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Compensation of Rate-Dependent Hysteresis Nonlinearities in a Piezo Micro-Positioning Stage

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Abstract— **Piezo micro-positioning actuators have been widely used in micro-positioning applications due to the fast expansion, high force generation, and unlimited resolution. However, these actuators exhibit some rate-dependent hysteresis effects which affect the accuracy of these micro-positioning systems and may even lead to system instability. In this paper, the rate-dependent Prandtl-Ishlinskii model is employed to characterize the rate-dependent hysteresis nonlinearities of a piezo micro-positioning stage. The analytical inverse of the rate-dependent Prandtl-Ishlinskii model is then formulated using the initial loading curve concept. This inverse is utilized as a feedforward compensator to compensate for the hysteresis nonlinearities of a piezo micro-positioning stage under excitation in the 1–50 Hz frequency range.**

I. Introduction

PIEZO micro-positioning actuators offer nanometer resolution, high stiffness, and fast response when subject to a varying electrical field [1]. Because of these advantages, these actuators are increasingly used in micro-positioning applications [2-7]. These smart actuators, however, show hysteresis nonlinearities that increase as the excitation frequency of the input voltage increases. The hysteresis effects are known to cause inaccuracies and oscillations in the system responses, which is especially severe for the micro-positioning case. Various control strategies have been proposed to compensate the hysteresis effects and thus enhance the tracking and positioning performances of smart actuators [8- 14]. These could be broadly classified into two groups based on the methodology. The first group employs a hysteresis compensator based on inversion of the hysteresis model, and has been applied for control of various smart actuators, such as piezoceramic, shape memory alloy, and magnetostrictive actuators [10-12,14,15]. The compensation strategies used in studies within the second group do not require the hysteresis inversion, and include the PID feedback controller, robust adaptive control, energy-based control, etc. [13].

The identification of an effective hysteresis model, and its inverse, however, involves many challenges, particularly when the rate-dependent hysteresis nonlinearity is considered. Considerable efforts have been made towards modeling of hysteresis properties of smart actuators and their inverse. A number of models have been evolved to describe the hysteresis nonlinearities [7]. The Preisach and Prandtl–Ishlinskii models are the most widely used phenomenological hysteresis models, which are based on the hysteresis operators. Since the Preisach is not analytically invertible, numerical algorithms have been proposed to invert these models [1,5,7]. However, Prandtl-Ishlinskii model is analytically invertible [8,19,20]. The Prandtl-Ishlinskii model, however, has not been explored to describe rate-dependence of the hysteresis nonlinearity.

The hysteresis nonlinearities of piezo-micro positioning actuators are known to be strongly dependent on the rate of the input. It has been shown that the hysteresis in the output displacement of piezo micro-positioning actuators increases with increase in the frequency of the applied voltage [16,17]. The Prandtl-Ishlinskii model thus yields considerable error under high frequency inputs. The application of the inverse rate-independent model would also yield considerable errors in the compensated outputs under high frequency inputs.

In this paper, the inverse rate-dependent Prandtl-Ishlinskii model is formulated analytically incorporating the rate-dependent hysteresis nonlinearity. The inverse is derived using the recently-proposed rate-dependent Prandtl-Ishlinskii model for describing the hysteresis nonlinearities of smart actuators over a wide range of input frequencies [16,17]. The analytical inverse is subsequently applied as a feedforward compensator for mitigating the hysteresis effects over a wide range of excitation frequencies. The results show the capability of the inverse model to compensate hysteresis at different excitation frequencies.

II. Background

In this section a brief description regarding the Prandtl-Ishlinskii model and its initial loading curve is presented. This section is essential to introduce the rate-dependent Prandtl-Ishlinskii model.

