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Ball Dribbling with an Underactuated Continuous-Time Control Phase

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***Abstract*— Ball dribbling is a central element of basketball. One main challenge for realizing basketball robots is to stabilize periodic motions of the ball. This task is nontrivial due to the discrete-continuous nature of the corresponding dynamics. The ball can be only controlled during ball-manipulator contact and moves freely otherwise. We propose a manipulator equipped with a spring that gets compressed when the ball bounces against it. Hence, we can have continuous-time control over this underactuated Ball-Spring-Manipulator system until the spring releases its accumulated energy back to the ball. This paper illustrates a systematic way of planning such a modified dribbling motion, computing an analytical transverse linearization and achieving orbital stabilization.**

***Index Terms*— Underactuated Mechanical Systems, Motion Planning, Orbital Stabilization, Virtual Holonomic Constraints**

I. INTRODUCTION

The control of rhythmic tasks has been an active research area over the last decades. The classical example is the so-called *juggling* that requires interaction with an object or multiple objects that would otherwise fall freely in the earths gravitational field [2]. For juggling tasks the continuous motion of the manipulator is used to control the continuous motion of the ball through an intermittent contact. Open-loop control of a vertically bouncing ball with repeated impacts was already studied in [5]. Orbital stabilization of juggling trajectories is discussed in [3], [6].

The *dribbling* task is, to some extent, comparable to the extensively studied juggling task. An example of robot dribbling was presented in [7] for experimental evaluation of a high-speed vision system. A comparison between juggling and dribbling is discussed in [1] with respect to local stability and parameter sensitivity. Furthermore, experimental tests were shown with and industrial robot dribbling a basketball for multiple periods of a desired cycle.

The novelty of this paper is the control of a ball-dribbling cycle with a continuous-time phase rather than an intermittent contact between manipulator and ball. For this to happen we need to add a spring to the manipulator so that an impact is avoided when the ball bounces against it. During a significant time interval, when the kinetic energy of the ball is converted into potential energy of the spring and its accumulated energy is eventually released back to the ball, a controller can take stabilizing action. Such an approach is clearly advantageous compared with a control law that is only applied at the time instant of ball-manipulator impact. Also a human basketball player keeps the ball on his hand for a relatively long time in order to control the dribbling task in a perfected fashion.

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The paper is organized as follows. A hybrid (discrete-continuous) model describing the ball-dribbling cycle is developed in Section II. During the controlled phase we have dynamics of an *underactuated Ball-Spring-Manipulator system*, which makes motion planning and control a challenging task. The *virtual holonomic constraints approach* [8], [9] is used as generic tool for shaping a desired ball-dribbling orbit (Section III) and its feedback stabilization (Section IV). Numerical simulations in Section V demonstrate that the structure and performance of the control system are suited for experimental tests. The paper ends with concluding remarks.

II. MODELING

In this section we develop a hybrid dynamical model that describes periodic motions of a robotic ball-dribbling system. It consists of a bouncing ball and a manipulator that is acting on the ball via a spring. The following assumptions are made:

- A1 The ball and the manipulator are modeled as single-degree-of-freedom (DOF) systems such that only motion along the vertical line is taken into account.
- A2 The deformation of the ball at contact with the ground and with the manipulator is neglible.
- A3 The ground impact of the ball is an instantaneous inelastic collision described by the coefficient of restitution [4].
- A4 Air resistance and rotational ball velocity are negligible.

A. Coupled Ball-Spring-Manipulator Dynamics

The ball-dribbling system presented in [1] is controlled by intermittent contact of the manipulator with the ball. We want to modify it by adding a compression spring to the manipulator as illustrated in Fig. 1(a). Thus, the ball acts on the spring and does not rebound. During a significant time interval the kinetic energy of the ball is converted into potential energy of the spring and eventually returned back to the ball. Consequently, we introduced a continuous-time control phase instead of instantaneous impact. This kind of modification resembles the actuation on the ball that humans perform by using a rather flexible "arm-hand manipulator".

In the real installation the manipulator is composed of a serial link structure that allows for controlling the position and velocity of the end effector $[x_M, \dot{x}_M]$. The schematic in

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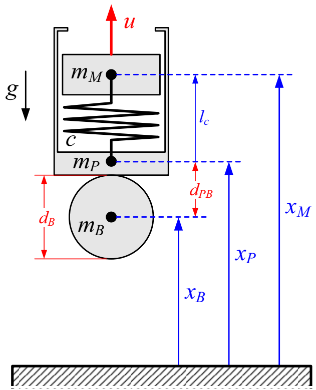


(a) Schematic.

