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Passive Dynamic Walking with Quadrupeds - Extensions towards 3D

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***Abstract*—In the present study, we applied the principles of passive dynamic walking onto the three dimensional motion of a simplified quadrupedal model. We extended the simulation framework of a planar system to include a rolling degree of freedom and searched for limit cycles that represent periodic gaits. Among the eight different gaits that we identified, were three kinds of trots and paces, as well as a lateral and diagonal single foot sequence. We could show that a distinct relation exists between the lateral spacing of the legs and the relative phase of the front and the back legs, and a certain trade-off between efficiency and dynamic stability. In agreement with established bipedal models, our results showed that the lateral rolling motion is invariably unstable.**

I. Introduction

PASSIVE Dynamic Walking [1, 2] refers to a class of mechanical mechanisms that, without using actuation or sensing of any kind, are able to walk down a shallow incline. Exploiting the nonlinear coupled pendula dynamics of their legs, these 'robots' exhibit a purely mechanical limit cycle that rejects small disturbances. For a well-selected set of mechanical parameters, such as the masses of the individual links or their inertial properties, deviations from the nominal periodic trajectory are damped out, enabling the walker to maintain a steady step length and forward speed. More complex systems based on these principles include walkers with knees [3] or mechanisms that move in three dimensions [4]. The gaits that emerge from these walkers and the parameter selections necessary to stabilize them result in models that resemble human bipedal walking, and researchers were able to show that the stabilizing effect is also utilized in human locomotion [5]. Applying these principles to robots, allows the construction of remarkably efficient devices [6, 7] which, through clever inclusion of actuation, are able to continuously walk on level ground.

When extending the principles of passive dynamic walking to quadrupedal locomotion [8, 9], a simple planar model is able to produce two distinct kinds of symmetric periodic gaits: *two-beat* gaits in which the front and back legs swing in phase, and *four-beat* gaits in which the leg pairs are acting 90º out of phase and the feet strike the ground in an evenly timed sequence (Fig. 1). Within the context of the present study, it is very important to note that apart from an *exact* two-beat gait in which the two swing legs strike the ground at exactly the same moment (a rather theoretical construct), there exist additionally two *inexact* two-beat gaits in which the two legs strike the ground in quick succession (both orders: front-back (1) and back-front (2) are possible). In a planar model the time between the two successive foot strikes accounts for only 0.2% of total step time. While the inexact two-beat gaits are dynamically stable, the four-beat gait has an unstable mode that, upon encountering a disturbance, will act on the phase-difference between the front and the back leg pair causing the four-beat gait to transition into one of the inexact two-beat gaits.

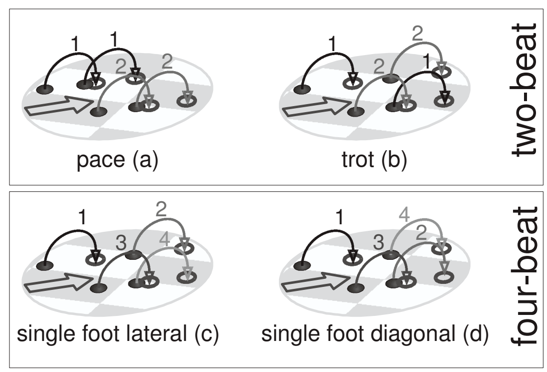

Fig. 1: In a planar quadrupedal model, no difference between the left and right side exists, which makes it possible to classify all passive dynamic gaits either as *two-beat*, or *four-beat*. If a third dimension is added, each of these gaits exists in two variations. For the two-beat gait, these are the *pace* shown in (a) and the *trot* in (b), and for the four-beat the *lateral single foot sequence* in (c) and the *diagonal single foot sequence* in (d).

