2010 IEEE International Conference on Robotics and Automation
Anchorage Convention District
May 3-8, 2010, Anchorage, Alaska, USA

# Design of Redundant Drive Joints with Double Actuation Using Springs in the Second Actuator to Avoid Excessive Active Torques

Kiyoshi Nagai, Yuichiro Dake, Yasuto Shiigi, Rui C. V. Loureiro and William S. Harwin

*Abstract*— **This paper discusses the design of a Redundant Drive Joint with Double Actuation (RDJ-DA) to produce controlled compliant motions over a higher bandwidth. First, our strategies on mechanical and controller designs to produce compliant motions are described, and the basic structure and impedance control of the RDJ-DA with internal serial structure explained. The standard form of the basic structure is introduced using a set of parameters which can be used to express the inertia property of RDJ-DA. Second, a problem statement for the design of RDJ-DAs is described after pointing out that required torques of the second actuator of the RDJ-DA could be large. Then, a basic idea of introducing springs into the second actuator in parallel is proposed as a part of the structural design of RDJ-DA to reduce the large torques. Simulations are conducted to find out a set of the design parameters of RDJ-DA which obtains a higher natural frequency of the output joint admittance satisfying both the desired output joint admittance and the limitations of two identical motors on their performance. Finally, a prototype design of RDJ-DA using the obtained set of the design parameters is presented.**

## I. I

Producing compliant motions is a key issue for neuro-rehabilitation robots to deliver upper and lower limb therapies after stroke [1]-[6]. As these rehabilitation robots are directly attached to the human body, it is important that the robots have good admittance properties in order to deliver gentle compliant motions while avoiding unexpectedly large forces. However, traditional design methods result in designs that have a large effective inertia and high friction as a result of typical solutions that use small motors coupled to transmissions with a large reduction ratio.

Introducing passive compliant mechanisms into robotic joints would be a good method to produce compliant motions appropriate to human-centered rehabilitation robots. Adjustable passive compliant motions have been investigated, such as, the NST (Nonlinear Spring Tensioner) [7] and the NLEM (Non-Linear Elastic Module) [8]. MIA (Mechanical Impedance Adjuster) [9]-[10] have also been introduced to produce an adjustable impedance based on a passive joint that could, in theory, be infinite. These passive compliant mechanisms are also explained in a review paper [11].

However, we think that establishing a systematic approach to produce desired compliant motions is required in which reducing effective joint inertia and producing compliant joint motions with good quality are realized separately. Therefore, we have employed systematic design strategies for Redundant Drive Joints (RDJs) with kinematical redundancy which consists of 1) a mechanical design to reduce joint resultant inertia, and 2) a controller design to change the joint admittance. This idea originally came from the validities that adopting an impedance control scheme for robotic manipulators with kinematical redundancy would be useful to produce compliant motions [12]-[13].

In this paper, we describe our strategies on mechanical and controller designs to produce compliant motions in Section 2. The basic structure and impedance control of the RDJ-DA are explained whereby a double-input-single-output mechanism with one internal Degree of Freedom (DOF) results in a specified impedance with a reduction of joint inertia. In Section 3, a problem statement for RDJ-DA design is described after pointing out that required torques of the second actuator of RDJ-DA could in actual fact, be large. Then, in Section 4, a basic idea consisting of the introduction of springs into the second actuator in parallel is proposed as a structural design of RDJ-DA to solve the problem. Simulations are conducted to find out a set of the design parameters of RDJ-DA. Finally, in Section 5, a prototype design of RDJ-DA using the obtained set of the design parameters is presented.

## II. R D J (RDJ) P C M

### A. Strategies to Produce Compliant Motions

Let us explain our strategies to produce compliant motions using the typical impedance control scheme. If the required torques for the impedance control can be produced, the admittance property $G(s)\Big(\equiv X(s)/F_E(s)\Big)$ will be theoretically the classic ordinary second-order system, that is:

$$G(s) = \frac{1}{M_d s^2 + D_d s + K_d} = \frac{1}{K_d}\frac{{\omega_n}^2}{s^2 + 2\zeta\omega_n s + {\omega_n}^2} \qquad (1)$$

where $F_E(s)$ and $X(s)$ denote the external force and the displacement expressed in frequency domain, respectively. $M_d$, $D_d$ and $K_d$ are the desired values of the inertia, the viscosity and the stiffness, respectively. The inherent inertia $M$ is often used instead of using $M_d$, because if $M_d$ is set apart from $M$, the required driving forces become larger at higher frequencies and desired properties cannot be realized. The natural frequency $\omega_n(=\sqrt{K_d/M_d})$ and the viscous damping coefficient $\zeta(= D_d/2\sqrt{K_d M_d})$ are often used to deal with this second-order system.

K. Nagai is with Department of Robotics, College of Science and Engineering, Ritsumeikan University, Kusatsu, Shiga 525-8577, Japan

Y. Dake and Y. Shiigi are with Advanced Science and Engineering Major, Graduate School of Science and Engineering, Ritsumeikan University, Kusatsu, Shiga 525-8577, Japan

R. C. V. Loureiro and W. S. Harwin are with School of Systems Engineering, University of Reading, Reading RG6 6AY, United Kingdom

978-1-4244-5040-4/10/$26.00 ©2010 IEEE 

When $M = M_d$, the bode diagram for Eq. (1) is shown in Fig. 1. The bode response at lower frequencies is mainly determined by the compliance which is reciprocal of the stiffness. At the higher frequencies the characteristic is mainly determined by inertia. In order to obtain a good admittance property, we consider the following two considerations:

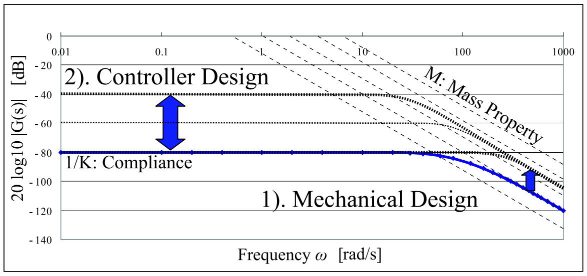



Fig. 1. Strategies to get compliant motions over a higher bandwidth [15]

1) Mechanical Design
2) Controller Design

For 1), the Mechanical design, it is important to reduce the effective joint inertia and the authors have already proposed a basic structure for a Redundant Drive Joint (RDJ) with kinematical redundancy [14] and have illustrated a RDJ with Adjustable Stiffness and Damping as a second actuator [15]. As for 2), the controller design, it is necessary to produce the specified compliance at the output joint. In order to satisfy this requirement, the authors proposed an impedance control scheme for the RDJ with Double Actuation (RDJ-DA) [16]. These approaches are explained in detail in the following part of this section.

### *B. Basic Structure and Inertia Property of RDJ-DA*

In this section, the basic structure of RDJ-DAs and how their resultant joint inertia can be reduced are presented as the mechanical design consideration.

The most important RDJ's feature with kinematical redundancy [14]-[16] is that such RDJs are adopting a double-input-single-output joint structure with 1 DOF kinematical redundancy. Here, we call such joint mechanisms RDJ-DA in a broad sense regardless that the two actuators are active or passive.

