T M(q_d)\dot{x}_2 + x_3^T \dot{x}_3 \qquad (48)$$

where we have utilized the fact that $\dot{M}(q_d) = 0$ for regulation control.

Upon substituting (46) into (48), it follows that

$$\dot{V}_*\big|_{(46)} = -x_3^T K_d \mathrm{Sig}(x_3)^{\alpha_2} \qquad (49)$$

From (17), we can conclude that $V_*\big|_{(46)} \le 0$. Since $V_*\big|_{(43)} \equiv 0$ means $x_3 \equiv 0$. By LaSalle's invariant principle [19], we have the conclusion that the equilibrium of the closed-loop system (46) is asymptotically stable.

Next, from the definition of the hyperbolic tangent function and (21), we have

$$z(\varepsilon^{r_1}x_1) = o(\varepsilon^{r_1}) \qquad (50)$$

Together with the fact that $M^{-1}(x_1+q_d)$ and $C(x_1+q_d, x_2)$ are smooth and $\kappa < 0$ [12], [23], we have

$$\begin{aligned}&\lim_{\varepsilon\to 0} -\frac{M^{-1}(\varepsilon^{r_1}x_1+q_d)}{\varepsilon^{\kappa+r_2}}\Big[C(\varepsilon^{r_1}x_1+q_d, \varepsilon^{r_2}x_2)\varepsilon^{r_2}x_2 + D\varepsilon^{r_2}x_2 \\ &\qquad\qquad + g(\varepsilon^{r_1}x_1+q_d) - g(q_d) + K_i z(\varepsilon^{r_1}x_1)\Big] \\ &= -M^{-1}(q_d)\Big[\big(C(q_d,0)+D\big)x_2 \lim_{\varepsilon\to 0}\varepsilon^{-\kappa} + K_i \lim_{\varepsilon\to 0} o(\varepsilon^{r_1-\kappa-r_2})\Big] \\ &= 0\end{aligned} \qquad (51)$$

Applying the mean value theorem to each entry of $\widetilde{M}(x_1, q_d)$, it follows that [12], [23]

$$\widetilde{M}(\varepsilon^{r_1}x_1, q_d) = M^{-1}(\varepsilon^{r_1}x_1+q_d) - M^{-1}(q_d) = o(\varepsilon^{r_1}) \qquad (52)$$

As a result, we have

$$\lim_{\varepsilon\to 0} -\frac{\widetilde{M}(\varepsilon^{r_1}x_1, q_d)K_p \mathrm{Sig}(\varepsilon^{r_1}x_1)^{\alpha_1}}{\varepsilon^{\kappa+r_2}} = \lim_{\varepsilon\to 0} o(\varepsilon^{r_1-\kappa-r_2}) = 0 \qquad (53)$$

Moreover, from the definition given in (15), we have

$$\mathrm{Sig}(\varepsilon^{r_3}x_3)^{\alpha_2} = o(\varepsilon^{r_3}) \qquad (54)$$

Hence we obtain

$$\begin{aligned}&\lim_{\varepsilon\to 0}\frac{-M^{-1}(\varepsilon^{r_1}x_1+q_d)K_d \mathrm{Sig}(\varepsilon^{r_3}x_3)^{\alpha_2}}{\varepsilon^{\kappa+r_2}} \\ &= -M^{-1}(q_d)K_d \lim_{\varepsilon\to 0} o(\varepsilon^{r_3-\kappa-r_2}) = -M^{-1}(q_d)K_d \lim_{\varepsilon\to 0} o(\varepsilon^{-\kappa}) = 0\end{aligned} \qquad (55)$$

Note that in the derivations of (51) and (53) we have utilized the fact that $r_1 - (\kappa + r_2) = 2(1-\alpha_1) > 0$ for $0 < \alpha_1 < 1$.

Thus, for any $x = (x_1^T\ x_2^T\ x_3^T)^T \in \Re^{3n}$, we get

$$\lim_{\varepsilon\to 0}\frac{\hat{f}_2(\varepsilon^{r_1}x_1, \varepsilon^{r_2}x_2, \varepsilon^{r_3}x_3)}{\varepsilon^{\kappa+r_2}} = 0 \qquad (56)$$

Similarly, for $x = (x_1^T\ x_2^T\ x_3^T)^T \in \Re^{3n}$, we have

$$\lim_{\varepsilon\to 0}\frac{\hat{f}_3(\varepsilon^{r_1}x_1, \varepsilon^{r_2}x_2, \varepsilon^{r_3}x_3)}{\varepsilon^{\kappa+r_3}} = \lim_{\varepsilon\to 0}\frac{B\varepsilon^{r_2}x_2}{\varepsilon^{\kappa+r_3}} = Bx_2 \lim_{\varepsilon\to 0}\varepsilon^{-\kappa} = 0 \qquad (57)$$

Therefore, according to Lemma 1, we have the global finite-time stability of the closed-loop system.

Finally, by invoking Lemma 2, we get the global finite-time stability. This completes the proof. ■

V. An Illustrative Example

Comparisons with the output feedback PID (OPID) control recently proposed by Su *et al.* [21] are conducted. The entries to model the robot manipulator can be found in [4] and [21].

