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# Control-oriented modelling of a hybrid AUV

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*Abstract*— **This paper introduces a control-oriented model for a class of autonomous underwater vehicles (AUVs) characterized by an actuation system that includes propellers, internal weight motion and ballast charge/discharge. Since this kind of actuation combines the actuation of both oceanic gliders and self-propelled AUVs, these vehicles are termed hybrid vehicles. The modelling results are employed in the definition of an adaptive backstepping control, using a fuzzy approximator in terms of Gaussian function in order to cope with model uncertainties and disturbances. Lyapunov stability of the control is proved and simulations are shown for a typical water sampling operation.**

## I. Introduction

During the last few years, underwater robotics, and in particular Autonomous Underwater Vehicles (AUVs) have received a great impulse due to the newest technologies available and the growing interest of scientific research centres and industries. As a consequence, performances and consumption considerations have become increasingly important in the evaluation of vehicle capabilities. In particular, there has been a renewed interest in oceanographic gliders, a class of AUVs designed for long-term oceanographic sampling [4]. Gliders navigate through change of buoyancy and attitude obtained through internal modification of ballast and movement of the vehicle centre of mass: the yo-yo movement in the water column obtained by these actuation systems is transformed in a net horizontal movement in the desired direction through the vehicle hydrodynamic surfaces (wings and rudder). The energy consumption of gliders is far less than those of propelled AUVs, and the efficacy of gliders as a component of an Autonomous Ocean Sampling Network has been demonstrated [2]; however, gliders clearly suffer from great limitations in terms of manoeuvrability and speed. Recently hybrid AUV/glider capability vehicles have been developed to fill the gap between consumption and manoeuvrability. In particular, the Folaga class vehicles [1] are torpedo-shaped ones, with no hydrodynamic surfaces as wings or fins, with jet-pump propulsion along the surge and sway axes, and with internal mass movement and buoyancy change for attitude and depth control. Suitable rigid-body dynamic models are available in literature for both gliders [3], [5] and propelled AUVs [7]. The main difference between these two class of models is that for propelled AUVs the invariance of mass and centre of mass produces equations that can be written in standard forms, which in turn can be efficiently used in vehicle control synthesis. Gliders rigid-body equations have an additional complexity that makes

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Fig. 1. FOLAGA Vehicle

them less amenable to standardization, and that may require a case-by-case analysis in order to define suitable stabilizing control laws [8]. The purpose of this paper is to derive an approximate, control-oriented, model of a hybrid vehicle expressed in a standard form similar to those presented in [7]. To this aim, some simplifying assumptions will be made in order to reduce the complexity of the rigid-body equations. Clearly, a model so obtained, a sort of trade-off between the AUV and Glider models, in terms of control will generate model mismatch with respect to the real vehicle. Therefore an adaptive control procedure is formulated to handle the mismatch. The adaptive control law is synthesized on the basis of the simplified standard form here proposed, and it combines a backstepping control approach with a fuzzy approximator consisting in Gaussian basis functions. Simulations are then shown in which the complete vehicle dynamics are considered, and it is shown how the proposed control law is able to maintain practical stability vs. model mismatch and external disturbances. It is expected that also different adaptive and/or robust control approaches, synthesized on the simplified model proposed, may exhibit the same practical stability properties.
The paper is organized as follows: after notation and definitions in Section II, vehicle modelling is pursued in Section III. Control synthesis on the basis of the derived model is presented in Section IV and simulative results with the complete vehicle dynamics are reported in Section V. Finally conclusions are given.

## II. Notation and Setting

Given two vectors $x, y \in \mathbb{R}^3$, the cross product $x \wedge y$ will be represented using the following matrix form:

$$x \wedge y = S(x)y = -S(y)x = -y \wedge x$$

where the operator $S(\cdot) : \mathbb{R}^3 \to \mathbb{R}^{3\times 3}$ is defined as follow:

$$S(x) = \begin{pmatrix} 0 & -x_1 & x_2 \\ x_1 & 0 & -x_3 \\ -x_2 & x_3 & 0 \end{pmatrix}$$

The transpose of a real matrix $A$ will be represented as $A^T$. A vector $p$ expressed in the reference frame $q$ will be identified as $p^q$. A positive definite matrix $A$ will be referred as $A \succ 0$. Vehicle motion will be represented using two frames: the navigation frame North East Down (NED) and the body frame.

