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SP-ID Regulation of Rigid-Link Electrically-Driven Robots with Uncertain Kinematics

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Abstract— **In this paper, the regulation problem of rigid-link electrically-driven (RLED) robotic manipulators with uncertain kinematics and dynamics is addressed. A task-space Saturated-Proportional Integral and Differential (SP-ID) based control approach is proposed using backstepping technique to deal with the uncertainties in actuator dynamics, robot dynamics and kinematics. The proposed method is structurally simple and easy for implementation. Sufficient conditions for choosing the feedback gains, approximate Jacobian matrix and motor torque constant matrix are provided to guarantee system stability. Simulation results demonstrate the effectiveness of the proposed approach.**

I. Introduction

Many control schemes for robotic manipulators have been developed in the literature during the past few decades. In most of the control methods [1]–[9], the controllers are designed at the torque input level and the actuator dynamics is neglected. As shown by Good $et.al.$ [10], the actuator dynamics constitutes an important part of the whole robotic system and may cause detrimental effects when neglected in the design procedure, especially in cases of high speed movement and highly varying loads.

Since then actuator dynamics has been explicitly included in control schemes and some control schemes for rigid-link robots have been developed to deal with this problem in joint space [11]–[17]. However, for most robot applications the desired position or path is specified in the task space. To design controllers in the joint space for such control tasks, inverse kinematics transformation should be carried out to obtain the desired joint-space position or trajectory. In order to avoid the problem of solving inverse kinematics, Takegaki and Arimoto [18] proposed a task-space controller for set-point regulation in Cartesian space using a transposed Jacobian matrix. Other task-space control schemes have been proposed later [19]–[21]. To apply these task-space control schemes, exact knowledge of the Jacobian matrix from joint space to task space is required. If uncertainties exist in the robot kinematics, degraded performance or even instability may occur as a result of employing the aforementioned task-space controllers. To deal with the problem of uncertain kinematics, several task-space feedback laws with uncertain kinematics from the joint space to the task space have been proposed [22]–[27]. However, in those works the actuator dynamics was not considered.

Recently, a control method [28] was presented which is able to deal with uncertain actuator transmission model but the the actuator dynamics has not been taken into consideration. The first task-space control scheme taking into account both uncertain kinematics and actuator dynamics for manipulator regulation problem is proposed in [29]. It's shown that regulation error convergence is achieved even with uncertainties in both kinematics and actuator dynamics at the same time. But there still exist certain constraints for this method: two Jacobian matrices (one adaptive and one approximate) are employed which complicates the control design and imposes a limit of estimation error for the approximate Jacobian matrix to guarantee system stability; the gravity is compensated using regressor which increases the design cost since the regressor structure varies with different configuration of manipulator used; one more adaptive regressor is used devoting to compensating the overall manipulator loop uncertainty.

To remove the above mentioned constrains, in this work we propose a new task-space SP-ID based method for the manipulator regulation problem. The new controller utilizes just one adaptive Jacobian matrix throughout the whole system design without assuming any estimation error limitations, and an integrator is use to compensate for gravity force and deal with all rest uncertainties in the manipulator loop without much concern of dynamics of the particular manipulator used. Hence the proposed controller is structurally simpler and more efficient for both design and implementation. The closed-loop system is shown to be asymptotic stable through Lyapunov analysis and simple sufficient conditions are presented to guarantee the system stability. Simulation results are provided to demonstrate the effectiveness of the proposed regulation approach.

II. Problem Formulation

In order to describe a task for the robot manipulator, the desired path for the end effector is usually specified in the task space. Let $X \in \mathbb{R}^m$ represents the position vector of the manipulator in task space defined by [19], [22]:

$$X = h(q), \tag{1}$$

where $q \in \mathbb{R}^n$ is the vector of generalized joint coordinates, $h(\cdot) \in \mathbb{R}^n \rightarrow R^m$ $(m \leq n)$ is generally a nonlinear transformation describing the relation between the joint space and the task space. The velocity vector $\dot{X}$ is therefore related to $\dot{q}$ as follows:

$$\dot{X} = J(q)\dot{q}, \tag{2}$$

where $J(q) \in \mathbb{R}^{m\times n}$ is the Jacobian matrix of mapping from the joint space to the task space.

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Property 1: The right hand side of equation (2) is linear in a set of kinematic parameters $\theta_J = (\theta_{J1}, \dots, \theta_{Jl})^T$, such as link lengths [29], so that it has

$$J(q)\dot{q} = Y_J(q,\dot{q})\theta_J \tag{3}$$

where $Y_J(q,\dot{q}) \in \mathbb{R}^{m\times l}$ is called the kinematic regressor matrix. Note that if the robot's kinematics is uncertain, θ_J is uncertain.

