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Moving Game Theoretical Patrolling Strategies from Theory to Practice: An USARSim Simulation

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Abstract— **Game theoretical approaches have been recently used to develop patrolling strategies for mobile robots. The idea is that the patroller and the intruder play a game, whose outcome depends on the combination of their actions. From the analysis of this game, an optimal strategy for the patrolling robot can be derived. Although game theoretical approaches are promising, their applicability in real settings is still an open problem. In this paper, we experimentally evaluate the practical applicability of the most general game theoretical approach for patrolling strategies, called *BGA model*. Experiments are conducted by using USARSim, with the goal of studying the behavior of the optimal patrolling strategy returned by the BGA model both in situations that violate its idealized assumptions and in comparison with other patrolling strategies that can be developed with much less computational effort.**

I. Introduction

The development of patrolling strategies for mobile robots is a topic that has received considerable attention in the last years [1], [2], [3], [4], [5], [6]. A *patrolling strategy* drives a robot around a known environment in order to prevent intrusions. Interesting situations arise when following a deterministic patrolling strategy (i.e., a fixed cyclic path) allows the intruder to attack successfully a target, and the patroller must resort to a series of randomized movements, in order to act unpredictably for an observing intruder. A new promising approach to develop such patrolling strategies considers a model of the intruder within a game theoretical framework [1], [2], [3], [5]. The idea is that the patroller and the intruder play a strategic game, whose outcome is influenced by the combination of their actions. Although game theoretical approaches to robotic patrolling are appealing and have already provided interesting theoretical results, they are based on a very idealized model of the real world and their applicability to real settings is still an open issue.

This paper has an experimental nature and aims at evaluating the practical applicability of the most general game theoretical approach for patrolling strategies, called *BGA model* [2], [7]. We start from the theoretical model and move toward an implemented system, addressing a number of issues involved in this passage. The experimental evaluation is conducted by using realistic simulation tools, like MOAST and USARSim, with a twofold goal. From the one hand, we verify if the optimal patrolling strategy returned by the BGA model performs well also in situations that violate the idealized assumptions of the theoretical model. From the other hand, we assess how much the optimal patrolling strategy returned by the BGA model outperforms other patrolling strategies that require much less computational effort for their development.

The most important original contribution of this paper is the implementation and the experimental evaluation of the robustness of a game theoretical patrolling strategy when the assumptions of the theoretical model do not hold. This kind of evaluation is not usually performed for game theoretical patrolling strategies presented in literature, which are often evaluated only with respect to their computational efficiency.

This paper is structured as follows. The next section surveys the mobile robot strategic patrolling literature. Section III overviews the BGA model that is experimentally tested in the setting described in Section IV. Experimental results are presented in Section V. Finally, Section VI concludes the paper.

II. State of the Art

A patrolling situation is characterized by one or more *patrollers*, a possible *intruder*, and some *targets* in an environment. The targets are areas with some interest for both patrollers and intruder. The development of *patrolling strategies* for mobile robots is a recent interesting scientific challenge [1], [4], [6]. In general, a patrolling strategy determines the next target to patrol given an history of previously visited targets. Usually, the patrollers adopt a patrolling strategy that randomizes over the targets trying to reduce the intrusion probability [1], [2], [8]. The unpredictability of a patrolling strategy is important, because it is often assumed that the intruder can observe the patroller for some time and derive a correct belief on its strategy. We note that similar strategic problems have been addressed in the pursuit-evasion field (e.g., [9], [10]). However, some assumptions, including the fact that the evader's goal is to avoid capture inside the environment and not to enter a target from outside the environment, make the pursuit-evasion strategies not directly applicable to our patrolling scenario.

Broadly speaking, two main approaches can be identified in the development of patrolling strategies for mobile robots. The first approach does not consider any explicit model of the intruder [4], [6], while the second approach does [1], [2], [3], [8], [11]. As shown in [3], strategies that consider a model of the adversary, like the one we consider in this paper, can provide the patroller a larger expected utility than strategies that do not. Two ways have been adopted to model an intruder: considering only its possible movements [1], [11] and considering also its preferences [2], [3], [8].