**Assumption n.** *1: NED will be assumed as inertial frame.*
Velocity expressed in the body frame $b$ can be decomposed in the navigation frame using the rotation matrix $R_b^n$, in particular:

$$v^n = R_b^n v_b$$

The body frame will be centred in the vehicle Centre of Buoyancy (COB). The class of vehicle considered is able to change dynamically the Centre of Gravity (COG) by acting on a ballast tanks (injecting/ejecting water) and by moving the internal battery packs along the surge axis. In the case of the Folaga class vehicles, as it may be expected in other hybrid vehicles with the same actuation principle, ballast variation and battery movements are slow compared to other system dynamics. In particular, the maximum internal mass rate of change within the Folaga is of 0.020 Kg/s, while the maximum speed for displacement of the battery pack is 0.0007 m/s [1]. The Folaga propulsion system is able to drive the vehicle at a speed of 1-2 m/s. The fact that COG is moving at slow speed with respect to vehicle speed is the basis of our approximate analysis in the following: in particular, in some equations, a quasi-static approximation will be used, neglecting the velocity of the COG.

## III. Modeling the Vehicle

To obtain the 6DOF model we are going to consider a vehicle with a variable mass $m(t)$ placed in the variable COG (see Fig. 2):

$$r_g^b(t) = \frac{\Lambda + \Upsilon(t)}{m(t)}, \quad \dot{r}_g^b(t) = \frac{\dot{\Upsilon}(t)}{m(t)} - \frac{\dot{m}(t)}{m(t)} r_g^b(t)$$

where $\Lambda$ and $\Upsilon(t)$ represent the static and dynamic contribution to the motion of the COG, respectively.

### A. Linear Motion

Velocity of the COG with respect to a point $o$ can be represented as:

$$v_g^b = v_o^b + \omega^b \wedge r_g^b, \quad v_g^n = R_b^n v_g^b = R_b^n \left(v_o^b + \omega^b \wedge r_g^b\right) \quad (1)$$

**Assumption n.** *2: In the previous equation the term $\dot{r}_g^b$ does not appear: as discussed in the previous section, we are assuming a quasi-static situation in which we can neglect $\dot{r}_g^b$ in the kinematic equation. A complete treatment, without this assumption, can be found in [5].*
Taking the derivative of Eq.(1):

$$\begin{aligned} \dot{v}_g^b &= \dot{v}_o^b + \dot{\omega}^b \wedge r_g^b + \omega^b \wedge \dot{r}_g^b \\ \dot{v}_g^n &= R_b^n \left(\omega^b \wedge \left(v_o^b + \omega^b \wedge r_g^b\right) + \dot{v}_o^b + \dot{\omega}^b \wedge r_g^b + \omega^b \wedge \dot{r}_g^b\right) \end{aligned}$$

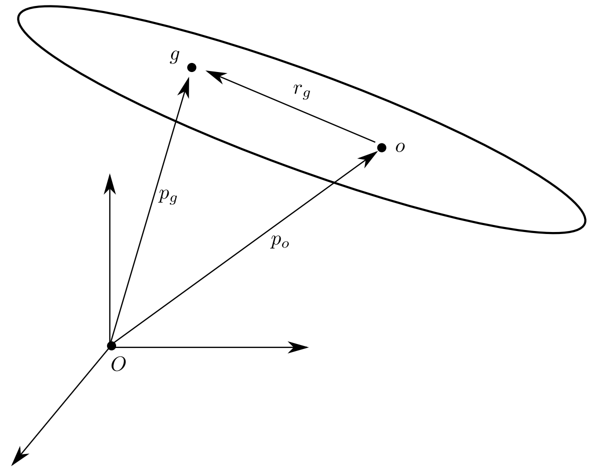

Fig. 2. Rigid Body Configuration

Since linear momentum can be expressed as $l_g^n = m v_g^n$, then by imposing that the rate of change of $l_g^n$ must be equal to the external forces we obtain:

$$\frac{dl_g^n}{dt} = m\dot{v}_g^n + \dot{m} v_g^n = \mathcal{F}_{ext}^n, \quad \mathcal{F}_{ext}^n = R_b^n \mathcal{F}_o^b$$

$$\begin{aligned} m &\left[\omega^b \wedge \left(v_o^b - r_g^b \wedge \omega^b\right) + \dot{v}_o^b - r_g^b \wedge \dot{\omega}^b + \omega^b \wedge \dot{r}_g^b\right] + \\ &+ \dot{m}\left(v_o^b - r_g^b \wedge \omega^b\right) = \mathcal{F}_o^b \end{aligned}$$

which can be rewritten in the following matrix form as:

$$\begin{aligned} \mathcal{F}_o^b = &\left( \begin{array}{c|c} mI & -mS(r_g^b) \end{array} \right) \left( \begin{array}{c} \dot{v}_o^b \\ \hline \dot{\omega}^b \end{array} \right) + \\ &+ \left( \begin{array}{c|c} mS(\omega^b) & -mS(\omega^b)S(r_g^b) \end{array} \right) \left( \begin{array}{c} v_o^b \\ \hline \omega^b \end{array} \right) + \\ &+ \left( \begin{array}{c|c} mS(\omega^b) & v_o^b - S(r_g^b)\omega^b \end{array} \right) \left( \begin{array}{c} \dot{r}_g^b \\ \hline \dot{m} \end{array} \right) \end{aligned} \quad (2)$$