We propose to use the following expression to deal with RDJ-DAs:

$$\dot{q} = \boldsymbol{J}\begin{bmatrix}\dot{q}_{A1}\\ \dot{q}_{A2}\end{bmatrix},\ \boldsymbol{J} = \begin{bmatrix}J_1 & J_2\end{bmatrix} \tag{2}$$

$$\dot{q} = \boldsymbol{J}\begin{bmatrix}\dot{q}_{A1}\\ \dot{q}_{A2}\end{bmatrix},\ \boldsymbol{J} = \beta\begin{bmatrix}(1-\alpha) & \alpha\end{bmatrix} \tag{3}$$

$$\alpha = \frac{J_2}{J_1 + J_2},\ \beta = (J_1 + J_2) \tag{4}$$

where $\dot{q}$ is the output joint velocity, and $\dot{q}_{A1}$, $\dot{q}_{A2}$ are the input actuator velocities. $\alpha$ represents the ratio of the second actuator velocity compared to the resultant actuator velocity, and $\beta$ the magnification of the two actuator velocities. These parameters not only determine the kinematics of RDJ-DAs but also identify the inertia property of RDJ-DAs which will be described in this subsection. We call Eq. (2) with $J_1$ and $J_2$ the generalized form of RDJ-DAs and Eq. (3) with $\alpha$ and $\beta$ the standard form of RDJ-DAs. As RDJ-DAs represented by using Eq. (2) or Eq.(3) can produce the output joint velocity as the sum of the actuator velocities, the two actuators in RDJ-DAs are considered to be connected in serial. Zinn et al. have introduced a joint mechanism with force redundancy such that the output joint driving force can be produced as the sum of the actuator driving forces [17]. It means that theoretically the two actuators to drive the same joint are connected in parallel.

A set of examples of RDJ-DAs with translational joints are shown in Fig. 2. The kinematics of these examples can be described below:

$$\dot{x} = \boldsymbol{J}\dot{\boldsymbol{x}}_A,\quad \boldsymbol{J} = \begin{bmatrix}\frac{L_2}{L_1+L_2} & \frac{L_1}{L_1+L_2}\end{bmatrix} \tag{5}$$

where $\dot{\boldsymbol{x}}_A = [\dot{x}_{A1} \quad \dot{x}_{A2}]^{\mathrm{T}}$ is the set of actuator velocities, $\dot{x}$ is the output joint velocity, $\boldsymbol{J}$ is the Jacobian matrix, where the positions of the output link from the two actuators, $L_1$ and $L_2$, are treated signed values such that when the output link is located inward is plus and outward minus from the respective actuators. Eq. (5) can be transformed to the standard form of Eq. (3) with the following $\alpha$ and $\beta$.

$$\alpha = \frac{L_1}{L_1 + L_2},\quad \beta = 1$$

In the two RDJ-DAs shown in Fig. 2, although the two actuators seem to be located in parallel, they are serial mechanisms as their kinematics can be described by Eq. (2).

Next, we describe how the RDJ-DAs reduce resultant joint inertia. Let $\gamma$ denote the inertia ratio between A and B as follows:

$$\gamma \equiv \frac{M_{A2}}{M_{A1}} \tag{6}$$

Then, the resultant joint inertia $M$ can be represented in the following equation when the inertia of the internal DOF part can be neglected:

$$M \approx M_{J0} + \frac{1}{\beta^2}\left(\frac{\gamma}{(1-\alpha)^2\gamma + \alpha^2}\right)M_{A1} \tag{7}$$

The derivation of $M$ is mentioned in the next section.

When $\rho \equiv M_{A1}/M_{J0}$ and $\bar{M} \equiv M/M_{J0}$, $\bar{M}$ can be represented as a function of $\alpha$, $\beta$, $\gamma$ and $\rho$. Figure 3 shows $\bar{M}$ represented in Eq. (7) when $\beta = 1$ and $\rho = 1$. $\bar{M}$ is desirable to have a value close to 1 because $\bar{M}$ denotes the total joint inertia normalized by the link inertia.

Based on the above property of $\bar{M}$ with $\beta = 1$ and $\rho = 1$, $M < (M_{J0} + M_{A1})$ holds when $\bar{M} < 2.0$. Therefore, $\alpha$ has to satisfy the following inequality to hold $M < (M_{J0} + M_{A1})$.

$$\alpha < 0 \quad \text{or} \quad \frac{2\gamma}{1+\gamma} < \alpha$$

When $\gamma$ is smaller, the range of $\alpha$ that satisfies $M < (M_{J0}+M_{A1})$ becomes larger, and $M$ with the same $\alpha$ becomes smaller.

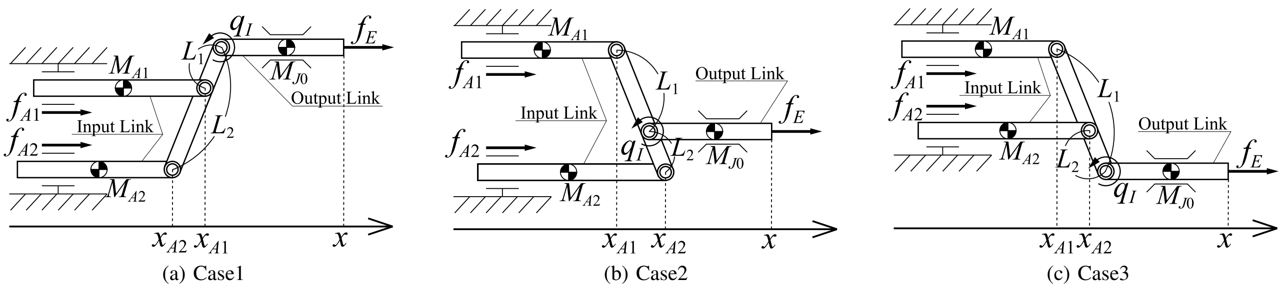



Fig. 2. Examples of RDJ-DA with translational joints

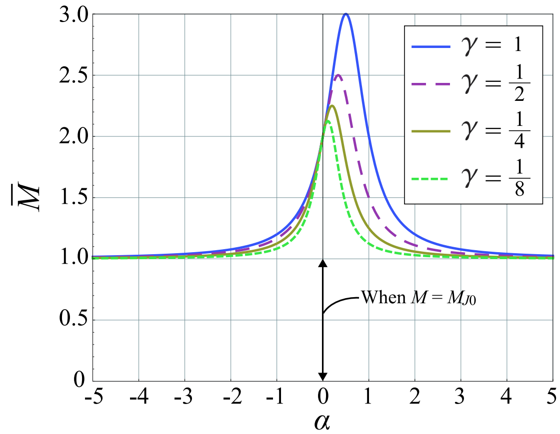



Fig. 3. Total joint inertia normalized by the link inertia

## C. Basic Equations of RDJ-DA

In this section, the basic equations of RDJ-DA are presented. The kinematics and statics of RDJ-DA are summarized as follows:

$$\dot{q} = \boldsymbol{J}\dot{\boldsymbol{q}}_A, \quad \boldsymbol{J} = \begin{bmatrix} J_1 & J_2 \end{bmatrix} \tag{8}$$