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We focus on works of the second kind, which exploit a *game theoretical* framework to explicitly model the preferences of the intruder. The idea is to define a game in which the patroller and the intruder compete against each other and to find their optimal strategies by computing the equilibrium of such game. Hence, the optimal patrolling strategy is the strategy that the patroller plays at the equilibrium. For example, consider the following strategic form game [12], where the players act simultaneously:

	C	D
A	$3,3$	$5,1$
B	$2,0$	$4,1$

Let us suppose that the patroller is the row player (its actions are $\{A, B\}$) and that the intruder is the column player (its actions are $\{C, D\}$). Each element of the table corresponds to a game outcome determined by the actions undertaken by the two players and described by the payoffs. For example, if the patroller plays A and the intruder plays D, the patroller gets 5 and the intruder gets 1. The only equilibrium in pure strategies (without randomizing over actions) of this game is $\langle A, C\rangle$. This outcome is called *Nash equilibrium* and is such that no player has an incentive to unilaterally deviate from it [12]. To model patrolling situations, a particular class of strategic form games is employed, called *leader-follower games*. In a leader-follower game players still act simultaneously but, before playing, the leader declares its strategy to the follower that considers it when deciding its strategy. The patroller is the leader and the intruder is the follower. This assumption amounts to suppose that the intruder can observe the movements of the patroller and derive a correct belief on the patrolling strategy. Indeed, the patroller, being observable, implicitly declares its strategy and the intruder considers the observed patrolling strategy in deciding how to act. In the above example, suppose that the patroller decides to commit to action B and that the intruder knows it. The intruder will decide to play action D since this action maximizes its revenue given that the adversary will play B. Therefore, the equilibrium outcome of the game will now be $\langle B, D\rangle$.

A leader-follower game model is used in [5], [8], where the authors deal with the problem of patrolling n areas by using a single patroller such that the time it would spend to patrol all the areas is strictly larger than the penetration time d of the intruder, i.e., the time needed by the intruder to enter an area. The actions available to the patroller are all the possible routes of d areas, while the intruder chooses a single area to enter. The optimal patrolling strategy, in which the patroller maximizes its expected utility, is determined by computing the leader-follower equilibrium of the corresponding game.

Another work that follows the same game theoretical approach is reported in [2], [7] and will be referred to as *BGA model*. Since in this paper we experimentally evaluate the practical applicability of the BGA model, we discuss it in detail in the following section.

All these game theoretical models have been experimentally evaluated in very idealized settings that, for example, do not consider uncertainty in the movements. Moreover, to the best of our knowledge, there have been no attempts (with the partial exception of [11]) to study the patrolling strategies derived from game theoretical approaches in situations that are "outside" the models, when the assumptions of the models do not hold.

III. The BGA Model

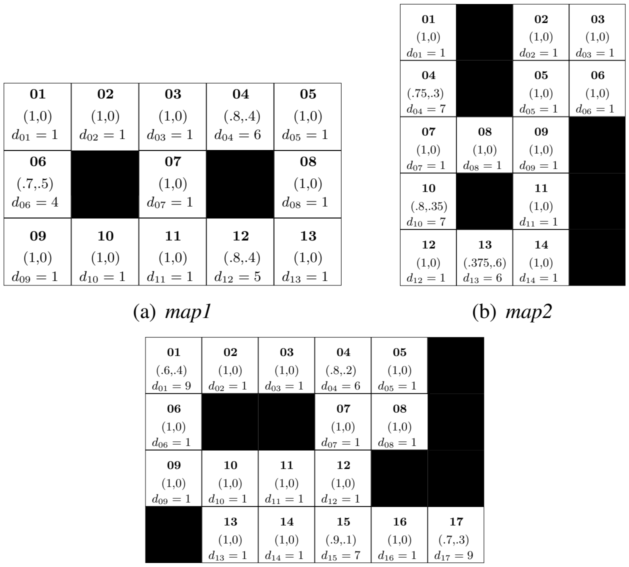
Preliminarily, we motivate the choice to consider the BGA model. As shown in [2], it is the most general game theoretical patrolling model currently available. It generalizes the models in [1] (by allowing any kind of environment, not only perimeter-like) and in [5], [8] (by considering environments with complex topologies, not necessarily fully-connected).