The dynamics of the rigid-link electrically driven robot manipulator include two coupled loops: the manipulator loop and the actuator loop. The dynamic equation of the RLED system with n degrees of freedom are described as follows [17]

$$M(q)\ddot{q} + (B + C(q,\dot{q}))\dot{q} + g(q) = K_N I, \tag{4}$$

$$L\dot{I} + RI + K_E\dot{q} = u. \tag{5}$$

where $I \in \mathbb{R}^n$ is the vector of armature currents and $u \in \mathbb{R}^n$ denotes the vector of armature voltages. $M(q) \in \mathbb{R}^{n\times n}$ denotes a positive definite inertia matrix; $B \in \mathbb{R}^{n\times n}$ denotes a positive definite damping matrix; $C(q,\dot{q})\dot{q} \in \mathbb{R}^n$ is the centrifugal and coriolis force and $g(q) \in \mathbb{R}^n$ denotes the gravitational force vector. $L \in \mathbb{R}^{n\times n}$ represents the actuator inductance matrix; $R \in \mathbb{R}^{n\times n}$ is the actuator resistance matrix, $K_E \in \mathbb{R}^{n\times n}$ is the matrix characterizing the voltage constant of the actuator and $K_N \in \mathbb{R}^{n\times n}$ is the motor torque constant diagonal matrix which characterizes the electromechanical conversion between current and torque. L, R, K_E and K_N are positive definite constant diagonal matrices.

Some important properties of the robot dynamics that will be used in the control analysis are as following [30]:

Property 2: $M(q)$ is symmetric and positive definite. Furthermore, since each entry of $M(q)$ is constant or a trigonometric function of components of q, there are positive constants ς_m and ς_M such that

$$\varsigma_m I \leq M(q) \leq \varsigma_M I, \tag{6}$$

where I denotes the $n \times n$ identity matrix.

Property 3: The matrix $C(q,\dot{q})$ and the time derivative $\dot{M}(q)$ of the inertia matrix satisfy:

$$\dot{q}^T[\frac{1}{2}\dot{M}(q) - C(q,\dot{q})]\dot{q} = 0 \quad \forall\ q, \dot{q} \in \mathbb{R}^n \tag{7}$$

In this study, it's assumed that X, q, $\dot{q}$ and I are measurable; the exact values of the dynamics parameters in $M(q)$, $C(q,\dot{q})$ and $g(q)$ and the actuator dynamic coefficient matrices L, R, K_E and K_N are not available; Jacobian matrix $J(q)$ is not known exactly due to uncertainties in robot kinematics. The regulation task for this RLED robot manipulator is hence to drive the manipulator endpoint to the desired position defined in task space with the existence of uncertainties in actuator dynamics, robot kinematics and dynamics.

III. Control Scheme Development Based on Backstepping Technique

In this section, we propose a task-space control scheme for the regulation problem of RLED robots. Firstly, based on the second-order manipulator subsystem dynamics (4), a desired armature current signal I_d is designed to ensure that the task-space position errors converge with the presence of uncertainties in kinematics and motor torque constant matrix. Then based on the actuator subsystem dynamics (5), a backstepping procedure is used to design a voltage control input u to guarantee that the actual armature current I tracks the desired current signal I_d in spite of the uncertain actuator coefficient matrices. Two Lyapunov functions are proposed for stability analysis of the control scheme proposed.

A. Desired Armature Current Design

Using the desired armature current I_d, the manipulator subsystem dynamics (4) can be rewritten as

$$M(q)\ddot{q} + (B + C(q,\dot{q}))\dot{q} + g(q) = K_N I_d + K_N\tilde{I} \tag{8}$$

where $\tilde{I} = I - I_d$ represents a current perturbation to the rigid-link robot dynamics. The revised subsystem (8) can be viewed as controlled by $K_N I_d$ with an input disturbance $K_N\tilde{I}$ which will be handled by the input control voltage designed in next subsection.

Let $\hat{J}(q)$ be the adaptive Jacobian matrix and $\hat{K}_N$ be the adaptive motor torque transmission matrix whose uncertain parameters $\hat{\theta}_J$ and $\hat{\theta}_K$ are updated by the updating laws (12) and (13) respectively as in following. Note that $\hat{\theta}_K$ is a vector containing the diagonal elements of $\hat{K}_N$.