Manipulator	$m_M = 2\,\mathrm{kg}$ $m_P = 1\,\mathrm{kg}$
Ball	$m_B = 0.62\,\mathrm{kg}$ $d_B = 0.242\,\mathrm{m}$ $d_{PB} = 0.171\,\mathrm{m}$
Spring	$c = 500\,\mathrm{N/m}$ $l_{c,0} = 0.15\,\mathrm{m}$ $l_{c,\min} = 0.05\,\mathrm{m}$
Gravitational constant	$g = 9.81\,\mathrm{m/s^2}$

(b) Physical parameters.

Fig. 1. Underactuated Ball-Spring-Manipulator system. The position of the mass m_M is physically controlled by a manipulating robot arm.

Fig. 1(a) represents an abstraction to a 2-DOF Ball-Spring-Manipulator system that is only actuated at the mass m_M, which is installed at the wrist of the end effector. When the ball acts on the the plate m_P, the spring gets compressed and we temporarily have system dynamics of the form

$$\begin{aligned} m_M \ddot{x}_M &= u - c(x_M - x_B - d_{PB} - l_{c,0}) - m_M g \\ \tilde{m}_B \ddot{x}_B &= c(x_M - x_B - d_{PB} - l_{c,0}) - \tilde{m}_B g\,, \end{aligned} \tag{1}$$

where $\tilde{m}_B = m_B + m_P$ is the merged mass of ball and plate, g is the acceleration due to gravity, and the relation $x_P = x_B + d_{PB}$ holds. The spring with the coefficient c is restricted in its working range $l_c = x_M - x_B - d_{PB}$, $l_c \in [l_{c,\min}, l_{c,0}]$ for experimental reasons to a minimum length $l_{c,\min}$ and the equilibrium length $l_{c,0}$. In that way the spring deflection is less or equal to zero, $l_c - l_{c,0} \le 0$. Required model parameters are given in Fig. 1(b). A state-space representation corresponding to (1) is formulated as

$$\dot{x} = f(x(t), u(t))\,, \quad x = [x_M, x_B, \dot{x}_M, \dot{x}_B]^T\,. \tag{2}$$

B. Decoupled Dynamics

The continuous-time dynamics of only the ball when it is freely moving in the gravitational field is governed by

$$m_B \ddot{x}_B = -m_B g\,, \tag{3}$$

which is written in state space form as

$$[\dot{x}_B, \ddot{x}_B]^T = f_B(x_B(t), \dot{x}_B(t))\,. \tag{4}$$

The continuous-time dynamics of the manipulator together with the spring and the connected plate are given by

$$\begin{aligned} m_M \ddot{x}_M &= u - c(x_M - x_P - l_{c,0}) - m_M g \\ m_P \ddot{x}_P &= c(x_M - x_P - l_{c,0}) - m_P g\,. \end{aligned} \tag{5}$$

However, for simplicity we assume a lumped rigid body during the time when the ball is not in contact, which yields collapsed dynamics to a single degree of freedom[1]

$$\tilde{m}_M \ddot{x}_M = u - \tilde{m}_M g \tag{6}$$

[1] In practice one might utilize an electo-mechanical lock for the plate when the equilibrium length of the spring is reached.

with the merged mass $\tilde{m}_M = m_M + m_P$. The corresponding state space representation is written as

$$[\dot{x}_M, \ddot{x}_M]^T = f_M(x_M(t), \dot{x}_M(t), u(t))\,. \tag{7}$$

During the time when the ball is freely moving the dynamics (7) must be controlled such that a desired state is reached for catching the ball again. When that happens it is required that the spring is at equilibrium length $l_c - l_{c,0} = 0$ for practical reasons. Considering (5) there should be no problem to plan those motions that have insignificant plate dynamics at the end.