Figure 2 compares these results to quadrupedal gaits found in nature. It shows a so called *gait graph*, (adapted from Hildebrand [10]) which illustrates the range of *symmetrical gaits* as defined by "more than 1,000 plots for 156 genera of tetrapods". Symmetrical gaits are gaits in which the left and right side of the quadruped perform the same motion but with a phase-lag of half a stride. The graph therefore excludes gaits such as bounding, pronking, or galloping. Moreover, when mirrored at the saggital plane, the second half of the stride of a symmetrical gait is equal to the first half, which means that it is possible to limit the analysis to a half-stride; a technique that was employed in [8] and [9].

In both these studies the legs were assumed to be perfectly rigid, meaning that exactly two legs are in ground contact at

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all times. The percentage of stride that each foot is on the ground (also called *duty cycle*) is therefore inevitably 50 %. This means that all possible gaits are situated on the center line of the gait graph in figure 2. The second variable that constitutes the graph refers to the relative phase of front and back legs, or -more precisely- the percentage of stride after which the forefoot falls after the hindfoot on the same side of the quadruped. For a two-beat gait, this number is either 0 % (if the two legs on each side of the quadruped swing together, a gait which is called *pace*), or 50 % (with diagonal pairs swinging together, called *trot*). For a four-beat gait, this number can be 25 % or 75 %, depending on whether the feet strike the ground in *lateral single foot sequence*, or *diagonal single foot sequence*. A visual definition of these gaits is given in Figure 1.

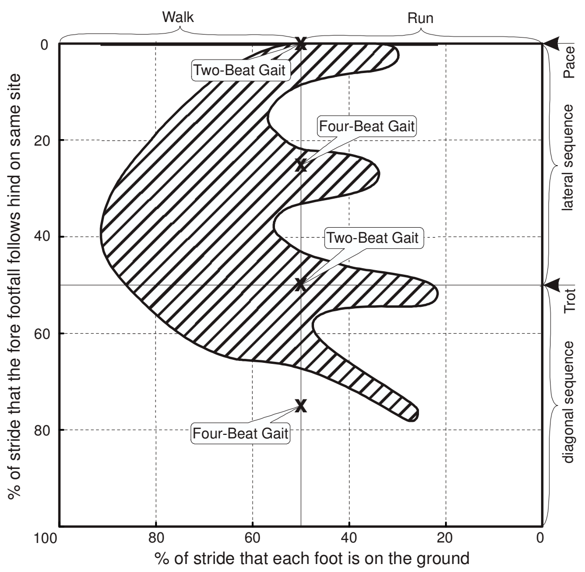

Fig. 2: Passive dynamic gaits of quadrupeds represent only a few distinct solutions in the large continuous range of symmetric gaits found in nature. In 2D, the models are also unable to explain the strong bias towards lateral gait sequences that can be observed in nature. The shaded region was adapted from [10] and shows the distribution "of more than 1000 plots for 156 genera of tetrapods."

In total, the passive dynamic gaits account for four points in the gait graph, indicated by black crosses in Figure 2. The shaded area in the same figure illustrates the considerably larger range of gaits found in actual quadrupeds. Nature utilizes this wide variety to adapt to various conditions, such as different body properties, locomotion speeds, or environments. Passive dynamics and the limited number of gaits produced by them might be an explanation for the fourfold indention of this range, but fail to account for the considerably larger variety of phase shifts and duty cycles found in nature, as well as for nature's strong bias towards walking gaits in which the legs fall in a lateral sequence. To be able to understand this bias, we have to recall that in a planar model with no true notion of left and right the top and bottom half of the gait graph cannot be distinguished. It is hence obvious, that the underlying effect can only be observed and be explained if we expand the model to three dimensions. In fact, in the biological literature, this bias has been attributed to the better static stability properties of the lateral single foot sequence in three dimensions. The support polygon that arises in such a lateral sequence has the beneficial property that throughout the entire gait cycle it naturally supports the quadruped's center of mass. In contrast thereto, large compensating movements are necessary to allow statically stable walking with a diagonal single foot gait [10]. However, this explanation is only valid as long as at least three feet of the quadruped are on the ground at all times. This excludes the rightmost three quarters of the graph (with a duty cycle smaller than 75%), for which the bias is still clearly visible.