Based on the adaptive motor torque transmission matrix and Jacobian matrix, the desired armature current I_d is proposed as

$$I_d = -\hat{K}_N^{-1}(\hat{J}^T(q)K_p s(e) + K_v v + K_I\int_0^t y(\tau)d_\tau), \tag{9}$$

$$\dot{v} = -\Lambda v + \dot{q} + \alpha\hat{J}^T(q)s(e), \tag{10}$$

$$y = \dot{q} + \alpha\hat{J}^T(q)s(e), \tag{11}$$

where $s_i(\cdot), i = 1, \cdots, n$ are saturated functions of e as will be explained later; $e = X - X_d = (e_1, \cdots, e_m)^T$ is a positional deviation from a desired task-space position $X_d \in \mathbb{R}^m$, X is obtained through camera or other positioning systems (laser, ultrared sensor etc.); $K_p = k_p I_E$, K_v, K_I, $\Lambda \in \mathbb{R}^{n\times n}$ are positive definite diagonal feedback gains, I_E represents the identity matrix and α is a positive constant.

The uncertain parameters $\hat{\theta}_K$ and $\hat{\theta}_J$ in $\hat{K}_N$ and $\hat{J}(q)$ are updated online by

$$\dot{\hat{\theta}}_J = \Phi_1 L_1 Y_J^T(q,\dot{q})s(e), \tag{12}$$

$$\dot{\hat{\theta}}_K = -\Phi_2 L_2 Q\hat{J}^T(q)s(e), \tag{13}$$

where $L_1 \in \mathbb{R}^{l\times l}$, $L_2 \in \mathbb{R}^{n\times n}$ are constant positive definite updating gain matrices, $Q = diag\{\dot{q}\} \in \mathbb{R}^{n\times n}$. Φ_1 and Φ_2 are diagonal projection operators defined as

$$\Phi_{1i} = \begin{cases} 0, & \text{if } \hat{\theta}_{Ji} \geq \theta_{imax} \text{ and } \{Y_J^T(q,\dot{q})s(e)\}_i \geq 0 \\ 0, & \text{if } \hat{\theta}_{Ji} \leq \theta_{imin} \text{ and } \{Y_J^T(q,\dot{q})s(e)\}_i \leq 0 \\ 1, & \text{otherwise,} \end{cases} \tag{14}$$

$$\Phi_{2i} = \begin{cases} 0, & \text{if } \hat{\theta}_{Ki} \leq 0 \text{ and } \{-Q\hat{J}^T(q)s(e)\}_i \leq 0 \\ 1, & \text{otherwise,} \end{cases} \quad (15)$$

where θ_{max}, θ_{min} denote the estimated upper and lower bound of kinematics parameter vector θ_J, the projection operators Φ_1 and Φ_2 guarantee that $\hat{\theta}_J$ lies in $[\theta_{min}, \theta_{max}]$ and $\hat{K}_N$ is positive definite [31].

Remark 1: In the desired current I_d design, an auxiliary variable v is proposed instead of using joint or task space damping $\dot{q}$ or $\dot{X}$ directly. Hence acceleration measurement is avoided in next step of the backstepping design as can be seen in equation (36). And for the projection operators, they are used simply to guarantee the boundedness of the adaptive parameters in the theoretical analysis and hence their ranges could be set quite flexibly without losing system stability in practical implementation plus the fact that it's not actually difficult to obtain a rough guess of the real physical limits of these parameters. ♢

For the saturated function, let us define a scalar function $S_i(e)$ and its derivative $s_i(e)$ as shown in Figure 1 with the following properties [30]:

1) $S_i(e) > 0$ for $e \neq 0$ and $S_i(0) = 0$.
2) $S_i(e)$ is twice continuously differentiable, and the derivative $s_i(e) = \frac{dS_i(e)}{de}$ is strictly increasing in e for $|e| < \gamma_i$ with some γ_i and saturated for $|e| \geq \gamma_i$, i.e. $s_i(e) = \pm s_i$ for $e \geq \pm\gamma_i$, and $e \leq -\gamma_i$ respectively where s_i is a positive constant.
3) There are constants $\hat{c}_i > 0$ such that for $e \neq 0$,

$$S_i(e) \geq \hat{c}_i {s_i}^2(e). \quad (16)$$

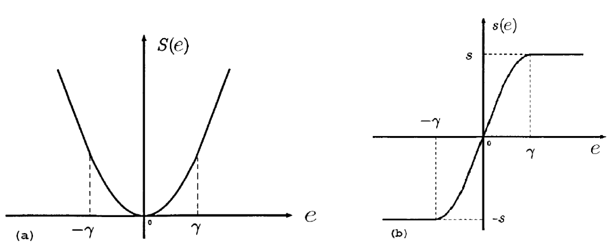

Fig. 1. (a)Quasi-natural potential: $S(e)$ (b)Derivative of $S(e)$: $s(e)$

Some examples of the saturated function can be found in [26], [28], [30].