$$\dot{q}_I = \boldsymbol{J}_I\dot{\boldsymbol{q}}_A, \quad \boldsymbol{J}_I = \begin{bmatrix} J_{I1} & J_{I2} \end{bmatrix} \tag{9}$$

$$\boldsymbol{\tau}_A = \boldsymbol{J}^{\mathrm{T}}\tau \tag{10}$$

where $\dot{\boldsymbol{q}}_A = [\dot{q}_{A1} \quad \dot{q}_{A2}]^{\mathrm{T}}$ is the set of actuator velocities, $\dot{q}$ the joint velocity, $\dot{q}_I$ the internal velocity, $\boldsymbol{\tau}_A = [\tau_{A1} \quad \tau_{A2}]^{\mathrm{T}} = [N_1\tau_{m1} \quad N_2\tau_{m2}]^{\mathrm{T}}$ the set of actuator driving force that is defined without loss by inertia forces, where $\tau_{m1}$ and $\tau_{m2}$ are the first and second motor torques, respectively. $\tau_{m1}$ and $\tau_{m2}$ are transmitted to become the actuator torques $\tau_{A1}$ and $\tau_{A2}$ being magnified with the reduction ratios $N_1$ and $N_2$, respectively. $\tau$ the joint driving force. $\boldsymbol{J}$ and $\boldsymbol{J}_I$ are the Jacobian matrices.

When $q_{A1}$, $q_{A2}$ are assumed as generalized coordinates and $\tau_{A1}$, $\tau_{A2}$ assumed as generalized forces, Lagrangian $\mathcal{L}$ can be obtained as follows by using Eqs. (8) and (9).

$$\mathcal{L} = \frac{1}{2}\Big\{M_{A1}\dot{q}_{A1}^2 + M_{A2}\dot{q}_{A2}^2 + M_{I0}(J_{I1}\dot{q}_{A1} + J_{I2}\dot{q}_{A2})^2 + M_{J0}(J_1\dot{q}_{A1} + J_2\dot{q}_{A2})^2\Big\} \tag{11}$$

where $M_{A1}$ and $M_{A2}$ are the first and second actuator inertias, respectively, $M_{I0}$ the inherent inertia of Pulley I, and $M_{J0}$ the inherent inertia of Output Link. Substituting this $\mathcal{L}$ into Lagrange's equation of motion, the following equation of motion can be obtained.

$$\boldsymbol{M}_A\ddot{\boldsymbol{q}}_A = \boldsymbol{\tau}_A \tag{12}$$

$$\boldsymbol{M}_A = \boldsymbol{M}_{A0} + \boldsymbol{J}^{\mathrm{T}}M_{J0}\boldsymbol{J} + \boldsymbol{J}_I^{\mathrm{T}}M_{I0}\boldsymbol{J}_I, \quad \boldsymbol{M}_{A0} = \begin{bmatrix} M_{A1} & 0 \\ 0 & M_{A2} \end{bmatrix} \tag{13}$$

$$M_{A1} = M_{A01} + M_{01}, \quad M_{A2} = M_{A02} + M_{02}$$

where $M_{A01}$ and $M_{A02}$ are the inherent inertia of $q_{A1}$ and $q_{A2}$ spaces, and $M_{01}$ and $M_{02}$ the actuator inertia represented in $q_{A1}$ and $q_{A2}$ spaces, respectively. Here, $\boldsymbol{M}_{A0}$ is defined the inertia which consists of those of the actuators and the reduction mechanisms.

When the external forces $\tau_E$ are applied to the output joint, the equation of motion represented at $\boldsymbol{q}_A$ space becomes below:

$$\boldsymbol{M}_A\ddot{\boldsymbol{q}}_A = \boldsymbol{\tau}_A + \boldsymbol{J}^{\mathrm{T}}\tau_E \tag{14}$$

The above equation of motion can be converted to the output joint space using Eqs. (8) and (10) as follows:

$$M\ddot{q} = \tau + \tau_E, \quad M = (\boldsymbol{J}\boldsymbol{M}_A^{-1}\boldsymbol{J}^{\mathrm{T}})^{-1} \tag{15}$$

When the inherent inertia of the Pulley I, $M_{I0}$, can be neglected, the resultant joint inertia $M$ becomes below:

$$\begin{aligned} M &\approx M_{J0} + \left(M_1^{-1} + M_2^{-1}\right)^{-1} \\ M_1 &= J_1^{-2}M_{A1} = \beta^{-2}(1-\alpha)^{-2}M_{A1} \\ M_2 &= J_2^{-2}M_{A2} = \beta^{-2}\alpha^{-2}\gamma M_{A1} \end{aligned} \tag{16}$$

where $M_1$ and $M_2$ are the converted inertia of $M_{A1}$ and $M_{A2}$ to the output joint space. Eq. (16) can be transformed to the form of Eq. (7) by using $\alpha$, $\beta$, $\gamma$ and $\rho$. Therefore, the resultant joint inertia expressed in Eq. (16) can be reduced by selecting the radii of the Pulley A1 and Pulley A2, $R_1$ and $R_2$.

Now, based on the Eq. (2), we introduce the first and second actuator velocities converted to the output joint space, $\dot{q}_1$ and $\dot{q}_2$, respectively, as follows:

$$\dot{q}_1 \equiv J_1\dot{q}_{A1}, \quad \dot{q}_2 \equiv J_2\dot{q}_{A2} \tag{17}$$

Then, the output joint velocity of RDJ-DA $\dot{q}$ can be expressed as follows:

$$\dot{q} = \dot{q}_1 + \dot{q}_2 = \begin{bmatrix} 1 & 1 \end{bmatrix}\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} \tag{18}$$

Equation (18) represents that the two actuators in RDJ-DA are connected serially showing its kinematical redundancy clearly.

On the other hand, the external force $\tau_E$ satisfy the following equation that can be derived by using Eq. (18):

$$\begin{bmatrix} \tau_{E1} \\ \tau_{E2} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \tau_E = \begin{bmatrix} \tau_E \\ \tau_E \end{bmatrix} \tag{19}$$

where, $\tau_{E1}$ and $\tau_{E2}$ are the external forces applied to the first and second actuators represented at the output joint space, respectively.

### D. Impedance Control of RDJ-DA [16]

In this section, the outline of the control scheme for the RDJ-DA to produce compliant motions is explained.

We apply the concept of the impedance control scheme of redundant macro-micro manipulators [12] to assign one actuator to a lower frequency domain and the other to a higher frequency domain.