C. Hybrid Dynamics

The dynamical system that describes the cyclic ball dribbling task is of hybrid nature consisting of continuous-time dynamics and jumps due to instantaneous updates of the states. The evolution chart of the state variables, introduced by (2), (4), (7) subject to discrete-time events, is depicted in Fig. 2. Instantaneous jumps along the periodic solution are defined by the state update law

$$F^{(i)} : \Gamma_-^{(i)} \to \Gamma_+^{(i)}\,, \quad i = 1, \ldots, 4 \tag{8}$$

as mappings between pairs of hyper-surfaces in the state space of the system. Here we have the following events:

- At "ball catch" (time $t^{(0)}$) the controlled phase of the ball-dribbling cycle starts. The ball contacts the manipulator after bouncing from the ground so that a switch from decoupled dynamics to coupled Ball-Spring-Manipulator dynamics occurs

$$\begin{aligned} \Gamma_-^{(4)} &= \left\{ x \in \mathbb{R}^4 : x_M - x_B = d_{PB} + l_{c,0} \right\} \\ \Gamma_+^{(4)} &= \Gamma_-^{(4)},\ F^{(4)} = I_4\,. \end{aligned}$$

  The switching surface $\Gamma_-^{(4)}$ is reached when the distance between ball and manipulator position reduces to the constant $d_{PB} + l_{c,0}$ (zero spring deflection). If a desired ball catching height $x_B = h_c$ is specified, the manipulator must be moved such that the contact point is met at a certain time $t_-^{(0)} = t^{(4)}$.
- A "switch from ball catching phase to pushing phase" is required (at time $t^{(1)}$) in order to change the control law for u. This is done when the ball velocity has been reduced to zero, i.e. at this point the ball has no kinetic energy left:

$$\Gamma_-^{(1)} = \Gamma_+^{(1)} = \left\{ x \in \mathbb{R}^4 : \dot{x}_B = 0 \right\},\ F^{(1)} = I_4\,.$$

- At "ball release" (time $t^{(2)}$) the ball starts falling to the ground and the controlled phase of the ball-dribbling cycle ends. A switch to decoupled dynamics occurs

$$\begin{aligned} \Gamma_-^{(2)} &= \left\{ x \in \mathbb{R}^4 : x_M - x_B = d_{PB} + l_{c,0} \right\} \\ \Gamma_+^{(2)} &= \Gamma_-^{(2)},\ F^{(2)} = I_4\,. \end{aligned}$$

  The manipulator position must be controlled such that the spring is at equilibrium when a desired release height $x_B = h_r$ is reached. After that the manipulator is moved back to the catching point while the ball falls to the ground and bounces back.

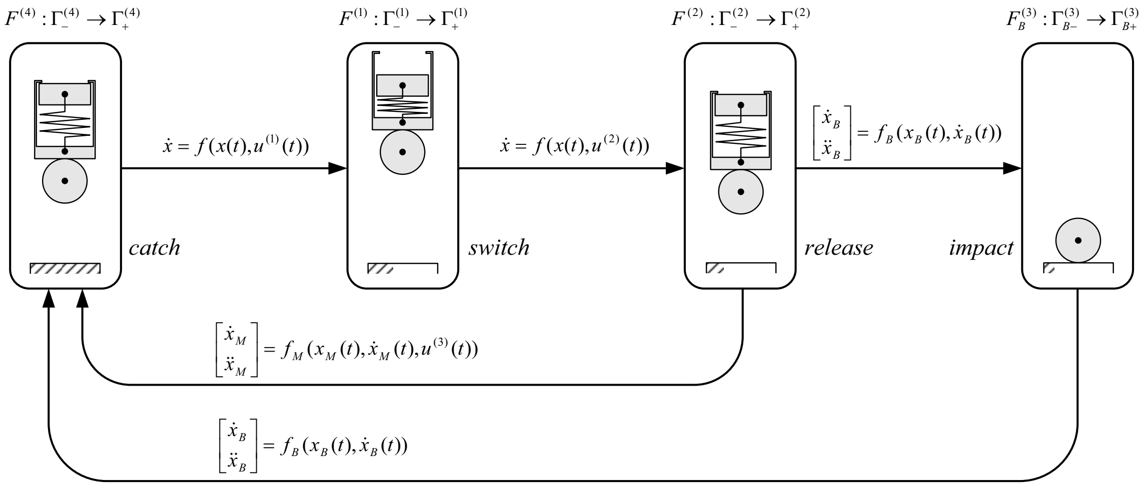


Fig. 2. Evolution chart for the hybrid dynamical system describing a full ball-dribbling cycle.

- The "ground impact of ball" happens at $x_B = d_B/2$ (at time $t^{(3)}$)

$$\begin{aligned}\Gamma_{B-}^{(3)} &= \Gamma_{B+}^{(3)} = \{[x_B, \dot{x}_B] :\, x_B = d_B/2\}\\ F_B^{(3)} &= \begin{bmatrix} 1 & 0 \\ 0 & -\mu_r \end{bmatrix}.\end{aligned}$$

  The instantaneous rebound and the loss of kinetic energy is described by the coefficient of restitution μ_r.