To better investigate the underlying principles, and to develop guidelines for the creation of robotic gaits, we extended a model of a passive dynamic walking quadruped towards three dimensions.

II. Methods

The model used in this study consists of five rigid links with distributed mass (Fig. 3). Four legs were connected to the front and back end of a main body through single-DoF rotational joints. Each pair of legs shared a common rotational axis normal to the sagittal plane of the model. Due to the rigid nature of the segments, it can be assumed that exactly two legs are in contact with ground at all times, while the other two legs are swinging freely. The feet were assumed to have no geometrical extension. We assumed that the system has no joint-friction, which means that energy is only lost during the ground contact collisions.

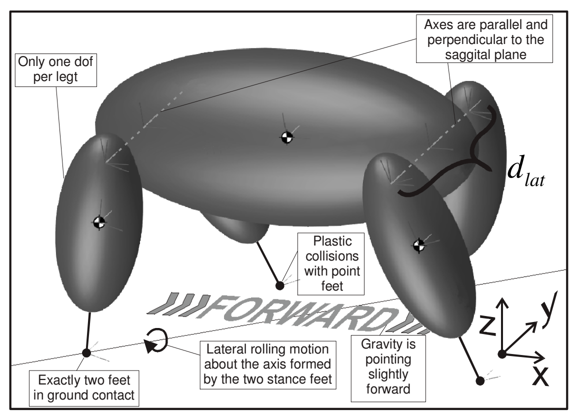

Fig. 3: Model of a three dimensional passive dynamic quadruped. While the internal dynamics of the model are planar (all legs swing in parallel planes), it can rotate freely about the axis defined by the stance feet on the ground, which adds an additional degree of freedom in comparison to the planar model described in [9]

We used normalized units [11] to scale all parameters with respect to overall weight M and leg length l. The gravitational constant was set to 1. To simulate walking on a shallow incline, gravity $\mathbf{g} = (\sin(1^o)\ 0\ -\cos(1^o))$ was pointing slightly into the positive x-direction, which was the direction of forward motion of the model. The main body contributed $0.8M$ to the total mass, was 1.5 leg lengths long, and had a radius of gyration of $1.5l\sqrt{1/10}$ about the mediolateral and dorsoventral axes and a radius of gyration of $1.5l\sqrt{1/25}$ about the anteroposterior axis. The COM was centered. Each of the four legs weighed $0.05M$, the center of mass was positioned $1/3l$ below the joints, and the radius of gyration was $\frac{2}{3}\sqrt{1/10}l$ about the two short principal axes and $1/10l$ about the long principal axis. These parameters were selected to roughly resemble the properties of a Merino sheep. All parameters were fixed throughout the study. Variations were only studied for the lateral spacing d_{lat} between the legs.

The equations of motion (EoM) of this system can be stated in matrix form according to:

$$\mathbf{M}(\mathbf{q})\cdot\ddot{\mathbf{q}}-\mathbf{h}(\mathbf{q},\dot{\mathbf{q}})=\sum_{i=1}^{k}\mathbf{J}_i^T(\mathbf{q})\cdot\boldsymbol{\lambda}_i=\tilde{\mathbf{J}}^T(\mathbf{q})\cdot\tilde{\boldsymbol{\lambda}}, \qquad (1)$$

with the coordinate set $\mathbf{q}=(x,y,z,\theta,\vartheta,\psi,\alpha_{1-4})^T\in\mathbb{R}^{10x1}$ (composed of the positions (x,y,z) and Euler rotations (θ,ϑ,ψ) of the main body, and the four leg angles (α_{1-4})), the mass matrix $\mathbf{M}\in\mathbb{R}^{10x10}$, and the differentiable force vector $\mathbf{h}\in\mathbb{R}^{10x1}$. Each of the k feet that are in contact with the ground exerts a Cartesian force $\boldsymbol{\lambda}_i\in\mathbb{R}^{3x1}$, which is projected into the generalized coordinates with the Jacobian $\mathbf{J}_i=\frac{\partial\mathbf{r}_i}{\partial\mathbf{q}}\in\mathbb{R}^{3x10}$ of the contact point $\mathbf{r}_i$. Additionally,