A. Prandtl-Ishlinskii model

Prandtl-Ishlinskii model, integrates play operator and density function to characterize hysteresis nonlinearities. Play operator is continuous and rate-independent hysteresis operator [18]. This hysteresis operator is constructed using single threshold r, the previous state of the output w, and the input v. For any input $v(t) \in C_m[0, T]$, where $0 = t_0 < t_1 < \ldots < t_N = T$ are intervals in $[0, T]$ such that the function v is monotone on each of the sub-intervals $[t_i, t_{i+1}]$, the output of the play operator $w(t)$ over each interval, $t_i \le t < t_{i+1}$, is

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analytically expressed as [18]:

$$\Gamma_r[v](t) = \begin{cases} \max\{v(t) - r, w(t_i)\} & for \quad v(t) > v(t_i) \\ \min\{v(t) + r, w(t_i)\} & for \quad v(t) < v(t_i) \\ w(t_i) & for \quad v(t) = v(t_i) \end{cases} \tag{1}$$

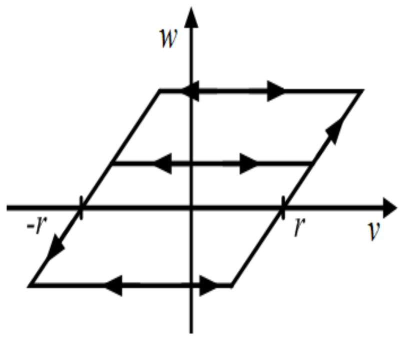

Fig. 1. Play hysteresis operator.

The argument of the operator Γ_r is written in square brackets to indicate the functional dependence, since it maps a function to another function. Owing to the unity slope between the input v and the output w, the play operator Γ_r is a Lipschitz continuous operator. The play operator Γ_r can thus be extended to a Lipschitz continuous operator such that operator Γ_r: $C\ [0, T] \times \Re \rightarrow C\ [0, T]$ [18]. The output of the Prandtl-Ishlinskii model can be presented as [18]:

$$\Pi[v](t) = \sigma\ v(t) + \int_0^\infty \theta_r(r)\Gamma_r[v](t)dr \tag{2}$$

where θ_r is an integrable density function, identified from experimental data and satisfying $\theta_r > 0$, and σ is a positive constant. The Prandtl-Ishlinskii model was applied to characterize and to compensate hysteresis nonlinearities in piezo micro-positioning stage [11]. The Prandtl-Ishlinskii model, however, is limited to rate-independent hysteresis nonlinearities, which is attributed to the rate-independent output of the play hysteresis operator. The model thus cannot be applied for predicting rate-dependent hysteresis properties, which is invariably observed in piezo micro-positioning stages.

B. Initial loading curve

The initial loading curve is used as an alternative description of the Prandtl-Ishlinskii model and to derive its analytical inverse in [6]. The curve is physically described by stress-strain relation, corresponding to an increasing load from zero to a final value, which describes the possible hysteresis loops generated by the Prandtl-Ishlinskii model. The initial loading curve for the Prandtl-Ishlinskii model Π can be expressed as [18]:

$$\varphi(r) = \sigma r + \int_0^r \theta_r(\vartheta)(r - \vartheta)d\vartheta \tag{3}$$

The alterative description of Prandtl-Ishlinskii model (2) using initial loading curve (3) can be expressed analytically as:

$$\Pi[v](t) = \varphi'(0)v(t) + \int_0^\infty \varphi''(r)\Gamma_r[v](t)dr \tag{4}$$

III. Rate Dependent Prandtl-Ishlinskii model

Rate-dependent Prandtl-Ishlinskii model is employed in this paper to characterize hysteresis nonlinearities of a piezo micro-positioning actuator over different excitation frequencies [16]. The rate-dependent Prandtl-Ishlinskii model Φ is formulated upon integrating the rate-dependent play operator and the density function. The output of the rate-dependent play operator $\Gamma_s(v(t))$ can be expressed as:

$$y(t) = \Gamma_s(v(t)) \tag{5}$$

The output of the rate-dependent play operator $\Gamma_s(v(t))$ with envelope function $\eta(v): \Re \rightarrow \Re$ is defined as:

$$\Gamma_s(v(t)) = \begin{cases} \max\{\ \eta(v(t)) - s, y(t_i)\} & for \quad v(t) > v(t_i) \\ \min\{\ \eta(v(t)) + s, y(t_i)\} & for \quad v(t) < v(t_i) \\ y(t_i) & for \quad v(t) = v(t_i) \end{cases} \tag{6}$$

where η is strictly continuous increasing function. The rate-dependent play operator could be utilized to realize the rate-dependent hysteresis in a piezo micro-positioning actuator. The rate-dependent play operator is subsequently integrated to the Prandtl–Ishlinskii model together with a density function to predict hysteresis properties as a function of the rate of the input. The output of the rate-dependent Prandtl-Ishlinskii model can be expressed as:

$$\Phi(v(t)) = \sigma\ \eta(v(t)) + \int_0^\infty \theta_s(s)\Gamma_s(v(t))ds \tag{7}$$

where θ_s is a dynamic density function. The model could be applied to characterize the rate-dependent hysteresis properties in piezo micro-positioning actuators [16]. The inverse rate-dependent Prandtl-Ishlinskii model is proposed to compensate the rate-dependent hysteresis of the piezo micro-positioning stage.

IV. Rate Dependent Prandtl-Ishlinskii model

The inverse model-based hysteresis compensation method that employs a cascade of a hysteresis model and its inverse to compensate for the hysteresis effects is employed in this paper to compensate the rate-dependent hysteresis effects.

This method, however, necessitates the formulation of the hysteresis model inverse, which is often a challenging task. The concept of an open-loop inverse control system for compensation of hysteresis effects is shown in Fig. 2, where u_d is the desired input, v is the control input, and u is the output of the inverse compensation.

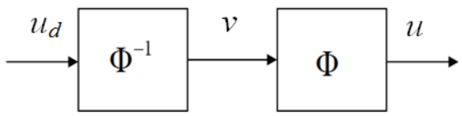

Fig. 2. Illustration of hysteresis inversion.

The objective of the inverse compensation is to obtain identity mapping between the desired input $u_d(t)$ and the desired output $u(t)$, such that :

$$u(t) = \Phi \; o \; \Phi^{-1}(u_d(t)) \tag{8}$$

where o denotes the composition operator. Analytical inverse of the rate-dependent Prandtl-Ishlinskii is presented analytically in this section. In other words, the exact inverse of the rate-dependent model can be obtained, consequently making it attractive for real-time applications. For rate-dependent Prandtl-Ishlinskii model (7), if both the inverse of the function η and the inverse the initial loading curve exist; the inverse of the rate-dependent model can be analytically expressed as:

$$\Phi^{-1}\big(v(t)\big) = \eta^{-1}(\sigma^{-1} \; v \;(t) + \int_0^{\infty} \hat{\theta}_z(z)\Gamma_z(v(t))dz) \tag{9}$$

where σ^{-1} is a positive constant, z is the dynamic threshold of the inverse rate-dependent Prandtl-Ishlinskii model, and $\hat{\theta}(z)$ is the dynamic density function of the inverse rate-dependent Prandtl-Ishlinskii model.

V. Numerical Implementation

In this section the numerical implementation for the rate-dependent Prandtl-Ishlinskii model and it is inverse are presented. The weights of the dynamic density function can be defined as:

$$\theta(s_j) = \theta_s(s_j)(s_j - s_{j-1}) \tag{10}$$

The rate-dependent Prandtl-Ishlinskii model can be defined numerically as:

$$\Phi(v(t)) = \sigma \; \eta(v(t)) + \sum_{j=1}^{J} \theta(s_j)\Gamma_{s_j}(v(t)) \tag{11}$$

where J is the number of the rate-dependent play operators that employed in the rate-dependent Prandtl-Ishlinskii model and $\theta(s_j)$ are weights of the model. The rate-dependent initial loading curve for rate-dependent Prandtl-Ishlinskii model Φ (9) can be expressed numerically as:

$$\delta(s) = \sigma \; s + \sum_{j=1}^{J} \theta(s_j)(s - s_j) \tag{12}$$

The derivative of the rate-dependent initial loading curve with respect to the dynamic threshold s is expressed as:

$$\delta'(s) = \sigma + \sum_{j=1}^{J} \theta(s_j) \tag{13}$$

The density function of the rate-dependent model is defined as:

$$\delta''(s) = \theta(s_j) \tag{14}$$

The output of the rate-dependent Prandtl-Ishlinskii model can be defined via the rate-dependent initial loading curve as:

$$\Phi(v(t)) = \delta'(0) \;\; \eta(v(t)) + \sum_{j=1}^{J} \delta''(s_j) \;\; \Gamma_{s_j}(\eta(v(t))) \tag{15}$$

The output of the inverse rate-dependent Prandtl-Ishlinskii model can be expressed as:

$$\Phi^{-1}\big(u_d(t)\big) = \eta^{-1}(\sigma^{-1} u_d(t) + \sum_{j=1}^{J} \hat{\theta}(z_j)\Gamma_{z_j}(u_d(t))) \tag{16}$$

where the weights of the inverse rate-dependent Prandtl-Ishlinskii model can be defined as:

$$\hat{\theta}(z_j) = \theta_z(z_j)(z_j - z_{j-1}) \tag{17}$$

The weights of the rate-dependent Prandtl-Ishlinskii model and its inverse can be expressed as:

$$\begin{gathered} \theta(s_j) = \theta_j \\ \hat{\theta}(z_j) = \hat{\theta}_j \end{gathered} \tag{18}$$

The inverse of the rate-dependent initial loading curve can be expressed numerically as:

$$\delta^{-1}(z) = \hat{\sigma} \; z + \sum_{j=1}^{J} \hat{\theta}_j \;\; (z - z_j) \tag{19}$$

where J is the number of the rate-dependent play operators of the inverse rate-dependent Prandtl-Ishlinskii model. The relationship between the rate-dependent initial loading curve and its inverse can be defined as:

$$\delta'^{-1}(z) = \frac{1}{\delta'(s)} \tag{20}$$

To obtain the weights of the inverse model, Equation (20) can be represented in the following manner:

$$\hat{\sigma}+\sum_{j=1}^{J}\hat{\theta}_j=\frac{1}{\sigma+\sum_{j=1}^{J}\theta_j} \tag{21}$$

The positive constant of the inverse rate-dependent Prandtl-Ishlinskii model is defined as:

$$\hat{\sigma}=\frac{1}{\sigma} \tag{22}$$

Equation (21) can be expressed as:

$$\begin{aligned}\hat{\sigma}+\hat{\theta}_1&=\frac{1}{\sigma+\theta_1}\\ \hat{\sigma}+\hat{\theta}_1+\hat{\theta}_2&=\frac{1}{\sigma+\theta_1+\theta_2}\\ &\vdots\\ \hat{\sigma}+\hat{\theta}_1+\hat{\theta}_2+\ldots+\hat{\theta}_n&=\frac{1}{\sigma+\theta_1+\theta_2+\ldots+\theta_n}\end{aligned} \tag{23}$$

It can be concluded that the weights of the model can be obtained as:

$$\hat{\theta}_j=-\frac{\theta_j}{(\sigma+\sum_{i=1}^{j}\theta_j)(\sigma+\sum_{i=1}^{j-1}\theta_j)} \tag{24}$$

The dynamic threshold z_k of the inverse rate-dependent Prandtl-Ishlinskii model can be defined using the weights and the thresholds of rate-dependent model (11) as:

$$z_j=\sigma\, s_j+\sum_{i=1}^{j-1}(\theta_i)\ (s_j-s_i) \tag{25}$$

VI. Modeling Rate- Dependent Hysteresis in a Piezo Micro-Positioning Stage

In this section, modeling rate-dependent hysteresis of a piezo micro-positioning stage is carried out via the rate-dependent Prandtl-Ishlinskii model at different excitation frequencies. The inverse model is applied in Section VII to compensate the rate-dependent hysteresis effects at different excitation frequencies.