The kinematics and statics of RDJ-DA required to develop the control scheme are as follows:

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q} \end{bmatrix} = \tilde{\boldsymbol{J}} \dot{\boldsymbol{q}}_A, \quad \tilde{\boldsymbol{J}} = \begin{bmatrix} \tilde{\boldsymbol{J}}_1 \\ \boldsymbol{J} \end{bmatrix}, \quad \tilde{\boldsymbol{J}}_1 = \begin{bmatrix} J_1 & 0 \end{bmatrix} \tag{20}$$

$$\begin{bmatrix} \ddot{q}_1 \\ \ddot{q} \end{bmatrix} = \tilde{\boldsymbol{J}} \ddot{\boldsymbol{q}}_A \tag{21}$$

$$\boldsymbol{\tau}_A = \tilde{\boldsymbol{J}}_1^{\mathrm{T}} \tau_1 \tag{22}$$

In our control scheme, the desired mechanical impedance is specified at the output joint, and the desired mechanical dynamics at the first actuator is specified for tracking the desired trajectories, as follows:

$$\tilde{\boldsymbol{M}}_d \begin{bmatrix} \ddot{q}_1 \\ \ddot{q} \end{bmatrix} + \tilde{\boldsymbol{D}}_d \begin{bmatrix} \dot{q}_1 \\ \dot{q} \end{bmatrix} + \tilde{\boldsymbol{K}}_d \begin{bmatrix} q_{1e} \\ q_e \end{bmatrix} = \begin{bmatrix} \tau_{E1} \\ \tau_E \end{bmatrix} \tag{23}$$

$$\tilde{\boldsymbol{M}}_d \in \boldsymbol{R}^{2\times 2}, \ \tilde{\boldsymbol{D}}_d = \begin{bmatrix} D_{1d} & 0 \\ 0 & D_d \end{bmatrix}, \ \tilde{\boldsymbol{K}}_d = \begin{bmatrix} K_{1d} & 0 \\ 0 & K_d \end{bmatrix} \tag{24}$$

where $q_{1d}$ and $q_d$ are the desired angles of $q_1$ and $q$, $q_{1e}(= q_1 - q_{1d})$ and $q_e(= q - q_d)$ the positional errors of $q_1$ and $q$, respectively. $\tilde{M}_d$, $\tilde{D}_d$ and $\tilde{K}_d$ are the desired matrices of the inertia, viscous friction and stiffness in the $[q_1 \ q]^{\mathrm{T}}$ space, and $D_{1d}$, $K_{1d}$, $D_d$ and $K_d$ are the desired values of the inertia, coefficient of viscous friction and stiffness of the first actuator and the output joint, respectively.

The set of $q_{1d}$ and $q_{2d}$ is determined as follows.

(a) determine $q_{2d}$ as:

$$q_{2d} = 0 \tag{25}$$

(b) determine $q_{1d}$ as it satisfies the equation below:

$$q_{1d} = q_d - q_{2d} \tag{26}$$

Then, by using Eqs. (26) and (25), $q_{1d}$ can be set as follows:

$$q_{1d} = q_d \tag{27}$$

The reason for introducing the condition (a) is to assign the desired motions in lower frequencies to the first actuator.

By using Eqs. (14), (21) and (23), we can derive the following impedance control law:

$$\boldsymbol{\tau}_A = \boldsymbol{M}_A \tilde{\boldsymbol{J}}^{-1} \tilde{\boldsymbol{M}}_d^{-1} \left( \begin{bmatrix} \tau_{E1} \\ \tau_E \end{bmatrix} - \tilde{\boldsymbol{D}}_d \begin{bmatrix} \dot{q}_1 \\ \dot{q} \end{bmatrix} - \tilde{\boldsymbol{K}}_d \begin{bmatrix} q_{1e} \\ q_e \end{bmatrix} \right) - \boldsymbol{J}^{\mathrm{T}} \tau_E \tag{28}$$

Now, let us determine the desired inertia matrix $\tilde{\boldsymbol{M}}_d$. By using Eqs. (10), (21), (22) and (14), the resultant output joint inertia $M$ and the first actuator inertia at the output joint space $\tilde{M}_1$ can be obtained by mapping the inertia matrix $\boldsymbol{M}_A$: as follows:

$$M = (\boldsymbol{J}\boldsymbol{M}_A^{-1}\boldsymbol{J}^{\mathrm{T}})^{-1}, \ \tilde{M}_1 = (\tilde{\boldsymbol{J}}_1 \boldsymbol{M}_A^{-1} \tilde{\boldsymbol{J}}_1^{\mathrm{T}})^{-1} \tag{29}$$

Then, $\boldsymbol{M}_A$ can be mapped in the $[q_1 \ \ q]^{\mathrm{T}}$ space to set the desired inertia $\tilde{\boldsymbol{M}}_d$ as follows:

$$\tilde{\boldsymbol{M}}_d = \begin{bmatrix} M_{1d} & 0 \\ 0 & M_d \end{bmatrix} = \begin{bmatrix} \tilde{M}_1 & 0 \\ 0 & M \end{bmatrix} \tag{30}$$

where, $M_{1d}$ and $M_d$ are the desired values of the inertia of the first actuator and the output joint, respectively.

## III. B I M D RDJ-DA

### A. Technical Trade-off on RDJ-DA

In our mechanical design strategy, the inertia of the second actuator should be small to make the bandwidth of the joint-compliant motions higher. However, trying to make the actuator inertia small often leads to a small reduction ratio and to reduce actuator torque. In addition, trying to set $\alpha$ to make the resultant joint inertia small might require large actuator torques on the second actuator. Moreover, as the two actuators in RDJ-DA are connected in serial, a required joint torque might directly determine a required large actuator torque. The above requirements might cause a technical trade-off problem requiring a small inertia and large actuator torques. This trade-off makes the design of RDJ-DA difficult.

The above technical trade-off problem can be summarized as follows: 1) Having a small inertia 2) Producing large actuator torques

### B. Basic Ideas to Overcome Technical Trade-off

On contrary to the technical trade-off explained in the previous subsection, the adopted impedance control scheme of RDJ-DA does not require large motions on the second actuator.

Therefore, the basic requirements on the second actuator include the above 1) and 2), on the other hand, 3) Producing small motions.

The details of the concept will be described below.

Figure 4 shows the basic concept for the mechanical design of RDJ-DA. First, the set of the four pulleys, Pulley 1, Pulley I1, Pulley 2, Pulley I2, are introduced to construct the basic structure of RDJ-DA. Here, Pulley I1 and Pulley I2 are connected to a single axis ($q_I$). The kinematical relationships of this structure are described as follows:

$$\dot{q} = \boldsymbol{J}\dot{\boldsymbol{q}}_A, \quad \boldsymbol{J} = \begin{bmatrix} J_1 & J_2 \end{bmatrix} = \begin{bmatrix} \frac{r_1}{r_1+r_2} & \frac{r_2}{r_1+r_2} \end{bmatrix} \tag{31}$$

$$\dot{q}_I = \boldsymbol{J}_I \dot{\boldsymbol{q}}_A, \quad \boldsymbol{J}_I = \begin{bmatrix} J_{I1} & J_{I2} \end{bmatrix} = \begin{bmatrix} \frac{r_1 r_2}{r_1+r2} & -\frac{r_1 r_2}{r_1+r2} \end{bmatrix} \tag{32}$$