$$\begin{aligned} r_i^z &= 0 \\ \dot{\mathbf{r}}_i &= \mathbf{J}_i\cdot\dot{\mathbf{q}}=\mathbf{0} \end{aligned} \qquad (2)$$

must hold for the vertical distance r_i^z and the relative velocity with respect to the ground $\dot{\mathbf{r}}_i$. To this end, a uni-directional force element that allows no slippage ($\ddot{r}_i^x=\ddot{r}_i^y=0$), no penetration ($\ddot{r}_i^z\geq 0$), and is limited in normal direction to positive forces ($\lambda_i^z\geq 0$) is assumed as contact law. The complementary condition ($\lambda_i^z\cdot\ddot{r}_i^z=0$) combines the requirements that a contact is either active ($\ddot{\mathbf{r}}_i=0$) or opening ($\ddot{r}_i^z>0,\boldsymbol{\lambda}_i=0$).

While the opening of a contact is already included in this formulation, the collision that occurs if a swinging leg (with index m) is about to penetrate into the ground ($r_m^z=0$ and $\dot{r}_m^z<0$) and a contact is closed, requires some additional consideration. Due to arising external impulsive forces $\tilde{\boldsymbol{\Lambda}}$, the occurrence of a collision implies instantaneous changes in velocities. To compute these, the equations of motion (1) have to be integrated over the instance of the collision:

$$\int_{\{t_0\}}\left\{\mathbf{M}\ddot{\mathbf{q}}-\mathbf{h}-\tilde{\mathbf{J}}^T\cdot\tilde{\boldsymbol{\lambda}}\right\}dt=\mathbf{M}(\dot{\mathbf{q}}^+-\dot{\mathbf{q}}^-)-\tilde{\mathbf{J}}^T\cdot\tilde{\boldsymbol{\Lambda}}=0. \qquad (3)$$

As integration is performed over an infinitesimally short time span, the bounded differentiable force vector $\mathbf{h}$ disappears and only the impulsive forces and the velocity changes remain. Assuming a perfect inelastic collision with a Newton type collision law [12, 13] , all foot points that are considered part of the collision instantaneously come to a rest (or remain motionless) ($\Lambda_i^z\geq 0,\dot{\mathbf{r}}_i^+=0$). All others must leave the ground right after the collision ($\boldsymbol{\lambda}_i=0,\dot{r}_i^{z+}\geq 0$). These two alternatives are expressed in the complementary description of the collision in normal direction:

$$\begin{aligned} \dot{\mathbf{r}}^+-\dot{\mathbf{r}}^- &= \tilde{\mathbf{J}}\cdot(\dot{\mathbf{q}}^+-\dot{\mathbf{q}}^-)\overset{(3)}{=}\tilde{\mathbf{J}}\mathbf{M}^{-1}\tilde{\mathbf{J}}^T\cdot\tilde{\boldsymbol{\Lambda}} \\ \dot{r}_i^{z+}\geq 0, &\quad \Lambda_i^z\geq 0,\quad \dot{r}_i^{z+}\cdot\Lambda_i^z=0 \end{aligned} \qquad (4)$$

Using a *full* set of coordinates $\mathbf{q}$ allows for a concise representation of the EoM and also for the most convenient computation of the contact collisions. However, in a steady gait (and hence for all reasonable initial conditions) exactly two legs are in contact with the ground at all times and only four *independent* degrees of freedom exist. To facilitate the limit cycle analysis and to restrict it to meaningful states, it is therefore desirable to switch to a set of *minimal* coordinates $\mathbf{p}\in\mathbb{R}^{4x1}$ with

$$\begin{aligned} \mathbf{q} &= \mathbf{q}(\mathbf{p}) \\ \dot{\mathbf{q}} &= \frac{\partial\mathbf{q}}{\partial\mathbf{p}}\cdot\dot{\mathbf{p}}=\mathbf{Q}\cdot\dot{\mathbf{p}},\quad \mathbf{Q}\in\mathbb{R}^{10x4} \end{aligned} \qquad (5)$$