A. Experimental setup

The experiments were performed on a piezo micro-positioning stage (P-753.31C) from Physic Instrumente Company. The actuator provided maximum displacement of 100 μm from its static equilibrium position, and it integrates a capacitive sensor (sensitivity = 1μm/V; resolution ≤ 0.1 nm) for measurement of stage displacement response. The excitation module compromises a voltage amplifier (LVPZT, E-505) with a fixed gain of 10, which provides the excitation voltage to the actuator. Fig. 3 illustrates a schematic of the experimental setup for the experiment, where the input was directly applied to the actuator.

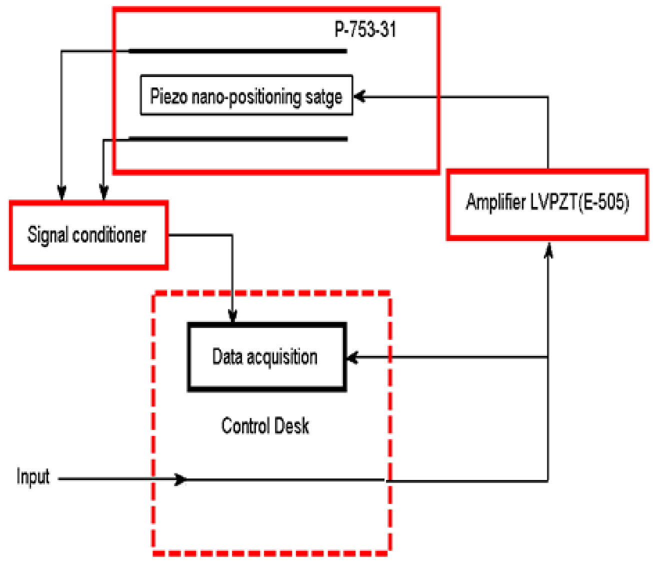

Fig. 3. Schematic of the experimental setup used for characterization hysteresis of the piezo micro-positioning stage.

B. Experimental results

The measurements with the piezo micro-positioning stage were performed under a harmonic input, $v(t)$= $40\sin(2\pi ft)$ at 3 different excitation frequencies (1, 25, and 50 Hz). The input voltage and output displacement signals were acquired using dSpace ControlDesk data acquisition card at a sampling frequency of 10 kHz. The resulting hysteresis loops relating displacement responses to the input voltage are shown in Fig. 4. The measured data were further analyzed to quantify hysteresis and displacement attenuation as a function of the applied excitation frequency. It is evident that the micro-positioning stage exhibit highly rate-dependent nonlinear effects between the input voltage and the output displacement. From the experimental results it can be concluded that the use of the inverse rate-independent hysteresis models could yield high positioning errors in the output displacement.

C. Parameters identifications

Measured rate-dependent hysteresis loops between the applied voltage and the output displacement shown in Fig. 4 are used to identify the parameters of the rate-dependent Prandtl-Ishlinskii model. The dynamic threshold of the rate-dependent play operator can be presented as:

$$s_j=g(\dot{v})j \quad j=1,\ 2,\ \ldots,J \tag{26}$$

where g is a positive function.

On the basis of the observed rate-dependent hysteresis of the piezo micro-positioning actuator, the function g of dynamic threshold (26) is proposed as:

$$g(\dot{v}) = \sum_{l=1}^{L} \alpha_l \ln\left(\beta_l + \lambda_l |\dot{v}|\right) \tag{27}$$

where α_l, $\beta_l > 1$, and λ_l are positive constants. The order of the rate-dependent threshold is determined by the positive integer L. Owing to symmetric rate-dependent hysteresis nonlinearities the following linear function is applied in the rate-dependent play operator:

$$\eta(v) = c\ v \tag{28}$$

where c is a positive constant. The weights of the rate-dependent Prandtl-Ishlinskii model are chosen as:

$$\theta(s_j) = \rho e^{-\zeta(\tau s_j - \mu)^2} \tag{29}$$

where $\rho > 0$, $\tau > 0$, and μ are constants.