Since the ball dynamics between "ball release" and "ball catch" is not directly controlled, we can derive algebraic relations for the state of the ball by integrating (4). From the evolution chart in Fig. 2 we determine the state of the ball just before "ground impact"

$$\begin{aligned} x_{B-}^{(3)} &= d_B/2 \\ t^{(3)} - t^{(2)} &= \frac{\dot{x}_{B-}^{(2)}}{g} + \sqrt{\left(\frac{\dot{x}_{B-}^{(2)}}{g}\right)^2 - 2\frac{x_{B-}^{(3)} - x_{B-}^{(2)}}{g}} \\ \dot{x}_{B-}^{(3)} &= -g\left(t^{(3)} - t^{(2)}\right) + \dot{x}_{B-}^{(2)} \end{aligned} \tag{9}$$

and just after "ball catch"

$$\begin{aligned} \begin{bmatrix} x_{B+}^{(3)} \\ \dot{x}_{B+}^{(3)} \end{bmatrix} &= F_B^{(3)} \begin{bmatrix} x_{B-}^{(3)} \\ \dot{x}_{B-}^{(3)} \end{bmatrix} \\ x_{B+}^{(4)} &= h_c \\ t^{(4)} - t^{(3)} &= \frac{\dot{x}_{B+}^{(3)}}{g} - \sqrt{\left(\frac{\dot{x}_{B+}^{(3)}}{g}\right)^2 - 2\frac{x_{B+}^{(4)} - x_{B+}^{(3)}}{g}} \\ \dot{x}_{B+}^{(4)} &= -g\left(t^{(4)} - t^{(3)}\right) + \dot{x}_{B+}^{(3)}. \end{aligned} \tag{10}$$

Furthermore, we can assume that the manipulator dynamics (7) is fast enough to reach a desired state at "ball catch" starting from "ball release" during the time that the ball falls to the ground and bounces back.

From the system analysis perspective it makes sense to consider the ball-dribbling cycle only for the controlled phase involving the dynamics of the underactuated Ball-Spring-Manipulator system (2). Hence, the evolution chart can be modified by incorporating (9), (10) into a new mapping $\tilde{F}^{(2)} : \Gamma_-^{(2)} \to \Gamma_+^{(4)}$ from "release" to "catch".

III. Motion Planning

A. Virtual Holonomic Constraints

During the ball-dribbling cycle there are two continuous-time phases that provide control over the ball, namely "ball catching" and "ball pushing". These motions must be carefully planned.

A desired continuous-time motion of the Ball-Spring-Manipulator system (2) can be described by the time evolution of its generalized coordinates

$$\Big\{x_M = x_{M\star}(t),\, x_B = x_{B\star}(t)\Big\}, \quad t \in [t_b, t_e],\, t_b < t_e\,. \tag{11}$$

However, the fact that our dynamical system (2) is underactuated makes the planning of desired motions nontrivial.

Let us introduce a set of geometric relations among the generalized coordinates alternatively to (11) such that the same motion is now given by

$$\Big\{x_M = \phi_1(\theta),\, x_B = \phi_2(\theta)\Big\}, \quad \theta = \theta_\star(t),\, t \in [t_b, t_e] \tag{12}$$

with a scalar variable $\theta \in [\theta_b, \theta_e]$ that is used as trajectory generator for parameterizing the time evolution. Geometric functions among the generalized coordinates as introduced by (12) are known as *virtual holonomic constraints* [8], [11]. A convenient choice for our system is the following:

$$\begin{bmatrix} x_M \\ x_B \end{bmatrix} := \Phi(\theta) = \begin{bmatrix} \phi(\theta) \\ \theta \end{bmatrix}. \tag{13}$$

Suppose that there exists a control law $u_\star$ for the input force u that makes the virtual holonomic constraint (13) invariant, then the overall closed-loop system can be generally represented by reduced order dynamics of the form [8]

$$\alpha(\theta)\,\ddot{\theta} + \beta(\theta)\,\dot{\theta}^2 + \gamma(\theta) = 0\,. \tag{14}$$