These coordinates inherently fulfill the geometric constraints

$$\dot{\mathbf{r}}=\tilde{\mathbf{J}}\cdot\mathbf{Q}\cdot\dot{\mathbf{p}}=\mathbf{0},\quad\forall\dot{\mathbf{p}}\qquad\Rightarrow\tilde{\mathbf{J}}\cdot\mathbf{Q}=\mathbf{0}, \qquad (6)$$

and allow us to state (1) without ground contact forces as:

$$\mathbf{Q}^T\mathbf{M}\mathbf{Q}\cdot\ddot{\mathbf{p}}-\mathbf{Q}^T\cdot\mathbf{h}=0. \qquad (7)$$

The specific set of minimal coordinates $\mathbf{p}$ used in this study was defined by the X and Y components of a body-fixed Z-Y-X rotation of the back stance leg with respect to a purely vertical pose and the relative angles of the main body with the two swing legs.

The equations of motion for this multi-body system were numerically integrated using MATLAB/Simulink and the SimMechanics toolbox (The MathWorks, Natick, MA) [14, 15]. To detect numerical errors, kinetic and potential energy were closely monitored throughout the course of the simulation. As the model was not provided with knees, the contact collision detection was limited to states in which the swing leg was in front of the corresponding stance leg (i.e. showed a larger angle with respect to the main body) to make sure that the swing motion was completed and to avoid foot scuffing. The pre-impact velocities and the positions were retrieved from the SimMechanics model and the collision complementary problem (4) was solved by cycling through all possible contact configurations until the unique solution that fulfills the complementary condition was found. After processing the collision, the roles of stance and swing foot were switched, the new relative position of front and back stance foot registered, and the integration re-started with the new geometric constraints and the new initial velocities.

Based on this model, we identified periodic gaits by searching for limit cycles in the 8-dimensional state space of $\mathbf{p}$. To this end, a Poincaré section was defined at the instance of back foot ground contact and periodic limit cycles were identified by numerically searching for stationary points in the resulting Poincaré map. We used the results of the planar model (for which stationary solutions were know), to initiate the root search on a flat model in three dimensions ($d_{lat} = 0$), and gradually increased lateral leg spacing. As we were only investigating symmetrical gaits, it was possible to limit the analysis to half a stride.

To assess stability, Floquet multipliers were computed for the previously identified limit cycles. These eigenvalues of the linearized stride-to-stride transfer function indicate the rate at which the disturbance of a certain mode vanishes (eigenvalue < 1) or grows (one eigenvalue > 1) [1].

III. Results

With the three dimensional model we were able to identify all eight expected gaits. Depending on the order of contact, what was known as four-beat gait in the planar model turned into a *lateral single foot sequence* or a *diagonal single foot sequence* and what was known as two-beat gaits (one exact, two inexact) turned into three different kinds of *paces* and *trots*. Other periodic gaits were not identified.

A. Lateral vs. Diagonal Sequence

In contrast to our expectation, absolutely no difference between the lateral and diagonal single foot sequence was observed with regard to stability and walking velocity. The magnitude of all eigenvalues (Fig. 4) and the walking velocities (Fig 6) of both gaits remained exactly identical over a wide range of d_{lat}. From a dynamical point of view, these gaits have identical properties, although they are in fact truly different gaits, with clearly distinguishable support patterns. The same observation was made for the inexact trots and paces. The order in which the feet strike the ground had no impact on limit cycle stability and velocity.

