

# Continuous Path Tracing by a Cable-Suspended, Under-Actuated Robot: The Winch-Bot

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**Abstract**—A simple, under-actuated robotic winch, called the “Winch-Bot,” is developed for surface inspection of a large object. The Winch-Bot, placed over an object surface, has only one actuator for tracing a free geometric path in a vertical plane. The cable length is controlled in relation to the direction of the cable so that the inspection end-effector hanging at the tip of the cable can follow the path dynamically despite the lack of full degrees of freedom. We analyze the tracing dynamics, address under what conditions a given geometric path can be traced (traceability conditions), and prove under what conditions the tracing motion is repetitive. A controller utilizing partial feedback linearization is proposed, and simulations are used to validate the explored traceability criteria and to confirm the controller’s performance improvement.

## I. INTRODUCTION

THERE is a need for inspecting the surface of a large object by moving an instrument along its surface. An aircraft body, for instance, needs to be inspected occasionally for security and safety. The Winch-Bot, consisting of a single-axis winch placed at a fixed point over a large object, is a simple, economic solution to those tasks where an end-effector must be moved along a large surface. Unlike a gantry crane or a long-arm rigid manipulator, the Winch-Bot does not require a large structure or many-degrees-of-freedom servoed joints. Under-actuated dynamics allow the end-effector to trace a surface continually.

Several prior works are relevant to the Winch-Bot design and control. The aforementioned gantry crane’s dynamics have been analyzed to allow for more control of the end-point’s path including input shaping to dampen residual oscillations [1], [2]. Casting robots were investigated so that a fixed, rotating arm can excite the oscillations of a pendulum which is then extended such that the end effector lands at a desired location [3]. Multi-cable cranes have been analyzed whereby controlling six cable lengths, a Stewart platform can be controlled to make arbitrary motions and rotations [4]. Finally, pseudo-mobile robots (such as a simple brachiating robot [5]) aim to move around a workspace to work outside their immediate reach when needed. Whereas these works are similar in application or theory, to our knowledge there is no work on continuous

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path tracing by a cable-suspended, under-actuated robot.

In the following, we will use the under-actuated dynamics of the system to define path tracing, generate criteria for path traceability, and simulate a controller utilizing feedback linearization to track various geometric paths and observe the results.

## II. CONCEPT

In principle, an under-actuated robot is unable to track an arbitrary time trajectory. It may track a limited class of trajectories that conform to the under-actuated dynamics, e.g. particular solutions to the dynamic equations, but in general it cannot track an arbitrary one. Tracking an arbitrary time trajectory requires the same number of independent servoed joints as the dimension of the trajectory, or to be fully actuated. This requirement, however, can be removed if the task is to trace a geometric path without specification of tracing speed. For many inspection tasks, tracing speed is not an important variable to regulate. As long as the speed is lower than a certain limit, or within an acceptable range, variation in tracing speed does not affect inspection performance.

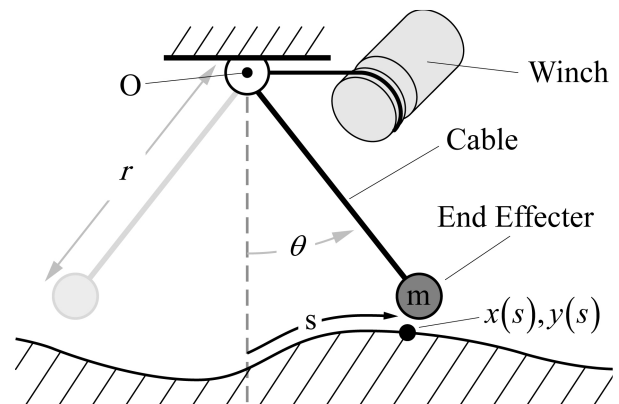


Fig. 1. Shown are the parameters important in our simple pendulum whose length is controlled by a smart winch.

Consider a two-dimensional geometric path, as illustrated in Fig. 1. Let  $x(s)$  and  $y(s)$  be a parametric representation of a geometric path, where  $s$  is path length. Path following is a standard problem if a robot has two independent servoed joints, e.g. a gantry robot with two prismatic joints. The challenge is to trace a two-dimensional path,

$$\{x(s), y(s) | s_0 \leq s \leq s_f\} \quad (1)$$

with only one servoed joint, where the tracing speed,  $ds/dt$ , is unspecified.