In the BGA model, time is discretized in turns and the environment is represented as a directed graph $\mathcal{G} = (V, A)$ where $V = \{c_1, \ldots, c_n\}$ is the set of nodes to be patrolled, and arcs in A define the topology, i.e., if $(c_i, c_j) \in A$ then nodes c_i and c_j are adjacent and the patrolling robot can directly move from c_i to c_j in one turn. Each node is associated to a pair of non-negative values that represent the importance of that node for the patroller and the intruder, as explained later. All the nodes with positive values for both patroller and intruder are called *targets*, $T \subseteq V$. Every attempted intrusion in a target requires the intruder some time to be successfully completed. Therefore, every node c_i is characterized by a *penetration time* d_i. When attempting to intrude in a target c_i, the intruder stays in that node and is exposed to the patroller's detection capabilities, for d_i turns. Note that graph-based representations of environments are quite common in mobile robotics and can be, for example, surveillance graphs, as in [13]. Some (self-explanatory) examples of environments that can be straightforwardly reduced to a graph representation are shown in Fig. 1.

At each turn, the patroller can move between two adjacent nodes and patrol the arrival one (extending the range of the patroller's sensors is possible [14]). The *patrolling strategy* is defined as a set of probabilities $\{\alpha_{i,j}\}$ with which the patroller reaches node c_j if its current node is c_i and $(c_i, c_j) \in A$.

The intruder can stay outside the environment for some turns observing the patroller and, when it decides to attack a target c_i, it stays there for d_i turns, without the possibility to observe the patroller and to take any other action. The intruder can directly enter in any target $c_i \in T$ at any time (it is also possible to force the intruder to follow a path for reaching a target [14]). More precisely, actions available to the intruder are *enter-when*(c_i, c_j), i.e., attack c_i at the next turn after having observed the patroller in c_j, and *stay-out*, i.e., never enter the environment.

The outcome of the game, when the intruder attempts to enter in target c_i at turn t and the patroller visits c_i in the time interval $[t, t+d_i)$, is *intruder-capture*. A second possible outcome is *penetration*-c_i, where the intruder successfully completes the intrusion in c_i. Finally, the last possible outcome is *no-attack*, and reflects the situation in which the



(a) *map1* (b) *map2* (c) *map3*

Fig. 1. The environments used in experiments, for every cell c_i the penetration time d_i and the payoffs (X_i, Y_i) are reported

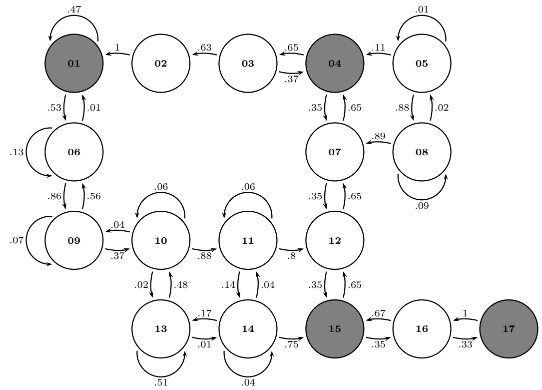


Fig. 2. The optimal patrolling strategy for *map3* of Fig. 1

intruder never tries to enter in the environment (it plays the *stay-out* action).

The preferences of the players over the environment's nodes are defined by the game payoffs. Each node c_i is associated to two values X_i and Y_i that denote the payoffs to the patroller and the intruder, respectively, when the outcome is *penetration-*c_i. X_0 and Y_0 are the corresponding payoffs when the outcome is *intruder-capture*. When the outcome is *no-attack*, the payoff to the patroller is X_0 and the payoff to the intruder is 0. (The rationale is that, when the intruder never enters, it gets nothing and the patroller preserves the values of all the nodes.) These values can be set freely, with the constraints that $0 \leq X_i < X_0$ and $Y_0 \leq 0 < Y_i$ for all $c_i \in T$, and $X_i = X_0$ and $Y_i = 0$ for all $c_i \in C/T$.