$$r_1 = \frac{R_1}{R_{I1}}, \ r_2 = \frac{R_2}{R_{I2}} \tag{33}$$

$$\alpha = \frac{r_2}{r_1 + r_2}, \ \beta = 1$$

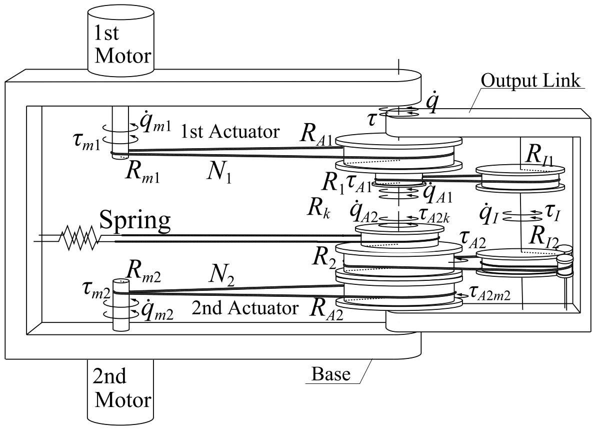


Fig. 4. Basic Structure of RDJ with Double Actuation

where $R_1$ [m] and $R_{I1}$ [m] are the radii of the Pulley $R_1$ and Pulley $R_{I1}$, $R_2$ [m] and $R_{I2}$ [m] the radii of the Pulley $R_2$ and Pulley $R_{I2}$, respectively. $r_1$ and $r_2$ are the ratios of the corresponding radii: The Pulley $R_{I2}$ is connected to rotate in the opposite direction to the rotation of Pulley $R_2$ in order to make the value of $\alpha$ properly.

Second, belts and wires are used to transmit the actuator driving forces in order to avoid frictions that may decrease the performances of the produced compliant motions. The kinematics and statics hold in the actuators are as follows:

$$\dot{\boldsymbol{q}}_{Am} = \boldsymbol{N}^{-1}\dot{\boldsymbol{q}}_m,\ \boldsymbol{N} = \begin{bmatrix} N_1 & 0 \\ 0 & N_2 \end{bmatrix} \tag{34}$$

$$\boldsymbol{\tau}_{Am} = \boldsymbol{N}^{\mathrm{T}}\boldsymbol{\tau}_m,\ \boldsymbol{\tau}_{Am} = \begin{bmatrix} \tau_{A1m1} \\ \tau_{A2m2} \end{bmatrix} \tag{35}$$

Where, $\dot{\boldsymbol{q}}_m = [\dot{q}_{m1}\ \dot{q}_{m2}]^{\mathrm{T}}$ is the motor velocity vector, $\boldsymbol{\tau}_m = [\tau_{m1}\ \tau_{m2}]^{\mathrm{T}}$ the motor torque vector. $N_1$ and $N_2$ are the first and second reduction ratios, respectively, and $\boldsymbol{N}$ the reduction ratio matrix.

Finally, springs are introduced into the second actuator. Figure 4 illustrates that the spring mechanism is connected to the second actuator pulley in parallel.

The kinematics and statics of the second actuator after introducing the springs are described as follows:

$$\begin{bmatrix} \dot{q}_{A2m2} \\ \dot{q}_{A2k} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \dot{q}_{A2} \tag{36}$$

$$\tau_{A2} = \tau_{A2m2} + \tau_{A2k} \tag{37}$$

where $\dot{q}_{A2m2}$ and $\dot{q}_{A2k}$ are the velocities of the second motor and the springs converted to the $q_{A2}$ space. The $\dot{q}_{A2}$ is the velocity of the pulley A2, and $\tau_{A2}$ the torque around it produced by the second actuator. Equations (36) and (37) come from that the springs and the second motor are connected in parallel.

Figure 5 shows how the springs are connected to Pulley 2. $l_0$ the original spring length, $l_1$ the offset length introduced to use the tension spring. $q_{A2ke}$, $x_{k1e}$ and $x_{k2e}$ the deviations caused by the deviation of $q_2$ from $q_{2d}$. The $R_k$ the radius of the pulley. The $F_{k1}$ and $F_{k2}$ the reaction forces of the tension springs, the $\tau_{A2k}$ the resultant torque produced by the reaction forces of the springs. $K$ the stiffness of the tension

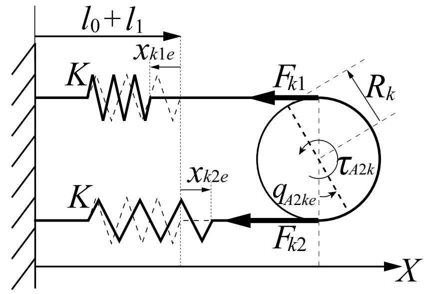


Fig. 5. Conceptual Spring Mechanism

springs, $K_r$ the stiffness around the pulley axis caused by the spring stiffness. The relations among the above variables are described as follows:

$$\begin{bmatrix} x_{k1e} \\ x_{k2e} \end{bmatrix} = \begin{bmatrix} -R_k \\ R_k \end{bmatrix} q_{A2ke},\ \tau_{A2k} = \begin{bmatrix} -R_k & R_k \end{bmatrix} \begin{bmatrix} F_{k1} \\ F_{k2} \end{bmatrix} \tag{38}$$

$$\tau_{A2k} = -K_r q_{A2ke},\ K_r = 2R_k^2 K \tag{39}$$

Note that $M_{A2}$ is the inertia of the second actuator after introducing additional spring part.

Then, the equation of motion of RDJ-DA is modified as follows:

$$\boldsymbol{M}_A \ddot{\boldsymbol{q}}_A = \boldsymbol{\tau}_A + \boldsymbol{J}^{\mathrm{T}}\tau_E \tag{40}$$

$$\boldsymbol{\tau}_A = \boldsymbol{\tau}_{Am} + \boldsymbol{\tau}_{Ak},\ \boldsymbol{\tau}_{Ak} = \begin{bmatrix} 0 \\ \tau_{A2k} \end{bmatrix} \tag{41}$$

where $\boldsymbol{\tau}_{Ak}$ is the reaction torque caused by the deviations of the springs. Note that the actuator torque is now produced as the sum of the motor torque and the reaction torque caused by the deviations of the springs.

After introducing the springs, the impedance control scheme of RDJ-DA will be modified as follows:

$$\boldsymbol{\tau}_{Am} = \boldsymbol{M}_A \tilde{\boldsymbol{J}}^{-1} \tilde{\boldsymbol{M}}_d^{-1} \left( \begin{bmatrix} \tau_{E1} \\ \tau_E \end{bmatrix} - \tilde{\boldsymbol{D}}_d \begin{bmatrix} \dot{q}_1 \\ \dot{q} \end{bmatrix} - \tilde{\boldsymbol{K}}_d \begin{bmatrix} q_{1e} \\ q_e \end{bmatrix} \right) - \boldsymbol{\tau}_{Ak} - \boldsymbol{J}^{\mathrm{T}}\tau_E \tag{42}$$

The proposed RDJ-DA with springs and the modified control scheme for that can be expected to decrease the required motor torques of the second motor.

## IV. S M D P RDJ-DA

### A. Poblem Statement

In this section, a mechanical design problem of RDJ-DA is described and solved. The mechanical design problem of RDJ-DA was set as follows:

**Mechanical Design Problem of RDJ-DA**:
Find a set of design parameters $\alpha$, $N_1$ $N_2$ that realizes a desired admittance property at the output joint with a higher natural frequency for a given external torque, while satisfying both the limitations of the motors on the torques, velocities and powers and the limitation of the springs on the deviations.