Additionally to the unstable phase-mode eigenvalue (magnitude of 2.383) that was already present in the planar model, a new (highly) unstable eigenvalue was observed. It corresponds to the 3D lateral rolling motion. Being most unstable for a quadruped with zero lateral spacing, its magnitude decreases for wider configurations of the quadruped (from 14.880 to 2.142). However, it never gets smaller than 1, meaning that the rolling motion is invariable unstable.

B. Width-phase relationship

A very distinct relation exists between the lateral leg spacing and the relative phase of back and front legs (Fig. 5). For the four-beat gaits the phase of the lateral sequence is increased and the phase of the diagonal sequence reduced when the spacing is widened, such that both gaits are converging towards a trotting gait. At the same time, both inexact trots undergo the opposite development, i.e. the very quick succession in which two feet strike the ground is extended, which creates a gait with a more evenly spaced rhythm and four clearly distinguishable foot strikes. At a leg spacing width of $\hat{d}_{lat} = 0.49l$ the four-beat gaits and inexact trots show the exact same phase shifts and hence become identical. This critical width also marks the maximally possible leg spacing for the aforementioned gaits. All these gaits cease to exist for a leg spacing wider than $\hat{d}_{lat}$, which explains why the graph in Figure 4 is limited to a leg spacing of $d_{lat} \le \hat{d}_{lat}$.

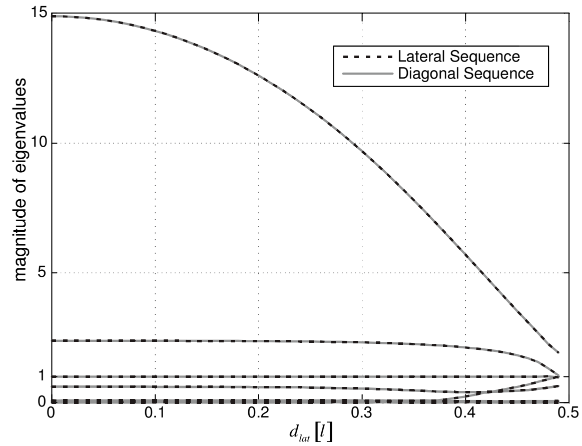

Fig. 4 Eigenvalue magnitudes for varying width of the main body (lateral leg spacing) for the lateral and diagonal single foot sequences are absolutely identical. The two gaits are equivalent with respect to stability.

The observations made here also explain the behavior of the unstable phase-mode eigenvalue in Figure 4. At a leg spacing of $d_{lat} \approx 0.35l$ the four-beat gaits start converging towards two-beat gaits, which means that the (unstable) phase-mode of the four-beat gait is also converging towards the (stable) phase-mode of the two-beat gait. At the point of critical leg spacing $\hat{d}_{lat}$, the phase-mode magnitude is 1, making the mode meta-stable. The three pace-gaits are not affected by this phase shift.

C. Walking velocities

As the nominal cost of transportation (energy consumption per distance traveled normalized by weight) is always equal to the inclination of the slope on which the passive dynamic walker moves, we utilize forward speed to compare the efficiency of different configurations and of different gaits on the same slope. This is a valid indicator as the energy losses are proportional to the overall walking speed, which means that on the same slope a faster walker is conceptually more efficient.

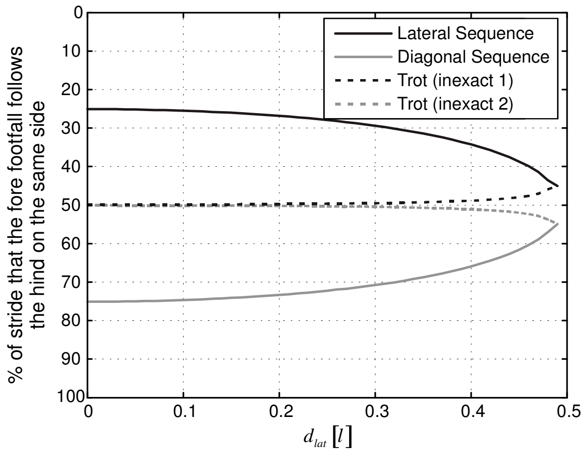

Fig. 5 Phase shift (in percent of a total stride) between the back and front legs on the same side of the quadrupedal model as a function of lateral leg spacing. For increasing lateral leg spacing, the four-beat gaits (in lateral and diagonal sequence) converge towards a trotting gait, while the inexact trots diverge towards more pronounced four-beat gaits. At the critical leg spacing of $0.49l$ four-beat and inexact trots become identical and cease to exist for even wider leg spacing.