Given this game model, the optimal patrolling strategy is the set of probabilities $\{\alpha_{ij}\}$ that guarantees the patroller the maximum expected utility when the intruder knows the patroller's strategy and acts as a best responder (i.e., as an utility maximizer). Such optimal strategy can be found by computing a *leader-follower equilibrium* of the patrolling game, resorting to multiple mathematical programming problems. For full details on the computation please refer to [2], [7]; here we give only an example: the optimal patrolling strategy computed with the BGA model for the *map3* of Fig. 1 is shown in Fig. 2.

In this paper we aim at providing a contribution to assess the real applicability of the patrolling strategies returned by the described game theoretical approach. A number of issues must be addressed when moving from the theoretical model to a real implementation. In particular, some idealistic assumptions of the BGA model must be challenged, including the following ones.

- The intruder is supposed to be a best responder, namely a rational agent that maximizes its utility, given the strategy of the patroller. This amounts to suppose that the patroller is facing the *strongest* intruder that can play the game. However, in real settings, the patroller can also face weaker intruders.
- The movements and the localization of the patrolling robot are supposed to be error-free. This is obviously not true in a real setting.
- The penetration times d_i are known perfectly by both the players. In real settings, the values d_i can be estimated, for example, analyzing the integrity of windows and doors in nodes c_i. Hence, values d_i may be not precisely known by the players.
- The intruder knows exactly the patroller's strategy. This assumption is realistic if we suppose, for example, that the intruder can compute the optimal patrolling strategy in the same way the patroller does or that it can steal the robot's control software. However, if we assume that the intruder derives such knowledge only by observation, then its knowledge of the patrolling strategy will be approximated (at best), otherwise an infinite observation time would be required. Note the knowing the patrolling strategy $\{\alpha_{i,j}\}$ does not mean to know the next action of the patroller, but only the probability distribution with which this action will be selected.

Other issues must be considered in a real implementation (sensors, battery, ...), but we deem that those listed above, and analyzed in the following, are among the most important ones and represent significant elements to assess the practical applicability of the BGA model.

IV. Experimental Setting

We decided to avoid some of the problems of dealing with a real robotic deployment and we used a realistic simulator. We exploited the MOAST framework [15] for developing the patrolling robot controller, that embeds the patrolling strategies, and we performed experiments within the USARSim robotic simulator [16]. In what follows, we illustrate how we have translated the model described in the previous section in the simulator.

Let us start with the graph-based environments. The environments are 3D models with a flat ground floor and

Fig. 3. A simulated patrolling environment

vertical walls. Nodes of the graph are associated to 3 m $\times$ 3 m squared cells on the floor that are associated to penetration times and payoffs (Fig. 3). We used such large cells both because our map is not intended to represent an accurate model of the environment but to represent the areas of interest and because the computation of the optimal patrolling strategy with the BGA model grows exponentially with the number of nodes (cells), see [2], [7] for further details. This complexity limits the application of the BGA model to environments with few dozens of cells.

A Pioneer P2AT has been used as patrolling robot. The intruder has been simulated through another mobile robot equipped with a RFID tag. Correspondingly, the patrolling robot is equipped with an RFID sensor that senses the presence of the intruder within a given range, that has been set to (approximately) cover a cell. The choice of RFID is motivated by the fact that we are concerned with patrolling strategies and not with the important, but different, problems of detecting intruders. Note that the same technique is employed in robotic virtual search and rescue to detect the presence of victims.