In this paper, obtaining a higher bandwidth for the compliant motions is the single seeking issue. However, it depends on the limitations on the actuators which consist of motors and springs. It also depends on the magnitude of the external forces. Moreover, It also depends on the settings on the desired admittances.

First, the external force $\tau_E(\omega)$[Nm] is assumed as follows:

$$\tau_E(\omega) = \begin{cases} 50\sin\omega_i t, & \omega_i = 2^{-\frac{14-i}{4}} \quad i = 1,\cdots,14 \\ 50\omega_i^{\epsilon}\sin\omega_i t, & \omega_i = 2^{-\frac{14-i}{4}} \quad i = 15,\cdots,53 \end{cases} \tag{43}$$

Here, a sine wave is used as the external force, and the magnitude of 50 [Nm] is used for $\tau_E(\omega)$ when the frequency is equal to or less than 1 [rad/s], and the power of $\tau_E(\omega)$ is 50 [W] is used otherwise, as $\epsilon = -1$.

Second, both the limitations of the motors on the torques, velocities and powers and the limitation of the springs on the deviations are given as follows:

$$|q_{A2}| \le \frac{L_s}{2R_k},\ L_s = 2.25\times 10^{-2}\ [\text{m}] \tag{44}$$

$$|\dot{q}_{Ai}| \le N_i^{-1}\dot{q}_{mi\max},\ \dot{q}_{mi\max} = 4.05\times 10^{3}\ [\text{rad/s}] \tag{45}$$

$$|\tau_{Ai}| \le N_i\tau_{mi\max},\ \tau_{mi\max} = 6.92\times 10^{-1}\ [\text{Nm}] \tag{46}$$

$$|P_{Ai}| \le P_{mi\max},\ P_{mimax} = 50[\text{W}] \tag{47}$$

where, i=1,2 for Eqs. (44)-(47). $L_s$[m] is the stroke of the spring, $R_k = 3.40\times 10^{-2}$ [m], $\dot{q}_{mi\max}$[rad/s] the maximum velocity of the $i$-th motor, $\tau_{mi\max}$[Nm] the maximum torque of the $i$-th motor, $P_{mi\max}$[W] the maximum power of the $i$-th motor, The desired stiffness at the output joint $K_d$ is set such that $K_{dmin} = 50$ and $K_{dmax} = 500$[Nm/rad], for the minimum and the maximum values. This means that the stiffness at the endpoint will be $K_{xd} = 200$[N/m] and $K_{xdmax} = 2000$[N/m] correspondingly, when the length of the virtual link is 0.5[m]. Now, the following relationships are considered when the desired stiffness is stiff;

$$\frac{1}{K_{1dmax}} + \frac{1}{K_2} = \frac{1}{K_{dmax}} \tag{48}$$

$$K_{1dmax} = K_2 \tag{49}$$

Then, $K_{1d\max}$[Nm/rad] the maximum value of $K_d$[Nm/rad], and $K_2$[Nm/rad] the stiffness of the second actuator which is determined by the springs, are determined as $K_{1d\max} = K_2 = 1.00\times 10^3$[Nm/rad].

Next, the following relationship is considered when the desired joint stiffness is compliant:

$$\frac{1}{K_{1dmin}} = \frac{1}{K_{dmin}} - \frac{1}{K_2} \tag{50}$$

By substituting the values of $K_{d\min}$ and $K_2$, $K_{1d\min} = 5.26\times 10^1$[Nm/rad]. Note that the joint stiffness can be changed by changing the stiffness of the first actuator.

In case of RDJ-DA, $K_{1d}$ can be adjusted as a function of the impedance controller, while $K_2$ is determined by the springs and cannot be adjusted.

When $M_d$ and $M_{1d}$ are set by using Eq. (31), the minimum and maximum values of the natural frequencies $\omega_n(=\sqrt{K_d/M_d})$ and $\omega_{n1}(=\sqrt{K_{1d}/M_{1d}})$ are determined by the above stiffness:

$$\omega_{n\min} = 64.38\ [\text{rad/s}],\ \omega_{n1\min} = 33.55\ [\text{rad/s}]$$
$$\omega_{n\max} = 203.46\ [\text{rad/s}],\ \omega_{n1\max} = 146.22\ [\text{rad/s}]$$

The viscous friction coefficients $\zeta(= D_d/2\sqrt{MK_d})$ and $\zeta_1(= D_{1d}/2\sqrt{M_1K_{1d}})$ are set as follow:

$$\zeta = 0.7,\ \zeta_1 = 0.7$$

*B. Solving the Problem*

Figures 6 and 7 represent the simulation results in which the proposed RDJ-DA with springs is used. Here, $K_{d\min}$ is used because the required torques of the motors would be large when $K_d$ is small. We have adopted a three-stage reduction mechanism for the first actuator and a two-stage reduction mechanism for the second actuator.

The obtained design parameters through the simulations of the controlled RDJ-DA are as follows:

$$\alpha = 2/3,\ N_1 = 60,\ N_2 = 24$$

The other parameters have the following values when the design parameters have the above values: Here, $M_{J0} = 1.00\times 10^{-2}$ [kgm$^2$] is assumed as the link inertia.

$$\gamma = M_{A2}/M_{A1} = 0.18$$
$$M_1 = 4.68\times 10^{-2}\ [\text{kgm}^2],\ M_2 = 2.16\times 10^{-3}\ [\text{kgm}^2]$$
$$M_2/M_1 = (1-\alpha^2)\gamma/\alpha^2 = 4.61\times 10^{-2}$$
$$M_{I0} = 9.58\times 10^{-5}\ [\text{kgm}^2]$$
$$M = 1.21\times 10^{-2}\ [\text{kgm}^2]$$

Here, The desired inertia is set to have the same values as the original values. The desired stiffness is set as follows. The desired damping is set such that the coefficients of the viscous damping is 0.7.

The deviation of $q$ is the sum of the deviations of $q_1$ and $q_2$ because Eq. (17) holds, and the external forces $\tau_{E1}$ and $\tau_{E2}$ are equal to $\tau_E$ as shown in Eq. (23). Therefore, the admittances $Y(s)(= q(s)/\tau_E(s))$, $Y_1(s)(= q_1(s)/\tau_{E1}(s))$ and $Y_2(s) = q_2(s)/\tau_{E2}(s)$ holds the equation below:

$$Y_2(s) = Y(s) - Y_1(s) \tag{51}$$

This above relation among the admittances means that the first and the second actuators are connected in serial. This relation also appears in Fig. 6(a). In the figure, it is found that the second actuator with a smaller inertia contributes to produce compliant motions in higher frequencies.