The Winch-Bot, originally developed for point-to-point positioning tasks [6], can perform this type of tracing task for a wide class of path geometries. The Winch-Bot consists of one servoed joint, two position sensors, a cable, and an end-effector, e.g. an inspection instrument, as shown in Fig. 1. The body of the Winch-Bot is fixed at a point over a surface, and the cable length  $r$  is controlled by the servoed joint, and  $\theta$  is the angle of the cable taken from the vertical.

If the cable length is fixed, the system is merely a pendulum, tracing a circular path back and forth. It can trace other paths when the cable length is controlled in relation to the cable angle. To trace a straight horizontal line, for example, the cable length must be coordinated with the cable angle such that:

$$r(\theta) = \frac{h}{\cos\theta}, \quad -\frac{1}{2}\pi < \theta < \frac{1}{2}\pi \quad (2)$$

where  $h$  is the distance from the origin  $O$ . See Fig. 2. Suppose that the end-effector of mass  $m$  starts at the center point  $C$  with initial velocity  $V_0$ . As the end-effector moves horizontally in the  $+x$  direction, it receives a horizontal restoring force,  $-mgx/h$ , which pushes the end-effector backwards. A simple calculation reveals that the speed of the mass becomes zero at the distance  $x_{\max} = V_0 \sqrt{h/g}$ , at which point the end-effector begins to move backwards. If the system is loss-less, the mass regains the initial velocity when passing the center point  $C$ , this time traveling in the  $-x$  direction. The motion is completely symmetric with respect to the center line  $OC$ , and the mass moves back and forth between  $x_{\max}$  and  $-x_{\max}$ .

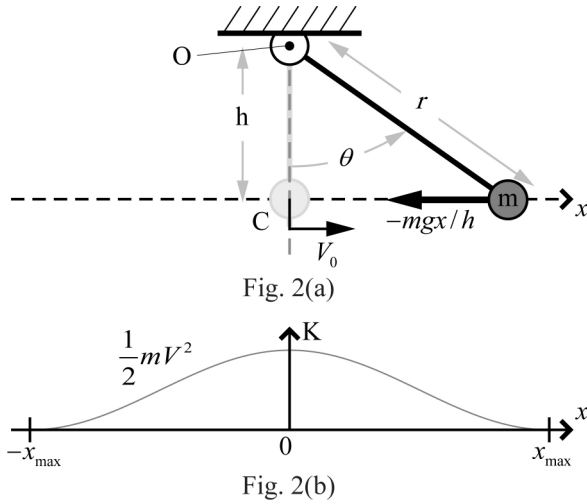


Fig. 2(a)

Fig. 2(b)

Fig. 2. Tracing a horizontal line is illustrative of many of the Winch-Bot's properties. Fig. 2(b) shows that the kinetic energy  $K$  dictates the maximum distance the end-effector travels.

This straight line tracing example illustrates that

- The Winch-Bot can trace a class of paths by coordinating its cable length with the cable angle, and
- Under a loss-less assumption the motion can be repetitive.<sup>1</sup>

<sup>1</sup> While this assumption may be unrealistic in real applications, the degree of energy loss due to air drag is very small compared to the energy

The objective of the following sections is to assure these properties based on a dynamic model. We will:

- Analyze when a given geometric path is traceable for the Winch-Bot (traceability conditions), and
- Prove that the Winch-Bot motion is repetitive if the path is traceable, is geometrically symmetric, and has an appropriate initial velocity.

In the following we will first obtain dynamic equations, analyze traceability, prove repetitiveness, and show implementation techniques, followed by numerical and experimental results.

### III. MODELING AND ANALYSIS

#### A. Dynamic Equations

The governing dynamic equations of the Winch-Bot are derived in this section. We assume that the end-effector is a point mass and that the cable is mass-less. We also ignore the longitudinal elasticity of the cable and aerodynamic effects on the end-effector and the cable. Experiments using a prototype to be discussed in Section IV have justified these assumptions.