### B. Angular Motion

The angular momentum $h_g^b$ can be represented as:

$$\begin{aligned} h_g^b &= h_o^b - m r_g^b \wedge v_g^b = I_o \omega^b + m r_g^b \wedge v_o^b - m r_g^b \wedge v_g^b = \\ &= I_o \omega^b + m r_g^b \wedge \left(r_g^b \wedge \omega^b\right) \end{aligned}$$

$$\begin{aligned} \dot{h}_g^b = &\dot{I}_o \omega^b + I_o \dot{\omega}^b + \dot{m} r_g^b \wedge \left(r_g^b \wedge \omega^b\right) + m \dot{r}_g^b \wedge \left(r_g^b \wedge \omega^b\right) + \\ &+ m r_g^b \wedge \left(\dot{r}_g^b \wedge \omega^b\right) + m r_g^b \wedge \left(r_g^b \wedge \dot{\omega}^b\right) \end{aligned}$$

Since

$$h_g^n = R_b^n h_g^b, \quad \dot{h}_g^n = R_b^n \left(\omega^b \wedge h_g^b + \dot{h}_g^b\right)$$

then, imposing that rate of change of angular momentum must be equal to the external moment applied:

$$\begin{aligned} \frac{dh_g^n}{dt} &= R_b^n \left(\omega^b \wedge h_g^b + \dot{h}_g^b\right) = \mathcal{M}_{ext}^n \\ \mathcal{M}_{ext}^n &= R_b^n \left(\mathcal{M}_o^b - r_g^b \wedge \mathcal{F}_o^b\right) \end{aligned}$$

which yields to:

$$\begin{aligned}&\omega^b \wedge \left[I_o\omega^b + mr_g^b \wedge \left(r_g^b \wedge \omega^b\right)\right] + \dot{I}_o\omega^b + I_o\dot{\omega}^b + \\ &\quad + \dot{m}r_g^b \wedge \left(r_g^b \wedge \omega^b\right) + m\dot{r}_g^b \wedge \left(r_g^b \wedge \omega^b\right) + \\ &\quad + mr_g^b \wedge \left(\dot{r}_g^b \wedge \omega^b\right) + mr_g^b \wedge \left(r_g^b \wedge \dot{\omega}^b\right) = \mathcal{M}_o^b - r_g^b \wedge \mathcal{F}_o^b\end{aligned}$$

Considering the term:

$$\begin{aligned}-r_g^b \wedge \mathcal{F}_o^b = &- r_g^b \wedge \left\{m\left[\omega^b \wedge \left(v_o^b - r_g^b \wedge \omega^b\right)\right.\right. \\ &\left. + \dot{v}_o^b - r_g^b \wedge \dot{\omega}^b + \omega^b \wedge \dot{r}_g^b\right] + \\ &\left. + \dot{m}\left(v_o^b - r_g^b \wedge \omega^b\right)\right\}\end{aligned}$$

and since:

$$\begin{aligned}-r_g^b \wedge \left[m\left(-r_g^b \wedge \dot{\omega}^b\right)\right] &= mr_g^b \wedge \left(r_g^b \wedge \dot{\omega}^b\right) \\ -r_g^b \wedge \left[m\left(\omega^b \wedge \dot{r}_g^b\right)\right] &= mr_g^b \wedge \left(\dot{r}_g^b \wedge \omega^b\right) \\ -r_g^b \wedge \left[\dot{m}\left(-r_g^b \wedge \omega^b\right)\right] &= \dot{m}r_g^b \wedge \left(r_g^b \wedge \omega^b\right) \\ -r_g^b \wedge \left[m\omega^b \wedge \left(-r_g^b \wedge \omega^b\right)\right] &= \omega^b \wedge \left[mr_g^b \wedge \left(r_g^b \wedge \omega^b\right)\right] \\ m\dot{r}_g^b \wedge \left(r_g^b \wedge \omega^b\right) &= - mr_g^b \wedge \left(\omega^b \wedge \dot{r}_g^b\right) + \\ &\quad + m\omega^b \wedge \left(r_g^b \wedge \dot{r}_g^b\right)\end{aligned}$$

we can conclude that:

$$\begin{aligned}&\omega^b \wedge I_o\omega^b + \dot{I}_o\omega^b + I_o\dot{\omega}^b - mr_g^b \wedge \omega^b \wedge \dot{r}_g^b + m\omega^b \wedge r_g^b \wedge \dot{r}_g^b + \\ &\quad + mr_g^b \wedge \left(\omega^b \wedge v_o^b\right) + mr_g^b \wedge \dot{v}_o^b + \dot{m}r_g^b \wedge v_o^b = \mathcal{M}_o\end{aligned}$$

