Abstract—This paper proposes a method of determining two pairs of vanishing points from an H-pattern, which consists of two parallel lines and another line perpendicular to the two, in a spherical image. In contrast to the conventional methods which determine a vanishing point (a pair of vanishing points at a spherical image) from a group of parallel lines, the proposed method first determines a pair of vanishing points by the two parallel lines and then compute the other pair of vanishing points from the remaining line in terms of the inherent relation of the H-pattern. The experimental result of estimating the pose of a camera mounted at the rear of a vehicle is given to show the effectiveness of the proposed method.

I. INTRODUCTION

A rear camera is widely used not only for avoiding accidents of backing a vehicle, but also for assisting to park a vehicle at the center of the division of a parking lot. To assist to park a vehicle, generating a top-view image and showing it to the driver is helpful. However, to generate a top-view, we need to know not only the intrinsic parameters of the rear camera, but also the camera’s pose relative to the ground.

While the intrinsic parameters of a camera can be measured and written into the memory of the camera during the process of the manufacture, the camera pose must be estimated when a user mounts a rear camera purchased from a store. The conventional method to estimate camera pose is to provide users a special calibration pattern and to let users operate the calibration process in terms of the instruction of a manual. If the camera pose can be determined by observing some natural patterns, it will lighten the users’ burden.

One of the natural patterns is the marking at a parking lot. Fig. 1 shows a typical image captured by a wide field-of-view rear camera at a parking lot. In Fig. 1 there are a group of parallel lines, called side lines in this paper, which are used to fit the two sides of the vehicle to be parked, and another line, called a rear line in this paper, which is used to fit the rear side of the vehicle. In 3D space the rear line is not only coplanar with the side lines, but also perpendicular to them. These lines constitute a pattern like the shape of the character ‘H’, called H-pattern in this paper. Can we use such H-patterns to determine the camera pose?

In this paper we propose a method of estimating camera pose from such an H-pattern by determining two pairs of vanishing points from an H-pattern in a spherical image. In contrast to the conventional methods which determine a vanishing point (a pair of vanishing points at a spherical image) from a group of parallel lines, the proposed method first determines a pair of vanishing points by the two parallel lines and then compute the other pair of vanishing points from the remaining line in terms of the inherent relation of the H-pattern. The proposed algorithm is simple and easy to be implemented. The experimental results of real-world environment show the effectiveness of the proposed method.

The rest of this paper is organized as follows. In the next section, we introduce the related research. Then, we describe the method of determining the two pairs of vanishing point from an H-pattern in section 3. The experimental results are given in section 4. Finally, we conclude in the last section.

II. RELATED RESEARCH

A vanishing point is defined as the intersection of a group of parallel lines at space at infinite. Fig. 2 shows an image of a cube; there are three groups of parallel lines in the cube; it results in three vanishing points in a perspective image. Since in man-made environments there are many parallel lines, a vanishing point is an important cue for understanding the structure of environments [10][8][7]. Vanishing points can be computed from architectural scenes or moving humans, and used for camera calibration [3][1][2][4][6][9]. Given the intrinsic parameters of camera, the vanishing points can be used for estimating camera pose [11].

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To determine the pose of a camera relative to a calibration pattern, at least two vanishing points are necessary [10]. In all the above research every vanishing point is determined as the intersection of a group of parallel lines at space. However, in some cases it is difficult to find two groups of parallel lines at space.

Fig. 3 shows a sketch at a junction of a corridor in indoor environments. There are four lines in the scene, where the points $A, B, C$ and $D$ are co-linear. While the two vertical lines, $EB$ and $FC$, are parallel to each other, the other two lines, $AD$ and $BG$, are perpendicular to the two vertical lines, but not parallel to each other. This means that there is only one group of parallel lines in the scene. By the conventional method, only one vanishing point can be found from the scene. Thus, we cannot estimate the camera pose by using the one vanishing point.

In this paper we define a $H$-pattern as a group of three lines in which two lines are parallel to each other and the other is perpendicular to the parallel two lines at space. In terms of the above definition, $AD$, $EB$ and $FC$ compose a $H$-pattern, and $BG$, $EB$ and $FC$ the other one in Fig. 2. Can we find two vanishing points from such an $H$-pattern to estimate camera pose?