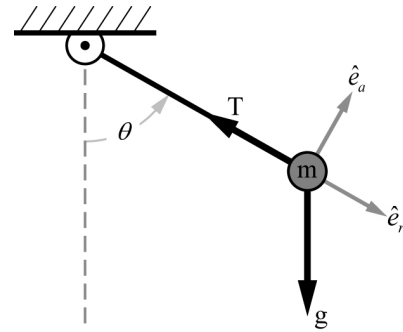


Fig. 3. The well-known free-body diagram of a point at the end of a rigid cable applies to our system's design.

Starting with the basic free-body diagram for a point mass, one can derive two equations of motion for our fixed-point/variable-length cable system, moving in a single plane. Let  $m$  be the mass of the end-effector and  $T$  be the tension of the cable. See Fig. 3. Assuming that the cable is taut, we can obtain the following equations of motion:

$$(mg \cos\theta - T)\hat{e}_r - (mg \sin\theta)\hat{e}_a = m \frac{d^2}{dt^2}(r\hat{e}_r) \quad (3)$$

where  $\hat{e}_r$  and  $\hat{e}_a$  are unit vectors pointing in the radial and angular directions, respectively. Expanding the right-hand side and taking a time derivative, we get

$$g \cos\theta - \frac{T}{m} = \ddot{r} - r\dot{\theta}^2 \quad (4)$$

in the radial direction, and

$$-g \sin\theta = 2\dot{r}\dot{\theta} + r\ddot{\theta} \quad (5)$$

or

$$\ddot{\theta} + \frac{2\dot{r}}{r}\dot{\theta} + \frac{g \sin\theta}{r} = 0 \quad (6)$$

in the system. Additionally, a non-holonomic controller has been designed to correct for these energy errors, but will be presented in future work.

in the angular direction. Both (4) and (6) together describe the degrees of freedom of the Winch-Bot system.

### B. Continuous Path Tracing

In order to perform path tracing, we must first address conditions under which the Winch-Bot can trace a path. A few necessary conditions for a geometric path to be traceable can be obtained immediately:

- The path must be twice differentiable with respect to path length  $s$ . Otherwise, an infinitely large acceleration is needed where the first-order derivative is discontinuous.
- Path length  $s$  and cable angle  $\theta$  must have a one-to-one correspondence, i.e.  $s$  is a single-valued function of  $\theta$ . As shown in Fig. 4, the end-effector cannot reach some segment of the surface if a single radial line from the point  $O$  intersects with the path at multiple points, such as points  $A, B, C$ . This implies that the path length  $s$  is a monotonically increasing function of  $\theta$  (or a decreasing function depending on the direction of  $s$ ).

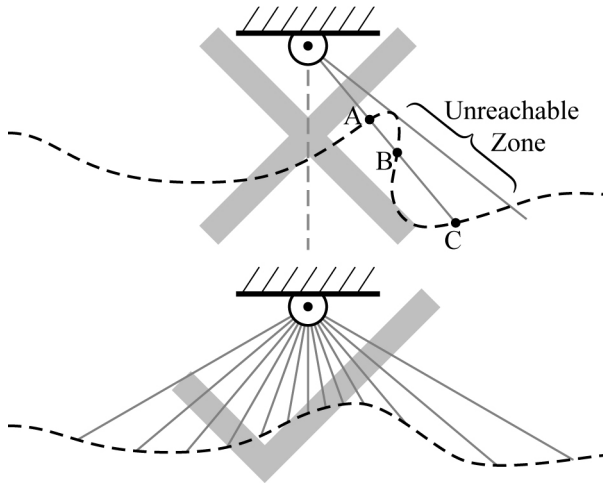


Fig. 4. The path to be traced must have a one-to-one correspondence with the angle of the string.

This second condition allows us to represent the cable length  $r$  as a function of  $\theta$ :

$$r = r(\theta). \quad (7)$$

Therefore, the first and second order time derivatives of  $r$  can be given by

$$\dot{r} = r' \dot{\theta}, \quad \ddot{r} = r'' \dot{\theta}^2 + r' \ddot{\theta}, \quad (8)$$

where

$$r' = \frac{dr}{d\theta}, \quad r'' = \frac{d^2r}{d\theta^2}. \quad (9)$$

Substituting these into dynamic equations (4) and (6) yield

$$T = mr' \ddot{\theta} + m(r'' - r) \dot{\theta}^2 + mg \cos \theta \quad (10)$$

and

$$\ddot{\theta} + \frac{2r'}{r} \dot{\theta}^2 + \frac{g \sin \theta}{r} = 0. \quad (11)$$

The last equation can be solved with initial conditions,  $\theta_0, \dot{\theta}_0$ , and a time-trajectory of length  $r$ . The tension  $T$  can

then be evaluated by substituting the solution  $\theta(t)$  into the first dynamic equation (10).