The intruder's controller has been developed as a separate application that constantly "observes" the simulation and, according to the strategy of the simulated intruder, decides when to attempt an intrusion and in which cell. When an intrusion is attempted, the intruder robot is inserted in the designated cell c_i. According to the theoretical model, the simulated intruder "appears" at c_i. Starting from that turn, if the penetration time d_i expires before the patrolling robot can sense the intruder's presence, then the intrusion is considered successfully completed and the outcome of the game is *penetration-*c_i. Otherwise, the intruder is detected and the outcome of the game is *intruder-capture*.

To quantify the advantages of the optimal patrolling strategy derived from the BGA model over simpler patrolling strategies, we tested four different patrollers. The *optimal patroller* moves according to the optimal strategy $\{\alpha_{ij}\}$ returned by the BGA model. At each turn, the next cell to reach is randomly chosen according the probability distribution defined by the $\{\alpha_{ij}\}$ values. Then, the patroller moves from the center of its current cell to the center of the destination cell. The *uniform patroller* determines the next cell to patrol extracting it from an uniform probability distribution over the cells adjacent to the current one. Formally, if we call R_i the set of cells that are adjacent to cell c_i, the strategy of the uniform patroller is defined as $\alpha_{ij} = 1/|R_i|$ if $c_j \in R_i$ and $\alpha_{ij} = 0$ otherwise. The *random patroller* selects the next cell according to a random probability distribution. Formally $\{\alpha_{ij}\}$ are randomly chosen with the constraint that $\sum_{c_j \in R_i} \alpha_{ij} = 1$ and $\alpha_{ij} = 0$ for every $c_j \notin R_i$. Finally, the *deterministic patroller* cyclically follows the shortest path that visits all the cells. The different patrollers can be distinguished with respect to the amount of knowledge about the patrolling setting they use to compute their strategy. The optimal patroller has full knowledge of the environment topology, of the payoffs, and of the penetration times. Differently, the other patrollers have only knowledge about the environment topology (for example, penetration times are not considered by patrollers different from the optimal one).

Three different intruders, with different intrusion strategies, have been defined. The *optimal intruder* is that assumed in the BGA model. It is the strongest intruder since it perfectly knows the strategy of the patroller and acts as a best responder. The *proportional intruder* does not know the patrolling strategy and, at a random turn, selects a target to attack according to a probability that is directly proportional to the value of that target for the patroller. Formally, the probability to attack target c_i is calculated as $X_i / \sum_{j \in T} X_j$. Finally, the *uniform intruder* selects, at a random turn, the target to attack with a uniform probability.

The patrolling games have been simulated in the three environments represented in Fig. 1.

We call configuration a combination of an environment to be patrolled, of a patrolling strategy, and of a type of intruder. For every configuration we simulated 100 patrolling games, each one with a randomly selected starting cell for the patroller. Every game ends either with the detection of the intruder or with a successful intrusion. In order to allow every type of intruder to actively participate in the game, we did not consider patrolling settings where the optimal strategy of the intruder is to never attack. At the end of a game, payoffs are assigned to the players as described in Section III.

We computed different metrics, averaging over the games played in each configuration. The most important metric we consider are the patroller's and intruder's average utilities (assigned payoffs), called U_p and U_i, respectively. The higher the average utility of a player, the better its strategy in the considered configuration. Moreover, we considered also the coverage percentage, which is calculated as the number of cells that are visited (at least once) by the patroller in a game, with respect to the total number of cells in the environment. This metric is related to the cost of patrolling, since the more cells a robot covers, the more it spends in terms of time and energy.

V. Experimental Results

The first set of experiments is focused on testing the behavior of the patroller against an intruder that is not optimal, as assumed in computing the optimal patrolling strategy with the BGA model. From the model, the expected patroller's utility for the optimal patrolling strategy is guaranteed to