The moment of inertia $I_o$ can be expressed as:

$$I_o = -mS(r_g^b)S(r_g^b)$$

It then follows that:

$$\dot{I}_o = -\dot{m}S(r_g^b)S(r_g^b) - m\dot{S}(r_g^b)S(r_g^b) - mS(r_g^b)\dot{S}(r_g^b)$$

and since

$$-m\dot{r}_g^b \wedge r_g^b \wedge \omega^b = mr_g^b \wedge \omega^b \wedge \dot{r}_g^b - m\omega^b \wedge r_g^b \wedge \dot{r}_g^b$$

the term $\dot{I}_o\omega^b$ can be rewritten as:

$$\begin{aligned}\dot{I}_o\omega^b = &- \dot{m}S(r_g^b)S(r_g^b)\omega^b - mS(\omega^b)S(r_g^b)\dot{r}_g^b + \\ &+ 2mS(r_g^b)S(\omega^b)\dot{r}_g^b\end{aligned}$$

and finally, after substitution, we can rewrite the previous equations in the following matrix form as:

$$\begin{aligned}\mathcal{M}_o = &\left(\begin{array}{c|c} mS(r_g^b) & -mS^2(r_g^b)\end{array}\right)\left(\frac{\dot{v}_o^b}{\dot{\omega}^b}\right) + \\ &+ \left(\begin{array}{c|c} mS(r_g^b)S(\omega^b) & -mS(\omega^b)S^2(r_g^b)\end{array}\right)\left(\frac{v_o^b}{\omega^b}\right) + \\ &+ \left(\begin{array}{c|c} mS(r_g^b)S(\omega^b) & S(r_g^b)v_o^b - S^2(r_g^b)\omega^b\end{array}\right)\left(\frac{\dot{r}_g^b}{\dot{m}}\right)\end{aligned} \quad (3)$$

### C. System actuation

We are considering a class of vehicles equipped with a ballast and a battery pack. We will use the state variables $\epsilon_b$ to identify the water mass contained in the ballast tank and $\epsilon_p^b$ to idenfity the position of the battery pack with respect to the body frame. In particular:

$$m(t) = m_s + \epsilon_b(t), \quad \Upsilon(t) = \epsilon_b r_b^b + m_p\epsilon_p^b$$

where $m_s$ is the static mass contribution, $r_b^b$ is the position of the ballast tank decomposed in the body frame and $m_p$ is the battery pack mass. With these assumptions, we can include this terms into the model:

$$\begin{aligned}\dot{r}_g^b &= \frac{r_b^b}{m}\dot{\epsilon}_b + \frac{m_p}{m}\dot{\epsilon}_p^b - \frac{r_g^b}{m}\dot{\epsilon}_b \\ \dot{m} &= \dot{\epsilon}_b\end{aligned}$$

or in an equivalent matrix form:

$$\left(\frac{\dot{r}_g^b}{\dot{m}}\right) = \left(\begin{array}{c|c} \frac{m_p}{m}I & \frac{1}{m}\left(r_b - r_g\right) \\ \hline 0^T & 1\end{array}\right)\left(\frac{\dot{\epsilon}_p^b}{\dot{\epsilon}_b}\right) \quad (4)$$

Now we can merge the previous equations (2),(3),(4) to obtain the 6DOF model:

$$M_{RB}(\epsilon)\dot{\nu} + C_{RB}(\nu,\epsilon)\nu + T(\nu,\epsilon)\dot{\epsilon} = \tau \quad (5)$$

where $\nu = \left(v_o^b, \omega^b\right)^T$, $\epsilon = \left(\epsilon_p^b, \epsilon_b\right)^T$ and $\tau = \left(\mathcal{F}_o^b, \mathcal{M}_o^b\right)$. Note that for constant $\epsilon = \bar{\epsilon}, \dot{\epsilon} = 0^T$, the model (5) becomes the same of [7].

## IV. Backstepping Control with fuzzy adaptation

By adding the hydrodynamics effects, restoring forces, disturbance and the kinematics equation, we obtain as result the final model:

$$M(\epsilon)\dot{\nu} + C(\nu,\epsilon)\nu + D(\nu)\nu + g(\eta,\epsilon) + T(\nu,\epsilon)\dot{\epsilon} + d = \tau \quad (6)$$

$$\dot{\eta} = J(\eta)\nu \quad (7)$$

where $d$ represents the effect of external disturbances and model uncertainties, $g(\eta,\epsilon)$ represents the contribution of gravity and buyoancy, $D(\nu)$ is the hydrodynamic damping and the terms $M(\epsilon) = M_{RB}(\epsilon) + M_A$, $C(\nu,\epsilon) = C_{RB}(\nu,\epsilon) + C_A(\nu)$ include the effects of added masses.