One critical condition for the Winch-Bot to be able to trace a path is that the tension  $T$  must be non-negative at all times. It can pull up the end-effector mass  $m$  at an acceleration that the winch actuator can generate. However, it cannot pull down the mass; the downward acceleration is limited by gravity. Therefore, we include the following third condition for traceability:

- The cable tension  $T$  given by (10) must be non-negative at all times.

Based on the dynamic equations (10) and (11), we can make the following observations regarding this third condition:

Remark 1: From (10) it follows that, if the end-effector mass moves slowly with small angular velocity and acceleration,  $|\dot{\theta}| \ll 1$ ,  $|\ddot{\theta}| \ll 1$ , the gravity term,  $mg \cos \theta > 0$ , dominates and keeps the tension positive.

Remark 2: From (11), as cable length  $r$  increases the angular velocity and acceleration tend to decrease,  $|\dot{\theta}| \rightarrow 0$ ,  $|\ddot{\theta}| \rightarrow 0$ , as  $r \rightarrow \infty$ , and thereby the tension is kept positive.

Remark 3: Condition c) is valid given an apparatus with a link between end-effector and winch that can only support tension. We can imagine a setup with a stiff, solid rod that is extended and retracted, with the mass at the end much larger than the mass of the rod. In that case, a negative tension could be supported by the rod, and therefore the first and second conditions only are sufficient for traceability.

### C. Divergence/Convergence of cyclic path tracing

In a typical pendulum, neglecting damping, the energy in the system stays constant and is merely transferred between potential and kinetic energy, arriving at the same state in which it began after one complete cycle. In the Winch-Bot, a motor is adding and removing energy through work done in the tension of the cable. While tracing a path, it is not intuitive whether the system will return to the initial conditions after a complete cycle or whether the energy will diverge or converge. In this section we will prove that due to the symmetric nature of path tracing, like a typical fixed-length pendulum, given a path that is geometrically symmetric, the state of the system will return to the same state after each complete cycle.

#### Proposition:

The dynamic equations given by

$$T = mr' \ddot{\theta} + m(r'' - r) \dot{\theta}^2 + mg \cos \theta \quad (12)$$

$$\ddot{\theta} = -\frac{2r'}{r} \dot{\theta}^2 - \frac{g \sin \theta}{r} \quad (13)$$

have a repetitive, periodic solution (which depends on the initial velocity alone) that perfectly traces that path while performing zero net-work on the system after each period as long as the path to be traced is symmetric about  $\theta=0$ . Additionally, this trajectory can be implemented on the Winch-Bot as long as the geometric path is traceable while

maintaining strictly-positive tension. In other words, given a symmetric path, the total work done by the winch in one part of the swing is cancelled by the work done on the winch in another, which results in a cyclic trajectory that neither diverges nor converges.

**Proof:**

By solving for the work done on the system by the winch, we can find the complete energy of the system at all points in a cycle. Here we can define work as force×distance, in this case there are two: the change in cable length  $r$  times the cable tension  $T$ , and the change in  $\theta$  times the moment about the origin  $O$  created by gravity. See Fig. 5. Because we know that the second term results in a potential energy, and we are returning to the same point, we know the net change in potential energy is zero and so we can remove that term. Because the change in  $\theta$  is perpendicular to the tension, the only remaining term is the tension and the change in cable length.