Substituting I_d as defined in (9) into the dynamic equation (8), we can get the closed-loop manipulator dynamic equation as

$$M(q)\ddot{q} + (B + C(q,\dot{q}))\dot{q} + g(q) + K_N\hat{K}_N^{-1}\hat{J}^T(q)K_p s(e) \\ +K_N\hat{K}_N^{-1}K_v v + K_N\hat{K}_N^{-1}K_I z = K_N\tilde{I}, \quad (17)$$

where z is an auxiliary variable defined as $z = \int_0^t y(\tau)d\tau$ so that $\dot{z} = y$. $K_N\hat{K}_N^{-1}K_I$ is a diagonal matrix since K_N, $\hat{K}_N^{-1}$ and K_I are all diagonal matrices.

We now study the the stability of the closed loop system equation (17) with $\tilde{I} = 0$ and the current perturbation $\tilde{I}$ will be considered in the overall Lyapunov function construction in next subsection. A Lyapunov function candidate V_1 is proposed with the form:

$$V_1 = \tfrac{1}{2}\dot{q}^T M(q)\dot{q} + \alpha\dot{q}^T M(q)\hat{J}^T(q)s(e) \\ +\textstyle\sum_{i=1}^m k_p S_i(e_i) + P(q) + \tfrac{1}{2}z^T K_N\hat{K}_N^{-1}K_I z \\ +\tfrac{1}{2}v^T K_N\hat{K}_N^{-1}K_v v + \tfrac{1}{2}k_p\Delta\theta_J^T L_1^{-1}\Delta\theta_J \\ +\tfrac{1}{2}k_p\Delta\theta_K^T\hat{K}_N^{-1}L_2^{-1}\Delta\theta_K, \quad (18)$$

where $\Delta q = q - q_d$, $\Delta\theta_J = \theta_J - \hat{\theta}_J$ and $\Delta\theta_K = \theta_K - \hat{\theta}_K$. $P(q)$ denotes the potential energy function of the manipulator so that $g(q) = \partial P(q)/\partial q$ [32].

From [30], it has

$$\tfrac{1}{4}\dot{q}^T M(q)\dot{q} + \alpha\dot{q}^T M(q)\hat{J}^T(q)s(e) + \tfrac{1}{2}\textstyle\sum_{i=1}^m k_p S_i(e_i) \\ = \tfrac{1}{4}(\dot{q} + 2\alpha\hat{J}^T(q)s(e))^T M(q)(\dot{q} + 2\alpha\hat{J}^T(q)s(e)) \\ -\alpha^2 s^T(e)\hat{J}(q)M(q)\hat{J}^T(q)s(e) + \tfrac{1}{2}\textstyle\sum_{i=1}^m k_p S_i(e_i) \\ \geq \textstyle\sum_{i=1}^m(\tfrac{1}{2}k_p\hat{c}_i - \alpha^2\lambda_m)s_i^2(e_i), \quad (19)$$

where α can be chosen small enough or k_p can be chosen large enough to to satisfy the inequality,

$$\frac{1}{2}k_p\hat{c}_i - \alpha^2\lambda_m > 0, \quad (20)$$

where $\lambda_m = \lambda_{max}[\hat{J}(q)M(q)\hat{J}^T(q)]$, $\lambda_{max}[A]$ denotes the maximum eigenvalue of a matrix A.

Substituting inequalities (19) into equation (18) and noting that $P(q)$ is always positive, we have

$$V_1 \geq \tfrac{1}{4}\dot{q}^T M(q)\dot{q} + \textstyle\sum_{i=1}^m(\tfrac{1}{2}k_p\hat{c}_i - \alpha^2\lambda_m)s_i^2(e_i) + P(q) \\ +\tfrac{1}{2}\textstyle\sum_{i=1}^m k_p S_i(e_i) + \tfrac{1}{2}z^T K_N\hat{K}_N^{-1}K_I z + \tfrac{1}{2}v^T K_N\hat{K}_N^{-1}K_v v \\ +\tfrac{1}{2}k_p\Delta\theta_J^T L_1^{-1}\Delta\theta_J + \tfrac{1}{2}k_p\Delta\theta_K^T\hat{K}_N^{-1}L_2^{-1}\Delta\theta_K > 0 \quad (21)$$

Hence, V_1 is positive definite with condition (20) satisfied.