In this paper we propose a method of determining two pairs of vanishing points from an $H$-pattern in a spherical image. In contrast to the conventional methods which determine a vanishing point (a pair of vanishing points at a spherical image) from a group of parallel lines, the proposed method first determines a pair of vanishing points by the two parallel lines and then compute the other pair of vanishing points from the remaining line in terms of the inherent relation of the $H$-pattern. The proposed algorithm is simple and easy to be implemented. The effectiveness of the proposed method is shown by the experiment results of estimating the camera pose of a rear camera of a vehicle from an $H$-pattern at a parking lot.

III. PROPOSED METHOD

A. Determining a Pair of Vanishing Points from a Group of Parallel Lines in a Spherical Image

Fig. 4 shows a group of parallel lines are observed at space by a virtual spherical camera. The group of parallel lines results in a pair of vanishing point in a spherical image [3][6]. The position of the pair of vanishing points in a spherical image changes with the orientation between the spherical camera and the group of parallel lines. The pair of vanishing points is at the opposite side of each other.

B. Determining a Pair of Vanishing Points as the Intersection of a Line with a Vanishing Line

A vanishing line is defined as the intersection of a group of parallel planes at space at infinite. In a spherical image a vanishing line corresponds to a great circle which is the intersection of the parallel plane passing through the center of the spherical image, as shown in Fig. 5. The pair of vanishing points of a line parallel with the planes can be determined as the intersection of the line with the vanishing line of the plane at space at infinite.
C. Determining Two Pairs of Vanishing Points from a H-pattern

Summarizing the above explanation, we know that the pair of vanishing points of a line can be determined either by a group of parallel lines including the line, or by a vanishing line of a plane parallel with the line. Based on these facts, we can determine the two pairs of vanishing points of a H-pattern. We first describe the idea, and then give the detailed algorithm.

Fig. 6 shows a H-pattern, which consist of two parallel lines, $l_{S1}$ and $l_{S2}$, and another line, $l_{R}$, perpendicular to the two. A pair of vanishing point, $V_{S}$ and $V_{S}'$, can be determined from the two parallel lines, $l_{S1}$ and $l_{S2}$, as shown in Fig. 6(a).

Then, with the normal vector $n$ pointing to $V_{S}$ from the origin we can determine a plane, $\Pi$, passing through the center of the spherical image. The intersection of this plane with the spherical image results in a great circle, $C_{V}$. The great circle $C_{V}$ is the vanishing line of the parallel planes.

Fig. 6 (a) The determination of the first pair of vanishing points, $V_{S}$ and $V_{S}'$, from the two parallel lines of an H-pattern. (b) The determination of the second pair of vanishing points, $V_{R}$ and $V_{R}'$, by using the vanishing line, $C_{V}$, and the perpendicular line of the H-pattern.
including \( \Pi \). Since line \( l_R \) is parallel with plane \( \Pi \), the intersection of line \( l_R \) with plane \( \Pi \), \( V R \) and \( V R' \), is the pair of vanishing points of line \( l_R \), as shown in Fig. 6(b). Thus, we acquire two pairs of vanishing points from an H-pattern.

Next, we give the detailed algorithm as follows.

Suppose the normal vectors of the three projection planes of the three lines, \( l_R \), \( l_{S1} \) and \( l_{S2} \), are \( n_{S1} \), \( n_{S2} \) and \( n_R \), respectively.

1. Firstly, compute the pair of vanishing points of the two parallel lines.

The pair of vanishing points, \( V_S \) and \( V_S' \), of the two parallel lines, \( l_{S1} \) and \( l_{S2} \), is computed as follows.

\[
\pm (n_{S1} \times n_{S2}) \quad (1)
\]

2. Secondly, compute the vanishing line of planes with the normal vector of \( (n_{S1} \times n_{S2}) \).

Let \( n = n_{S1} \times n_{S2} \). Suppose the plane passing through the origin and having the unit vector, \( n \), is \( \Pi \). The great circle, \( C_y \), is obtained as the intersection of the plane, \( \Pi \), with a unit spherical image.

\[
n_x \sin \theta \cos \phi + n_y \sin \theta \sin \phi + n_z \cos \theta = 0 \quad (2)
\]

Where \( n = [n_x, n_y, n_z] \).