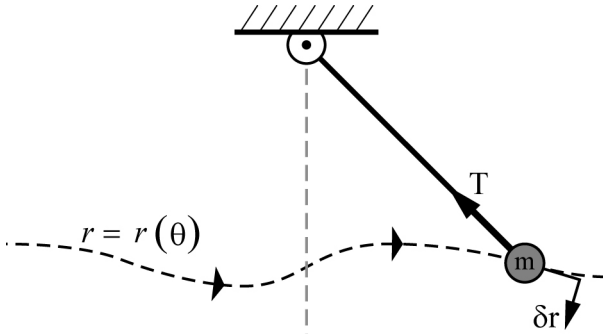


Fig. 5. The only net work over a cycle would be performed by the Tension because the gravity is a conservative force.

The complete differential work term can be written as

$$\delta Work = -T \delta r, \quad (14)$$

and around one complete cycle we can perform a closed-loop integration to find

$$Work = -\oint T(\theta, \dot{\theta}) dr. \quad (15)$$

Substituting the results from Eq. (12) and  $dr = r' d\theta$ , we write *Work* as

$$-\oint [mr'^2 \ddot{\theta} + (r'' - r)mr' \dot{\theta}^2 + mr'g \cos \theta] d\theta. \quad (16)$$

This can be abbreviated as

$$-\oint [A \cdot \ddot{\theta} + B \cdot \dot{\theta}^2 + C \cdot \cos \theta] d\theta \quad (17)$$

where

$$\begin{aligned} A &= mr'^2, \\ B &= (r'' - r)mr', \\ C &= mr'g \end{aligned} \quad (18)$$

This loop integral starts from a  $\theta_1$  and monotonically increases to  $\theta_2$ , then reverses back monotonically to  $\theta_1$  (as shown in Section III-B-b). Because of this, we can separate the loop integral into two line integrals, one for the monotonically increasing portion of time, namely:

$$-Work = \int_{\theta_1}^{\theta_2} [A|_{-\theta} \ddot{\theta} + B|_{-\theta} \dot{\theta}^2 + C|_{-\theta} \cos \theta] d\theta \quad (19)$$

and one for the monotonically decreasing portion, where we substitute  $-\theta$  for  $\theta$ :

$$+ \int_{\theta_2}^{\theta_1} [A|_{-\theta} \cdot (-\ddot{\theta}) + B|_{-\theta} \cdot (-\dot{\theta})^2 + C|_{-\theta} \cos(-\theta)] d\theta \quad (20)$$

Combining these and swapping the limits of integration, we get

$$\begin{aligned} -Work &= \int_{\theta_1}^{\theta_2} [A|_{\theta} \ddot{\theta} + B|_{\theta} \dot{\theta}^2 + C|_{\theta} \cos \theta] d\theta \\ &- \int_{\theta_1}^{\theta_2} [A|_{-\theta} \cdot (-\ddot{\theta}) + B|_{-\theta} \cdot (-\dot{\theta})^2 + C|_{-\theta} \cos(-\theta)] d\theta \end{aligned} \quad (21)$$

We can now solve for the coefficients from (18) and find

$$\begin{aligned} A|_{-\theta} &= m \left( \frac{dr}{-d\theta} \right)^2 = mr'^2 = A|_{\theta} \\ B|_{-\theta} &= \left( \frac{d^2r}{(-d\theta)^2} - r|_{-\theta} \right) m \frac{dr}{-d\theta} = -(r'' - r|_{-\theta})mr'. \quad (22) \\ C|_{-\theta} &= m \left( \frac{dr}{-d\theta} \right) g = -mr'g = -C|_{\theta} \end{aligned}$$

Clearly,  $A$  is an even function, and  $C$  is odd, but  $B$  isn't obvious. So now we substitute and can rewrite (21) and cancel most of the terms, finding

$$-Work = \int_{\theta_1}^{\theta_2} \left[ \begin{aligned} &m(r'^2 - r'^2) \ddot{\theta} \\ &+ (r|_{-\theta} - r|_{\theta})mr' \dot{\theta}^2 \\ &+ (r'' - r'')mr' \dot{\theta}^2 \\ &+ (\cos \theta - \cos \theta)mr'g \end{aligned} \right] d\theta. \quad (23)$$

Therefore as long as  $r|_{\theta} = r|_{-\theta}$  (and thus symmetric),  $Work_{loop} = 0$ ,

Q.E.D.

Here we have solved for a sufficient criterion for repetitive tracing, but not necessary. There may be other ways to maintain repetitiveness, such as a non-holonomic controller which strays from the path to be traced and does net work, but such a controller will be discussed in future work.