Differentiating equation (18) with respect to time and substituting equations (10), (11) and (17) (with $\tilde{I} = 0$) into it, we have:

$$\frac{d}{dt}V_1 = -W_1 \quad (22)$$

where

$$W_1 = \dot{q}^T B\dot{q} + v^T K_N\hat{K}_N^{-1}K_v\Lambda v \\ -k_p\dot{q}^T[J^T(q) - \hat{J}^T(q)]s(e) - k_p\dot{q}^T(I - K_N\hat{K}_N^{-1})\hat{J}^T(q)s(e) \\ +\alpha s^T(e)\hat{J}(q)[k_p K_N\hat{K}_N^{-1}\hat{J}^T(q)s(e) + g(q)] \\ +\alpha\{s^T(e)\hat{J}(q)[B - \dot{M}(q) + C(q,\dot{q})]\dot{q} \\ -\dot{s}^T(e)\hat{J}(q)M(q)\dot{q} - s^T(e)\dot{\hat{J}}(q)M(q)\dot{q}\} \\ +k_p\Delta\theta_J^T L_1^{-1}\dot{\hat{\theta}}_J + k_p\Delta\theta_K^T\hat{K}_N^{-1}L_2^{-1}\dot{\hat{\theta}}_K \quad (23)$$

Following [30] and knowing that both $\hat{J}(q)$ and $g(q)$ are trigonometric functions of q, the following condition stands with specified constant $k_0 > 0$ for any q:

$$k_p s^T(e)\hat{J}(q)K_N\hat{K}_N^{-1}\hat{J}^T(q)s(e) \\ +s^T(e)\hat{J}(q)g(q) \geq k_0(k_p + 1)\|s(e)\|^2 \quad (24)$$

Also, since $s(e)$ and $\hat{\theta}_J$ are bounded, there exist constants $c_0 > 0$ and $c_1 > 0$ so that [30]:

$$\alpha\{s^T(e)\hat{J}(q)[B - \dot{M}(q) + C(q,\dot{q})]\dot{q} - s^T(e)\dot{\hat{J}}(q)M(q)\dot{q} \\ -\dot{s}^T(e)\hat{J}(q)M(q)\dot{q}\} \geq -\alpha c_0\|\dot{q}\|^2 - \alpha c_1\|s(e)\|^2. \quad (25)$$

Hence substituting inequalities (25) and (24) into (23), we can get:

$$W_1 \geq (\lambda_{min}[B] - \alpha c_0)\|\dot{q}\|^2 + \lambda_{min}[K_N\hat{K}_N^{-1}K_v\Lambda]\|v\|^2 \\ +\alpha(k_0 + k_0k_p - c_1)\|s(e)\|^2 - k_p\dot{q}^T[J^T(q) - \hat{J}^T(q)]s(e) \\ -k_p\dot{q}^T(I - K_N\hat{K}_N^{-1})\hat{J}^T(q)s(e) + k_p\Delta\theta_J^T L_1^{-1}\dot{\hat{\theta}}_J \\ +k_p\Delta\theta_K^T\hat{K}_N^{-1}L_2^{-1}\dot{\hat{\theta}}_K \quad (26)$$

According to Property 1, we have

$$k_p\dot{q}^T[J^T(q) - \hat{J}^T(q)]s(e) = k_ps^T(e)[J(q)\dot{q} - \hat{J}(q)\dot{q}] \\ = k_ps^T(e)Y_J(q,\dot{q})\Delta\theta_J = k_p\Delta\theta_J^TY_J^T(q,\dot{q})s(e), \quad (27)$$

also it has

$$k_p\dot{q}^T(I - K_N\hat{K}_N^{-1})\hat{J}^T(q)s(e) \\ = k_p\dot{q}^T(\hat{K}_N - K_N)\hat{K}_N^{-1}\hat{J}^T(q)s(e) \\ = -k_p\Delta\theta_K^T\hat{K}_N^{-1}Q\hat{J}^T(q)s(e). \quad (28)$$

Substituting (27) and (28) into (23) and using updating laws (12) and (13), we can get

$$W_1 \geq (\lambda_{min}[B] - \alpha c_0)\|\dot{q}\|^2 + \lambda_{min}[K_N\hat{K}_N^{-1}K_v\Lambda]\|v\|^2 \\ +\alpha(k_0 + k_0k_p - c_1)\|s(e)\|^2 \\ -k_p\Delta\theta_J^T(I - \Phi_1)Y_J^T(q,\dot{q})s(e) \\ -k_p\Delta\theta_K^T\hat{K}_N^{-1}(I - \Phi_2)[-Q\hat{J}^T(q)s(e)] \quad (29)$$

From the definitions of Φ in (14), (15) and following [31], it's easily verified that

$$k_p\Delta\theta_J^T(I - \Phi_1)Y_J^T(q,\dot{q})s(e) \geq 0, \quad (30)$$
$$k_p\Delta\theta_K^T\hat{K}_N^{-1}(I - \Phi_2)[-Q\hat{J}^T(q)s(e)] \geq 0, \quad (31)$$

so that the above inequality (29) has

$$W_1 \geq (\lambda_{min}[B] - \alpha c_0)\|\dot{q}\|^2 + \lambda_{min}[K_N\hat{K}_N^{-1}K_v\Lambda]\|v\|^2 \\ +\alpha(k_0 + k_0k_p - c_1)\|s(e)\|^2. \quad (32)$$

Hence if the following conditions are satisfied

$$\lambda_{min}[B] - \alpha c_0 > 0, \quad (33)$$
$$k_0 + k_0k_p - c_1 > 0, \quad (34)$$

then W_1 is positive definite in $\dot{q}$, v and $s(e)$.