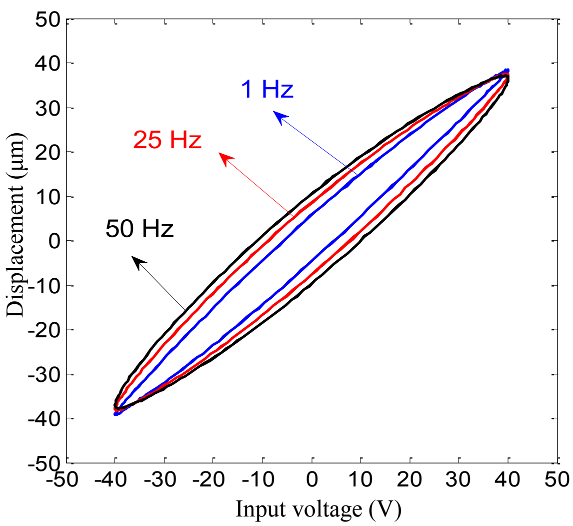

Fig. 4. Measured hysteresis loops relating displacement response of the piezoceramic actuator to the applied voltage at different frequencies.

The experimental data obtained for the piezoceramic actuator harmonic inputs at 1, 25, and 50 Hz are applied to identify the model parameters. The parameter vector $X = \{\alpha_1, \alpha_2, \beta_1, \beta_2, \lambda_1, \lambda_2, c, \rho, \zeta, \tau, \mu\}$, was identified through minimization of an error sum-squared function over a wide frequency range, given by:

$$Q(X) = \sum_{i=1}^{I} \left(\Phi(v(i)) - Y(i)\right)^2 \tag{30}$$

subject to:

$$\alpha, \lambda_1, \lambda_2, \rho, \tau, \zeta > 0$$

$$\beta_1, \beta_2 \geq 1$$

where Φ is the displacement response of the rate-dependent Prandtl-Ishlinskii model corresponding to a particular excitation frequency and Y is the measured displacement under the same excitation frequency. The error function is constructed through summation of squared errors over a range of input frequencies. The index i ($i = 1, \ldots, I$) refers to the number of data points considered to compute the error function Q for one complete hysteresis loop. 100 data points ($I = 100$) were available for each measured hysteresis loop. Three different excitation frequencies of 1, 25, and 50 Hz are used. The error minimization is performed using the MATLAB constrained optimization toolbox.

VII. Compensation of Rate-Dependent Hysteresis Nonlinearities of a Piezo Micro-positioning Stage

In this section, the effectiveness of the inverse model in compensating the rate-dependent hysteresis effects is investigated through simulation and laboratory experiments. The inverse rate-dependent Prandtl-Ishlinskii model is applied as a feedforward compensator to compensate for rate-dependent hysteresis effects.

A. *Inverse rate-dependent Prandtl-Ishlinskii model*

The inverse of the rate-dependent Prandtl-Ishlinskii hysteresis model was derived and employed as a feedforward compensator to compensate for rate-dependent hysteresis nonlinearities of the piezo micro-positioning stage. The parameters of the inverse model were identified using the relations in (24) and (25). The measured input-output characteristics of the inverse rate-dependent Prandtl-Ishlinskii model at 1, 25, and 50 Hz are shown in Fig. 5. The results show that the nonlinearity of the output of the inverse rate-dependent Prandtl-Ishlinskii model increases as the excitation frequency of the input voltage increases. Since the experimental results of the piezo micro-positioning stage show similar behavior, the inverse can compensate the hysteresis effects of the piezo micro-positioning stage at different excitation frequencies.