#### IV. IMPLEMENTATION

##### A. Partial Feedback Linearization

In implementation, it is our goal to define a controller for  $u$  that forces the system's closed-loop dynamics to trace a geometric path. This requires the cable length to have time trajectories of  $r_{desired}$ ,  $\dot{r}_{desired}$ , and  $\ddot{r}_{desired}$  where these are found from the definition of the path to be traced. Specifically,

$$r_{desired} = r(\theta) \quad (24)$$

$$\dot{r}_{desired} = \frac{dr}{d\theta} \dot{\theta} \quad (25)$$

$$\ddot{r}_{desired} = \frac{d^2r}{d\theta^2} \dot{\theta}^2 + \frac{dr}{d\theta} \ddot{\theta}. \quad (26)$$

We will use partial feedback linearization to force the acceleration of the cable length to be that of the desired

trajectory such that perfect tracing will occur. First we define error terms  $\tilde{r} = r - r_{desired}$ ,  $\dot{\tilde{r}} = \dot{r} - \dot{r}_{desired}$ , and  $\ddot{\tilde{r}} = \ddot{r} - \ddot{r}_{desired}$ . Replacing  $T/m$  by input  $u$  in our dynamic equation (4) yields

$$\ddot{r} = g \cos \theta + r \dot{\theta}^2 - u. \quad (27)$$

By defining our input as

$$u = -(\ddot{r}_{desired} - K_d \dot{\tilde{r}} - K_p \tilde{r}) + r \dot{\theta}^2 + g \cos \theta \quad (28)$$

(where  $K_d, K_p$  are negative constant) and then substituting it into our plant dynamics, we show that the nonlinear terms cancel leaving the final equation of the cable length,

$$\ddot{\tilde{r}} + K_d \dot{\tilde{r}} + K_p \tilde{r} = 0. \quad (29)$$

This system has two poles in the left half-plane and therefore  $\tilde{r} \rightarrow 0$ , and thus our cable length dynamics converge to the desired dynamics.

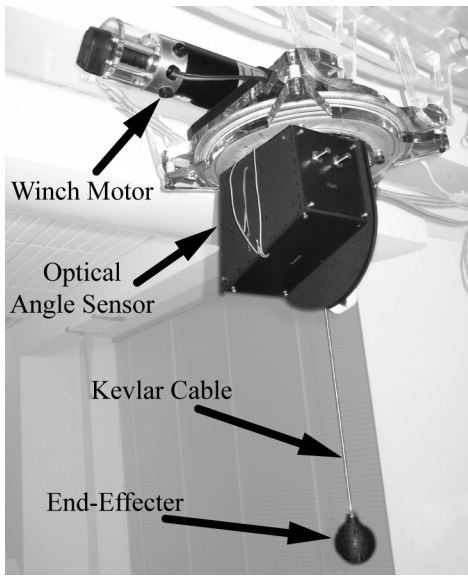


Fig. 6. Our Winch-Bot prototype included a rotating stage to investigate 3D motions, but ultimately was never used. These are the main components of the prototype.

### B. Prototyping

Whereas the idea of a single-axis winch controlling the length of a cable seems simple, the implementation had some problems to overcome. One such problem was how to measure the cable angle without introducing measurement dynamics, while obtaining a high enough measurement frequency to facilitate taking the time derivative mathematically for use in closed-loop control. In our prototype, this was done by casting a shadow of the cable onto an array of 1280 optical sensors spaced  $63.5 \mu\text{m}$  apart read by a high-speed FPGA. With this, a complete measurement of the angle could be obtained at a frequency of approximately 1 kHz, fast enough to take a time derivative mathematically.