Lemma *The closed-loop system described by equation (17) gives rise to the convergence of $X \to X_d$ and $\dot{q} \to 0$ as $t \to \infty$ if $\tilde{I} = 0$, the feedback gains K_p, α are chosen to satisfy conditions (20), (33) and (34).*

Proof Since V_1 is positive definite and W_1 is positive semi-definite, from equation (22) we have

$$\frac{d}{dt}V_1 = -W_1 \leq 0. \quad (35)$$

Hence, V_1 is a Lyapunov function whose time derivative is negative definite in $s(e)$, $\dot{q}$ and v. Since $W_1 = 0$ implies that $\dot{q} = 0,\ e = X - X_d = 0,\ v = 0$, by LaSalle's invariance Theorem, the proof is complete. $\Delta\Delta\Delta$

Remark 2: The stability conditions (20), (33) and (34) could be easily satisfied by tuning control parameters. In fact, a careful look at these conditions simply suggests the guideline of tuning as larger K_p or/and smaller α is/are requested if the system goes unstable. Moreover, these conditions are sufficient conditions and hence could be conservative. $\diamondsuit$

B. Input Control Voltage Design

We can now use the desired current input I_d as defined by (9) in previous subsection to design a voltage input u which will force armature current perturbation $\tilde{I}$ to zero. Since the actuator dynamic coefficient matrices L, R, K_E and K_N are unknown, we employ approximate models $\hat{L}$, $\hat{R}$, $\hat{K}_E$ and $\hat{K}'_N$ and propose the input voltage u as:

$$u = \hat{L}\dot{I}_d + \hat{R}I_d + \hat{K}_E\dot{q} - K_D\tilde{I} - N_N(y)\hat{\theta}_{K'_N}, \quad (36)$$
$$\dot{\hat{\theta}}_L = -L_LN_L(\dot{I}_d)\tilde{I}, \quad (37)$$
$$\dot{\hat{\theta}}_R = -L_RN_R(I_d)\tilde{I}, \quad (38)$$
$$\dot{\hat{\theta}}_{K_E} = -L_EN_E(\dot{q})\tilde{I}, \quad (39)$$
$$\dot{\hat{\theta}}_{K'_N} = L_NN_N(y)\tilde{I}. \quad (40)$$

where $y = \dot{q} + \alpha\hat{J}^T(q)s(e)$, $K_D \in \mathbb{R}^{n\times n}$ is a positive definite control gain, L_L, L_R, L_E, L_N are positive definite diagonal matrices as introduced in Section II. $N_L(\dot{I}_d) = diag\{\dot{I}_{d1},\cdots,\dot{I}_{dn}\}$, $N_R(I_d) = diag\{I_{d1},\cdots,I_{dn}\}$, $N_E(\dot{q}) = diag\{\dot{q}_1,\cdots,\dot{q}_n\}$, $N_N(y) = diag\{y_1,\cdots,y_n\}$ and hence $\hat{L}\dot{I}_d = N_L(\dot{I}_d)\hat{\theta}_L, \hat{R}I_d = N_R(I_d)\hat{\theta}_R, \hat{K}_E\dot{q} = N_E(\dot{q})\hat{\theta}_{K_E}, \hat{K}'_Ny = N_N(y)\hat{\theta}_{K'_N}$.

The approximate models $\hat{L}$, $\hat{R}$, $\hat{K}_E$ and $\hat{K}'_N$ are updated online by their updating laws respectively during the regulation. Note that $\hat{K}'_N$ here is different from the adaptive transmission matrix $\hat{K}_N$ in I_d and they are updated by different updating laws.

Substituting the input voltage (36) into the actuator dynamics (5), we have

$$L(\dot{I} - \dot{I}_d) + R(I - I_d) = (\hat{L} - L)\dot{I}_d + (\hat{R} - R)I_d \\ +(\hat{K}_E - K_E)\dot{q} - K_D\tilde{I} - N_N(y)\hat{\theta}_{K'_N} \quad (41)$$

which can be rewritten as

$$L\dot{\tilde{I}} + (R + K_D)\tilde{I} \\ = N_L(\dot{I}_d)(\theta_{\hat{L}} - \theta_L) + N_R(I_d)(\theta_{\hat{R}} - \theta_R) \\ +N_E(\dot{q})(\theta_{\hat{K}_E} - \theta_{K_E}) - N_N(y)\hat{\theta}_{K'_N} \quad (42)$$