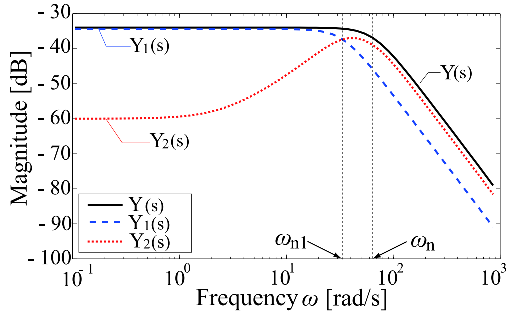
On the other hand, as the second motor and the introduced spring are connected in parallel, the relation among the impedances hold as follows:

$$Z_{A2m2}(s) = Z_{A2}(s) - Z_{A2k}(s) \tag{52}$$

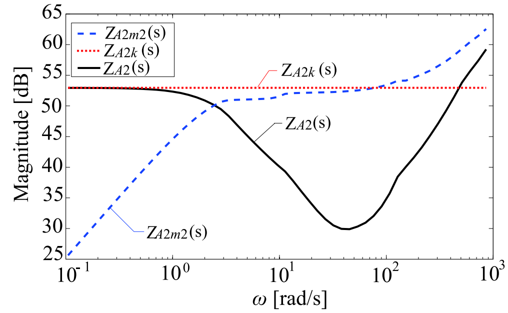
Here, $Z_{A2}(s)$, $Z_{A2k}(s)$ and $Z_{A2m2}(s)$ are the desired impedance of the second actuator determined by Eq. (51), the produced impedance of the spring and the required impedance to the second motor, respectively, and these impedances are represented at the $q_{A2}$ space.

This relation also appears in Fig. 6(b). In the figure, it is found that $Z_{A2m2}(s)$ is small in lower frequencies. This means that the required torque to the second motor is also small. Therefore, introducing the springs into the second actuator works well to avoid excessive active torques of the second motor.

Figure 7 shows the required performances of the two motors. Figure 7(a) also shows that the required deviation to the introduced spring is within the stroke of the springs. By comparing Fig. 7(c) and 7(d) with Fig. 8(a) and 8(b),

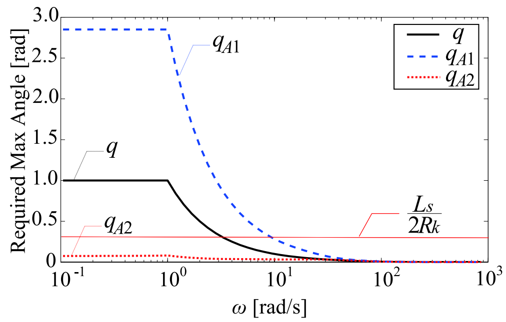



(a) Admittance

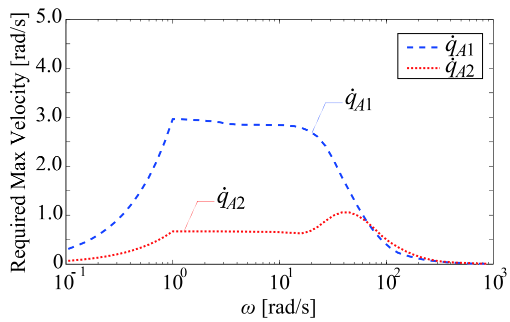



(b) Impedance

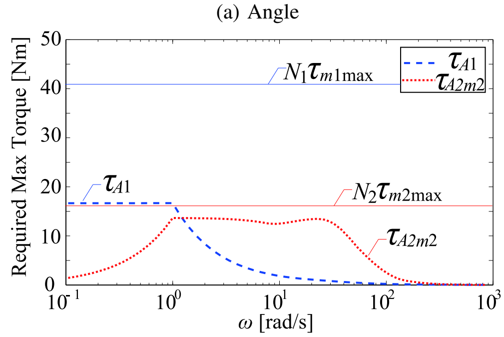
Fig. 6. Admittance and Impedance



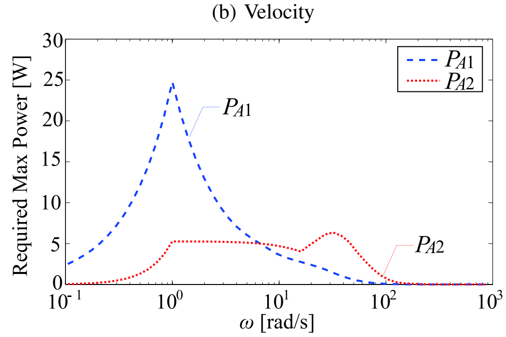

(a) Angle




(b) Velocity

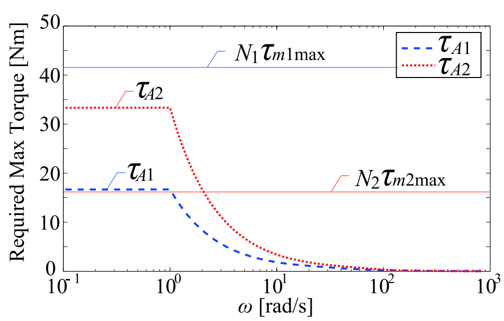



(c) Torque

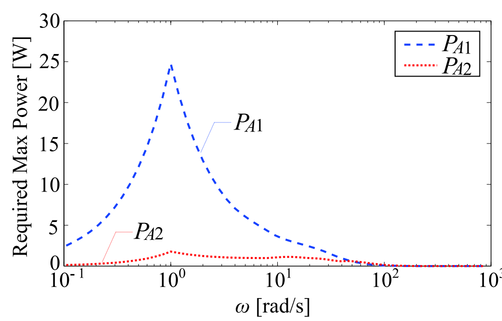



(d) Power

Fig. 7. Required performances of the prototype with the springs




(a) Torque




(b) Power

Fig. 8. Required performances of the prototype if the springs not introduced

introducing the springs into the second actuator makes the required torques and powers to the second motor smaller in lower frequencies. It also makes the required torques and powers to the second motor larger in higher frequencies in order to produce the required motions of the second actuator against the reaction forces of the springs.

## V. P D RDJ-DA

Fig. 9 shows the basic prototype design of the proposed RDJ-DA with springs. Two identical brushless DC motors with 50 [W] of power are used for the first and second actuators. Two compression springs with 192 [N/mm] of stiffness are being introduced in the second actuator. The reason for choosing compression type springs is that springs

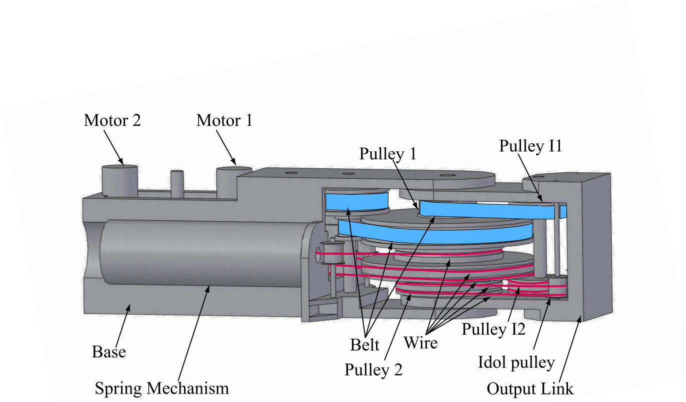
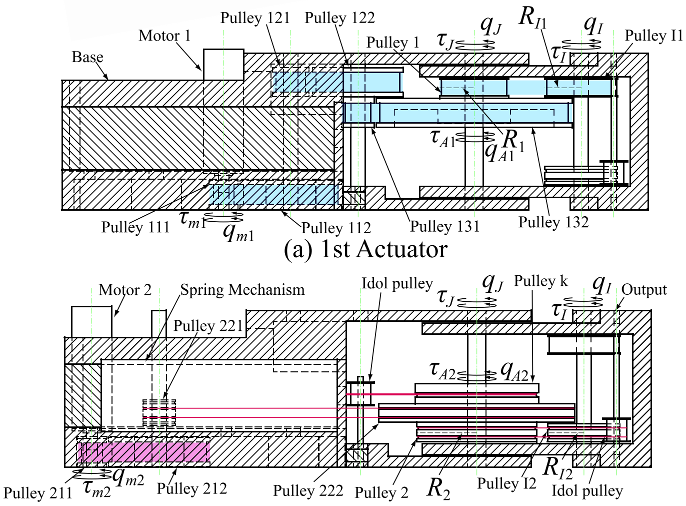