Another problem was the elasticity of the cable causing unwanted dynamics. The cable we used was thin Kevlar string, which has a high stiffness, thus virtually eliminating

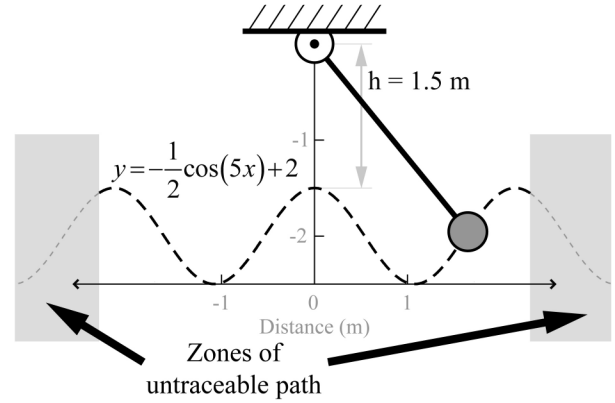


Fig. 7. While a path may be traceable in particular areas, there may be zones which invalidate the tracing, as shown here.

the dynamic effects due to the elasticity of the cable.

The end-effector was a 25 mm diameter steel sphere directly attached to the cable. The sphere had a much larger mass than the entirety of the Kevlar string, and so our lumped mass model is kept accurate. Our final implementation can be seen in Fig. 6.

## V. SIMULATION AND EXPERIMENTATION

Using MATLAB's differential equation solver, we were able to simulate this system to a high degree of accuracy. We used this to study some of the feasibility requirements discussed in Section III. We will focus on criteria b) and c).  
b) Path length  $s$  and cable angle  $\theta$  must have a one-to-one correspondence.

Shown in Fig. 7 is a particular path that is defined by the curve  $y = 1/2 \cdot \cos(5x) + 2$  that has a one-to-one correspondence within a range  $|\theta| < \theta_{max}$ , but beyond which the path is not reachable. When the initial conditions are low enough such that the range of motion never reaches the

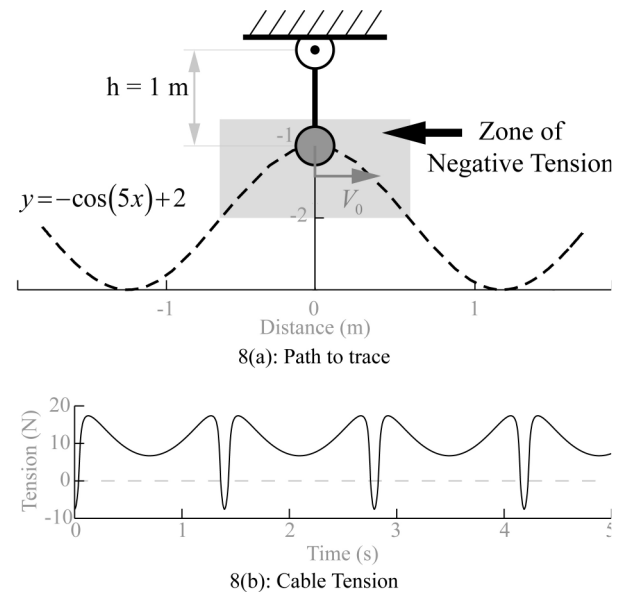


Fig. 8. Tracing a wave with initial conditions over a threshold results in tensions that become negative, as shown in Fig. 8(b).

untraceable areas, the path can be successfully traced.

c) The cable tension  $T$  must be non-negative at all times.

By using a similar path to be traced as was done in the previous simulation, we see that large changes in cable length will cause the tension to become negative in areas where the mass must be forced downward faster than gravity can provide. Fig. 8 is an example of a path that the Winch-Bot would be unable to trace. As can be seen in Fig. 8(b), the tension drops below zero cyclically. This is because immediately after time  $t_0$  the end-effector, with initial velocity  $V_0$ , is to move downward to trace the path. Here, path tracing demands acceleration downward greater than that of gravity, and so tension must be negative to drive the system along the path. As the end-effector stops, reverses, and returns to this same area, it is now moving upward so quickly that path tracing demands downward acceleration greater than that of gravity, thus the negative tension.