3. Finally, compute the pair of vanishing points of line \( l_R \).

The intersection of the projection plane of \( l_R \) with a unit spherical image is represented as follows.

\[
n_{Ra} \sin \theta \cos \phi + n_{Rb} \sin \theta \sin \phi + n_{Rc} \cos \theta = 0 \quad (3)
\]

Where \( n_R = [n_{Ra}, n_{Rb}, n_{Rc}] \).

Then, the pair of vanishing points, \( V_R \) and \( V_R' \), of \( l_R \) is computed as the solution of the set of equations (2) and (3).

By dividing the two side of (2) and (3) by \( \cos \theta \), and letting \( x = \tan \theta \cos \phi \) and \( y = \tan \theta \sin \phi \), we have

\[
\begin{align*}
n_a x + n_b y + n_c &= 0 \\
n_{Ra} x + n_{Rb} y + n_{Rc} &= 0
\end{align*}
\]

Solve (4),

\[
\begin{bmatrix}
    n_c & n_b \\
    n_{Rc} & n_{Rb} \\
    n_a & n_b \\
    n_{Ra} & n_{Rb}
\end{bmatrix} = \frac{n_{Ra} n_{Rb} - n_{Rb} n_{Ra}}{n_a n_{Rc} - n_{Ra} n_{Rc}},
\]

\[
\begin{bmatrix}
    n_a & n_c \\
    n_{Ra} & n_{Rc} \\
    n_a & n_b \\
    n_{Ra} & n_{Rb}
\end{bmatrix} = \frac{n_{Ra} n_{Rc} - n_{Rb} n_{Rc}}{n_a n_{Ra} - n_{Ra} n_{Ra}}.
\]

Thus,

\[
\begin{align*}
\tan \phi &= \frac{y}{x} = \frac{n_a n_{Rc} - n_{Ra} n_{Rc}}{n_a n_{Rb} - n_{Ra} n_{Rb}} \quad (6)
\end{align*}
\]

Using the value of \( \phi \), we can compute the value of \( \theta \), and then determine the pair of vanishing points of line \( l_R \).

D. Estimating the orientation of the camera from the two pairs of vanishing points

Using the two pairs of vanishing point computed from an H-pattern, mentioned above, we can estimate the orientation of a camera relative to the H-pattern. By setting the world coordinate system to the H-pattern, as shown in Fig. 6, the relative orientation matrix, \( R_{WC} \), between the camera coordinate system and the world coordinate system can be estimated by the cue of the two pairs of vanishing points as follows.

\[
V_S = R_{WC} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}; V_R = R_{WC} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}. \quad (11)
\]

Further using the projection of the origin of the world coordinate system in the image, the translational vector can be computed up to a scale factor.

IV. EXPERIMENT OF REAL-WORLD ENVIRONMENT

In this section we apply the proposed method to the estimation of the pose of a rear camera of a vehicle from an H-pattern usually existing at a parking lot, as shown in Fig. 1 in order to show the effectiveness of the proposed method. A rear camera is widely used not only for avoiding accidents of backing a vehicle, but also for assisting to park a vehicle at the center of the division of a parking lot. To assistant a driver to park his/her vehicle, the camera pose relative to the parking partition must be estimated continuously and used to control the motion of the vehicle in terms of the visual feedback.

In this experiment a rear view camera, Alpine HCE-C105, is mounted at the rear of a vehicle, Toyota NOAH, as shown in Fig. 7. The park environment used in this experiment is one of the parking lots of Tottori University.
Fig. 1 shows a typical image captured by the rear camera at the parking lot. In Fig. 6 there are a group of parallel lines, corresponding to the side lines of Fig. 1, which are used to fit the two sides of the vehicle to be parked, and the other line, corresponding to the rear line of Fig. 1 which is used to fit the rear side of the vehicle. In 3D space the rear line is not only coplanar with the side lines, but also perpendicular to them. These lines constitute an H-pattern. Here, we estimate the camera pose from such an H-pattern given the intrinsic parameters of the camera.