The solutions of this virtually constrained system define achievable motions with precise synchronization given by (13). It means that the whole motion is parameterized by the evolution of the chosen configuration variable θ. The smooth coefficient functions of (14) can be easily derived from the system dynamics (1) substituting (13):

$$\begin{aligned} \alpha(\theta) &= \tilde{m}_B\,, \quad \beta(\theta) = 0 \\ \gamma(\theta) &= -c(\phi(\theta) - \theta - d_{PB} - l_{c,0}) + \tilde{m}_B g\,. \end{aligned} \tag{15}$$

The reduced order dynamics of the form (14) is always integrable. Specifically, the integral function[2] [8]

$$I(\theta_b, \dot{\theta}_b, \theta, \dot{\theta}) = \dot{\theta}^2 - \dot{\theta}_b^2 + \int_{\theta_b}^{\theta} \frac{2\,\gamma(s)}{\alpha(s)}\, ds \tag{16}$$

preserves its zero value along the solution of (14), initiated at $(\theta(t_b), \dot{\theta}(t_b)) = (\theta_b, \dot{\theta}_b)$.

With the above arguments we can now convert the motion planning problem from a search of feasible orbits in the state space of (2) into a search for a parameterizing function $\phi(\theta)$ in (13) such that a desired solution of the reduced dynamics (14) is found. Here we use a Bézier polynomial [11]

$$\phi(\theta) = \sum_{k=0}^{M} a_k \frac{M!}{k!(M-k)!} s^k (1-s)^{M-k}, \; s = \frac{\theta - \theta_b}{\theta_e - \theta_b} \tag{17}$$

of degree $M = 3$ as geometric relation between the generalized coordinates. Consequently, we need to find the coefficients $a = [a_0, a_1, a_2, a_3]$ that yield a desired time evolution $\theta_\star(t)$ between the specified initial and final conditions $[\theta_b, \dot{\theta}_b, \theta_e, \dot{\theta}_e]$.

B. *Planning Ball-Catching and Ball-Pushing Motions*

For planning the ball-catching and ball-pushing motion we look at a particular ball-dribbling cycle experimentally studied in [1]: the ball-manipulator contact is instantaneous at a desired dribbling height. Instead of having a ball-manipulator impact, the cycle shall be modified to have the continuous-time control phases of "catching" and "pushing" embedded. This is clearly advantageous because one can perform stabilizing control action during a significantly large time interval. The modified cycle is shown as state-space plot for the ball in Fig. 3.

Planning a ball-catching motion is done as follows:

1. Specify initial and final conditions for the reduced dynamics (14) based on a desired orbit of the ball dynamics (from the *dashed-dot* curve in Fig. 3):

$$[\theta_b,\, \dot{\theta}_b]^{(1)} = [0.8,\, 1.721], \quad [\theta_e,\, \dot{\theta}_e]^{(1)} = [0.9,\, 0] \tag{18}$$

2. Solve an optimization problem for the coefficients $a^{(1)}$ of the parameterizing function (17) such that

$$\min_{a^{(1)}} \left(I(\theta_e, \dot{\theta}_e, \theta_b, \dot{\theta}_b) \right)^2$$

from (16) becomes zero subject to the following constraints:

$$\begin{aligned} & l_{c,\min} \le \phi(\theta) - \theta - d_{PB} \le l_{c,0} \\ & \max\left(|\ddot{x}_M|\right) = \max\left(\left| \phi''(\theta)\dot{\theta}^2 + \phi'(\theta)\ddot{\theta} \right| \right) < 3g\,. \end{aligned}$$

For the numerical search we use `fmincon` from MATLAB. The parameter $a_0^{(1)}$ is taken as the initial position of the manipulator such that $\phi(\theta_b) - \theta_b - d_{PB} = l_{c,0}$ is satisfied, meaning that the spring is at equilibrium length in the beginning.

[2]Notice that this function is greatly simplified in the case of $\beta(\theta) = 0$. In general we have $I(\theta_b, \dot{\theta}_b, \theta, \dot{\theta}) = \dot{\theta}^2 - \exp\left\{-2\int_{\theta_b}^{\theta} \frac{\beta(\tau)}{\alpha(\tau)}\, d\tau\right\} \dot{\theta}_b^2 + \int_{\theta_b}^{\theta} \exp\left\{-2\int_{s}^{\theta} \frac{\beta(\tau)}{\alpha(\tau)}\, d\tau\right\} \frac{2\,\gamma(s)}{\alpha(s)}\, ds$.