Velocity is the parameter that is most obviously affected by the choice of gait. In pacing gaits, the entire mass of the main body has to rock from side to side to maintain a continuous motion. For larger lateral leg spacing, this leads to increased losses in the contact collisions, as the angle at which the center of mass is redirected during this lateral motion (and the associated loss) is growing according to $2\tan\left(d_{lat}/l\right)$. This energy can not be used for forward movement, which means that for a lateral leg spacing of more than $0.37l$ walking in an *exact* pace becomes impossible on a 1^o slope. The *inexact* paces have slightly lower losses in the forward motion and can hence continue walking up to a leg spacing of $0.40l$.

Trotting gaits, on the other hand, remain nearly unaffected by changes in the lateral spacing. This can be explained by imagining a virtual leg with the same mechanical effect as the two stance legs. When trotting, the two stance legs are positioned at diagonal corners of the main body and the virtual leg will hence always act right at the center of the main body [16] . As a consequence, the main body will not undergo any rotational motion at all, and the entire motion becomes independent of the lateral spacing between the legs. Absolutely no changes in walking velocity can therefore be observed for an *exact* trot. As a consequence to the phase shifts described above, the walking velocities of the *inexact* trots are slightly affected by changes in leg spacing. The phase shifts have two effects on velocity that are partly canceling each other. While walking in a more equally timed rhythm reduces the collision losses of the forward motion, it also introduces some lateral rocking motion which leads to collision losses in a lateral motion.

Similar effects can be observed in the four-beat gaits: Walking velocity decreases for a leg spacing up to $d_{lat} = 0.45l$ due to the collision losses associated with the lateral rocking motion of the main body. If the leg spacing is increased further, the phase shift gives the four-beat gaits more and more the property of a trot, which means that these losses become smaller for a spacing larger than $d_{lat} > 0.45l$.

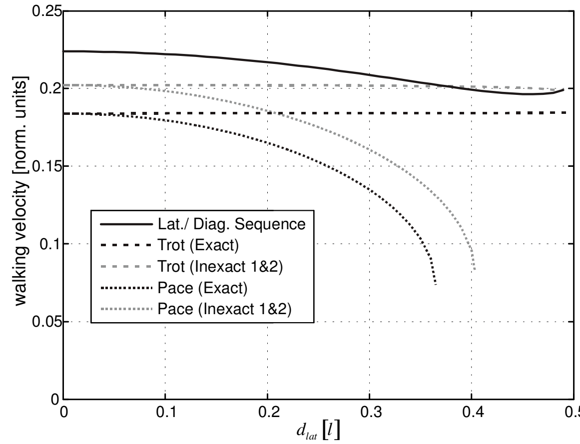

Fig. 6 Walking velocity (in normalized units $\sqrt{gl}$) for the different 3D gaits as a function of lateral leg spacing. No difference in walking speed exists between the lateral and diagonal single foot sequences as well as between the different kinds of inexact paces and trots. While the trots are nearly unaffected by the leg spacing, the lateral rocking motion in the paces dissipates energy slowing down the forward motion.