Fig. 9. Prototype of RDJ-DA




(a) 1st Actuator

(b) 2nd Actuator

Fig. 10. 1st and 2nd actuators in prototype of RDJ-DA

with large stiffness and compact size are needed for the given desired impedance. A three-stage reduction mechanism with a reduction ratio of 60 is adopted for the first motor. On the other hand, a two-stage reduction mechanism with a reduction ratio of 24 is adopted for the second motor. Thanks to the introduction of the springs, the reduction ratio of the second motor keeps small because no large forces are needed. The small reduction ratio of the second motor would be effective not only to keep the inertia small but also to avoid large friction forces of in the transmission. Wire/belt drive mechanisms are used for transmitting the motor torques to the main part of the RDJ-DA to avoid large friction and backlash. The first actuator adopts belt drive mechanisms because it needs to produce large motions. The second actuator adopts wire drive mechanisms in some parts because it needs high stiffness in the transmission while no large motions are needed.

## VI. C

This paper addresses the design of Redundant Drive Joint with Double Actuation (RDJ-DA) which has an internal DOF. The main results obtained in this paper are as follows.

1) RDJ-DAs have a structure that the two actuators are serially connected to produce the single output motion. The introduced parameters, $\alpha$, $\beta$, $\gamma$ and $\rho$, are useful to show how the inertia of the second actuator can reduce the total inertia of RDJ-DA at the output joint.
2) Introducing springs in the second actuator of RDJ-DA in parallel is useful to find a set of the design parameters of RDJ-DA, $\alpha$, $N_1$ and $N_2$ that realizes a given admittance property at the output joint with a higher natural frequency satisfying the limitations of the motors on the torques, velocities and powers. The introduced springs is found to have a function that reduces the large torques of the second motor. The obtained design parameters are being used in the real prototype design of RDJ-DA.

## R

[1] H. I. Krebs, et al.: "Rehabilitation Robotics: Pilot Trial of a Spatial Extension for MIT-Manus," *J. of NeuroEngineering and Rehabilitation*, Vol. 1, No. 5, 2004.
[2] W.S. Harwin, et al.: "The GENTLE/S Project: A New Method of Delivering Neuro-Rehabilitation," *Proc. of Association for the Advancement of Assistive Technology in Europe*, Vol. 10, pp. 36-41, 2001.
[3] R.C.V. Loureiro, et al.: "Upper Limb Mediated Stroke Therapy - GENTLE/s Approach," *Special Issue on Rehabilitation Robotics, J. of Autonomous Robots*, Vol. 15, No. 1, pp. 35-51, 2003
[4] T. Sasaki, et al: "TEM: Therapeutic Exercise Machine for Hip and Knee Joint of Spastic Patients," *Proc. of 1999 IEEE Int. Conf. on Rehabilitation Robotics*, pp. 183-186, 1999.
[5] G. Colombo, et al: "Treadmill training of paraplegic patients using a robotic orthosis," *J. of Rehabilitation Research and Development*, vol. 37, no. 6, pp. 693-700, 2000.
[6] K. Nagai, Y. Kojima, S. Yonemoto, T. Okubo, R.C.V. Loureiro and W.S. Harwin: "Structural Design of an Escort Type Rehabilitation Robot for Post-Stroke Therapies of Upper-Limb," *Proc. of 2007 IEEE Int. Conf. on Rehabilitation Robotics*, pp. 1121-1128, 2007.
[7] K. Hyodo and H. Kobayashi: "Kinematic and Control Issues on Tendon Controlled Wrist Mechanism," *J. of Robotics Society of Japan*, vol.10, no. 6, pp. 809-816, 1992.
[8] K. Koganewaza, M. Yamazaki and N. Ishikawa, "Mechanical Stiffness Control of Tendon-Driven Joints" *J. of the Robotics Society of Japan*, vol. 18, no. 7, pp. 1003-1010, 2000.
[9] T. Morita, S. Sugano: "Mechanical Softness and Compliance Adjustment," *J. of the Robotics Society of Japan*, vol. 17, no. 6, pp. 36-40, 1999.
[10] T. Morita and S. Sugano: "New Control Method for Robot Joint by Mechanical Impedance Adjuster-Proposition of Mechanisms and Application to Robot Finger," *J. of the Robotics Society of Japan*, Vol. 14, No. 1, pp. 131-136, 1996.
[11] R. Ham, T. Sugar, B. Vanderborght, K. Hollander, D. Lefeber: "Compliant Actuator Designs," *Robotics and Automation Magazine, IEEE*, vol. 16, pp81-94, September, 2009.
[12] K. Nagai and T. Yoshikawa: "Impedance Control of Redundant Macro-micro Manipulators," *Proc. of 2009 IEEE/RSJ Int. Conf. on Intelligent Robots and Systems*, pp. 1438-1445, 1994.
[13] K. Nagai and T. Yoshikawa: "Grasping and Manipulation by Arm/Multifingered-Hand Mechanisms," *Proc. of 1995 IEEE Int. Conf. on Robotics and Automation*, pp.1040-1047, 1995.
[14] K. Nagai, Y. Ikegami, R.C.V. Loureiro and W.S. Harwin: "Proposal of an Admittance Enhanced Redundant Joint Mechanism to Improve Backdrivability," *Proc. of 2008 IEEE/ASME Int. Conf. on Advanced Intelligent Mechatronics*, pp.504-509, 2008.
[15] Y. Ikegami, K. Nagai, R.C.V. Loureiro and W.S. Harwin: "Design of Redundant Drive Joint with Adjustable Stiffness and Damping Mechanism to Improve Joint Admittance," *Proc. of 2009 IEEE Int. Conf. on Rehabilitation Robotics*, pp. 202-210, 2009.
[16] K. Nagai, Y. Shiigi, Y. Ikegami, R.C.V. Loureiro and W.S. Harwin: "Impedance Control of Redundant Drive Joints with Double Actuation," *Proc. of 2009 IEEE Int. Conf. on Robotics and Automation*, pp. 1528-1534, 2009.
[17] M. Zinn, O. Khatib, B. Roth and J. Salisbury: "Large Workspace Haptic Devices - A New Actuation Approach," *Proc. of Symposium on Haptic Interfaces for Virtual Environments and Teleoperator Systems*, pp. 185-192, 2008.