**Assumption n.** *3: $det(J(\eta)) \neq 0, \forall\eta$, in particular, we are going to choose $J(\eta)$ as [7] and a roll passively stabilized vehicle.*

To derive the control law, we define the position error $\tilde{\eta} = \eta - \eta_d$ and:

$$\begin{aligned}z_1 &= \tilde{\eta} + \kappa\int_0^t \tilde{\eta}dt, \quad \kappa \succ 0 \\ \dot{z}_1 &= \dot{\tilde{\eta}} + \kappa\tilde{\eta} = J(\eta)\nu - \dot{\eta}_d + \kappa\tilde{\eta}\end{aligned}$$

and the following Lyapunov function:

$$\begin{aligned}V_1 &= \frac{1}{2}z_1^T z_1 \\ \dot{V}_1 &= z_1^T\left(J(\eta)\nu - \dot{\eta}_d + \kappa\tilde{\eta}\right)\end{aligned} \quad (8)$$

choosing $\nu = \alpha + z_2$ as virtual input, with

$$\alpha = J^{-1}(\eta)\left(\dot{\eta}_d - K_1 z_1 - \kappa\tilde{\eta}\right), \quad K_1 \succ 0$$
$$\dot{\alpha} = J^{-1}(\eta)\left(\ddot{\eta}_d - K_1 \dot{z}_1 - \kappa\dot{\tilde{\eta}}\right) +$$
$$+ (\dot{J^{-1}})(\eta)\left(\dot{\eta}_d - K_1 z_1 - \kappa\tilde{\eta}\right)$$

then the Eq.(8) becomes:

$$\dot{V}_1 = -z_1^T K_1 z_1 + z_1^T J(\eta) z_2 \tag{9}$$

Since $z_2 = \nu - \alpha$ and $\dot{z}_2 = \dot{\nu} - \dot{\alpha}$ and recalling that $x^T\left(\dot{M} - 2C\right)x = 0, \forall x \neq 0$, then we can define the following Lyapunov function:

$$V_2 = \frac{1}{2} z_2^T M(\epsilon) z_2 + \frac{1}{2}\sum_{i=1}^{6} \tilde{\beta}_i^T \tilde{\beta}_i \tag{10}$$

where the vectors $\beta_i$ will be defined in the sequel. Taking the time derivative of Eq.(10) we obtain:

$$\begin{aligned}
\dot{V}_2 &= z_2^T M(\epsilon)\left(\dot{\nu} - \dot{\alpha}\right) + \frac{1}{2} z_2^T \dot{M}(\epsilon) z_2 + \sum_{i=1}^{6} \tilde{\beta}_i^T \dot{\tilde{\beta}}_i = \\
&= z_2^T\left(\tau - C(\nu,\epsilon)\nu - D(\nu)\nu - g(\eta,\epsilon) - T(\nu,\epsilon)\dot{\epsilon} - d + \right. \\
&\left. + C(\nu,\epsilon) z_2 - M(\epsilon)\dot{\alpha}\right) + \sum_{i=1}^{6} \tilde{\beta}_i^T \dot{\tilde{\beta}}_i
\end{aligned} \tag{11}$$

**Assumption n.** *4: Each element $d_i$ of $d = \left(\ d_1, ..., d_6\ \right)^T$ is bounded:*

$$|d_i| \leq \gamma_i, \quad \gamma_i \in \mathbb{R}, \quad \gamma_i > 0,$$

According to the Universal Approximation Theorem [9] there exists a fuzzy system $\mathcal{F}_i^*$ based on gaussian membership functions, such that:

$$|\mathcal{F}_i^* - d_i| \leq \omega_i, \qquad \omega_i \in \mathbb{R}, \omega_i \geq 0$$

A natural choice for the fuzzy system can be [6]:

$$\mathcal{F}_i = \frac{\sum_{j=1}^{L} \mu_j(z_{2i})\beta_{ij}}{\sum_{j=1}^{L} \mu_j(x_i)} = Q(z_{2i})^T \beta_i$$
$$\mu_j(z_{2i}; \xi_j, \sigma_j) = \exp\left[-\left(\frac{z_{2i} - \xi_j}{\sigma_j}\right)^2\right]$$

where $L$ is the number of rules and $\beta_i \in \mathbb{R}^L$ are the weighting coefficients ("consequent terms" in the Fuzzy Logic jargon). Based on the previous definition, we can choose the following control law:

$$\begin{aligned}
\tau =& M(\epsilon)\dot{\alpha} - C(\nu,\epsilon) z_2 + C(\nu,\epsilon)\nu + D(\nu)\nu + g(\eta,\epsilon) + \\
&+ T(\nu,\epsilon)\dot{\epsilon} + \mathcal{F} - K_2 z_2 - J^T(\eta) z_1, \quad K_2 \succ 0
\end{aligned}$$

where $\mathcal{F} = \left[\mathcal{F}_1\ ,\ \ldots\ ,\ \mathcal{F}_6\right]^T$ and we can rewrite the Eq.(11) as:

$$\dot{V}_2 = -z_2^T K_2 z_2 + \sum_{i=1}^{6} z_{2i}\left(\mathcal{F}_i - d_i\right) + \sum_{i=1}^{6} \tilde{\beta}_i^T \dot{\tilde{\beta}}_i - z_1^T J(\eta) z_2 \tag{12}$$