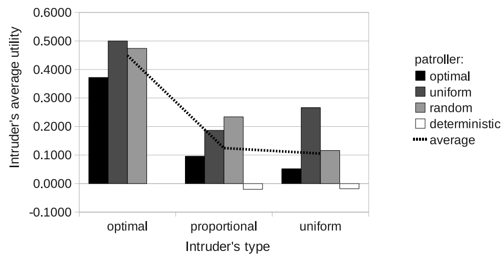


Fig. 4. Intruder's average utilities in *map1*

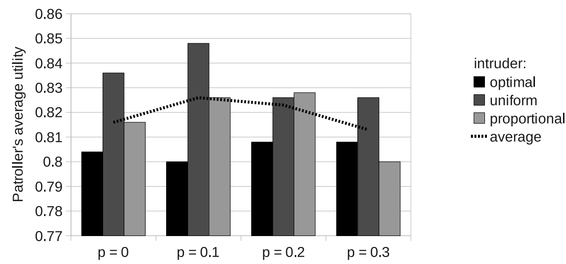


Fig. 5. Optimal patroller's average utilities in *map1* with movement errors

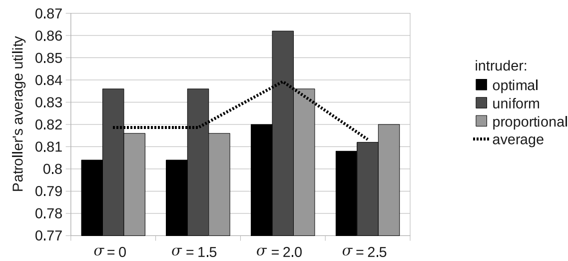


Fig. 6. Optimal patroller's average utilities in *map1* with uncertain d_i

be maximum when facing the optimal intruder; however, the model does not say anything for other types of intruders. We measured the performances of the optimal patroller when facing the proportional and the uniform intruders defined in Section IV. In Table I, the average (over 100 games) utility U_p for the patroller in the environments of Fig. 1 is reported with the corresponding variance (in parentheses). No significative worsening in U_p can be observed when changing the intruder's type. The optimal strategy is able to effectively protect the environment from intrusions even when facing intruders different from that assumed in its computation. In this sense, we can say that the optimal patrolling strategy computed with the BGA model is robust. The topology and the payoffs of *map2* are such that the optimal patrolling strategy tends to visit more frequently the targets with large X_i, explaining the worst performance against a uniform intruder. A somehow expected result is that the weaker the intruder, the more often it is detected by the optimal patroller, as shown in Fig. 4, where U_i for *map1* are reported for each type of intruder. A decrease in U_i (when facing the optimal patroller) can be observed when moving from the optimal intruder to the proportional one and to the uniform one. Similar trends have been obtained for other environments. These results further confirm the robustness of the optimal patrolling strategy calculated with the BGA model: it performs better and better as the intruder becomes weaker and weaker.

Environment	Intruder	U_p
map1	optimal	0.8020 (0.0004)
	proportional	0.8130 (0.0125)
	uniform	0.8520 (0.0108)
map2	optimal	0.8120 (0.0023)
	proportional	0.8397 (0.0635)
	uniform	0.7567 (0.0436)
map3	optimal	0.6200 (0.0077)
	proportional	0.8034 (0.0249)
	uniform	0.8360 (0.0228)

TABLE I

OPTIMAL PATROLLER'S AVERAGE UTILITIES

When dealing with real situations, other idealistic assumptions of the BGA model should be considered. For example, movement errors affect the performance of a real mobile robot. In order to start to evaluate the impact of these errors, we defined a probability value p as the probability with which the simulated patroller will not succeed in executing the movements prescribed by its patrolling strategy. More precisely, if at turn t the strategy prescribes to reach and patrol cell c_j from the current cell c_i, at turn $t+1$ the robot will move to cell c_j with probability $1-p$ while it will remain in its current cell c_i with probability p. No dramatic worsening in U_p is observed for increasing values of p, as shown for example in Fig. 5, where the results obtained in *map1* are reported.

As discussed in Section III, in a real setting, the penetration times d_i would be characterized by some degree of uncertainty. We studied the behavior of the optimal patrolling strategy when penetration times are random variables with a normal probability distribution with mean value d_i. This amounts to say that the penetration times $\bar{d}_i$ used in a simulated game are determined only when the intruder attacks, and their value is calculated as $\bar{d}_i = d_i + \epsilon$ where $\epsilon \sim \mathcal{N}(0, \sigma^2)$ is a random variable drawn from a zero mean normal distribution. As Fig. 6 shows, the optimal patrolling strategy performs well also in presence of increasingly uncertain d_i.