In addition to the feasibility examples, we simulated the

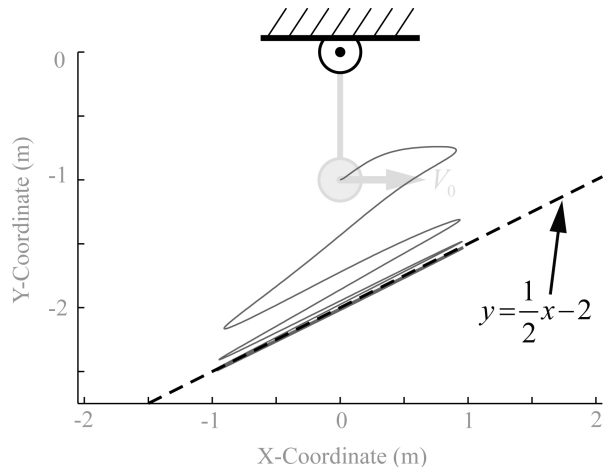


Fig. 9. With  $K_d=500, K_p=250$  and starting with initial length of 1 m and initial angular velocity of 2.25 rad/sec, this system converges to the desired path, a line at a diagonal to gravity define above.

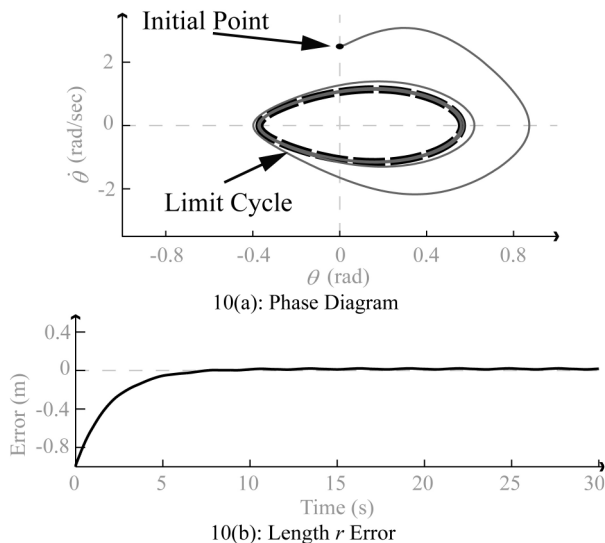


Fig. 10. With an angular limit cycle of a misshapen egg, the phase diagram above differs greatly from the traditional pendulum. Fig. 10(b) shows that the string length error quickly converges to zero.

partial feedback linearization controller described in Section IV. We simulated a system with a target geometric path defined by the diagonal line  $y=1/2 \cdot x-2$ , as shown in Fig. 9. It was started at  $\theta=0$  with  $\dot{\theta}>0$ , and with a cable length  $r = 1$  m such that it starts away from the path to be traced. These simulation results are shown in Fig. 9 and Fig. 10.

On our prototype Winch-Bot, we implemented a simplified controller with low gains to trace the geometry of a curve simulating a wing surface, shown in Fig. 11. Fig. 11(b) shows the resulting distance errors of the tracing.

## VI. CONCLUSION

By properly defining and studying the properties of arbitrary path tracing, we've shown that despite the Winch-Bot having only a single degree-of-freedom, we can trace an arbitrary geometric path in space by sacrificing the ability to control the time trajectory of the tracing. This is still very useful in the planned applications, and so we developed a controller to trace these geometric paths. Upon simulation, we validated our theoretical understanding of the feasibility of certain geometric paths, and were able to show our controller succeeds in converging toward our desired path to trace.

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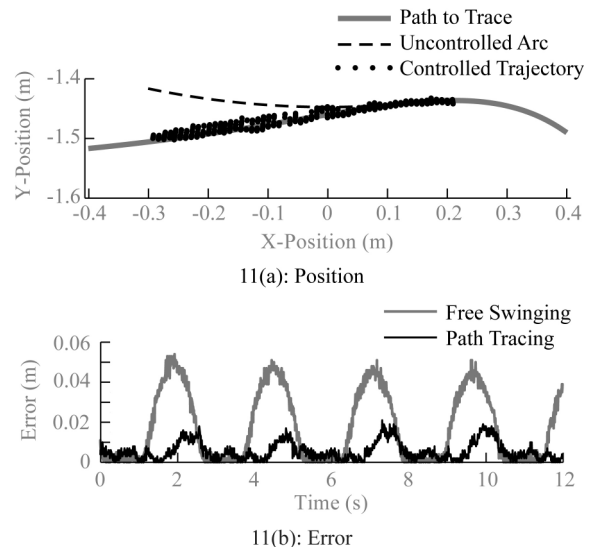


Fig. 11. Using an implemented controller with low gain, we were able to show path tracing of a wing-like surface.