Defining $\tilde{\beta}_i = \beta_i - \beta_i^*$ the Eq.(12) becomes:

$$\begin{aligned}
\dot{V}_2 =& -z_2^T K_2 z_2 + \sum_{i=1}^{6} z_{2i}\left(Q(z_{2i})^T \beta_i^* - d_i\right) + \\
&+ \sum_{i=1}^{6} \tilde{\beta}_i^T\left(\dot{\tilde{\beta}}_i + Q(z_{2i}) z_{2i}\right) - z_1^T J(\eta) z_2
\end{aligned} \tag{13}$$

Choosing the updating law:

$$\dot{\tilde{\beta}}_i = -Q(z_{2i}) z_{2i} \tag{14}$$

we obtain:

$$\begin{aligned}
\dot{V}_2 &= -z_2^T K_2 z_2 + \sum_{i=1}^{6} z_{2i}\left(Q(z_{2i})^T \beta_i^* - d_i\right) - z_1^T J(\eta) z_2 \\
&\leq -z_2^T K_2 z_2 + \sum_{i=1}^{6} |z_{2i}||Q(z_{2i})^T \beta_i^* - d_i| - z_1^T J(\eta) z_2 \\
&\leq -z_2^T K_2 z_2 + \sum_{i=1}^{6} |z_{2i}|\omega_i - z_1^T J(\eta) z_2
\end{aligned} \tag{15}$$

Combining Eq.(9),(15) we conclude that:

$$\dot{V} = \dot{V}_1 + \dot{V}_2 \leq -z_1^T K_1 z_1 - z_2^T K_2 z_2 + \sum_{i=1}^{6} |z_{2i}|\omega_i$$

Since the following inequalities hold:

$$\sum_{i=1}^{6} |z_{2i}|\omega_i \leq \varphi\|z_2\| \leq \varphi\|z\|$$
$$-z_1^T K_1 z_1 - z_2^T K_2 z_2 \leq -\varrho\|z\|^2$$

where $\varphi = 6\max_i \omega_i$ and $\varrho = \min_i \lambda_i(diag(K_1, K_2))$, then it follows that:

$$\dot{V} \leq \|z\|\left(\varphi - \varrho\|z\|\right) \tag{16}$$

which is negative definite in $z$ for all:

$$\|z\| > \frac{\varphi}{\varrho} \tag{17}$$

Condition (17) proofs the uniform ultimate boundedness of the trajectory error. Note that, increasing the controller gain ($\varrho$) or as the norm of the fuzzy approximation error ($\varphi$) decreases, the region of ultimate boundedness shrinks towards the origin.

## V. SIMULATION

The controller defined in the previous section is now tested against a complete dynamic model, in order to evaluate the robustness of the controller with respect to model mismatch and external disturbances. Physical parameters for the Folaga vehicle have been used. With respect to the control law, we assume to know the mass and COG rate of variation and to neglect the high order derivative. However these effects will be included in the generalized external disturbance $d$. In order to simulate the controller presented in the previous section, we need to control the $\epsilon$-subsystem and to access its state. For this purpose, it is possible to realize a controller

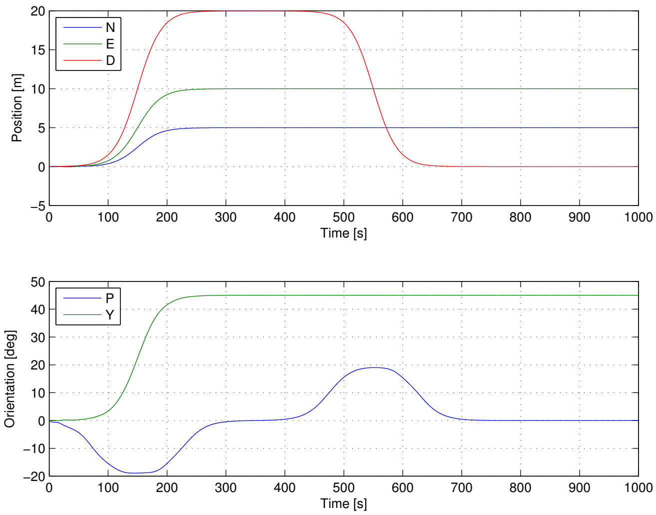


Fig. 3. Trajectory of the vehicle during the proposed sampling operation

$\tau$-dependent (control allocation) or alternatively, use an external controller connected directly to the depth and pitch error. In the following simulation we are going to follow the second policy, applying for both $\epsilon_p$ and $\epsilon_b$ subsystems a PI control. The following disturbance $f_c$ is applied to simulate the sea current:

$$f_c^b = R_n^b R_w^n f_c^w \tag{18}$$

and in addiction, to simulate the real vehicle, the neglected dynamics are included in the rigid-body equation, in particular defining:

$$f_m = m\left(\omega^b \wedge \dot{r}_g + \ddot{r}_g\right) + \dot{m}\dot{r}_g \tag{19}$$

and combining the Eq.(18) and (19) we obtain:

$$d = F_c + F_m \tag{20}$$

where $F_m = \left[f_m, r_g^b \wedge f_m\right]^T$ and $F_c = \left[f_c, r_g^b \wedge f_c\right]^T$. The purpose of this simulation is to sample data over different reference points. In particular, the vehicle moves towards a given position in geographical coordinates and depth, then is kept idle while performing environmental sampling, and finally re-surfaces moving on the vertical axis only. In details, reference is generated using sigmoidal functions:

$$\sigma_i(t) = \left[1 + e^{-\rho_i t}\right]^{-1},\ t \in \mathbb{R},\ \rho \in \mathbb{R}, \quad t \geq 0,\ \rho_i \geq 0$$

In particular, defining $\eta_d(t) = R^T\sigma(t)$ , with $\sigma = col(\sigma_i(t))$ and $R$ a real matrix of proper dimension, and computing the time derivative, it is possible to choose $\rho_i$ in order to provide a bound in terms of velocity. Results of this simulation are presented in Fig. 3,4,5,6,7 and 8. The results show that during the vehicle motion the $\epsilon$-subsystem, which is controlled by an external PI, will affect the model structure and will provide an unmeasurable disturbance. This problem will be handled by the adaptive control by changing the fuzzy consequent terms $\beta_i$.

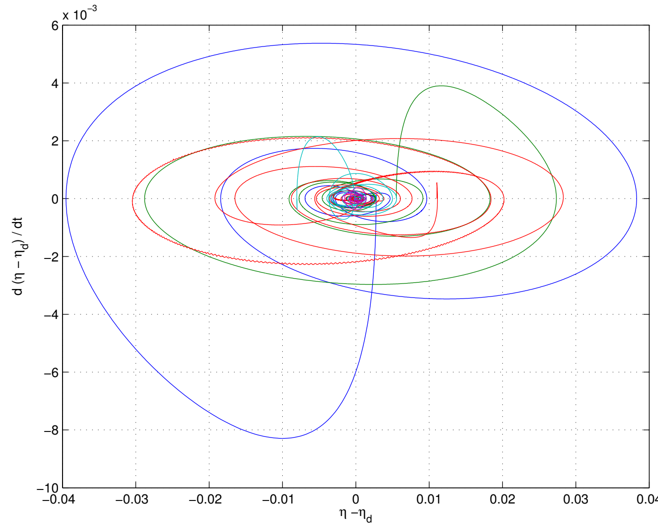


Fig. 4. Evolution of the position and velocity errors. As result of the control law selection, the trajectory of each element of $\tilde{\eta}$ and $\dot{\tilde{\eta}}_d$ goes towards the origin.

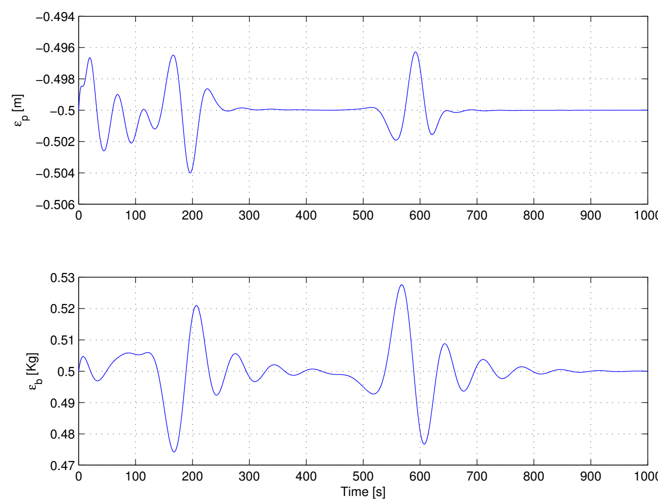


Fig. 5. Battery pack position along the surge axis $\epsilon_p$ and water mass contained in the ballast tank $\epsilon_b$ as function of time within the mission.