The next issue we consider in our experimental evaluation is the exact knowledge of the patrolling strategy that characterizes the optimal intruder. To investigate the behavior of the optimal strategy when this assumption is no longer valid, we defined an *approximated* optimal intruder, which is an intruder working with a noised knowledge about the patrolling strategy. In practice, the intruder knows a patrolling strategy that is obtained from the real one with the addition of a random noise from a normal distribution with $\mu = 0$ and $\sigma = 0.2$. Table II shows that the U_p of the

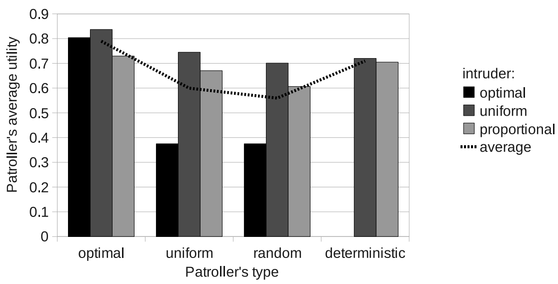


Fig. 7. Patroller's average utilities in *map2*

optimal patroller does not decrease when the optimal intruder has an imprecise knowledge of the patrolling strategy.

Intruder	U_p
optimal	0.6320
proportional	0.7980
uniform	0.8340
approximated	0.6420

TABLE II

OPTIMAL PATROLLER'S AVERAGE UTILITIES IN $map3$

The last set of experiments evaluates the performance of the optimal strategy returned by the BGA model when compared with other patrolling strategies. From the one hand, the optimal strategy is theoretically guaranteed to be the best one under the assumptions of the BGA model but, from the other hand, it requires a significant computational effort to be determined. (For example, the optimal strategy of Fig. 2 has been computed in about 30 minutes.) Therefore, it is important to assess if the adoption of the optimal strategy can bring significative advantages with respect to non-optimal strategies (like the uniform, random, and deterministic patrolling strategies) that are much easier to compute. Averaging over all configurations, the U_p of the optimal patroller is 13%, 15%, and 30% larger than those of the uniform, random, and deterministic patrollers, respectively. Fig. 7 shows, as a representative example, the results for *map2*. Note that the advantage of the optimal strategy over the other patrolling strategies is more evident with the optimal (strongest) intruder (this advantage is statistically significant, according to the one-way ANOVA test [17]). Note also that the deterministic strategy was not tested with the optimal intruder since, in these configurations, the optimal intruder will always attack as soon as the time needed by the patroller to reach the attacked target from its current position is larger than the penetration time of that target. The outcome of the game is therefore predetermined.

In order to understand from where the advantage of the optimal patrolling strategy comes from, it is interesting to look at how many cells a strategy covers. For example, in *map2* the optimal patrolling strategy was able to achieve its good performance with a coverage of about 60% of the cells of the environment, while the other strategies needed a 100% complete coverage (similar results have been obtained for other environments). The main reason behind this result is that the optimal patrolling strategy restricts its routes to an essential subset of interesting cells, avoiding to visit cells that do not contribute to prevent intrusions (for example, cells 05 and 08 of the *map3* of Fig. 2).

VI. CONCLUSIONS

In this paper we have implemented in USARSim the patrolling strategy returned by a game theoretical approach, called BGA model, to experimentally evaluate its practical applicability. The results have shown that the optimal patrolling strategies returned by the BGA model perform better than alternative, less computational demanding, strategies in a number of situations and continue to perform good even in situations that go beyond the assumptions of the game theoretical model.

Further work is required to reach the final goal of having a game theoretical-based patrolling robot. Among the most significant issues that will be addressed in the future there are the sensors for detecting intruders and more realistic movements for the patroller and the intruder.

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