We now propose another Lyapunov function candidate V_2 with the form as following:

$$V_2 = V_1 + \tfrac{1}{2}\tilde{I}^TL\tilde{I} + \tfrac{1}{2}(\theta_L - \hat{\theta}_L)^TL_L^{-1}(\theta_L - \hat{\theta}_L) \\ +\tfrac{1}{2}(\theta_R - \hat{\theta}_R)^TL_R^{-1}(\theta_R - \hat{\theta}_R) \\ +\tfrac{1}{2}(\theta_{K_E} - \hat{\theta}_{K_E})^TL_E^{-1}(\theta_{K_E} - \hat{\theta}_{K_E}) \\ +\tfrac{1}{2}(\theta_{K_N} - \hat{\theta}_{K'_N})^TL_N^{-1}(\theta_{K_N} - \hat{\theta}_{K'_N}) \quad (43)$$

where V_1 is defined as in last subsection and $\theta_L, \theta_R, \theta_{K_E}, \theta_{K_N}$ denotes the vector of true values in L, R, K_E, K_N respectively.

It's easy to see that V_2 is positive definite if the condition (20) is satisfied. Differentiating V_2 with respect to time and using equations (37)-(39),(42), we have:

$$\begin{aligned} \dot{V}_2 &= \dot{V}_1 + y^T K_N \tilde{I} - \tilde{I}^T (R + K_D)\tilde{I} - \tilde{I}^T N_N(y)\hat{\theta}_{K_N} \\ &\quad -\dot{\hat{\theta}}^T_{K'_N} L_N^{-1} \Delta\theta_{K_N} \\ &= -W_1 - \tilde{I}^T (R + K_D)\tilde{I} + \tilde{I}^T N_N(y)\Delta\theta_{K_N} \\ &\quad -\dot{\hat{\theta}}^T_{K'_N} L_N^{-1} \Delta\theta_{K_N} \end{aligned} \tag{44}$$

here we notice an additional term $y^T K_N \tilde{I}$ which is introduced by the differentiation of V_1 since the current perturbation $\tilde{I}$ is considered in this design step as from equation (17).

Substituting (40) into equation (44), we can get:

$$\dot{V}_2 = -W_1 - \tilde{I}^T (R + K_D)\tilde{I} \tag{45}$$

where W_1 is defined in last subsection. Hence, if the corresponding sufficient stability conditions are satisfied $\dot{V}_2$ is guaranteed to be negative.

We are now in a position to state the following Theorem:
Theorem *If the control input voltage u given by (9) and (36) is applied on the rigid-link robot dynamics (4), (5), then the closed-loop system gives rise to the convergence of $X \to X_d$, $\dot{q} \to 0$ and $I \to I_d$ as $t \to \infty$, provided that the feedback gains K_p, α are chosen to satisfy conditions (20), (33) and (34).*
Proof Since V_2 is positive definite, its time derivative $\dot{V}_2$ is negative definite in $s(e)$, $\dot{q}$, v, and $\tilde{I}$. $\dot{V}_2 = 0$ implies that $\dot{q} = 0, e = X - X_d = 0, v = 0, \tilde{I} = 0$, by LaSalle's invariance Theorem, the proof is complete. $\Delta\Delta\Delta$

IV. Simulation Results

In this section, we present the simulation results to illustrate the performance of the proposed control scheme. The simulation is based on a two-link RELD robot holding an uncertain object as shown in Fig. 2. The manipulator endpoint is required to move from an initial position $X_0 = (0.8, 0.32)$ to an desired position $X_d = (0.5, 0.55)$ defined in Cartesian space.

In the simulation, we set the true masses, lengths and gravity centers of link 1, 2 and the object to $m_1 = 17.4kg, m_2 = 4.8kg, m_o = 2kg, l_1 = 0.4318m, l_2 = 0.4318m, l_0 = 0.2m, l_{c1} = 0.068m, l_{c2} = 0.07m, l_{c0} = 0.1m$ respectively. The object grasping angle is set to $q_0 = 30°$. The true actuator dynamic coefficient matrices are defined as $L = diag(1\ 1), R = diag(15\ 15), K_E = diag(10\ 8), K_N = diag(10\ 8)$.

To test the ability of the proposed control scheme to deal with kinematics and actuator uncertainties, first we set the initial estimated kinematics parameters wrongly to $\hat{l}_1 = 0.5m, \hat{l}_2 = 0.5m, \hat{l}_0 = 0.25m, \hat{q}_0 = 45°$ and the initial approximated actuator models to $\hat{L} = diag(2.5\ 2.5), \hat{R} = diag(10\ 12), \hat{K}_E = diag(15\ 10), \hat{K}_N = \hat{K}'_N = diag(15\ 12)$ in the controller design. The system performance with control gains $K_p = 200I_E, K_v = 1000I_E, K_I = 260I_E, \alpha = 5, \Lambda = 12I_E$, $K_D = 300I_E$, $L_1 = L_2 = I_E$, $L_R = L_E = L_N = 3I_E$ are illustrated by Fig. 3 and 4.