## VI. Conclusion

An approach to the simplified dynamic modelling of AUVs with hybrid actuation systems, combining propellers with change of mass and mass displacement, has been proposed. The approach, based on a quasi-static approximation of the vehicle COG motion, leads to a general model expressed in the standard, compact form of [7]. The Folaga is used to be an example of such modelling approach. Partial knowledge of COG and mass variations is included in the rigid-body equations while the unmeasurable parts are encapsulated into the generalized external disturbance. Due to the model structure, analysis and control synthesis can also be approached in a standard, systematic way. An example of such design, a backstepping controller with fuzzy adaptation, has been presented and it has been shown through simulation that the

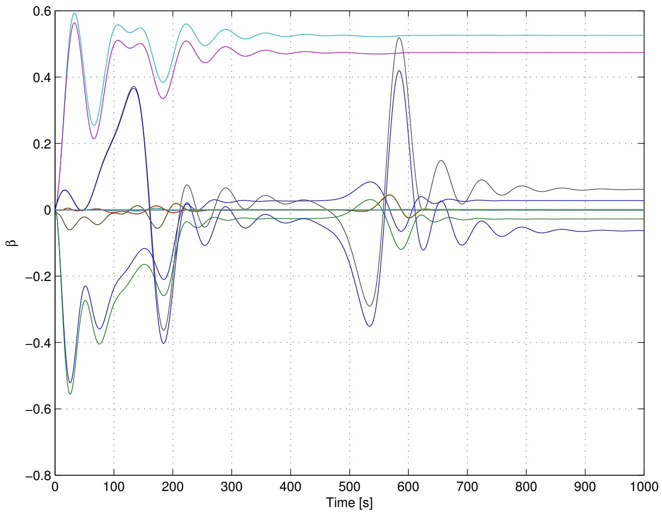


Fig. 6. Evolution of the coefficients $\beta$. In this example, a Fuzzy system with $L = 3$ is adopted. As shown in the picture, during vehicle motion, the adaptive system plays a fundamental role to contrast model uncertainties and external sea current disturbance. During sampling operation, since the contribution of $f_m$ becomes negligibile, the adaption process terminates with suitable value of $\beta$ to contrast sea currents disturbance only.

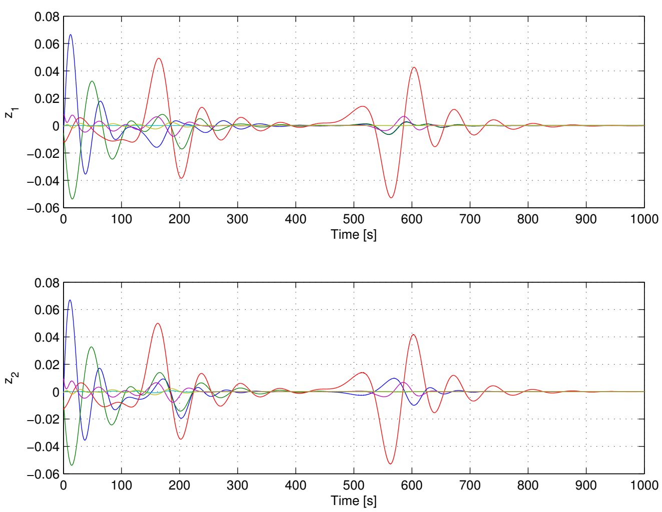


Fig. 7. Trajectory of $z_1$ and $z_2$

designed controller is able to account for the dynamic mismatch between the simplified model and the complete vehicle dynamics, as well as to cope with external disturbances.

## ACKNOWLEDGMENT

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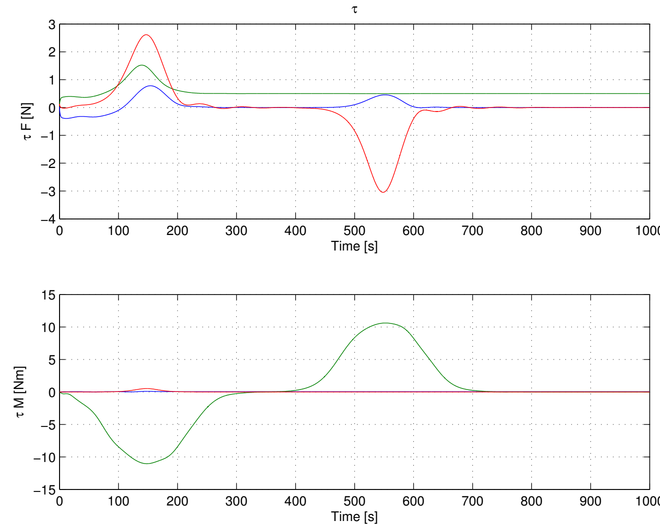


Fig. 8. Generalized forces $\tau$