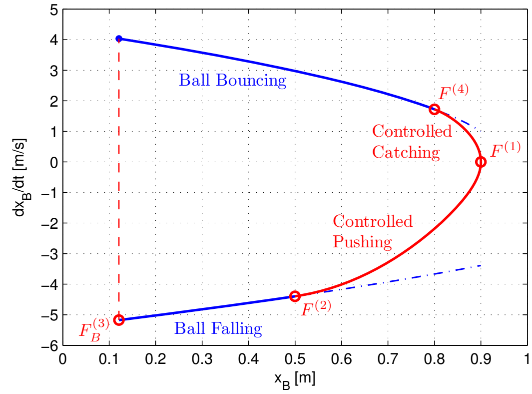


Fig. 3. Ball-dribbling cycle in the state space of the ball with a continuous-time control phase at manipulator contact. The *dashed-dot* curve represents the cycle of [1] with ball-manipulator impact at a dribbling height of $h_d = 0.9\,\text{m}$, where $[x_B, \dot{x}_B]^{(h_{d}-)} = [0.9\,\text{m}, 1\,\text{m/s}]$.

As result we obtain the coefficients

$$a^{(1)} = [1.1210,\, 1.1338,\, 1.1655,\, 1.1989]\,. \tag{19}$$

Planning a ball-pushing motion subsequent to the ball-catching motion is done in a similar way:

1. Specify initial and final conditions for the reduced dynamics (14) based on the orbit for the ball dynamics that are related to (18):

$$[\theta_b,\, \dot{\theta}_b]^{(2)} = [0.9,\, 0], \quad [\theta_e,\, \dot{\theta}_e]^{(2)} = [0.5,\, -4.3965] \tag{20}$$

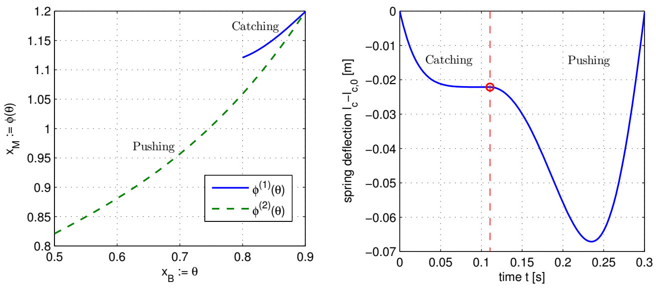
2. Solve an optimization problem, the same as for the catching-ball motion, but now for the coefficients $a^{(2)}$. The parameter $a_0^{(2)}$ is already defined by the end configuration of the catching motion, whereas the parameter $a_3^{(2)}$ must satisfy $\phi(\theta_e) - \theta_e - d_{PB} = l_{c,0}$ giving the equilibrium length of the spring.

The resulting coefficients are

$$a^{(2)} = [1.1989,\, 0.9826,\, 0.8955,\, 0.8210]\,. \tag{21}$$

The two virtual holonomic constraints that have been found are shown in Fig. 4(a). The functions are very smooth due to the low order of the polynomial and the acceleration constraints of the manipulator used in the search. The spring deflection from equilibrium $l_c - l_{c,0}$ along the two motions is shown in Fig. 4(b) as a function of time. Clearly, the spring reduces the energy from the incoming ball during the catching phase such that the desired dribbling height is met at zero velocity. After the switch the ball is pushed down with maximum manipulator acceleration such that the desired state of the ball can be reached at the release height, also returning the potential energy from the spring.

A combined state-space plot of the planned orbit is shown in Fig. 5. It can be seen that the trajectory of the manipulator



(a) Virtual holonomic constraints. (b) Spring deflection from equilibrium $l_c - l_{c,0}$ w.r.t. time.

Fig. 4. Virtual holonomic constraints and spring deflection for the planned ball-catching and ball-pushing motions. The spring deflection is zero at the ball catch height and the ball release height.

starts from non-zero velocity at "ball catch". From a practical point of view it would be desired to have the manipulator at rest just before the ball contact, but in that case the required manipulator acceleration violates the physical constraints. The next step is to design a controller that stabilizes the Ball-Spring-Manipulator dynamics in a vicinity of the desired trajectory.

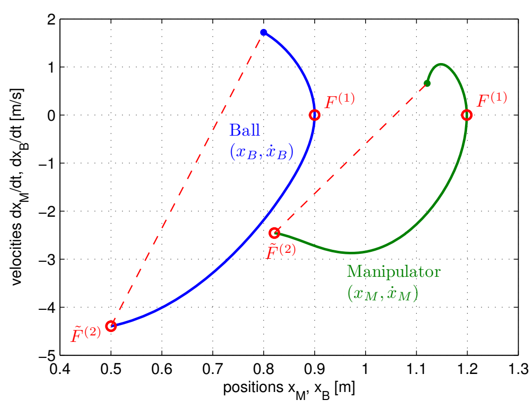


Fig. 5. Combined state-space plot of the planned orbit showing the continuous-time control phase at manipulator contact.