D. Stability

The lateral motion of the three dimensional model is invariably unstable. Even when the lateral leg spacing is going towards infinity (which is only possible for the exact trot) the corresponding eigenvalue asymptotically converges towards 1, without ever actually reaching it. Energy dissipation due to a lateral motion, as it was discussed in the previous section, is reducing the instability in the sense that the rolling-mode eigenvalue becomes smaller and disturbances are growing slower. A comparison of the rolling-mode eigenvalue magnitudes for all possible gaits is given in figure 7. It is interesting to note, that the 'rolling-mode' is largely decoupled from disturbances in other degrees of freedom. Only for pacing, an interaction can be observed, which results in a pair of complex eigenvalues for lateral spacings above $0.3l$.

IV. Discussion

With the help of a simplified passive dynamic quadrupedal model, we were able to show that a clear relationship exists between the phasing of the front and back leg pair and the lateral leg spacing, that the sequence of ground contacts (which seems to be of high importance in nature) has no influence on stability and efficiency for the investigated gaits, and that a certain trade-off exists between efficiency and dynamic lateral stability.

Similar to bipedal robots [17], the self-stabilizing effect of passive dynamic walking was limited to the sagittal plane, leaving the lateral motion invariably unstable and necessitating active control in an actual quadrupedal robot. Appropriate leg spacing might be able to aid such a controller, however, making the spacing as wide as possible as our results suggest, is most probably not the optimal solution. One should keep in mind that there is always a trade-off between stability and maneuverability, and a leg spacing that is too wide might adversely affect the ability to turn and move laterally.

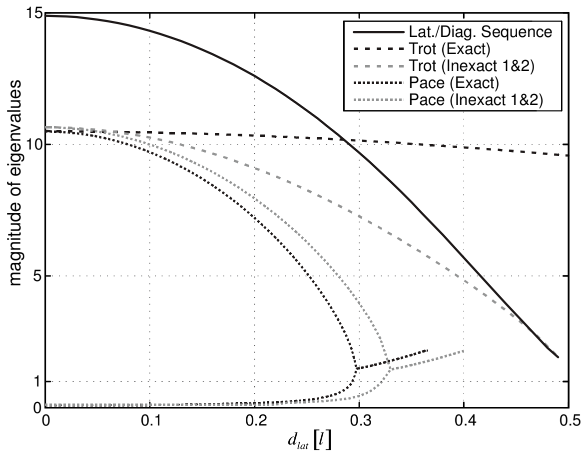

Fig. 7 Magnitude of the *roll-mode* for the different 3D gaits. This mode is unstable for all gaits and the possible range of lateral leg spacing. Gaits that show more pronounced energy losses due to lateral motion (such as the pace) have a 'less unstable' roll-mode, in the sense that disturbances grow slower.

As a main limitation, these results were obtained with a strongly simplified model that can certainly not represent the complex and far more detailed mechanics of an animal or an actual robot. This makes comparisons with biological systems challenging. However, some general tendencies, as for example the fact that only long-legged dogs (i.e. with small d_{lat}) are using pacing gaits [18] might already be inferred from this simplified model.

Creating a more detailed and sophisticated model (for example, with knees, springy legs, or more degrees of freedom in the hip and shoulder joints) might, on the other side, conceal the very basic aspects of locomotion this study was aiming at. Nevertheless, to investigate the full range of possible gaits (both symmetrical and asymmetrical) in a passive dynamic model, the use of springy legs will be imperative. Only elastic legs can allow for phases of multi-leg support and (through a 'powered' lift-off) airborne phases and thus create gaits with a duty factor other than 50%. It has already been shown that self stabilizing effects as they are utilized in passive dynamic locomotion can be employed in running robots with elastic legs [19, 20], but a more conceptual foundation for the underlying mechanisms would be desirable.

The role of all the different gaits that are possible for four-legged robots and that can be found in an even wider variety in nature is still a largely unanswered question. Yet, knowledge about the fundamental principles of gait selection is very important if we seek to build efficient robots that optimally exploit their inherent natural dynamics over a wide range of locomotion speeds and areas of application. In this context, we believe that the study of passive dynamic walking and running is a particularly fruitful approach as it reduces the problem of legged locomotion to the core questions of efficiency and stability.

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