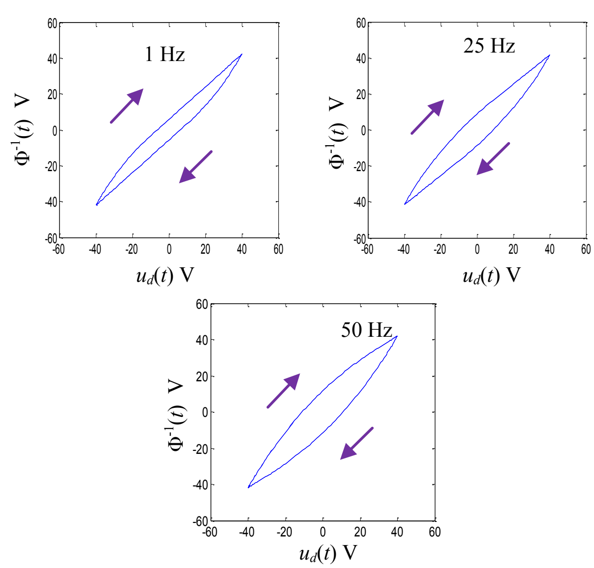

Fig. 5. Input-output characteristics inverse rate-dependent Prandtl-Ishlinskii model at different excitation frequencies.

B. *Compensation of rate-dependent hysteresis in a piezo micro-positioning stage*

In this section, the inverse rate-dependent Prandtl-Ishlinskii model presented in Fig. 5 is applied as a feedforward compensator to compensate for the rate-dependent hysteresis nonlinearities of the piezo micro-positioning stage. The measured output-input characteristics of the piezo micro-positioning stage with inverse compensator are illustrated in Fig. 6 at 1, 25, and 50 Hz. The results show that the inverse model can effectively compensate the hysteresis effects at different excitation frequencies. However, some deviations are also evident at 50 Hz. These deviations are attributed to small prediction errors between the output of the rate-dependent model and the measured displacement of the piezo micro-positioning stage.

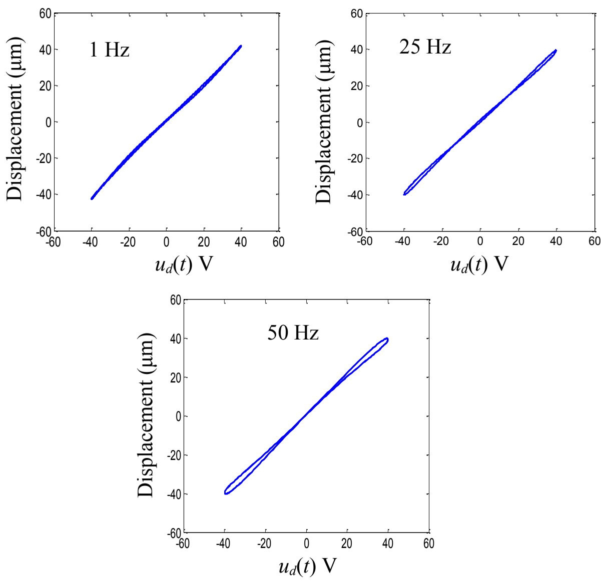

Fig. 6. Input-output characteristics of the piezo micro-positioning stage with inverse feedforward compensator at different excitation frequencies.

VIII. CONCLUSIONS

This paper presents inverse rate-dependent Prandtl-Ishlinskii model to compensate for rate-dependent hysteresis nonlinearities in a piezo micro-positioning stage. The rate-dependent hysteresis nonlinearities are modeled via the rate-dependent Prandtl-Ishlinskii model. An analytical inverse rate-dependent Prandtl-Ishlinskii model is then derived to compensate for hysteresis nonlinearities at different excitation frequencies. Furthermore, compensation of the rate-dependent hysteresis in a piezo micro-positioning stage is carried out experimentally. Experimental results show the capability of the inverse rate-dependent Prandtl-Ishlinskii model (inverse feedforward compensator) to compensate for hysteresis nonlinearities at different excitation frequencies.

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