IV. Control Design

A. Transverse Linearization

Given a continuous-time motion (11) for our Ball-Spring-Manipulator system (2) in the format of (12) allows for introducing new coordinates and velocities in the vicinity of the target orbit, which is depicted in Fig. 5:

$$\begin{aligned} x_M &= \phi(\theta) + y\,, \qquad & x_B &= \theta \\ \dot{x}_M &= \phi'(\theta)\dot{\theta} + \dot{y}\,, \qquad & \dot{x}_B &= \dot{\theta}\,. \end{aligned} \tag{22}$$

The dynamics of y, which is representing the synchronization error to the specified virtual holonomic constraint, can be computed from the system dynamics (2) by substituting (22):

$$\ddot{y} = r(\theta, \dot{\theta}, y, \dot{y}) + m_M u = v\,. \tag{23}$$

An auxiliary control signal v is introduced by a control transformation via partial feedback linearization [10].

The dynamics of θ is described by the scalar second order differential equation (14), (15) substituting (22):

$$\tilde{m}_B\ddot{\theta} - c(\phi(\theta) - \theta - d_{PB} - l_{c,0}) + \tilde{m}_B g = cy\,, \tag{24}$$

where the right-hand side equals to zero on the desired orbit. Eventually, we can represent our target motion (11) in the new generalized coordinates by

$$y_\star(t) \equiv 0, \quad \theta = \theta_\star(t), \quad t \in [t_b, t_e]\,. \tag{25}$$

The dynamical system (23), (24) has a natural choice of transverse coordinates $x_\perp = [I(\theta, \dot{\theta}, \theta_{b\star}, \dot{\theta}_{b\star});\ y\ ;\ \dot{y}]$ that describe the system's behavior away from a specified orbit [8]. We can analytically compute a transverse linearization along a continuous-time target motion (25) to be used for system analysis and control design:

$$\frac{d}{d\tau}\begin{bmatrix} I_\bullet \\ Y_{1\bullet} \\ Y_{2\bullet} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & \frac{2\dot{\theta}_\star(\tau)}{\tilde{m}_B}c & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}}_{A(\tau)} \underbrace{\begin{bmatrix} I_\bullet \\ Y_{1\bullet} \\ Y_{2\bullet} \end{bmatrix}}_{z} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}_{B} V_\bullet\,. \tag{26}$$

The linearized hybrid dynamics transverse to the entire orbit (see Fig. 5) consists of two continuous-time phases and two discrete jumps. A cyclic solution $z = z(\tau) = z(\tau + T)$ with time period $T = t^{(2)} - t^{(0)}$ and a switch at $T_s = t^{(1)} - t^{(0)}$ is defined by:

$$\begin{aligned} &\tfrac{d}{d\tau}z = A(s)z + BV_\bullet\,, \quad s = \tau \bmod T \\ &A(s) = \begin{cases} A^{(1)}(s), & s \in (0, T_s] \\ A^{(2)}(s), & s \in (T_s, T) \end{cases} \\ &z(\tau_{k+}) = \mathcal{F}\, z(\tau_{k-})\,, \quad \tau_k = kT,\ k \in \mathbb{N}\,. \end{aligned} \tag{27}$$

The linear operator $\mathcal{F}$ is a mapping from "ball release" to "ball catch" at the end of the controlled phase applying a linearized version of the update law $\tilde{F}^{(2)}$ and certain projections from the tangential planes on the switching surfaces to corresponding planes transverse to the vector field of the target motion [9].

B. Closed-Loop System

Consider the control law

$$V_\bullet = K(s)z\,, \quad K(s) = \begin{cases} K^{(1)}(s), & s \in (0, T_s] \\ K^{(2)}(s), & s \in (T_s, T) \end{cases} \tag{28}$$

for the hybrid linear system (27). It can be shown that asymptotic stability of the origin for the linear impulsive system is equivalent to exponential orbital stability of the periodic motion for the nonlinear one [9] using the control transformation (23) with $v(t) = K(s)x_\perp(t)$, where $s = \psi(\theta)$ associates any point in the vicinity of the target orbit with a time moment on it: $\theta = \theta_\star(t) \iff t = \psi(\theta)$ so that $\psi\big(\theta_\star(t)\big) = t$.