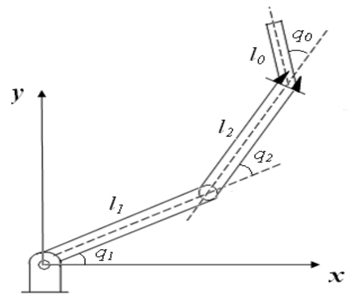

Fig. 2. 2-Link Robot Holding Object

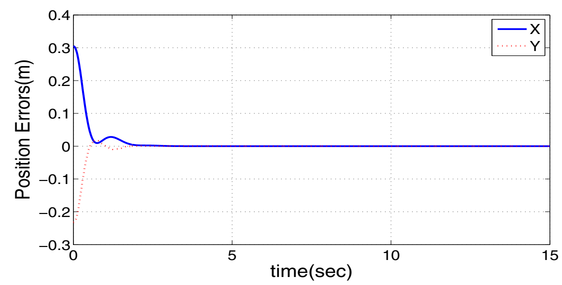

Fig. 3. Position Errors

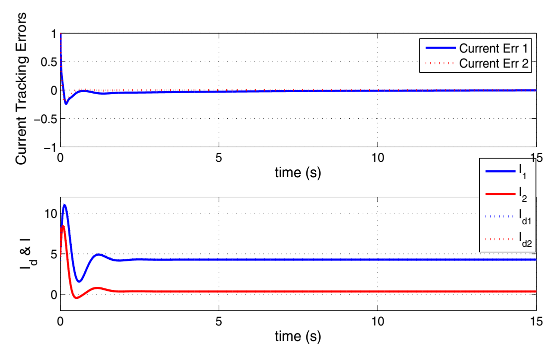

Fig. 4. Currents Tracking: Errors and Record (real v.s. desired)

To further test the robustness of the proposed control scheme, larger and hence more challenging kinematics and actuator uncertainties settings are used as $\hat{l}_1 = 0.6m, \hat{l}_2 = 0.6m, \hat{l}_0 = 0.35m, \hat{q}_0 = 60°$, $\hat{L} = diag(3.5\ 3.5), \hat{R} = diag(10\ 10), \hat{K}_E = diag(15\ 15), \hat{K}_N = \hat{K}'_N = diag(15\ 15)$. Figure 5 and 6 show the system performance with the the same control parameters as in the first simulation study. As shown by these simulation results, the proposed control scheme is efficient to achieve convergence of both regulation errors and armature current tracking errors even with the presence of kinematics and dynamics uncertainties in the RLED robot system.

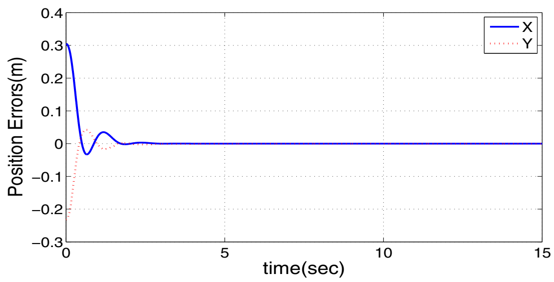

Fig. 5. Position Errors

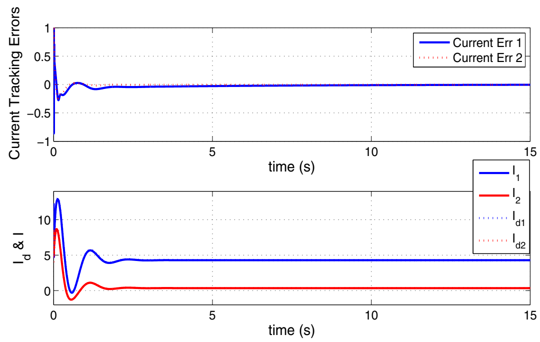

Fig. 6. Currents Tracking: Errors and Record (real v.s. desired)

V. Conclusions

In this paper, we proposed a task-space SP-ID control scheme for regulation problem of RLED robotic manipulators with uncertain kinematics. The proposed control scheme does not require exact knowledge of actuator dynamics, robot kinematics and dynamics. The novelty and main contribution of this work lies in that it is shown through rigorous theoretical analysis that PID type controller which is used widely in various applications beyond industry due to its simplicity is able to achieve regulation convergence even for complicated tasks like in RLED robot control. Sufficient conditions for choosing the feedback gains were given to guarantee the system stability which are easy to meet. The performance of the proposed control scheme was illustrated through simulation studies.

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