The output feedback PID (OPID) control is given by

$$\tau = -K_p\Delta q - K_i z - K_d \upsilon \qquad (61)$$

$$\dot{\upsilon} = -A\mathrm{Sig}(\upsilon)^{\alpha_2} + B\dot{q} \qquad (62)$$

where z was defined by (21), and K_p, K_i, K_d, A and B were defined by (20) and (22).

The desired positions were $q_d = [\pi/4,\ \pi/2]^T$ (rad). The sampling period was $T = 1\,\mathrm{ms}$. A white noise with an amplitude of $0.01\,\mathrm{rad}$ is added to the position signals to imitate the measurement noise. The initial parameters were all set as zero. The gains for the ONPID controller were chosen as $\varepsilon = 0.01$, $\alpha_1 = 0.5$, $K_p = \mathrm{diag}(380,\ 100)$, $K_i = \mathrm{diag}(300,\ 5)$, $K_d = \mathrm{diag}(75,\ 15)$, $A = \mathrm{diag}(150, 80)$, and $B = \mathrm{diag}(120, 60)$. For a fair comparison, gains of the OPID controller were chosen the same as the ones of the proposed ONPID controller.

Fig. 1 illustrates the position errors of the proposed approach and the OPID control. For a clear comparison, the requested input torques are shown in Figs 2 and 3. It can be seen that with the proposed ONPID controller, the robot successfully completed its movement at the desired final position, and after a transient due to errors in initial condition, the position errors tend to zero; while with the same gains, the OPID controller can not regulate the first link error to zero. Obviously a fast response of the ONPID controller is achieved over the OPID scheme presented in [21]. Moreover, the fast response is not achieved with expensive of much large torque.

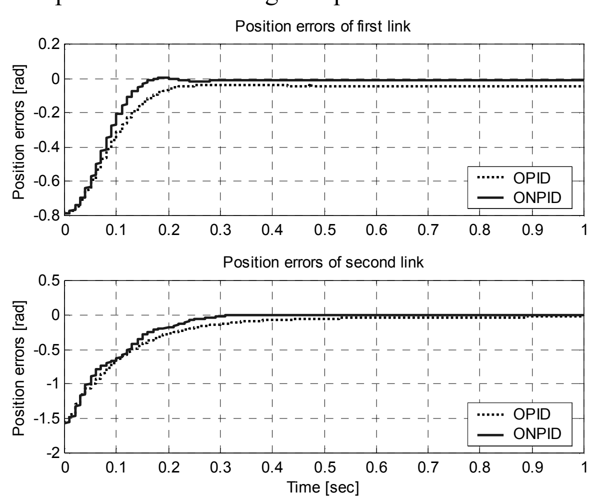

Fig. 1. Position errors.

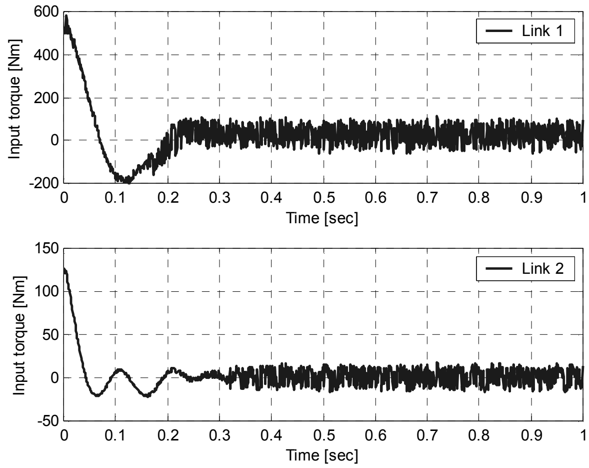

Fig. 2. Input torques of the proposed ONPID.

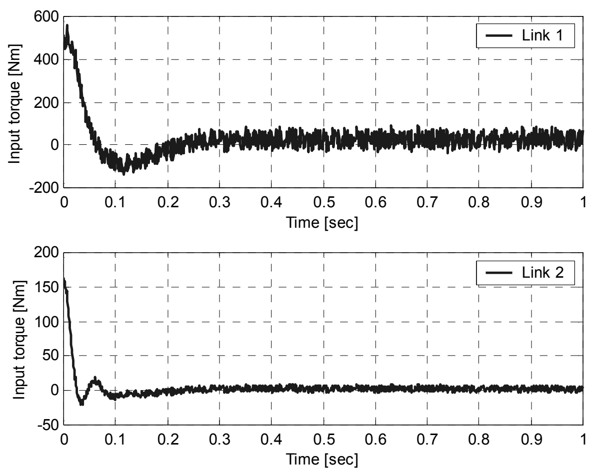

Fig. 3. Input torques of the OPID.

VI. Conclusion

We have resolved the global finite-time regulation of robot manipulators with simple nonlinear PID control and without velocity measurements. A nonlinear filter is developed to eliminate velocity measurements and a very simple output feedback nonlinear PID controller is proposed. The global finite-time stability is shown with Lyapunov's direct method and finite-time stability theory. The algorithm does not refer to model parameters and the control gains can be explicitly determined based on some well-known bounds extracted from the robot dynamics, and therefore, is easy to implement in practice. The developed approach offers an alternative approach for improving the design of the robot regulator, and also solves the global finite-time stabilization control problem for a large class of nonlinear systems without velocity measurement. Simulations on a two-DOF robot manipulator demonstrate the effectiveness of the proposed approach.

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