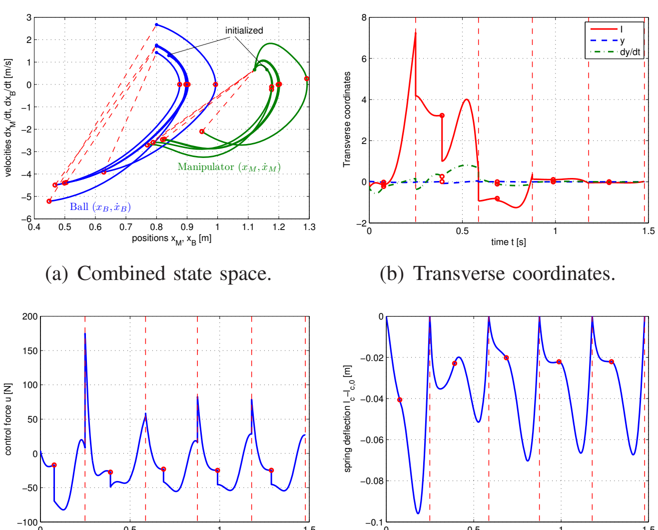
Three independent sets of initial conditions $z(0)$ can be used to construct a transition matrix from the linear closed-loop system (27), (28) as approximation for the Poincaré first return map. Here we compute the feedback gains in (28) via numerical search using `fminsearch` from MATLAB such that the eigenvalues of the transition matrix get minimized strictly inside the unit circle. For reasons of implementation only a constant feedback of the synchronization error $y = x_M - \phi(x_B)$ shall be used since these measurements are available. The following feedback gains have been obtained:

$$K^{(1)} = [0,\, 764,\, 0]\,, \quad K^{(2)} = [0,\, 89,\, 0]\,. \qquad (29)$$

Note that one also has to apply another control law $u^{(3)}$ for moving the manipulator from the "ball release" point to the desired manipulator state at "ball catch". The time at which the ball bounces to the desired catching height can be computed from (9), (10). There are many ways to plan a trajectory for the manipulator, preferably online with an update of the estimated catching time.

V. Numerical Simulation

In this section we discuss some simulation results of the controlled nonlinear ball-dribbling system for the nominal trajectory depicted in Fig. 5. The closed-loop performance was tested under a variation of initial conditions and model parameters and at the presence of measurement noise. An example is given in Fig. 6, where the ball was initially caught $4\,$cm too high with a $25\,\%$ lower velocity of the manipulator. The desired orbit is generally reached after a short time using the constant feedback gains (29). This is indicated by the transverse coordinates converging to zero. The main practical limitation is a violation of the minimum working length of the spring. However, the variation of initial velocities for the ball and the manipulator in a reasonable range of more than 25% can be compensated by the controller. An improvement of robustness and rate of convergence is expected when using a full state-feedback controller with possibly state-dependent coefficients. Moreover, a modification of the switching surface for the event "ball catch" might enlarge the region of attraction to the cycle: manipulator and ball velocities could be incorporated into the update law, which is however not very practical.



(a) Combined state space. (b) Transverse coordinates.

(c) Input force. (d) Spring deflection.

Fig. 6. Simulation of catching the ball too high with low velocity.

VI. Conclusions

We presented a novel approach for orbital stabilization of a human-like periodic ball-dribbling motion. Adding a spring to the manipulator allows us to control the ball in a continuous-time phase instead of having intermittent impact. During the controlled phase, in which the kinetic energy of the ball is converted into potential energy of the spring and its accumulated energy is eventually released back to the ball, we have dynamics of an underactuated Ball-Spring-Manipulator system.

The virtual holonomic constraints approach allows for systematic motion planning of a "controlled catching" and a "controlled pushing" phase such that a desired ball-dribbling orbit is shaped. The time interval of the entire control phase is about half the dribbling period, which permits effective stabilizing control action. It is necessary to switch the virtual constraint from "catching" to "pushing" because of the energy that gets lost during ball impact with the ground.

An orbitally stabilizing feedback controller is designed for the underactuated Ball-Spring-Manipulator system based on a transverse linearization along the desired motion. We chose a setting with constant feedback gain on the synchronization error between the ball and the manipulator since it can be implemented in a straightforward way. Numerical simulations demonstrate that the structure and performance of the control system are suited for experimental tests, which are currently in preparation.

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