The pose of a camera consists of the orientation and position of the camera, which correspond to the rotational matrix and translational vector, respectively. Using two pairs of vanishing points, we can compute the rotational matrix and recover the translational vector up to a scale factor [11].

Next, we give the actual steps of the processing of estimating the rear camera pose by the proposed method.

A. Extracting lines

The rear camera used in this experiment has about 138 degree horizontal field of view. Heavy distortion exists as shown in the image of Fig. 1. First, we calibrate the intrinsic parameter of the rear camera using the method of [5], and then undistort the input image, as shown in Fig. 8(a). Next, the edge points are classified into two groups according to the gradient cue of edge points by using the method of [7], as shown in Fig. 8(b) and (c). Note that in every group there is an H-pattern. The H-pattern is used to estimate the pose of the rear camera.

B. Computing the projection plane of lines

To estimate the orientation from a H-pattern by the proposed method we need to know the normal vector of the projection plane of a line. Here, we give the method of computing it from a calibrated perspective image. Suppose a found line is described by the parameter of \((d, \theta)\), where \(d\) is the distance of the principle point at the image to the line and \(\theta\) is the polar angle, as shown in Fig. 9.

Then, the equation of line \(l\) at the image is

\[ x \cos \theta + y \sin \theta - d = 0. \] (7)

Assume that the focal length of the camera is \(f\). The three-dimensional vector, \(m\), of the line direction at the coordinate system of the camera is

\[ m = \begin{pmatrix} -\sin \theta & \cos \theta & 0 \end{pmatrix}. \] (8)

Let the intersection of line \(l\) with the line passing through the principle point and perpendicular to line \(l\) be \(p\). Then, the three-dimensional vector of \(p\) is

\[ p = \begin{pmatrix} d \cos \theta & d \sin \theta & f \end{pmatrix}. \] (9)

Finally, the unit normal vector, \(n\), of the projection plane of line \(l\) is computed as follows.

\[ n = \frac{p \times m}{\|p \times m\|}. \] (10)

C. Generating the top-view from a rear camera

By using the algorithm of section 3, the two pairs of vanishing points of the every H-pattern of Fig. 8(b) and (c) can be determined. Further, the camera orientation can be
determined from the two pairs of vanishing points, and the translational vector can be computed up to a scale factor by using the projection of the origin of the world coordinate system in the image. Finally, we can generate the top-view from the image of the rear camera by using the estimated camera pose.

Fig. 10 shows the top-view generated from the image of Fig. 6. The top-view is generated by projecting the input image to the ground by aligning the horizontal and vertical axes with the rear and side lines, respectively. Since the virtual camera of the top-view is at the same position as the rear camera, the lower part of the top-view is empty. However, the generated top-view can be further to be shifted and rotated in terms of the need of the concrete tasks.

V. CONCLUSIONS
The contribution of this paper is as follows.

- In the conventional methods a vanishing point is determined as the intersection of a group of parallel lines. This paper proposes a method of determining two pairs of vanishing points from a H-pattern. A H-pattern consists of three lines, in which two lines are parallel with each other and the other line is perpendicular to the two. While one pair of vanishing points is determined from two parallel lines, the other pair of vanishing points is determined as the intersection of the other line with the vanishing line computed from the two.

- The proposed method is also used to estimate the pose of a rear camera of a vehicle. Using the estimated camera pose a top-view can be generated from the image of the rear camera to assist the parking of the vehicle. The experimental result shows the effectiveness and usefulness of the proposed method.

Camera pose can also be estimated from a set of n correspondences between 3D points and their 2D projections given the intrinsic parameters of camera, known as the PnP (Perspective-n-Point) problem [12][13][14]. While the metric information of 3D points is assumed to be known in the PnP problem, the proposed method is based on the qualitative properties of a H-pattern, and is a non-metric approach. Like the method for the PnP problem, the proposed method is effective as hypothesis generators in popular RANSAC paradigm [15] or can be used for initializing the bundle adjustment [16].

Since the straight lines are found by the Hough transform approach in this paper, the proposed algorithm works even if the painted lines are partially hidden. Evaluating its performance for various parking lot scenes under various luminosity is a future work to do.

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